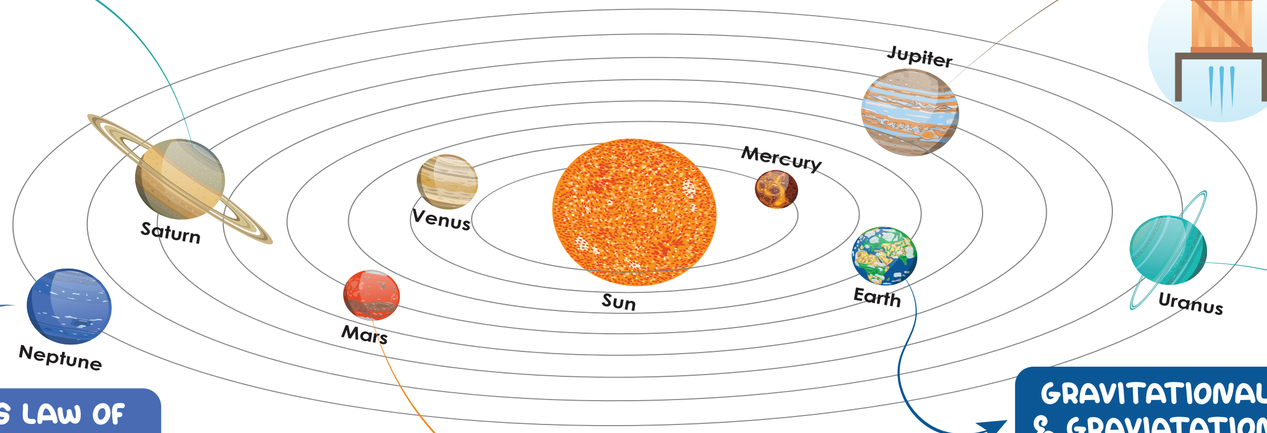
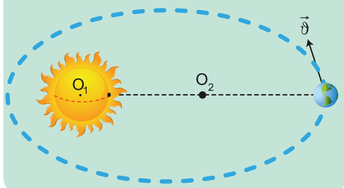
GRAVITATION

KEPLER'S LAW OF PLANETARY MOTION



LAW OF ORBIT

Every planet revolves around the sun in an elliptical orbit and sun is at it's one of the foci points.

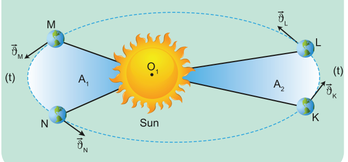


LAW OF AREA

(i) The line joins any planet to the sun sweeps equal area in equal intervals of time

(ii) $\frac{dA}{dt} = \frac{L}{2m}$

(iii) Areal velocity is constant

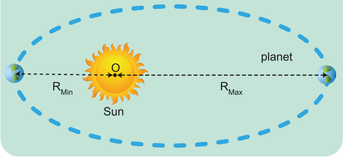


LAW OF PERIODS

(i) The square of time period of revolution of a planet is proportional to cube of semi – major axis of an ellipse

(ii) $T^2 \propto R^3$

(iii) $T^2 = \frac{4\pi^2 R^3}{Gm}$



NEWTON'S LAW OF GRAVITATION

The gravitational force acting between two bodies separated by distance 'r' is directly proportional to product of their masses and inversely proportional to square of distance between them

$$F = \frac{Gm_1m_2}{r^2}$$

$$\Rightarrow G = 6.67 \times 10^{-11} \frac{Nm^2}{Kg^2}$$

$\vec{r_1}$ = position of first particle

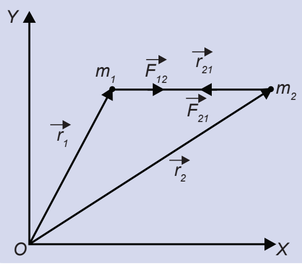
$\vec{r_2}$ = position of second particle

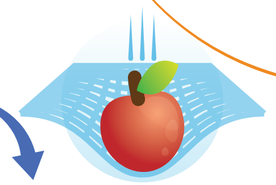
$\vec{r_{12}}$ = Force between them.

$$\vec{F_{12}} = \frac{Gm_1m_2}{|\vec{r_1} - \vec{r_2}|^2}\hat{r}_{12}$$

$$\vec{F_{12}} = \frac{Gm_1m_2(\vec{r_1} - \vec{r_2})}{|\vec{r_1} - \vec{r_2}|^3}$$

$$\boxed{\vec{F_{12}} = -\vec{F_{21}}}$$





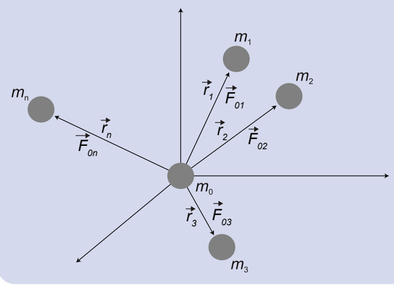
SUPERPOSITION PRINCIPLE IN VECTOR FORM

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + ... + \vec{F_{1n}}$$

SUPERPOSITION PRINCIPLE IN SCALAR FORM

Resultant force acting on a particle due to other particles is vector sum of forces exerted by individual particle in it

F1 = F12 + F13 + ……. + F1N

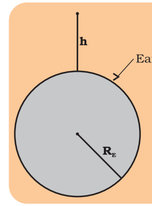


GRAVITATIONAL ACCELERATION

At surface of earth, $F_{gravitational}$ = weight

$$Mg = \frac{GmMe}{R_c^2}$$

$$gs = \frac{Gme}{R_c^2}$$

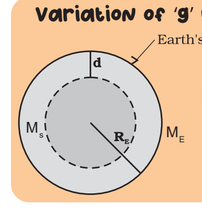


$$g_h = g_s\left(\frac{1+h}{Rc}\right)^{-2} \text{ (i)}$$

if h <<<< Rc (ii)

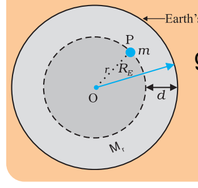
$$g_h = g_s\left(1 - \frac{2h}{Rc}\right) \text{ (iii)}$$

Variation of 'g' with depth



$$g_d = g_s\left(1 - \frac{d}{Rc}\right)$$

Variation of 'g' from equator to pole



$$g = g - Rw^2\cos^2$$

GRAVITATIONAL POTENTIAL & GRAVIATATION POTENTIAL

- Energy required to bring a mass from an infinite position to point under gravitational field of earth with constant velocity

$$U = \frac{Gm_1m_2}{r}$$

- Generally, infinite is reference point

Amount of work done in moving a unit test mass from ∞ - position to point under gravitational field of earth

$$V = \frac{Gm}{r} = \frac{U}{M}$$

(i) $r < R \rightarrow v = -GM\frac{(3R^2 - r^2)}{2R^3}$

(ii) $r = R \rightarrow v = -\frac{GM}{R}$

(iii) $r > R \rightarrow v = -\frac{GM}{r}$

Strength of Gravitational field applied per unit test mass is defined as Gravitational Field Intensity

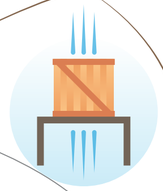
$$\vec{E} = \frac{-GM}{r^2}\hat{r}$$

Relation between Gravitational Potential & Intensity

(i) $E = \frac{dv}{dr}$

(ii) $\Delta V = \int \vec{E}.d\vec{r}$

WEIGHTLESSNESS



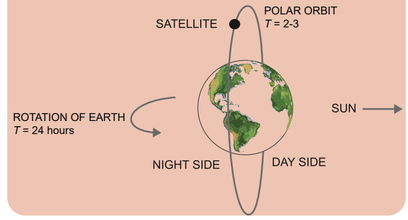
(1) During free fall under gravity in side a spacecraft or satellite, body is weightless.

(2) Effective weight of body becomes zero.

GEOSTATIONARY & POLAR SATELLITE

(1) GEOSTATIONARY SATELLITE
Height from earth'S surface = 36.000 km
RADIUS = 42.400 Km
Time Period = 24 hours.

(2) POLAR SATELLITE
Height from earth'S surface = 330 Km
Time Period = 84 Min
Orbital Velocity = 7.92 Km/S

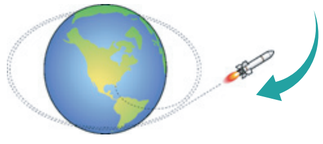


ESCAPE SPEED & ENERGY CONSERVATION

Minimum speed required by an object to escape gravitational field of Earth

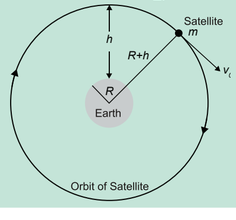
$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

Ve = 11.2 Km/S



(i) orbital velocity = $\sqrt{\frac{GM}{r}}$

(ii) Total energy of satellite = constant
K.E + P.E = constant

(iii) Total energy = $\frac{GMM}{2r} = \frac{GMM}{r}$

$= -\frac{GMM}{2r}$