Chapter 4 Matrices and Determinants

Ex 4.1

Answer 1e.

We know that the parent function of all quadratic functions is $f(x) = x^2$. The graph of $f(x) = x^2$ is a U-shaped curve called parabola.

Therefore, the graph of a quadratic function is called a parabola.

Answer 1gp.

STEP 1 We need to find some points to graph the function. For this, make a table of values for the given function.

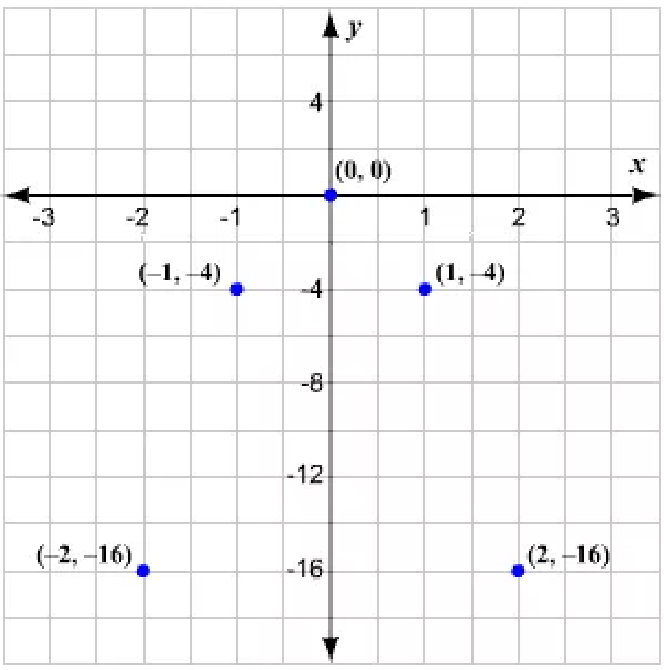
Substitute any value for x, say, -2 and evaluate y.

$$y = -4(-2)^2$$
$$= -4(4)$$
$$= -16$$

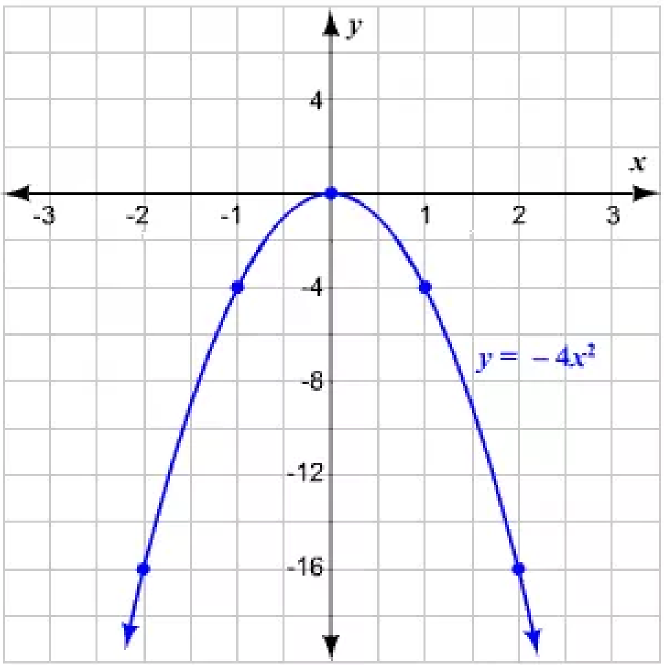
Choose some more x-values and find the corresponding y-values. Organize the results in a table.

x	-2	-1	0	1	2
y	-16	-4	0	-4	-16

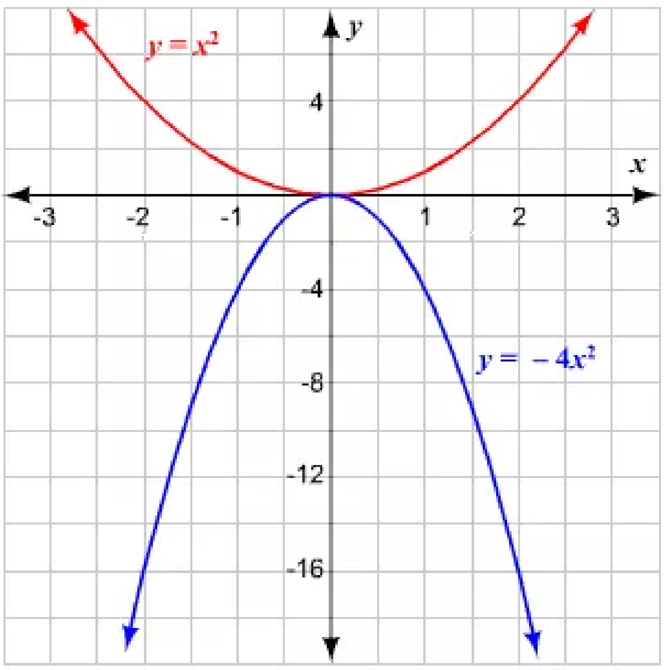
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.



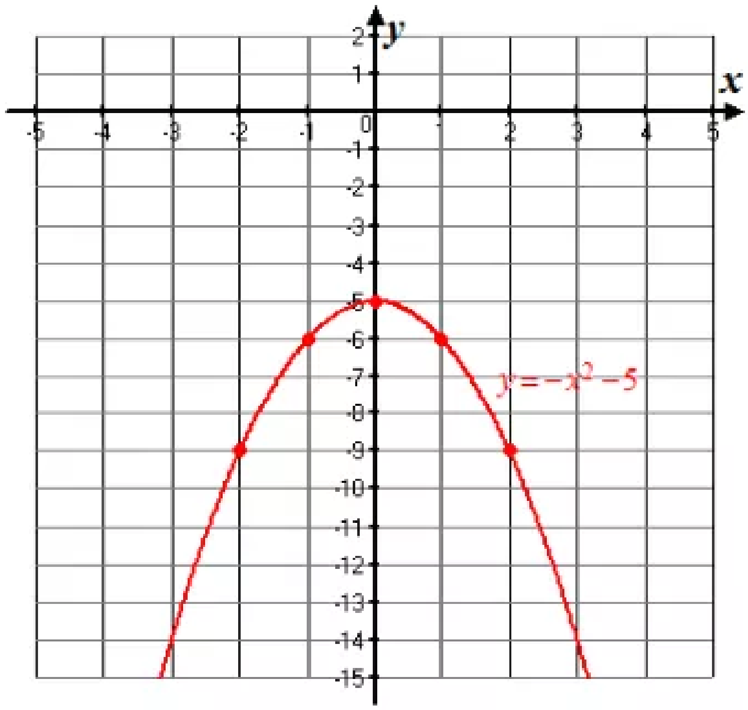
On comparing we get both the graphs have the same vertex and axis of symmetry.
The graph of $y = -4x^2$ opens down and is narrower than the graph of $y = x^2$.

Answer 2e.

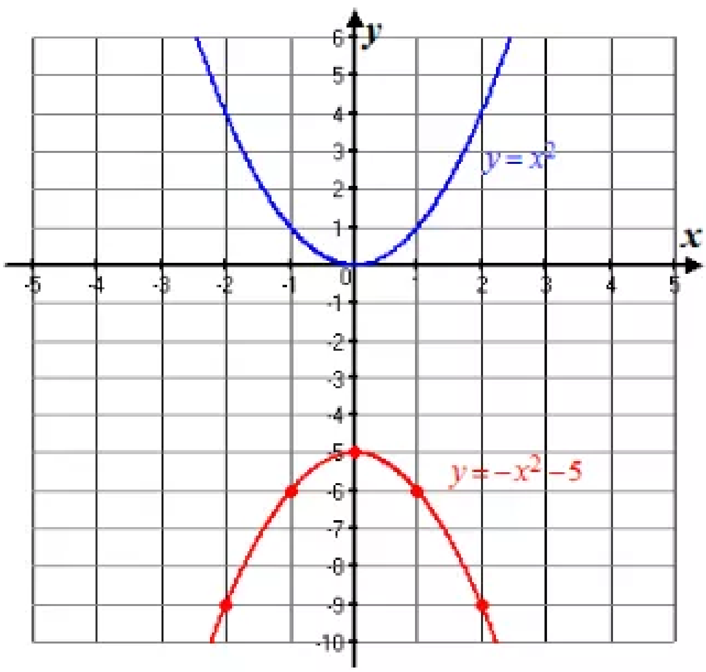
Consider a quadratic function.
Determine whether the quadratic function has a minimum value or a maximum value.
First identify the quadratic coefficient as positive or negative.
Because, if the quadratic coefficient is positive, then the quadratic function will have minimum value and if the quadratic coefficient is negative, then the quadratic function will have maximum value.
The quadratic function $ax^2 + bx + c$ has minimum value if $a > 0$ and has maximum value if $a < 0$.

Answer 2gp.

STEP 2:
Plot the points and draw a smooth curve from the table:



STEP 3:
Consider the graph of $y=x^2$ and the above graph:



In the above diagram, the graph of $y=-x^2-5$ is represented by red curve and the graph of $y=x^2$ is represented by blue curve.

The graph of $y=-x^2-5$ and $y=x^2$ are open downward and upward respectively and they have vertices $(0,-5)$ and $(0,0)$ respectively and they both have same axis of symmetry.
The vertex of $y=-x^2-5$ is 5 units down from the vertex of $y=x^2$.

Answer 3e.

First, substitute −2 for x in the given equation.
$y=4(-2)^2$

Evaluate.
$y = 4(4)$
$= 16$

Now, substitute 1 for x in the equation and evaluate.
$y = 4(1)^2$
$= 4(1)$
$= 4$

Similarly, evaluate y for the remaining x-values. The completed table is as shown.

x	−2	−1	0	1	2
y	16	4	0	4	16

Answer 3gp.

STEP 1 We need to find some points to graph the function. For this, make a table of values for the given function.

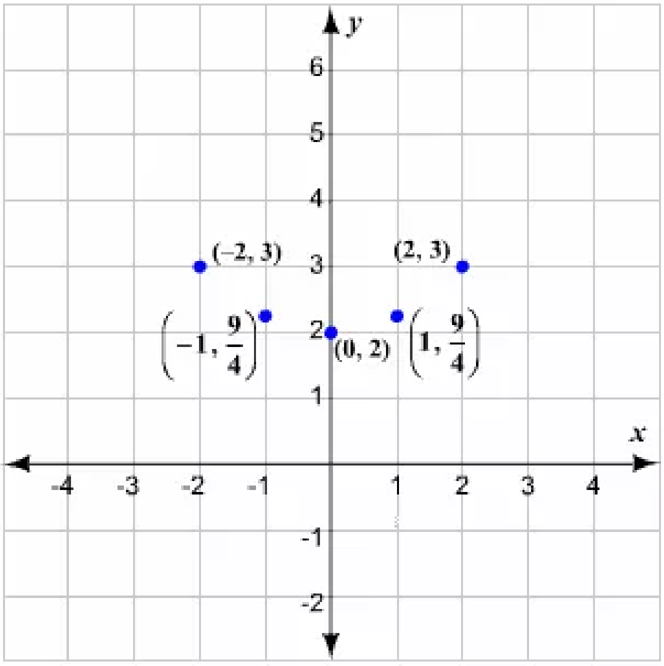
Substitute any value for x, say, −2 and evaluate y.

$$f(-2) = \frac{1}{4}(-2)^2 + 2$$
$$= \frac{1}{4}(4) + 2$$
$$= 3$$

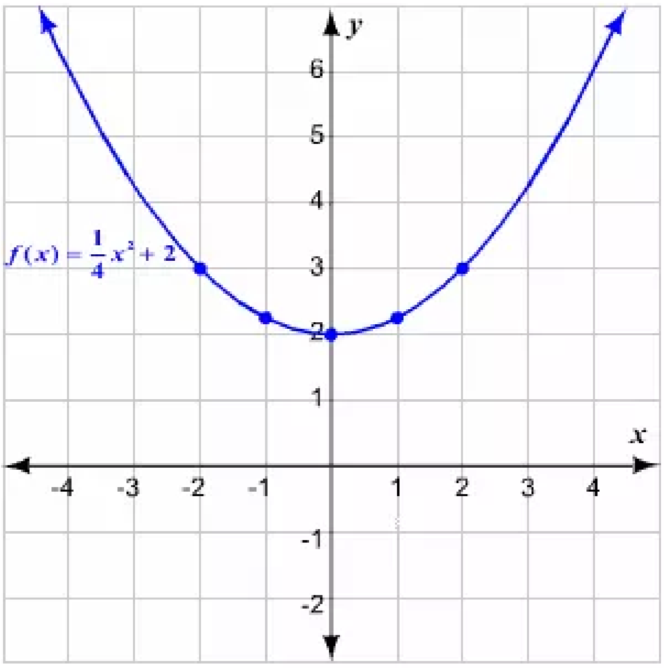
Choose some more x-values and find the corresponding y-values. Organize the results in a table.

x	−2	−1	0	1	2
$f(x)$	3	$\frac{9}{4}$	2	$\frac{9}{4}$	3

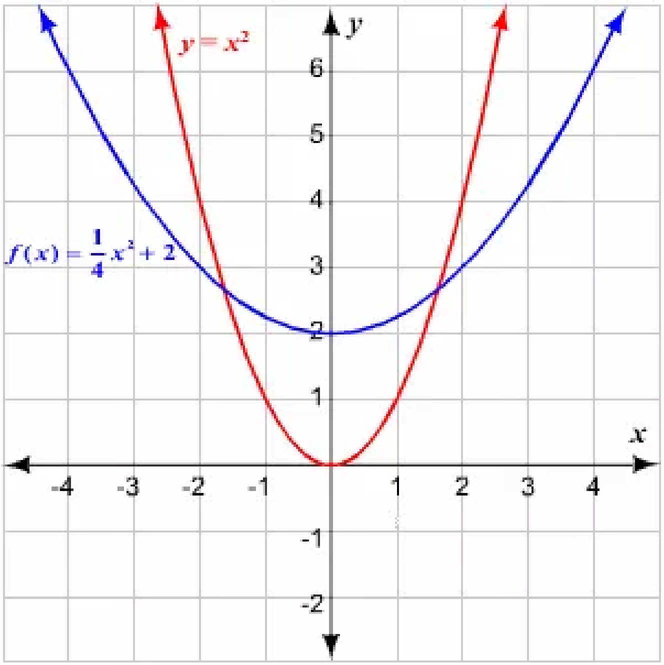
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.



On comparing we get both the graphs open up and have the same axis of symmetry.

The graph of $f(x) = \frac{1}{4}x^2 + 2$ is wider than the graph of $y = x^2$ and its vertex is 2 unit higher.

Answer 4e.

Consider the quadratic function $y = -3x^2$

Consider the table:

x	-2	-1	0	1	2
y	?	?	?	?	?

Complete the table of the values for the function $y = -3x^2$

Consider the function $y=-3x^2$

If $x=-2$ then

$$y=-3(-2)^2$$
$$=-3(4)$$
$$=-12$$

If $x=-1$ then

$$y=-3(-1)^2$$
$$=-3(1)$$
$$=-3$$

If $x=0$ then

$$y=-3(0)^2$$
$$=-3(0)$$
$$=0$$

If $x=1$ then

$$y=-3(1)^2$$
$$=-3(1)$$
$$=-3$$

If $x=2$ then

$$y=-3(2)^2$$
$$=-3(4)$$
$$=-12$$

Therefore,
The following table of values for the function is true.

x	-2	-1	0	1	2
y	-12	-3	0	-3	-12

Answer 4gp.

Consider the function $y=x^2-2x-1$

STEP 1:

Identify the coefficients of the function $y=x^2-2x-1$

Comparing the function $y=x^2-2x-1$ with $y=ax^2+bx+c$

The coefficients are $a=1, b=-2, c=-1$

Because $a>0$, the parabola opens up.

STEP 2:
Find the vertex.
Calculate the x-coordinate.

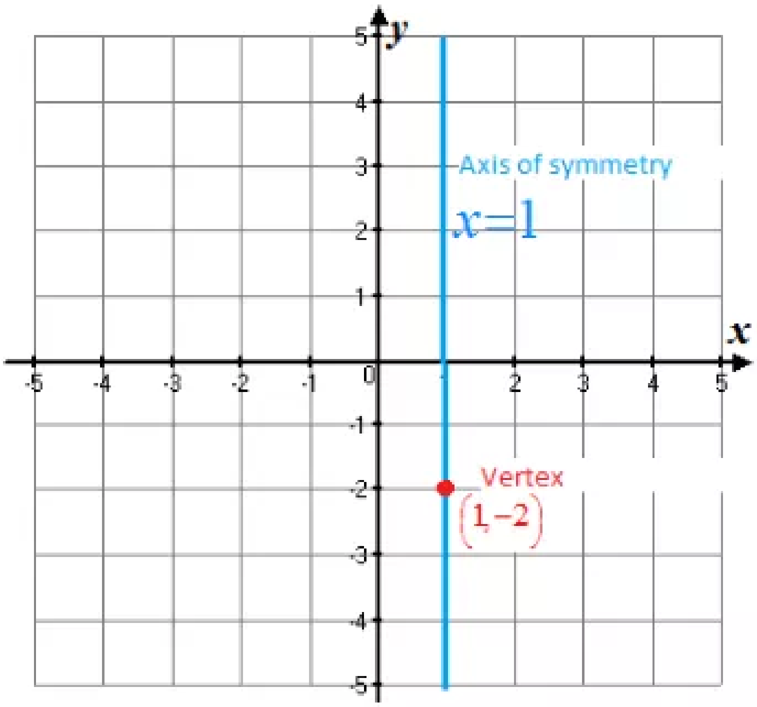
The vertex has x-coordinate$=-\dfrac{b}{2a}=-\dfrac{(-2)}{2(1)}=1$

Then find the y-coordinate of the vertex.

$y=1^2-2(1)-1=-2$

Therefore, the vertex is $(1,-2)$.

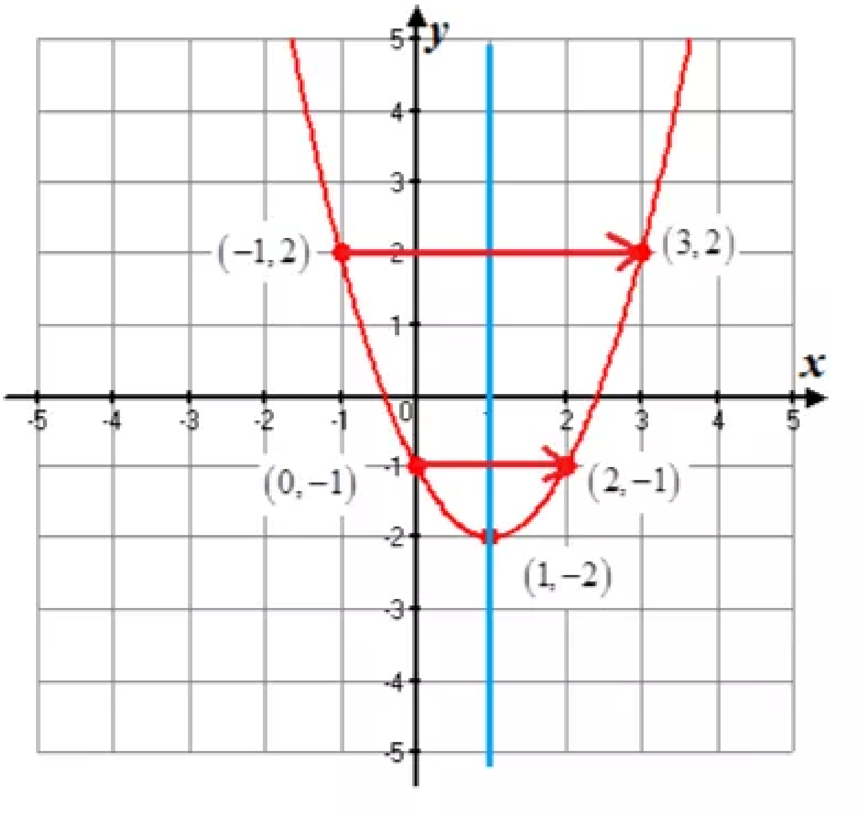
STEP 3:
Plot the point $(1,-2)$ and draw the axis of symmetry $x=1$.



STEP 4:
Identify the y-intercept c, which is -1. Plot the point $(0,-1)$. Then reflect this point in the axis of symmetry to plot another point, $(2,-1)$.
Evaluate the function for the another value of x, such as $x=3$.

$y=3^2-2(3)-1=2$

Plot the point $(3,2)$ and its reflection $(-1,2)$ in the axis of symmetry.
Draw the parabola through the plotted points



Therefore, the above graph is the required graph of the function $y = x^2 - 2x - 1$

Answer 5e.

First, substitute −4 for x in the given equation.

$$y = \frac{1}{2}(-4)^2$$

Evaluate.

$$\begin{aligned} y &= \frac{1}{2}(16) \\ &= 8 \end{aligned}$$

Now, substitute 2 for x in the equation and evaluate.

$$\begin{aligned} y &= \frac{1}{2}(2)^2 \\ &= \frac{1}{2}(4) \\ &= 2 \end{aligned}$$

Similarly, evaluate y for the remaining x-values. The completed table is as shown.

x	−4	−2	0	2	4
y	8	2	0	2	8

Answer 5gp.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y = ax^2 + bx + c$. On comparing, we have a is 2, b is 6, and c is 3. Since $a = 2 > 0$, the graph opens up.

STEP 2 Find the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has the x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute 2 for a, and 6 for b and evaluate.

$$-\frac{b}{2a} = -\frac{6}{2(2)}$$
$$= -\frac{6}{4}$$
$$= -1.5$$

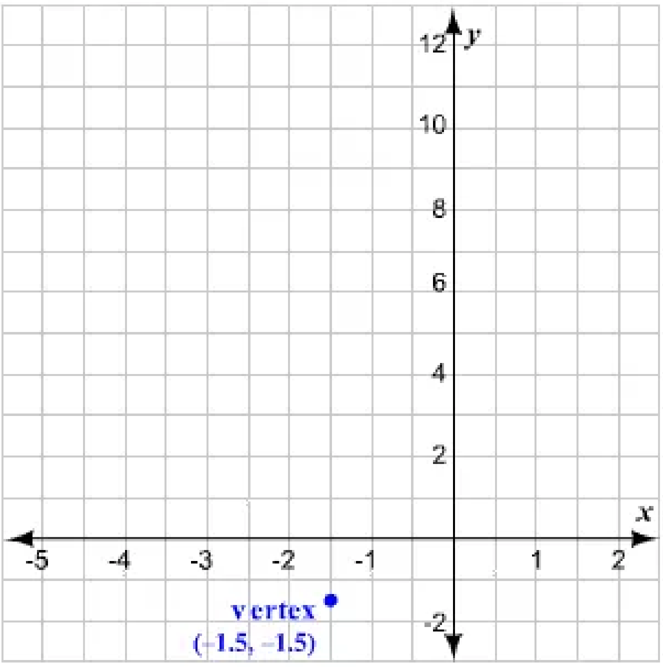
The x-coordinate of the vertex is -1.5.

Substitute -1.5 for x in the given function to find the y-coordinate.

$$y = 2(-1.5)^2 + 6(-1.5) + 3$$
$$= 4.5 - 9 + 3$$
$$= -1.5$$

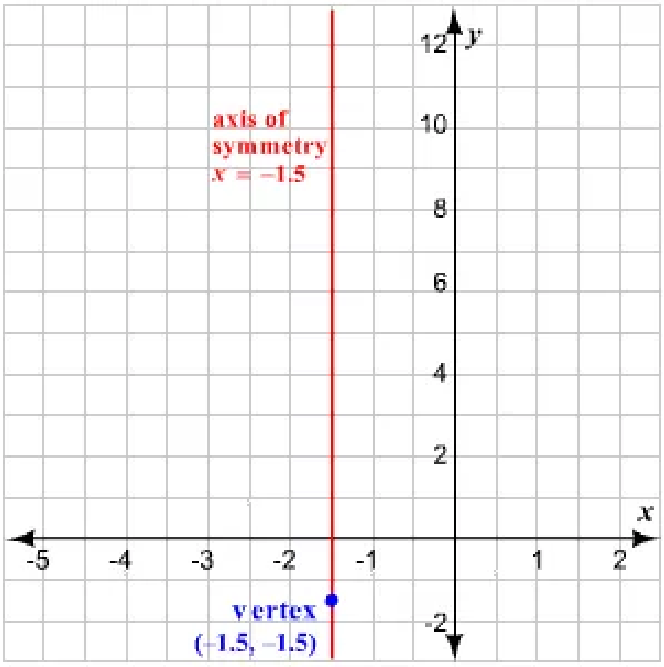
Thus, the vertex of the graph of the given function is $(-1.5, -1.5)$.

Plot the vertex on a coordinate plane.



STEP 3 We know that the axis of symmetry is $x = -\dfrac{b}{2a}$.

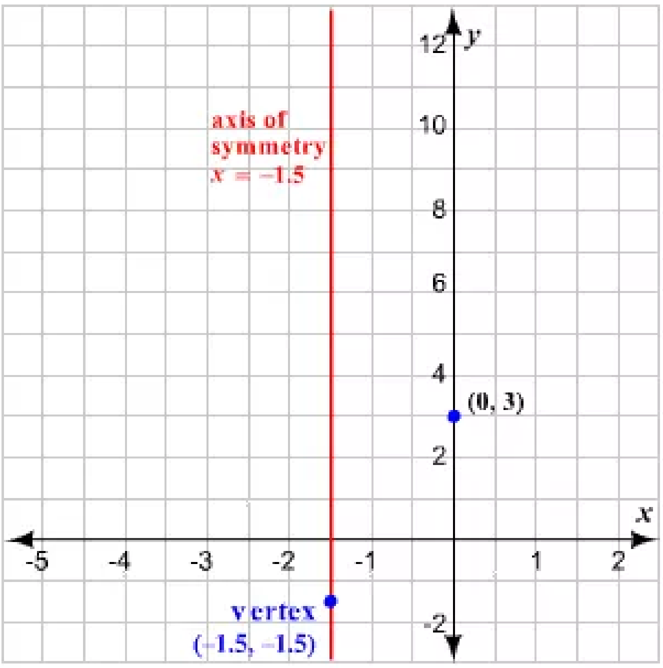
The axis of symmetry of the given function is the line $x = -1.5$.
Now, draw the axis of symmetry $x = -1.5$.



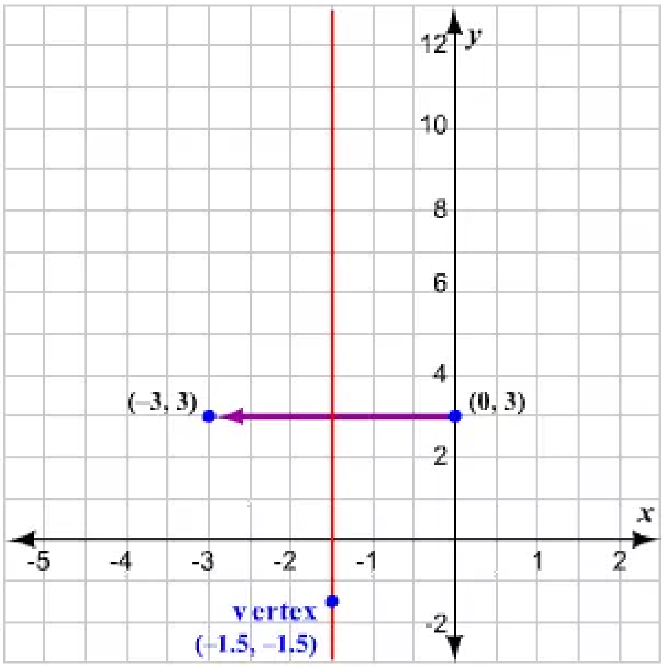
STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola.

Thus, the y-intercept of the given function is 3 and (0, 3) is on the parabola.

Plot the point (0, 3) on the same coordinate plane.



Now, reflect the point (0, 3) in the axis of symmetry to get another point.



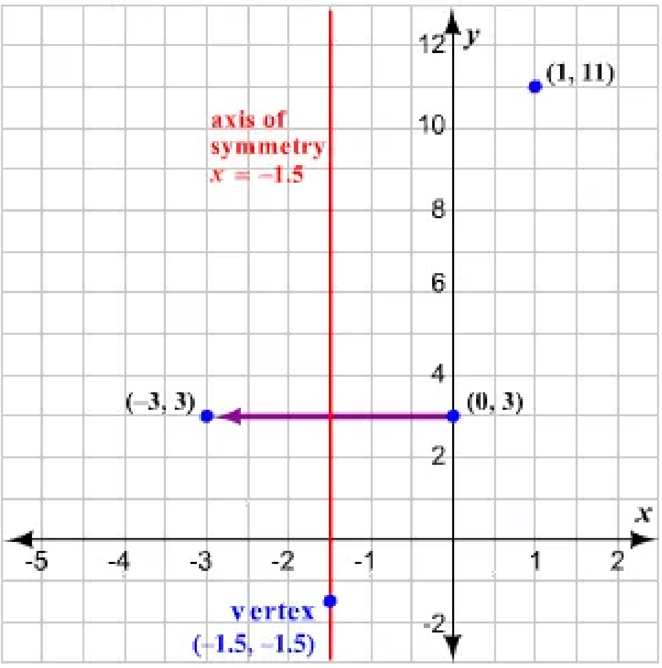
STEP 5 Evaluate the given function for another value of x, say, 1.

Substitute 1 for x in the function and simplify.

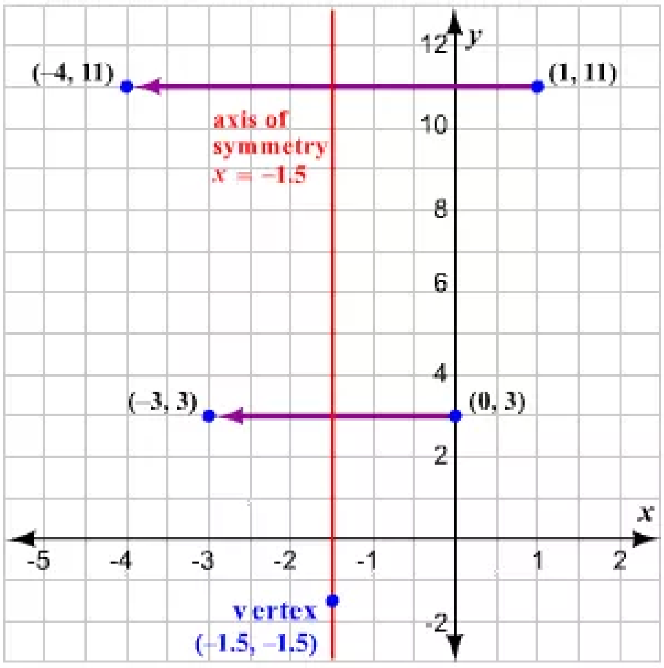
$$\begin{aligned} y &= 2(1)^2 + 6(1) + 3 \\ &= 2 + 6 + 3 \\ &= 11 \end{aligned}$$

Thus, the point (1, 11) lies on the graph.

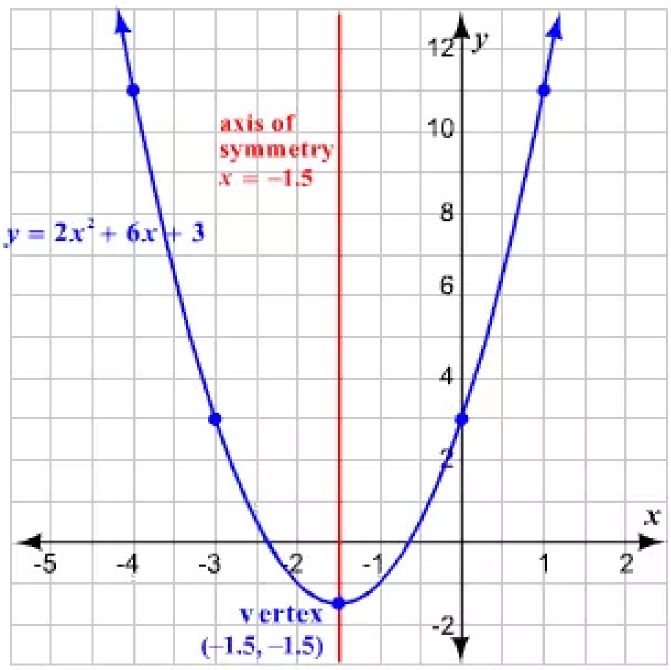
Plot the point on the coordinate plane.



Reflect (1, 11) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 6e.

Consider the quadratic function $y = -\frac{1}{3}x^2$

Consider the table:

x	-6	-3	0	3	6
y	?	?	?	?	?

Complete the table of the values for the function $y = -\frac{1}{3}x^2$

Consider the function $y=-\frac{1}{3}x^2$

If $x=-6$ then

$$y=-\frac{1}{3}(-6)^2$$
$$=-\frac{1}{3}(36)$$
$$=-12$$

If $x=-3$ then

$$y=-\frac{1}{3}(-3)^2$$
$$=-\frac{1}{3}(9)$$
$$=-3$$

If $x=0$ then

$$y=-\frac{1}{3}(0)^2$$
$$=-\frac{1}{3}(0)$$
$$=0$$

If $x=3$ then

$$y=-\frac{1}{3}(3)^2$$
$$=-\frac{1}{3}(9)$$
$$=-3$$

If $x=6$ then

$$y=-\frac{1}{3}(6)^2$$
$$=-\frac{1}{3}(36)$$
$$=-12$$

Therefore,

The following table of values for the function is true.

x	–6	–3	0	3	6
y	–12	–3	0	–3	–12

Answer 6gp.

Consider the function $f(x)=-\frac{1}{3}x^2-5x+2$

STEP 1:

Identify the coefficients of the function $f(x)=-\frac{1}{3}x^2-5x+2$

Comparing the function $f(x)=-\frac{1}{3}x^2-5x+2$ with $y=ax^2+bx+c$

The coefficients are $a=-\frac{1}{3}, b=-5, c=2$

Because $a<0$, the parabola opens down

STEP 2:

The axis of symmetry is

$$x=-\frac{b}{2a}$$
$$=-\frac{15}{2}$$

Find the vertex.

Calculate the x-coordinate.

The vertex has x-coordinate$=-\frac{b}{2a}=-\frac{(-5)}{2\left(\frac{-1}{3}\right)}=\frac{-15}{2}$

Then find the y-coordinate of the vertex.

$$f(x)=-\frac{1}{3}x^2-5x+2$$

$$f(x)=-\frac{1}{3}\left(\frac{-15}{2}\right)^2-5\left(\frac{-15}{2}\right)+2$$

$$=-\frac{1}{\cancel{3}}\left(\frac{\overset{75}{\cancel{225}}}{4}\right)+\frac{75}{2}+2$$

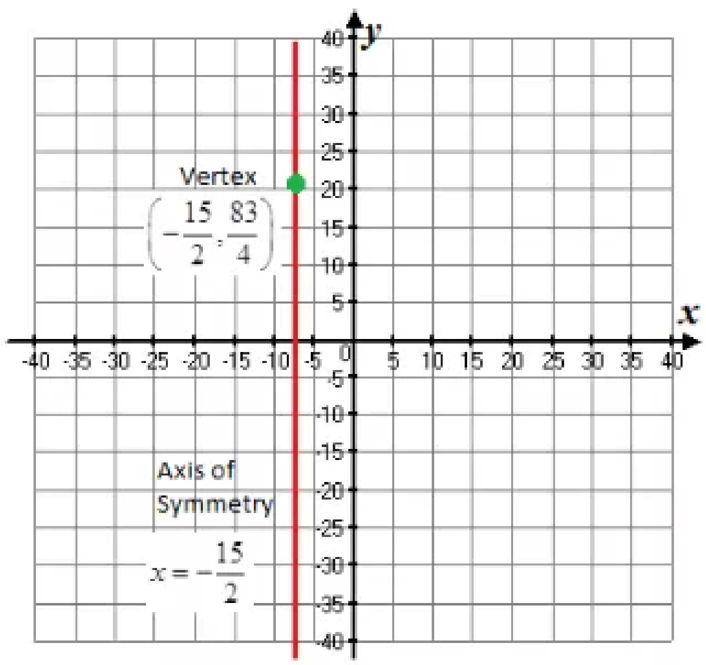
$$=\frac{-75+150+8}{4}$$

$$=\frac{83}{4}$$

Therefore, the vertex is $\left(-\frac{15}{2},\frac{83}{4}\right)$.

STEP 3:

Plot the point $\left(-\frac{15}{2},\frac{83}{4}\right)$ and draw the axis of symmetry $x=-\frac{15}{2}$.



STEP 4:

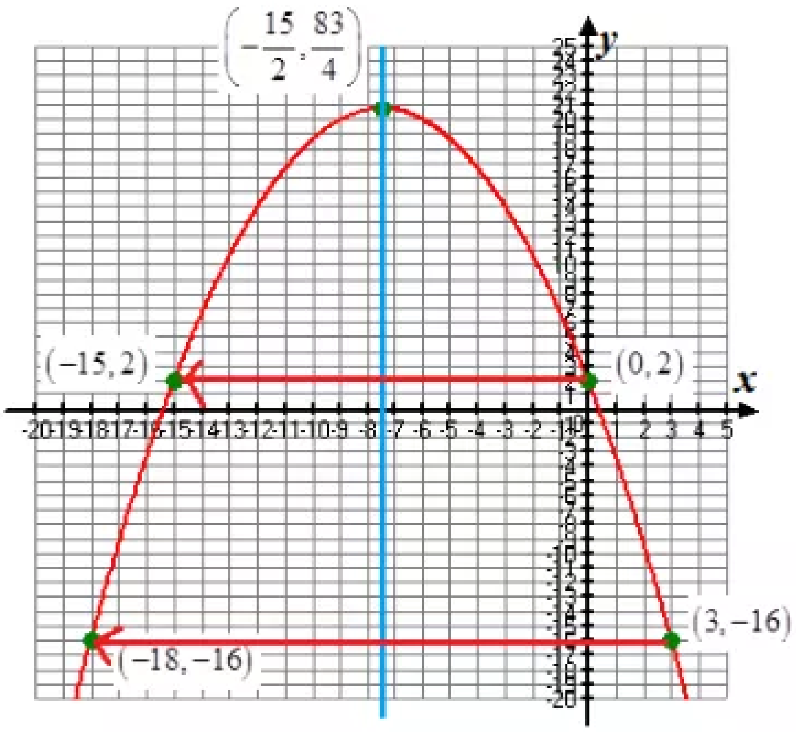
Identify the *y*-intercept *c*, which is 2 . Plot the point $(0,2)$. Then reflect this point in the axis of symmetry to plot another point, $(-15,2)$.

Evaluate the function for the another value of *x*, such as $x=3$.

$$f(x)=-\frac{1}{3}x^2-5x+2$$

$$\begin{aligned}f(x)&=-\frac{1}{3}(3)^2-5(3)+2\\&=-\frac{9}{3}-15+2\\&=-3-13\\&=-16\end{aligned}$$

Plot the point $(3,-16)$ and its reflection $(-18,-16)$ in the axis of symmetry.
Draw the parabola through the plotted points



Therefore, the above graph is the required graph of the function $f(x) = -\frac{1}{3}x^2 - 5x + 2$

Answer 7e.

STEP 1 We need to find some points to graph the function. For this, make a table of values for the given function.

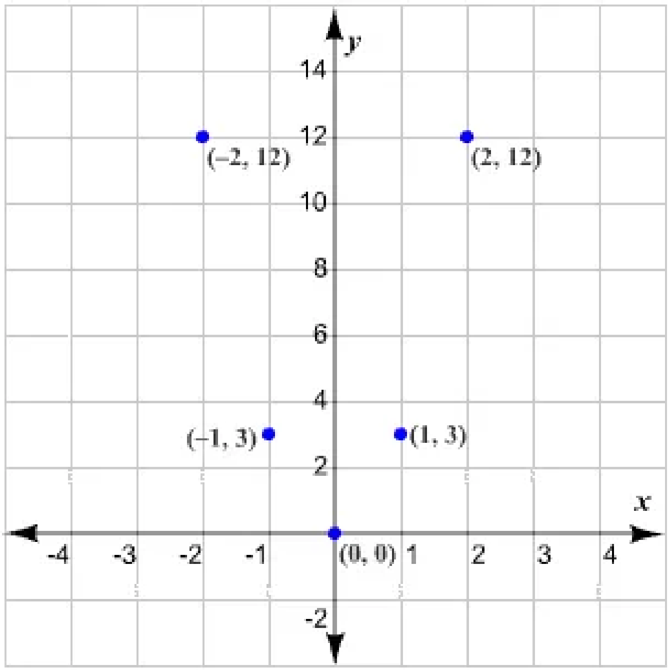
Substitute any value for x, say, -2 and evaluate y.

$$y = 3(-2)^2$$
$$= 3(4)$$
$$= 12$$

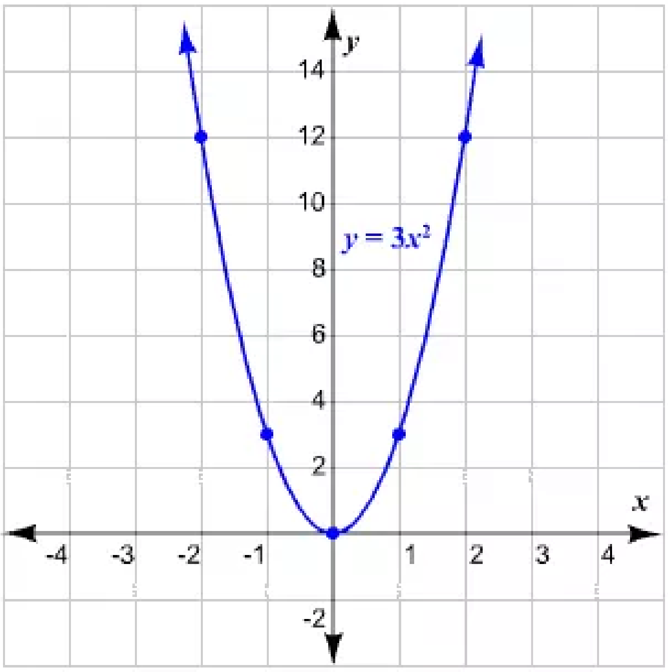
Choose some more x-values and find the corresponding y-values. Organize the results in a table.

x	-2	-1	0	1	2
y	12	3	0	3	12

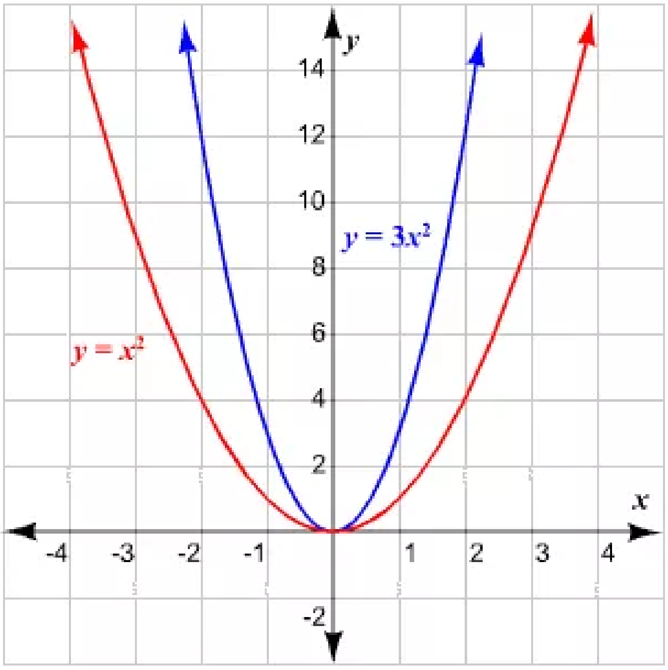
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.



On comparing we get both the graphs open up and have the same vertex and axis of symmetry.
The graph of $y = 3x^2$ is narrower than the graph of $y = x^2$.

Answer 7gp.

We know that for a function $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value if $a > 0$, and the maximum value if $a < 0$.

First, find the coordinates of the vertex.

The vertex of the graph of $y = ax^2 + bx + c$ has the x-coordinate as $-\frac{b}{2a}$. Find the x-coordinate by substituting 4 for a, and 16 for b and evaluate.

$$-\frac{b}{2a} = -\frac{16}{2(4)}$$
$$= -2$$

The x-coordinate of the vertex is -2.

Substitute -2 for x in the given function to find the y-coordinate.

$$y = 4(-2)^2 + 16(-2) - 3$$
$$= 16 - 32 - 3$$
$$= -19$$

Therefore, the minimum value of the function is -19.

Answer 8e.

Consider the function $y = 5x^2$

Graph the function $y = 5x^2$ and compare the graph with the graph $y = x^2$.

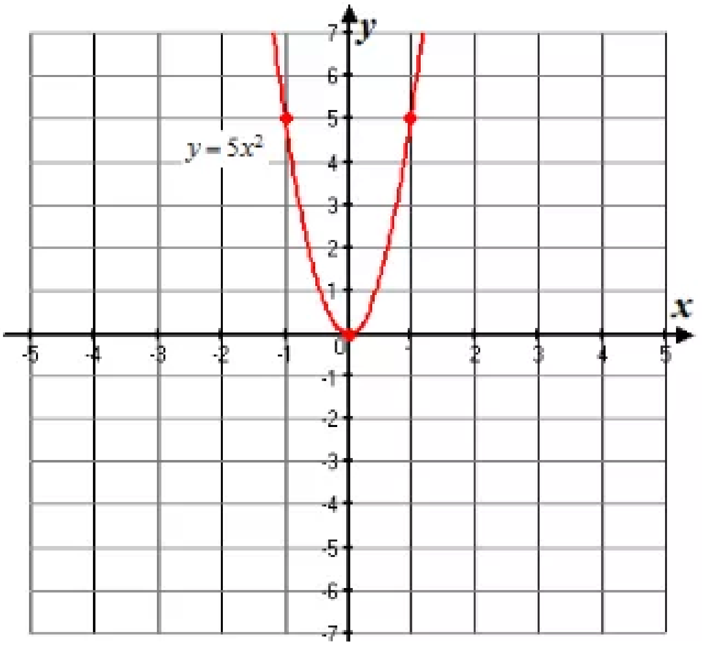
STEP 1:

Make a table of values for $y = 5x^2$

x	-2	-1	0	1	2
y	20	5	0	5	20

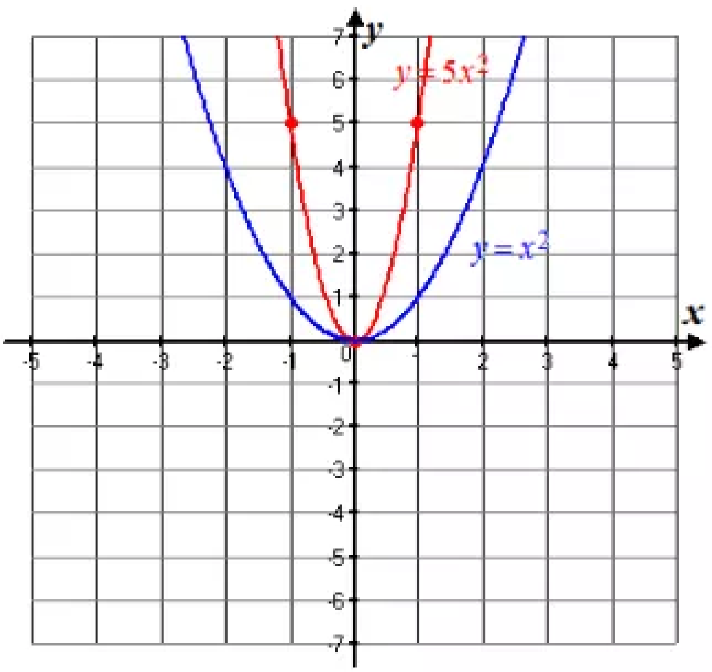
STEP 2:

Plot the points and draw a smooth curve from the table:



STEP 3:

Consider the graph of $y = x^2$ and the above graph:



In the above diagram, the graph of $y = 5x^2$ is represented by red curve and the graph of $y = x^2$ is represented by blue curve.

STEP 4:

Compare the graphs of $y=5x^2$ and $y=x^2$:

In the above diagram, the graph of $y=5x^2$ and $y=x^2$ both are open upward and they have vertices $(0,0)$ and $(0,0)$ respectively and they both have same axis of symmetry.

The graph of $y=x^2$ is wider than the graph of $y=5x^2$.

Answer 8gp.

STEP 2:

Write a verbal model. Then write and simplify a quadratic function.

Revenue (dollars) = Price (dollars/racer) · Attendance (racers)

$$R(x)=(35-x)(380+40x)$$

$$R(x)=13300+1400x-380x-40x^2$$

$$R(x)=-40x^2+1020x+13300$$

Here $a=-40$, $b=1020$, $c=13300$

STEP 3:

Find the coordinates $(x, R(x))$ of the vertex.

x-coordinate of the vertex is

$$x=-\frac{b}{2a}$$

$$=-\frac{1020}{2(-40)}$$

$$=\frac{51}{4}$$

$$=12.75$$

y-coordinate of the vertex is

$$R(12.75)=-40(12.75)^2+1020(12.75)+13300$$

$$=19802.5$$

Thus, the vertex is $(12.5, 19802.5)$

Therefore, the owner should reduce the price per racer by $\boxed{\$12.75}$ to increase the weekly revenue to $\boxed{\$19802.5}$.

Answer 9e.

STEP 1 We need to find some points to graph the function. For this, make a table of values for the given function.

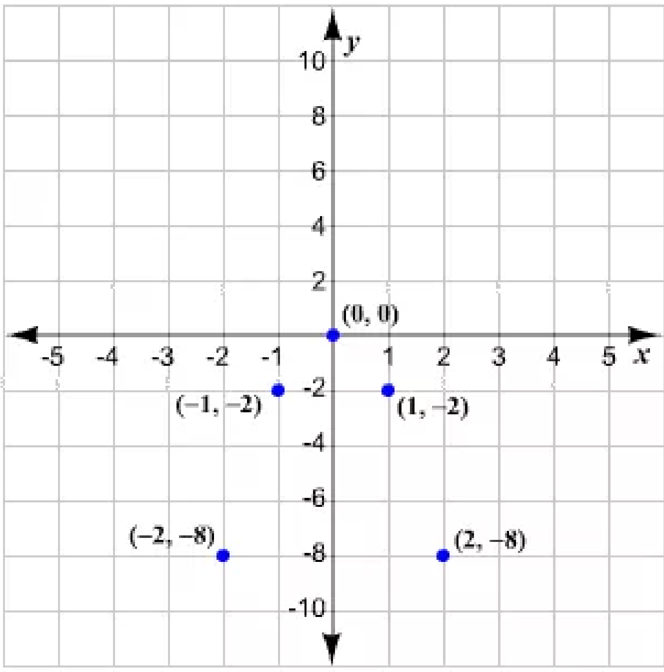
Substitute any value for x, say, -2 and evaluate y.

$$\begin{aligned} y &= -2(-2)^2 \\ &= -2(4) \\ &= -8 \end{aligned}$$

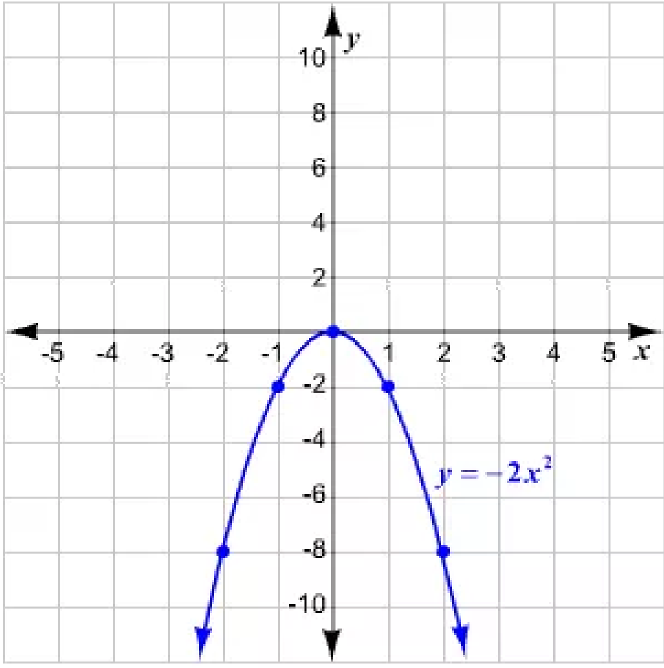
Choose some more x-values and find the corresponding y-values. Organize the results in a table.

x	-2	-1	0	1	2
y	-8	-2	0	-2	-8

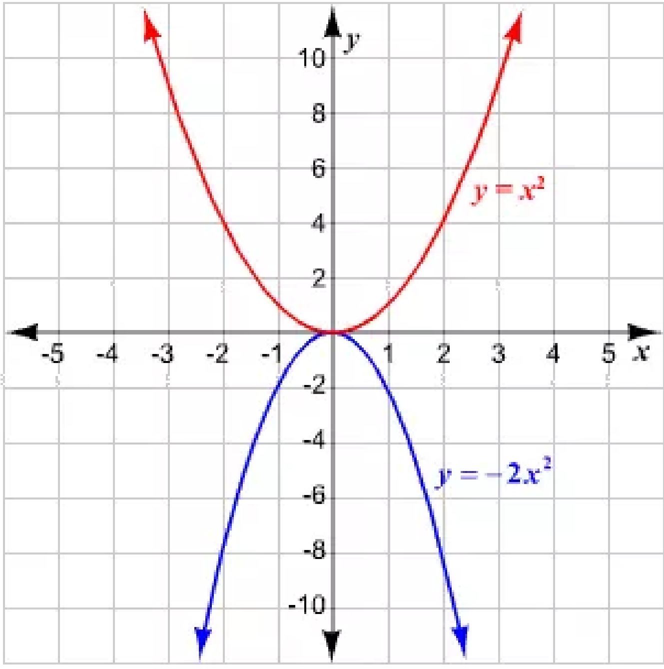
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



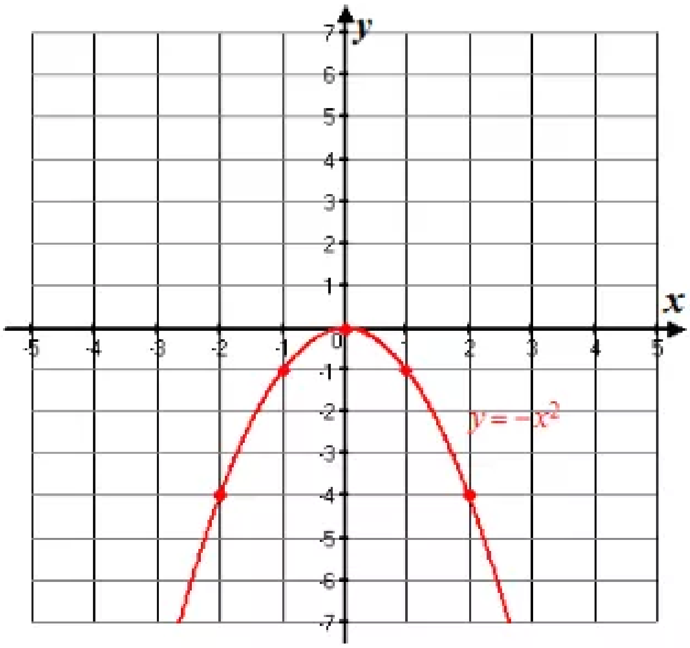
STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.



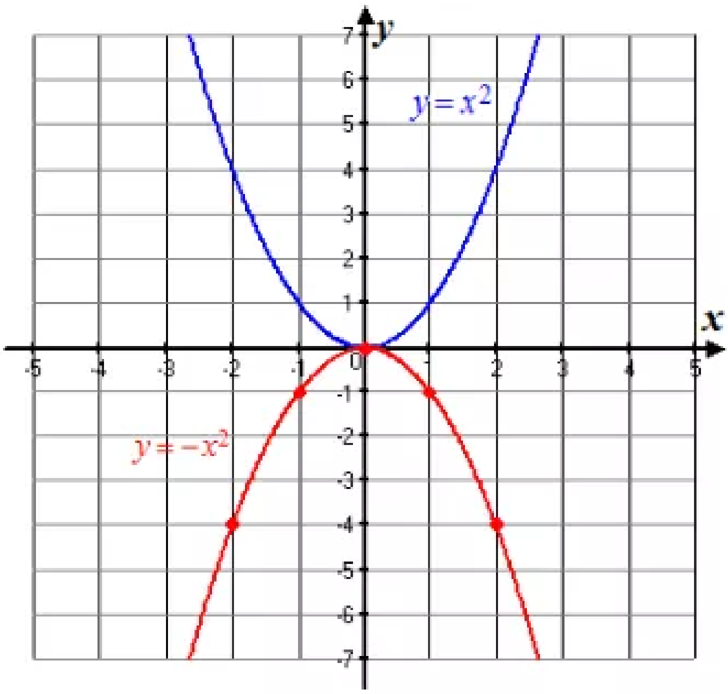
On comparing we get both the graphs have the same vertex and axis of symmetry.
The graph of $y = -2x^2$ opens down and is narrower than the graph of $y = x^2$.

Answer 10e.

STEP 2:
Plot the points and draw a smooth curve from the table:



STEP 3:
Consider the graph of $y = x^2$ and the above graph:



In the above diagram, the graph of $y = -x^2$ is represented by red curve and the graph of $y = x^2$ is represented by blue curve.

In the above diagram, the graph of $y=-x^2$ and $y=x^2$ are open downward and upward respectively and they have vertices $(0,0)$ and $(0,0)$ respectively and they both have same axis of symmetry.

Answer 11e.

STEP 1 We need to find some points to graph the function. For this, make a table of values for the given function.

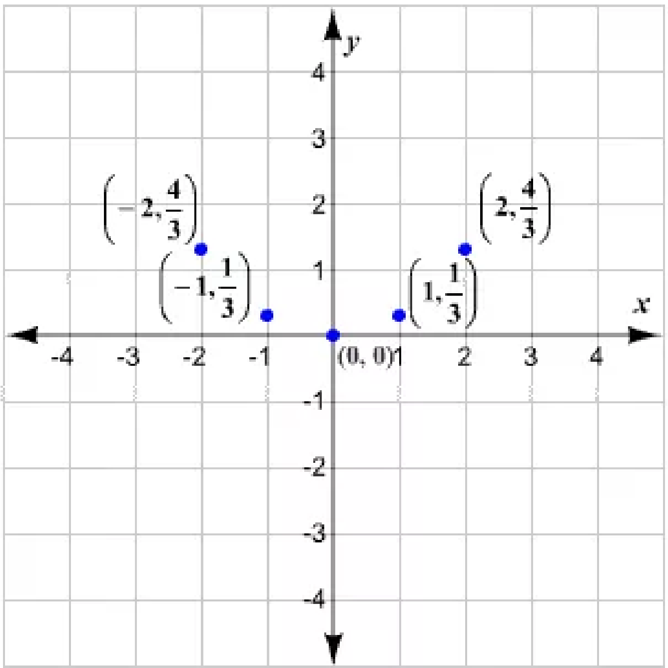
Substitute any value for x, say, -2 and evaluate y.

$$f(-2)=\frac{1}{3}(-2)^2$$
$$=\frac{1}{3}(4)$$
$$=\frac{4}{3}$$

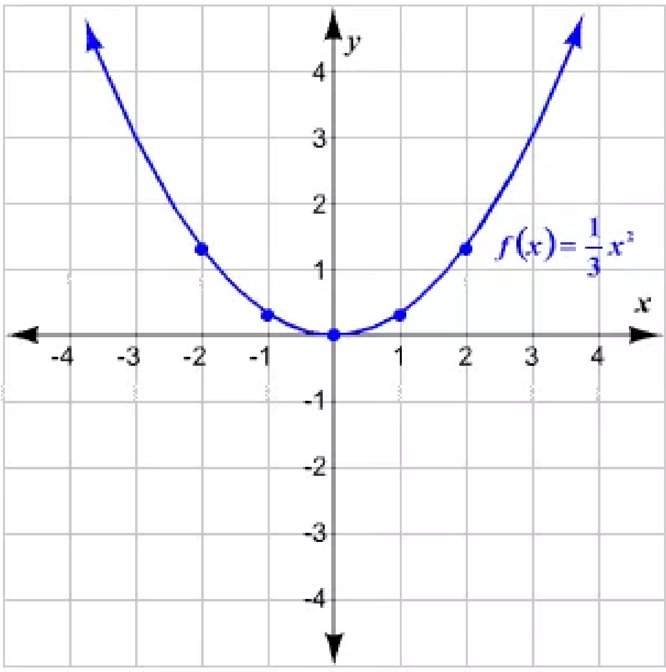
Choose some more x-values and find the corresponding y-values. Organize the results in a table.

x	-2	-1	0	1	2
$f(x)$	$\frac{4}{3}$	$\frac{1}{3}$	0	$\frac{1}{3}$	$\frac{4}{3}$

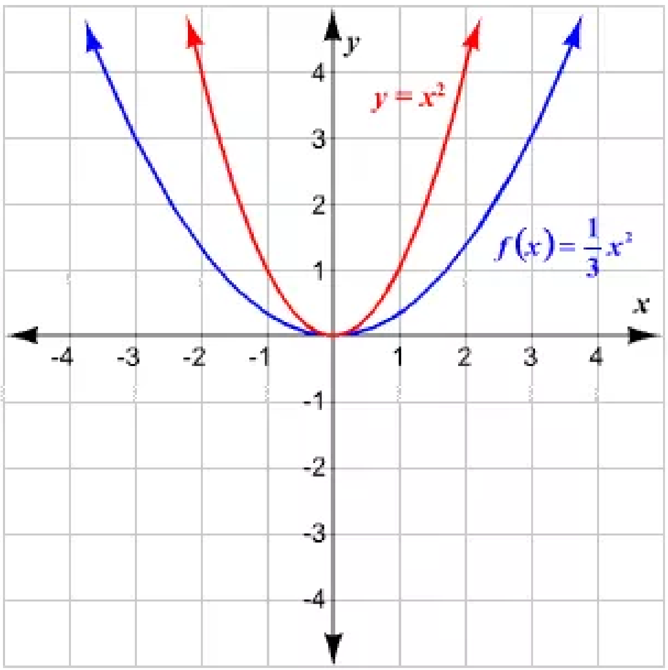
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.

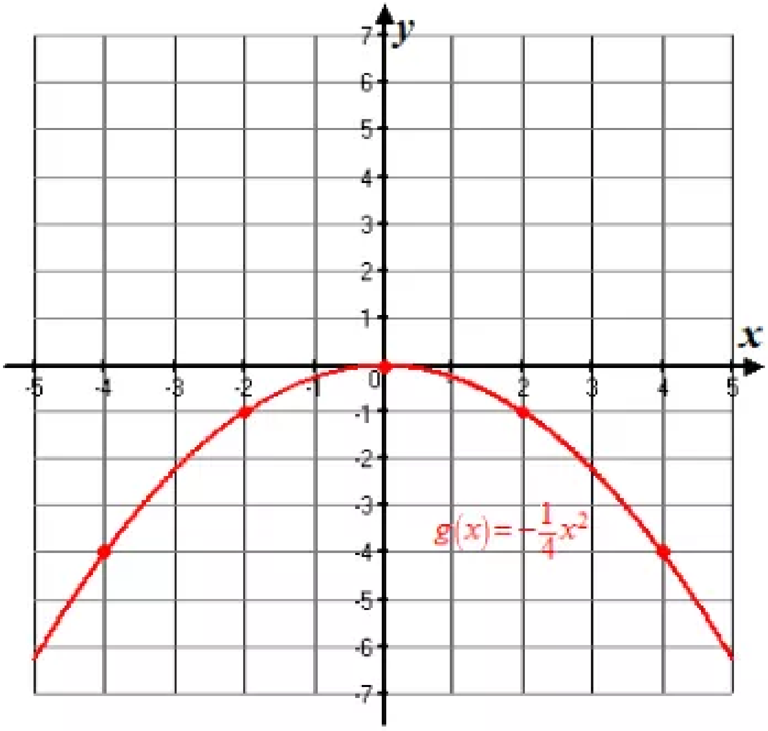


On comparing we get both the graphs open up and have the same vertex and axis of symmetry.

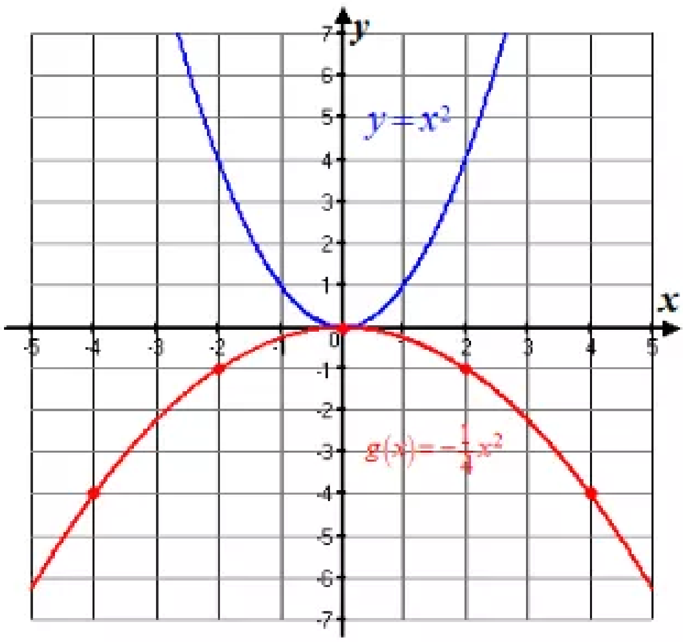
The graph of $f(x) = \frac{1}{3}x^2$ is wider than the graph of $y = x^2$.

Answer 12e.

STEP 2:
Plot the points and draw a smooth curve from the table:



STEP 3:
Consider the graph of $y=x^2$ and also the above graph:



In the above diagram, the graph of $g(x)=-\frac{1}{4}x^2$ is represented by red curve and the graph of $y = x^2$ is represented by blue curve.

In the above diagram, the graph of $g(x) = -\frac{1}{4}x^2$ and $y = x^2$ are open downward and upward respectively and they have vertices $(0,0)$ and $(0,0)$ respectively and they both have same axis of symmetry.

Answer 13e.

STEP 1 We need to find some points to graph the function. For this, make a table of values for the given function.

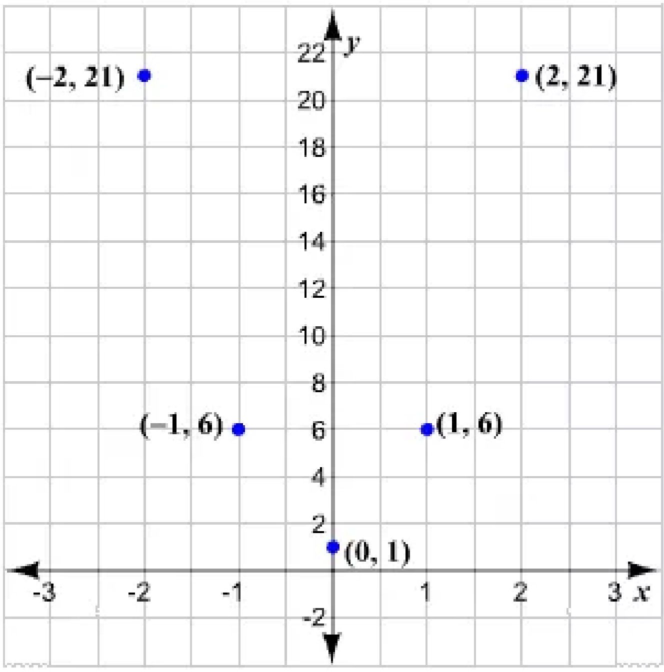
Substitute any value for x, say, -2 and evaluate y.

$$y = 5(-2)^2 + 1$$
$$= 5(4) + 1$$
$$= 21$$

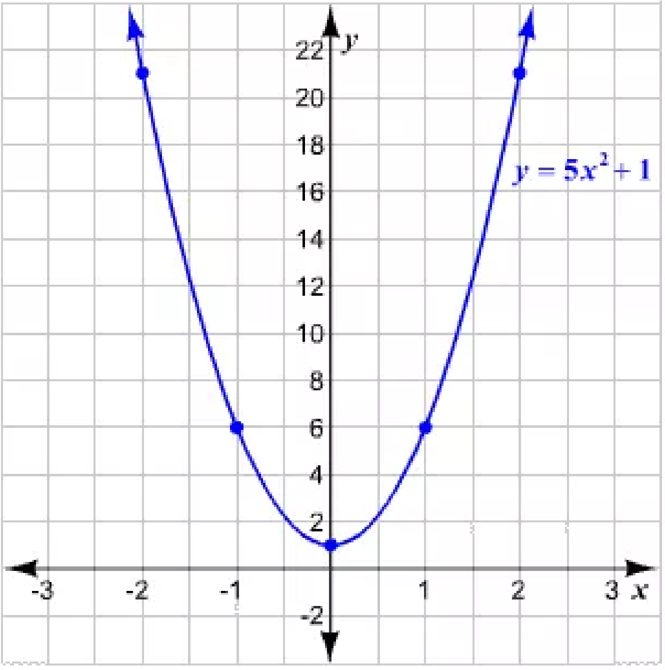
Choose some more x-values and find the corresponding y-values. Organize the results in a table.

x	-2	-1	0	1	2
y	21	6	1	6	21

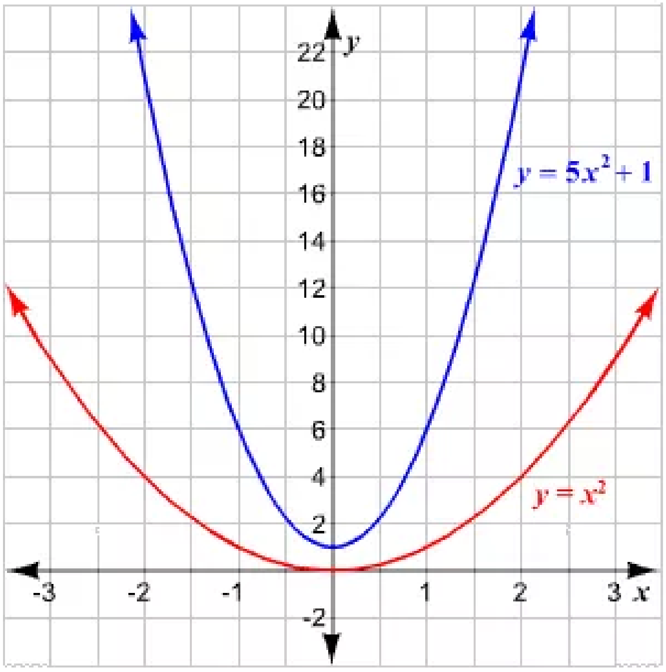
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.



On comparing we get both the graphs open up and have the same axis of symmetry.
The graph of $y = 5x^2 + 1$ is narrower than the graph of $y = x^2$ and its vertex is 1 unit higher.

Answer 14e.

Consider the function $y = 4x^2 + 1$

Graph the function $y = 4x^2 + 1$ and compare the graph with the graph $y = x^2$.

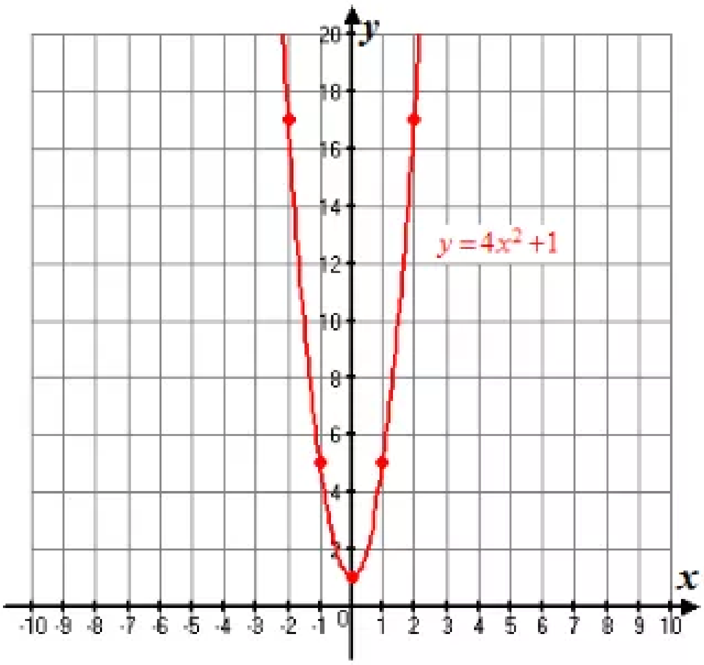
STEP 1:

Make a table of values for $y = 4x^2 + 1$

x	-2	-1	0	1	2
y	17	5	1	5	17

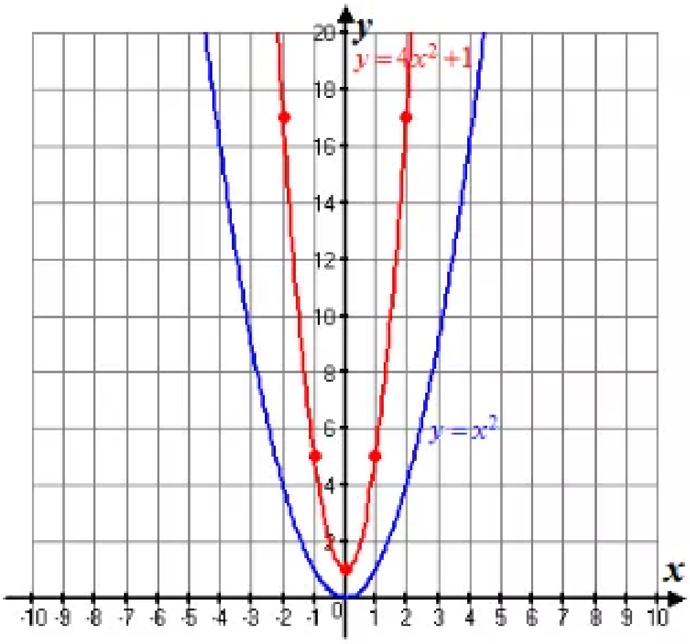
STEP 2:

Plot the points and draw a smooth curve from the table:



STEP 3:

Consider the graph of $y=x^2$ and the above graph:



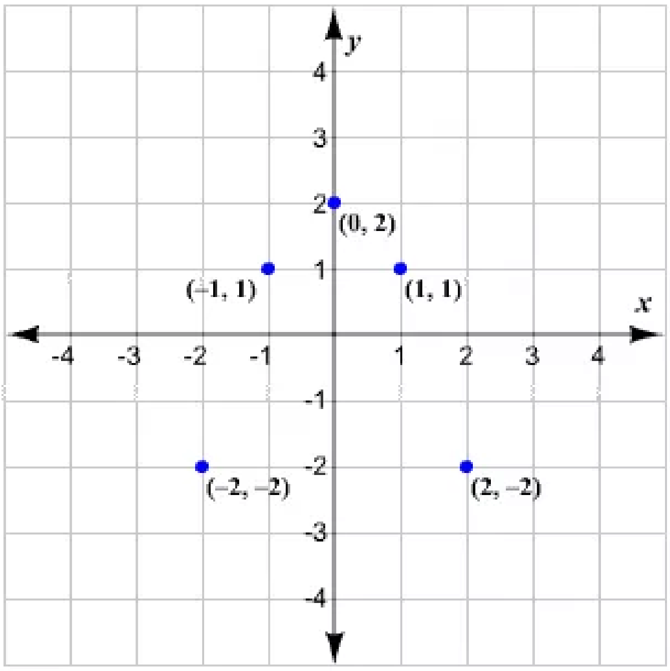
In the above diagram, the graph of $y=4x^2+1$ is represented by red curve and the graph of $y=x^2$ is represented by blue curve.

In the above diagram, the graph of $y=4x^2+1$ and $y=x^2$ are open upward respectively and they have vertices $(0,1)$ and $(0,0)$ respectively and they both have same axis of symmetry.

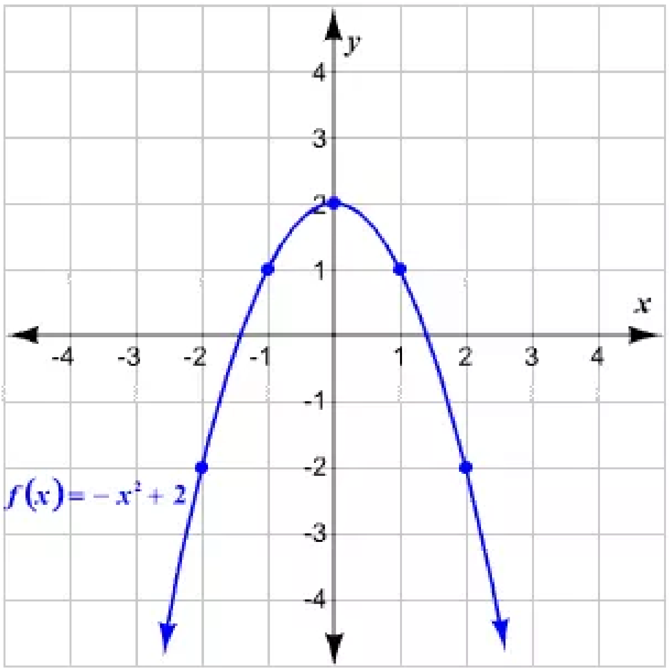
The vertex of $y=4x^2+1$ is 1 units up from the vertex of $y=x^2$.

Answer 15e.

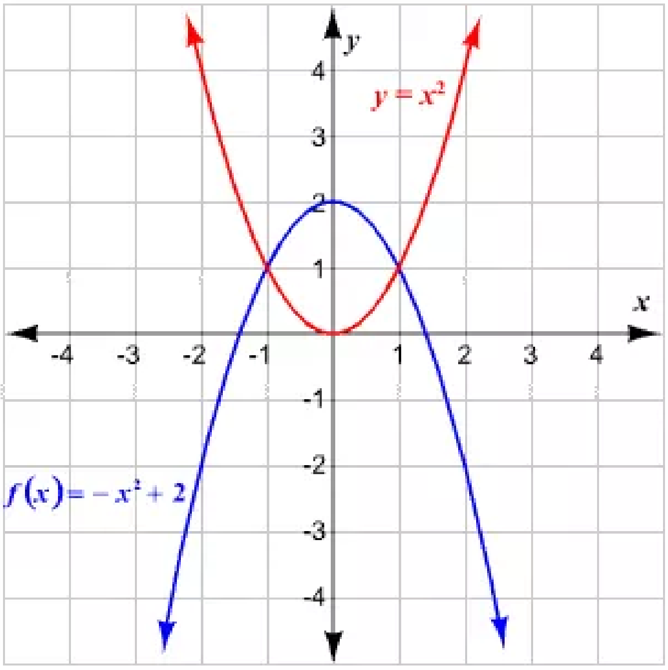
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



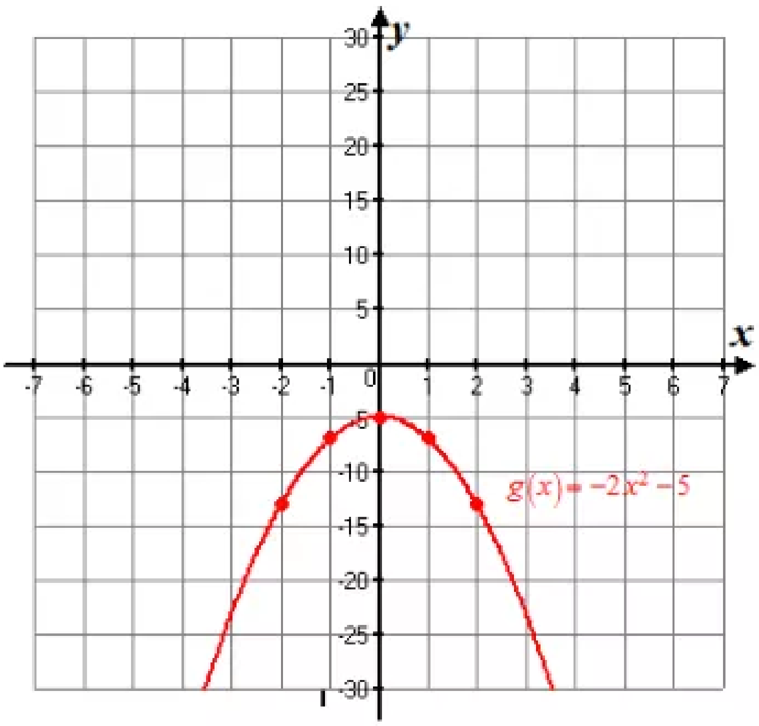
STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.



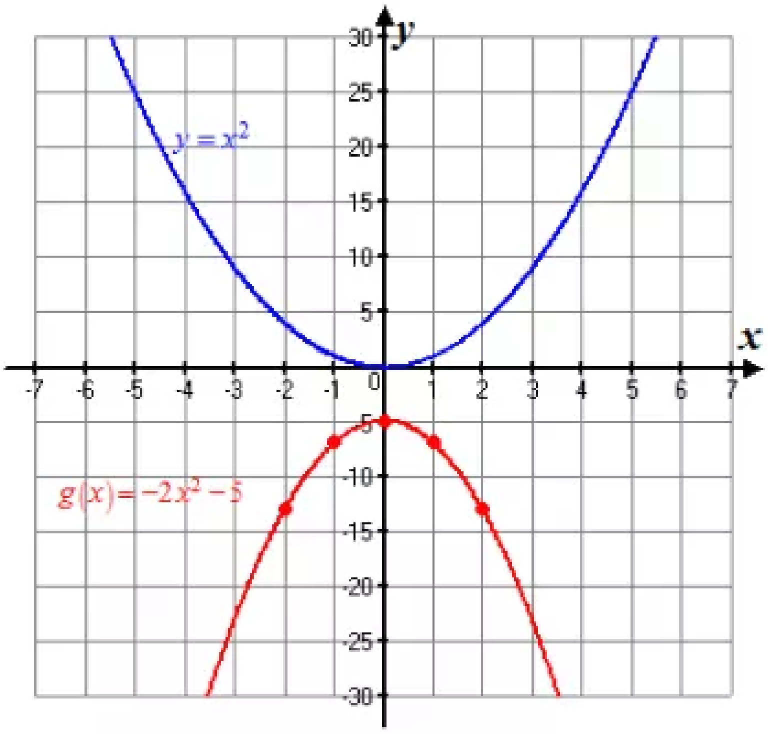
On comparing we get both the graphs have the same axis of symmetry. The graph of $f(x) = -x^2 + 2$ opens down and is wider than the graph of $y = x^2$ and its vertex is 2 units higher.

Answer 16e.

STEP 2:
Plot the points and draw a smooth curve from the table:



STEP 3:
Consider the graph of $y=x^2$ and the above graph:

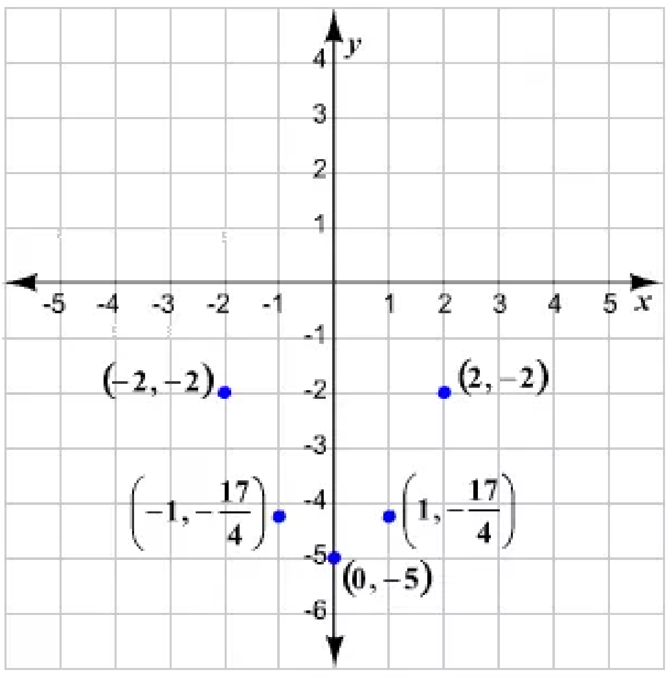


In the above diagram, the graph of $g(x)=-2x^2-5$ is represented by red curve and the graph of $y=x^2$ is represented by blue curve.

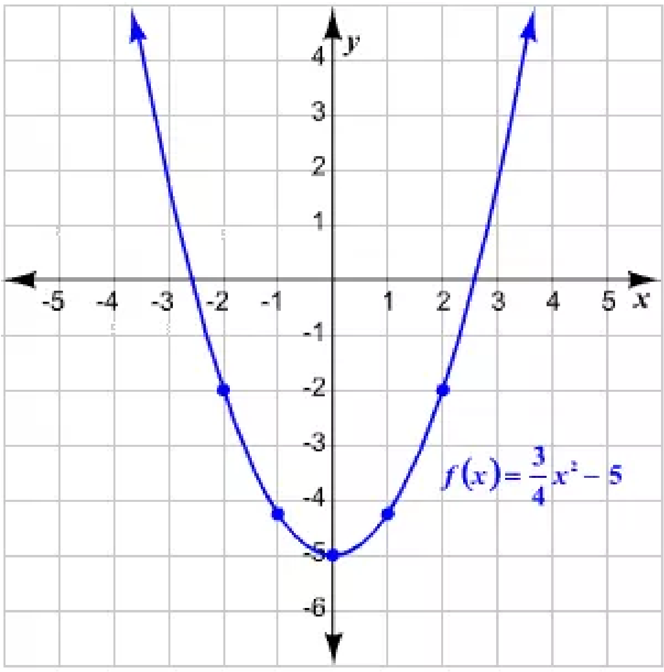
In the above diagram, the graph of $g(x)=-2x^2-5$ and $y=x^2$ are open downward and upward respectively and they have vertices $(0,-5)$ and $(0,0)$ respectively and they both have same axis of symmetry.
The graph of $y=x^2$ is wider than the graph of $g(x)=-2x^2-5$.
The vertex of $g(x)=-2x^2-5$ is 5 units down from the vertex of $y=x^2$.

Answer 17e.

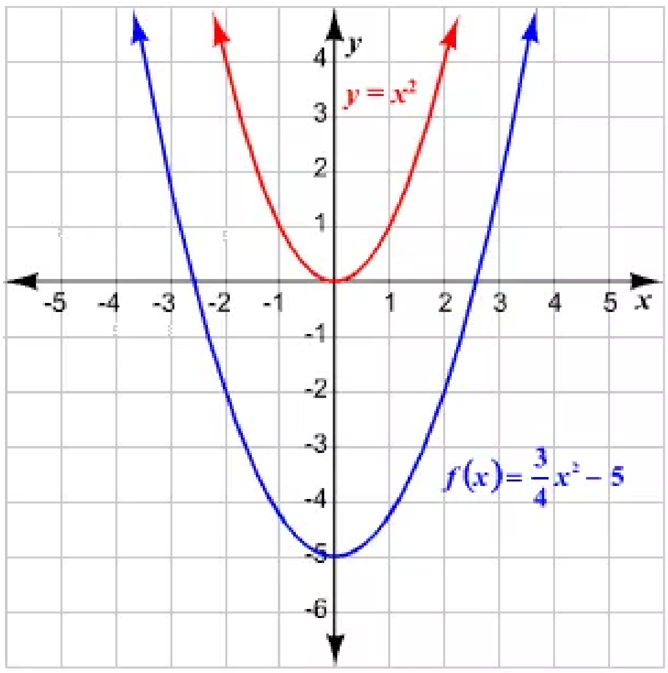
STEP 2 Plot the points from the table on a coordinate plane.



STEP 3 Connect the plotted points with a smooth curve.



STEP 4 Similarly, draw the graph of $y = x^2$ on the same coordinate plane.



On comparing we get both the graphs open up and have the same axis of symmetry.

The graph of $f(x) = \frac{3}{4}x^2 - 5$ is wider than the graph of $y = x^2$ and its vertex is 5 units lower.

Answer 18e.

Consider the function $g(x) = -\frac{1}{5}x^2 - 2$

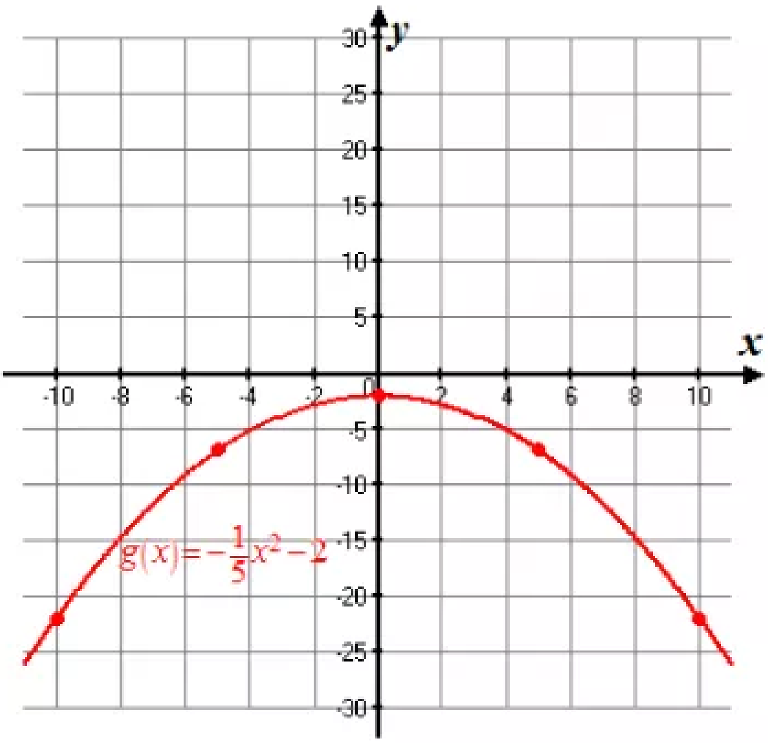
Graph the function $g(x) = -\frac{1}{5}x^2 - 2$ and compare the graph with the graph $y = x^2$.

STEP 1:

Make a table of values for $g(x) = -\frac{1}{5}x^2 - 2$

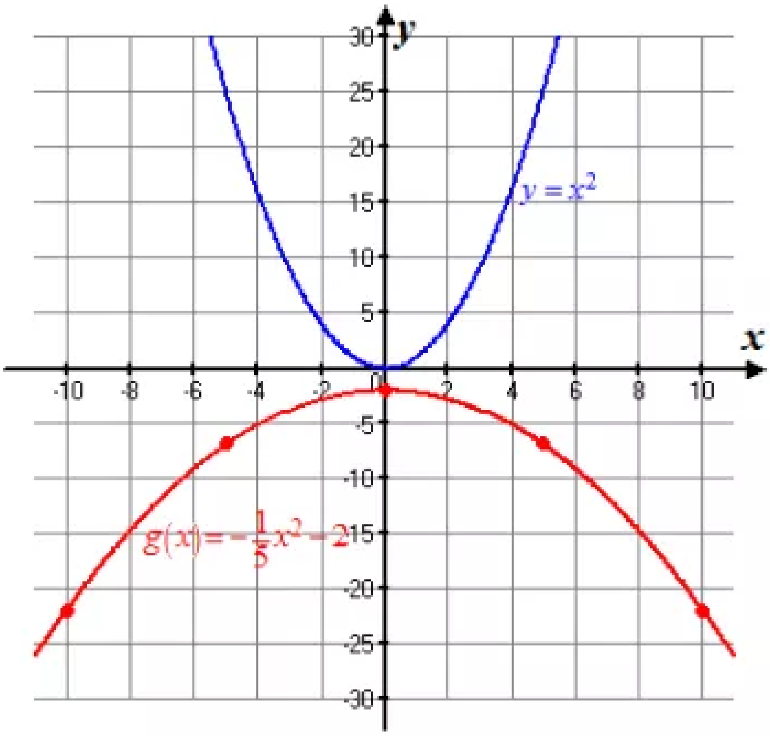
x	-10	-5	0	1	2
$y = g(x)$	-22	-7	-2	-7	-22

STEP 2:
Plot the points and draw a smooth curve from the table:



STEP 3:

Consider the graph of $y = x^2$ and the above graph:



In the above diagram, the graph of $g(x) = -\frac{1}{5}x^2 - 2$ is represented by red curve and the graph of $y = x^2$ is represented by blue curve.

In the above diagram, the graph of $g(x) = -\frac{1}{5}x^2 - 2$ and $y = x^2$ are open downward and upward respectively and they have vertices $(0, -2)$ and $(0, 0)$ respectively and they both have same axis of symmetry.

The graph of $y = x^2$ is wider than the graph of $g(x) = -\frac{1}{5}x^2 - 2$.

The vertex of $g(x) = -\frac{1}{5}x^2 - 2$ is 2 units down from the vertex of $y = x^2$.

Answer 19e.

We know that the formula for the x-coordinate of the vertex is $-\frac{b}{2a}$.

In the given problem, the formula used is $\frac{b}{2a}$ instead of $-\frac{b}{2a}$.

In order to correct the error, substitute 4 for a, and 24 for b in $x = -\frac{b}{2a}$.

$$x = -\frac{24}{2(4)}$$

Now, evaluate.

$$x = -\frac{24}{8}$$
$$= -3$$

Therefore, the x-coordinate of the vertex is -3.

Answer 20e.

Consider a quadratic function $y = 4x^2 + 24x - 7$

Assume the y-intercept of the graph is the value of c, which is 7.

The above statement is $\boxed{\text{false.}}$

Because, by comparing the quadratic function $y = 4x^2 + 24x - 7$ with $y = ax^2 + bx + c$.

We get, $a = 4, b = 24, c = -7$

Therefore, the y-intercept of the graph is the value of c, which is $\boxed{-7}$

Answer 21e.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y = ax^2 + bx + c$. On comparing, we have a is 1, b is 2, and c is 1. Since $a = 1 > 0$, the graph opens up.

STEP 2 Find the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has the x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute 1 for a, and 2 for b and evaluate.

$$-\frac{b}{2a} = -\frac{2}{2(1)}$$
$$= -1$$

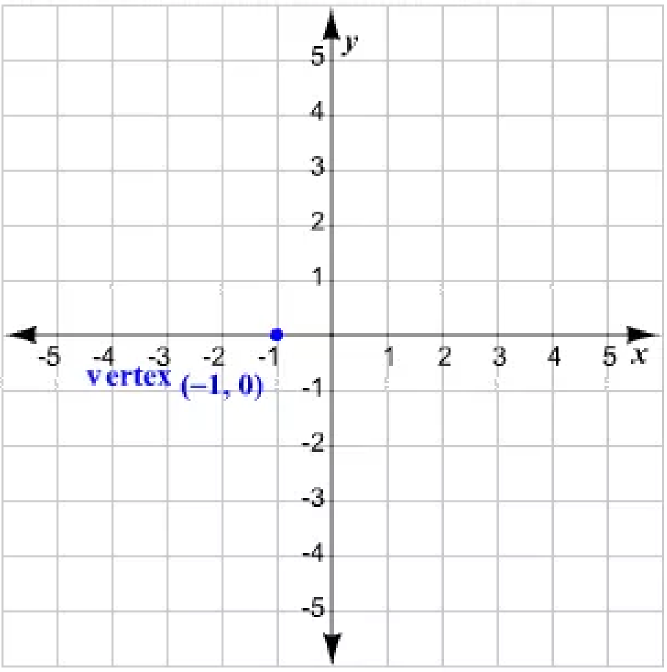
The x-coordinate of the vertex is -1.

Substitute -1 for x in the given function to find the y-coordinate.

$$y = (-1)^2 + 2(-1) + 1$$
$$= 1 - 2 + 1$$
$$= 0$$

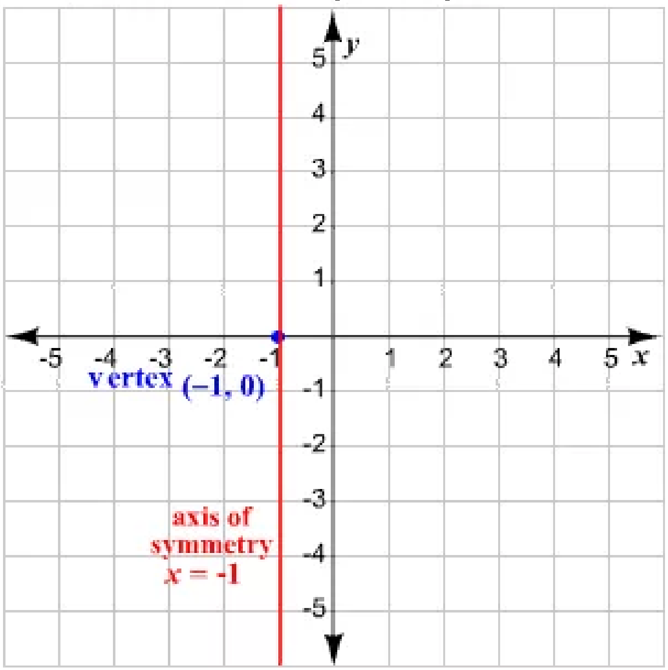
Thus, the vertex of the graph of the given function is $(-1, 0)$.

Plot the vertex on a coordinate plane.



STEP 3 We know that the axis of symmetry is $x = -\dfrac{b}{2a}$.

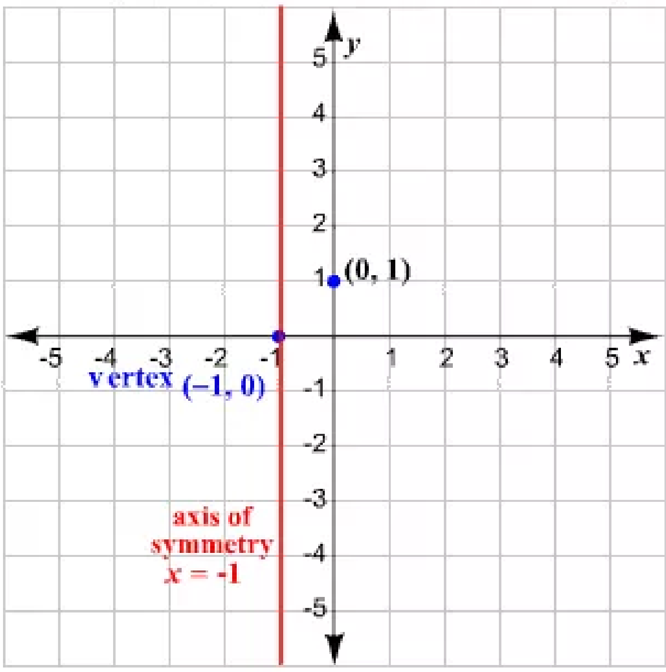
The axis of symmetry of the given function is the line $x = -1$.
Now, draw the axis of symmetry $x = -1$.



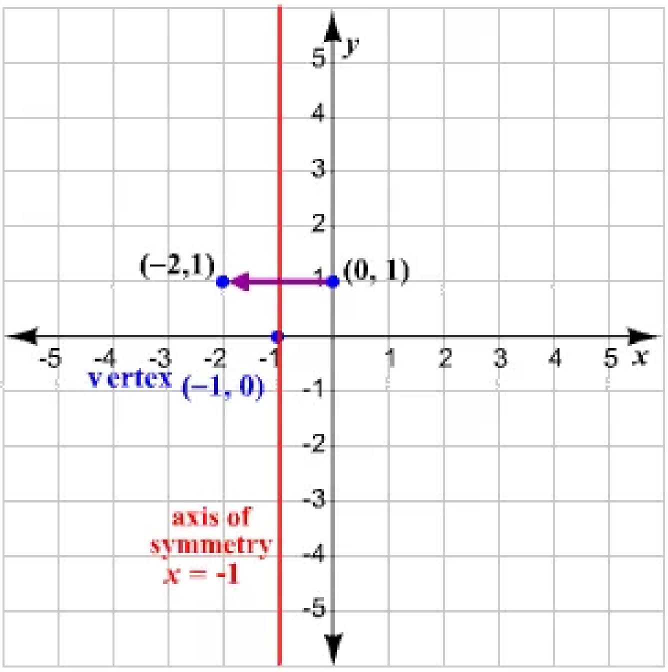
STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola.

Thus, the y-intercept of the given function is 1 and (0, 1) is on the parabola.

Plot the point (0, 1) on the same coordinate plane.



Now, reflect the point (0, 1) in the axis of symmetry to get another point.



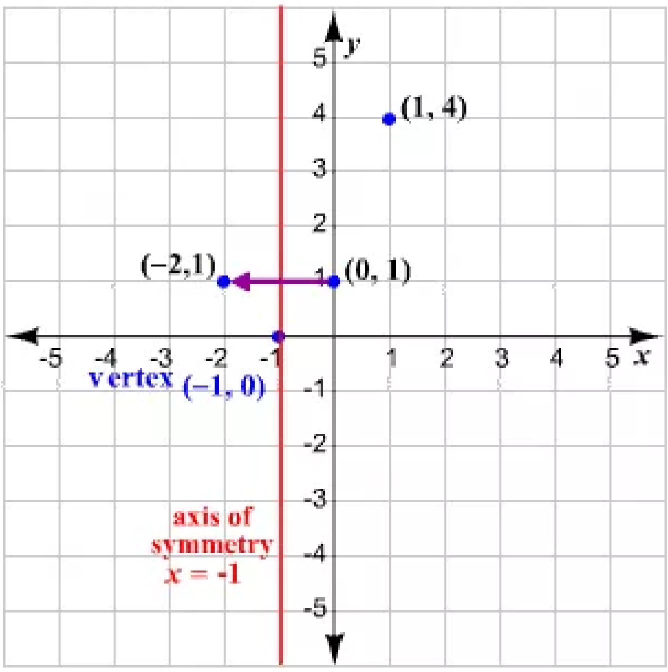
STEP 5 Evaluate the given function for another value of x, say, 1.

Substitute 1 for x in the function and simplify.

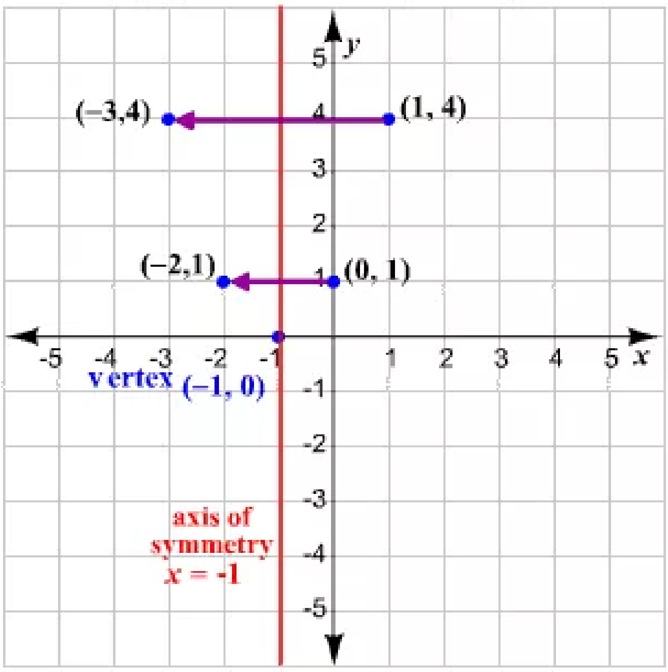
$$\begin{aligned} y &= 1^2 + 2(1) + 1 \\ &= 1 + 2 + 1 \\ &= 4 \end{aligned}$$

Thus, the point (1, 4) lies on the graph.

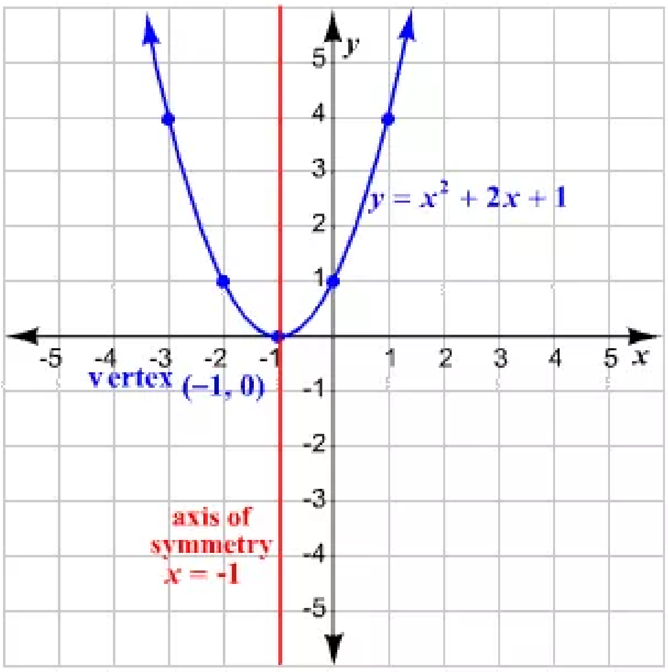
Plot the point on the coordinate plane.



Reflect (1, 4) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 22e.

Consider the function $y = 3x^2 - 6x + 4$

STEP 1:

Identify the coefficients of the function $y = 3x^2 - 6x + 4$

Comparing the function $y = 3x^2 - 6x + 4$ with $y = ax^2 + bx + c$

The coefficients are $a = 3, b = -6, c = 4$

Because $a > 0$, the parabola opens up.

STEP 2:

The axis of symmetry is $x = -\dfrac{b}{2a} = -\dfrac{-6}{2(3)} = 1$

Find the vertex.

Calculate the x-coordinate.

The vertex has x-coordinate$= = -\dfrac{b}{2a} = -\dfrac{-6}{2(3)} = 1$

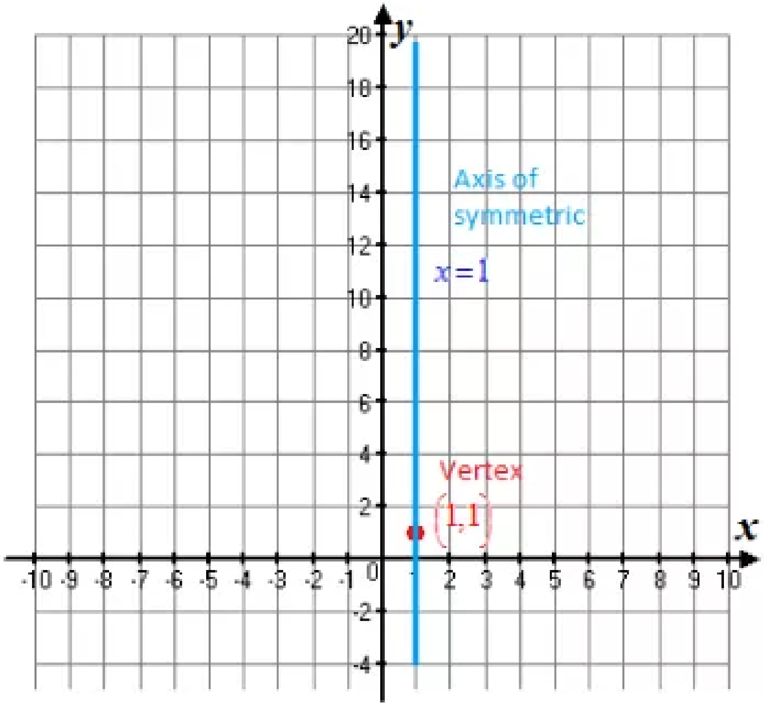
Then find the y-coordinate of the vertex.

$y = 3(1^2) - 6(1) + 4 = 1$

Therefore, the vertex is $(1,1)$.

STEP 3:

Plot the point $(1,1)$ and draw the axis of symmetry $x=1$.



STEP 4:

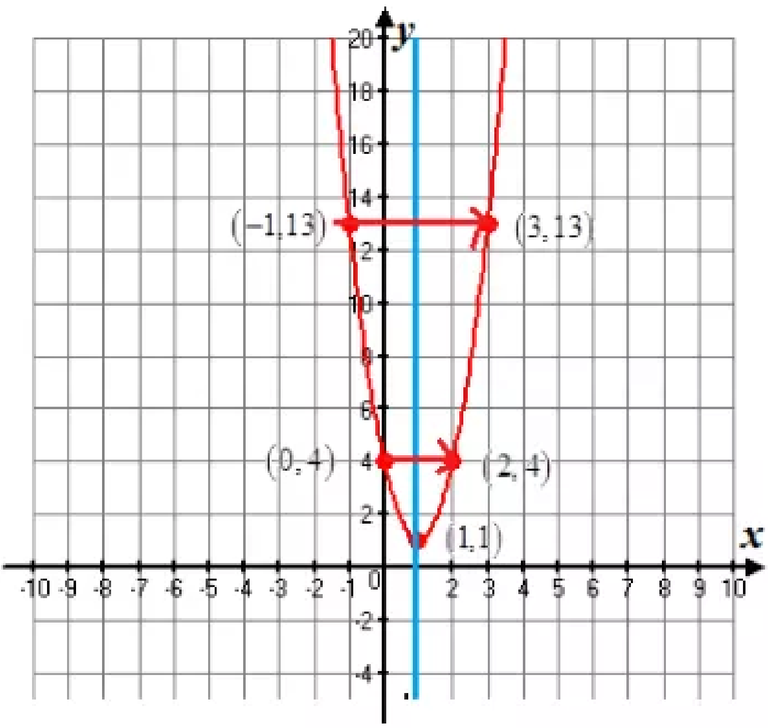
Identify the y-intercept c, which is 4. Plot the point $(0,4)$. Then reflect this point in the axis of symmetry to plot another point, $(2,4)$.

Evaluate the function for the another value of x, such as $x=-1$.

$y=3\left((-1)^2\right)-6(-1)+4=13$

Plot the point $(-1,13)$ and its reflection $(3,13)$ in the axis of symmetry.

Draw the parabola through the plotted points



Therefore, the above graph is the required graph of the function $y=3x^2-6x+4$

Answer 23e.

STEP 2 Find the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute -4 for a, and 8 for b and evaluate.

$$\begin{aligned} -\frac{b}{2a} &= -\frac{8}{2(-4)} \\ &= 1 \end{aligned}$$

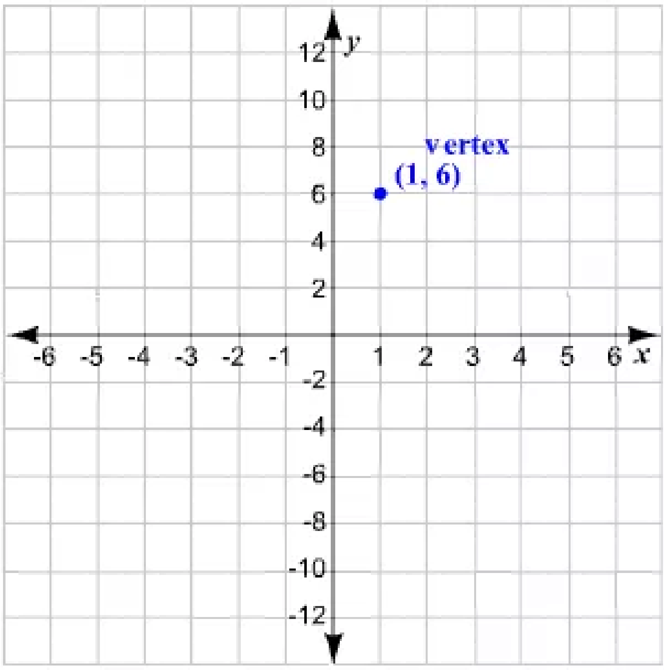
The x-coordinate of the vertex is 1.

Substitute 1 for x in the given function to find the y-coordinate.

$$\begin{aligned} y &= -4(1)^2 + 8(1) + 2 \\ &= -4 + 8 + 2 \\ &= 6 \end{aligned}$$

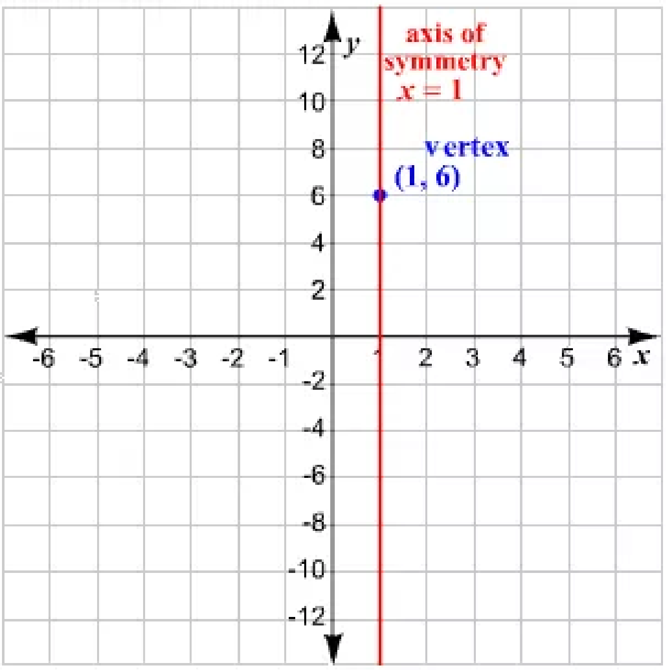
Thus, the vertex of the graph of the given function is (1, 6).

Plot the vertex on a coordinate plane.



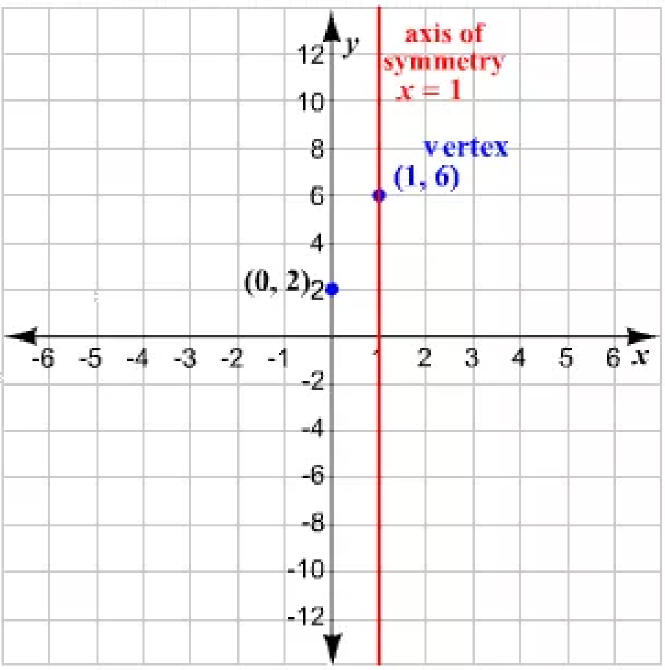
STEP 3 We know that the axis of symmetry is $x = -\dfrac{b}{2a}$.

The axis of symmetry of the given function is the line $x = 1$. Now, draw the axis of symmetry $x = 1$.

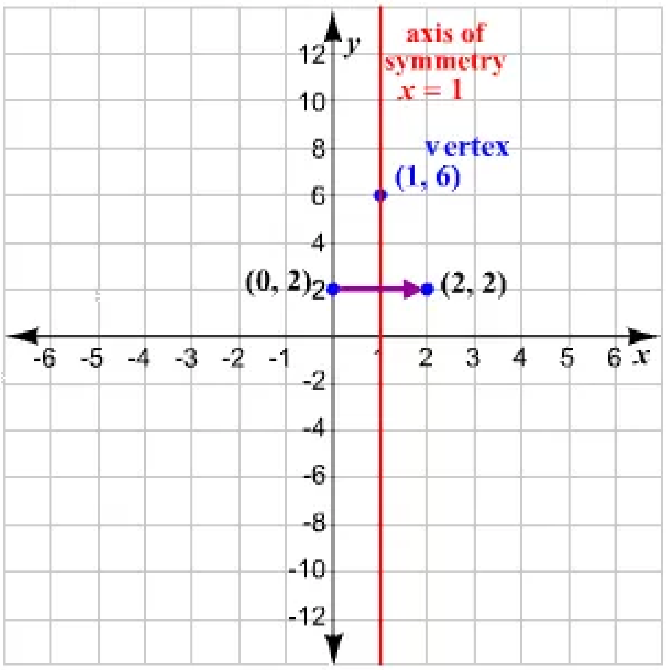


STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola. Thus, the y-intercept of the given function is 2 and (0, 2) is on the parabola.

Plot the point (0, 2) on the same coordinate plane.



Now, reflect the point (0, 2) in the axis of symmetry to get another point.



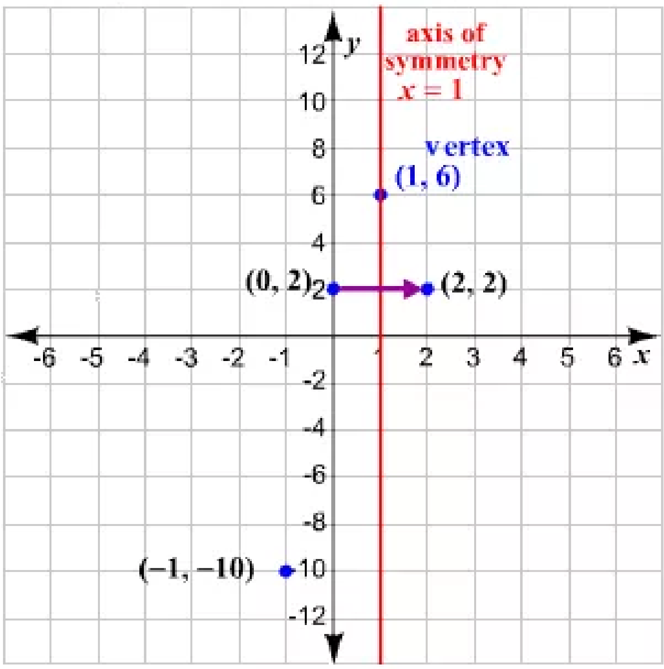
STEP 5 Evaluate the given function for another value of x, say, -1.

Substitute -1 for x in the function and simplify.

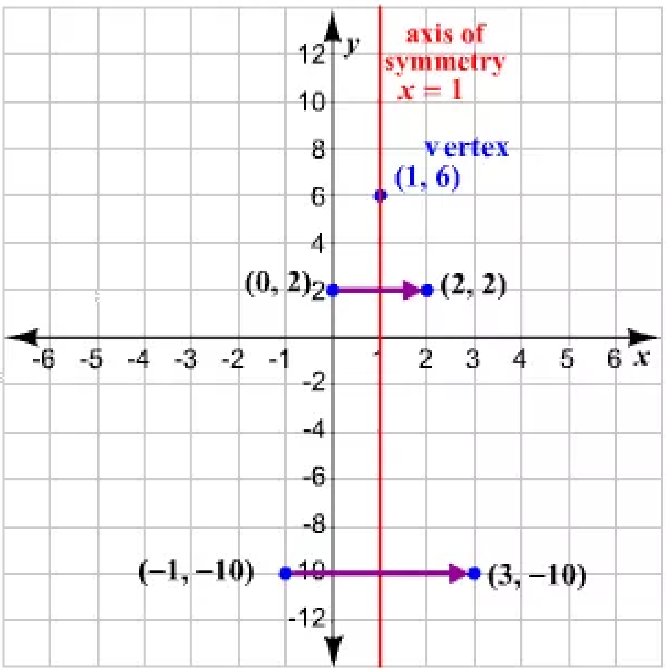
$$\begin{aligned} y &= -4(-1)^2 + 8(-1) + 2 \\ &= -4 - 8 + 2 \\ &= -10 \end{aligned}$$

Thus, the point $(-1, -10)$ lies on the graph.

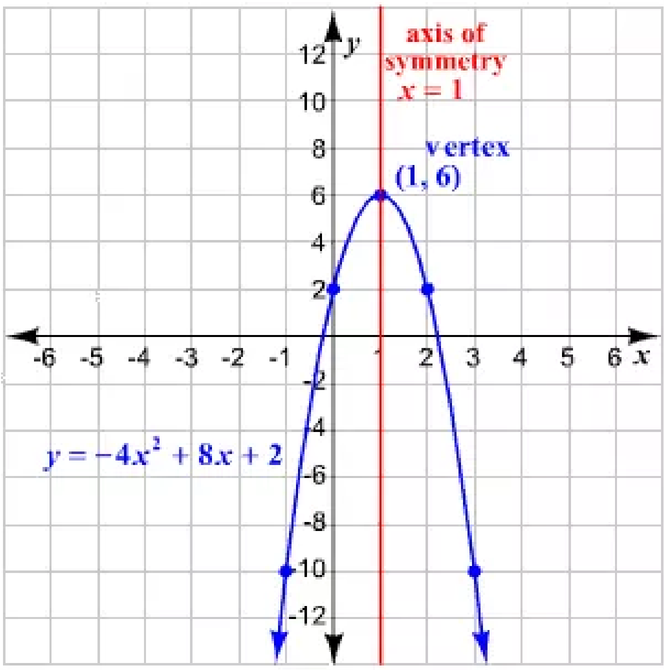
Plot the point on the coordinate plane.



Reflect $(-1, -10)$ in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 24e.

STEP 2:

The axis of symmetry is $x = -\frac{b}{2a} = -\frac{-6}{-2(2)} = \frac{-3}{2}$

Find the vertex.
Calculate the x-coordinate.

The vertex has x-coordinate$= = -\frac{b}{2a} = -\frac{-6}{-2(2)} = \frac{-3}{2}$

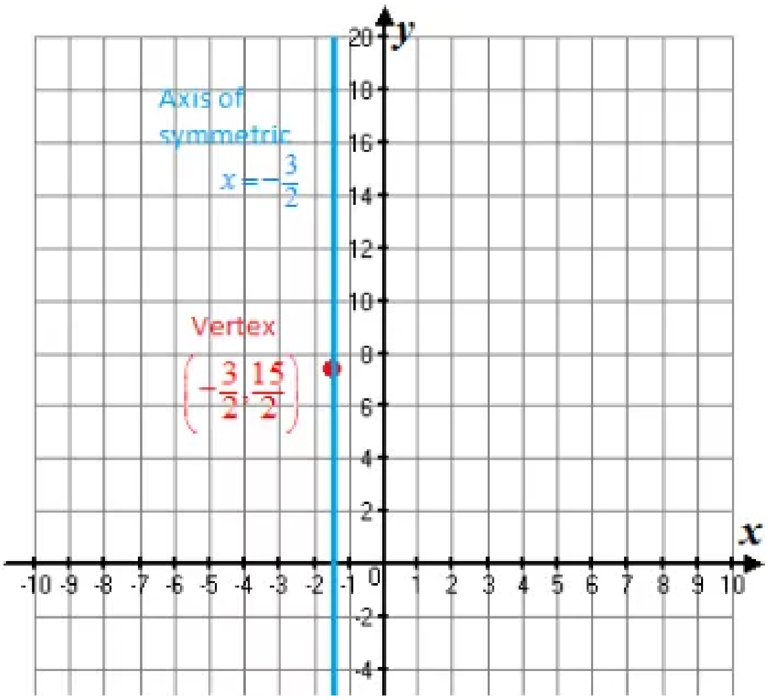
Then find the y-coordinate of the vertex.

$$y = -2\left(-\frac{3}{2}\right)^2 - 6\left(-\frac{3}{2}\right) + 3 = \frac{15}{2}$$

Therefore, the vertex is $\left(-\frac{3}{2}, \frac{15}{2}\right)$.

STEP 3:

Plot the point $\left(-\frac{3}{2}, \frac{15}{2}\right)$ and draw the axis of symmetry $x = -\frac{3}{2}$.



STEP 4:

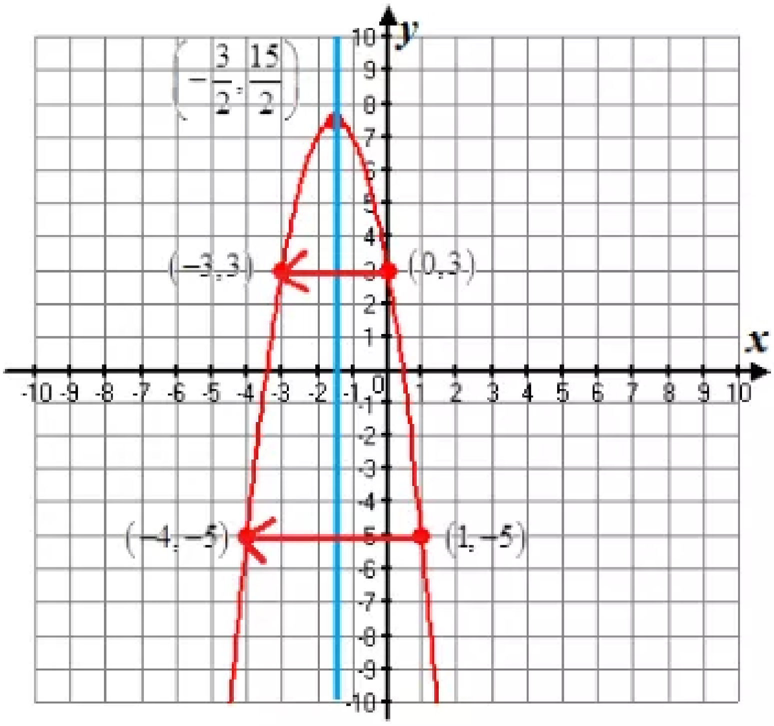
Identify the y-intercept c, which is 3. Plot the point $(0,3)$. Then reflect this point in the axis of symmetry to plot another point, $(-3,3)$.

Evaluate the function for another value of x, such as $x=1$.

$y=-2(1)^2-6(1)+3=-5$

Plot the point $(1,-5)$ and its reflection $(-4,-5)$ in the axis of symmetry.

Draw the parabola through the plotted points.



Therefore, the above graph is the required graph of the function $y=-2x^2-6x+3$

Answer 25e.

STEP 2 Find the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute -1 for a, and -2 for b and evaluate.

$$\begin{aligned} -\frac{b}{2a} &= -\frac{-2}{2(-1)} \\ &= -1 \end{aligned}$$

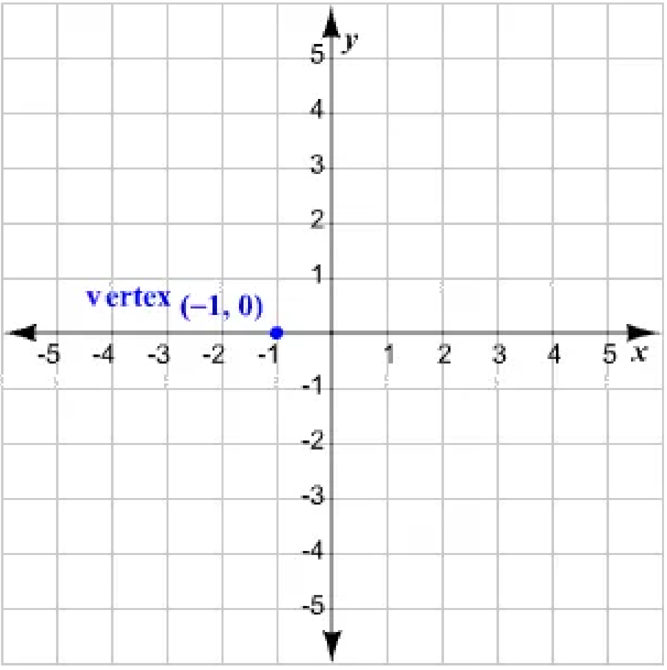
The x-coordinate of the vertex is -1.

Substitute -1 for x in the given function to find the y-coordinate.

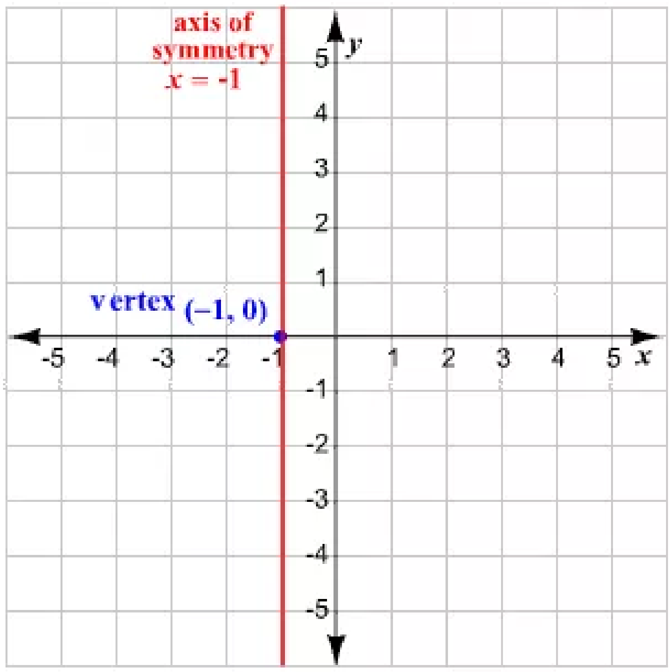
$$\begin{aligned} g(-1) &= -(-1)^2 - 2(-1) - 1 \\ &= -1 + 2 - 1 \\ &= 0 \end{aligned}$$

Thus, the vertex of the graph of the given function is $(-1, 0)$.

Plot the vertex on a coordinate plane.

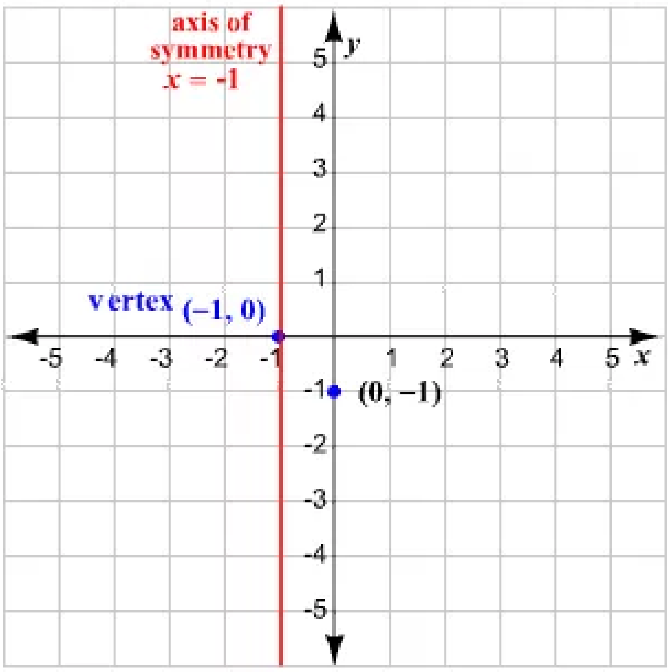


STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$. The axis of symmetry of the given function is the line $x = -1$. Now, draw the axis of symmetry $x = -1$.

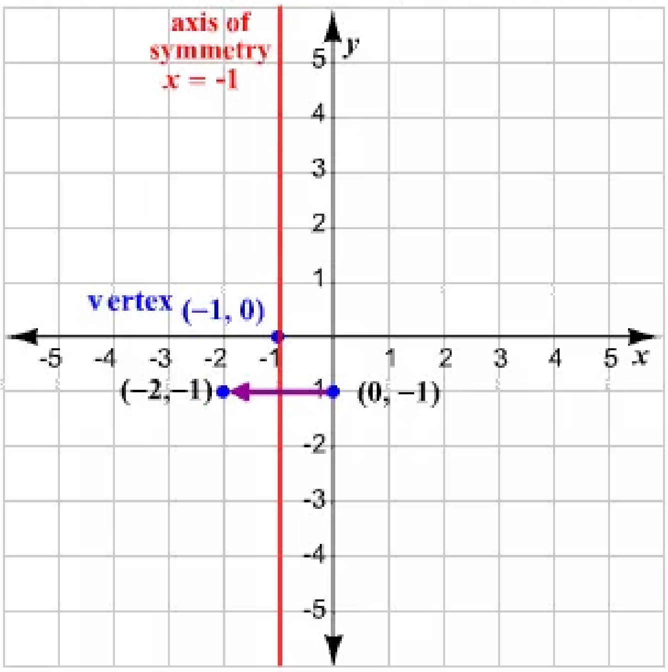


STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola. Thus, the y-intercept of the given function is -1 and $(0, -1)$ is on the parabola.

Plot the point $(0, -1)$ on the same coordinate plane.



Now, reflect the point (0, −1) in the axis of symmetry to get another point.

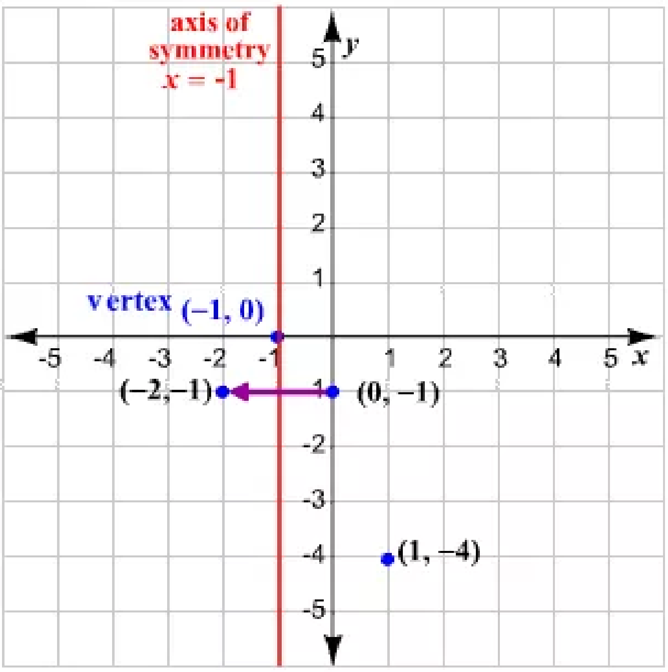


STEP 5 Evaluate the given function for another value of x, say, 1.

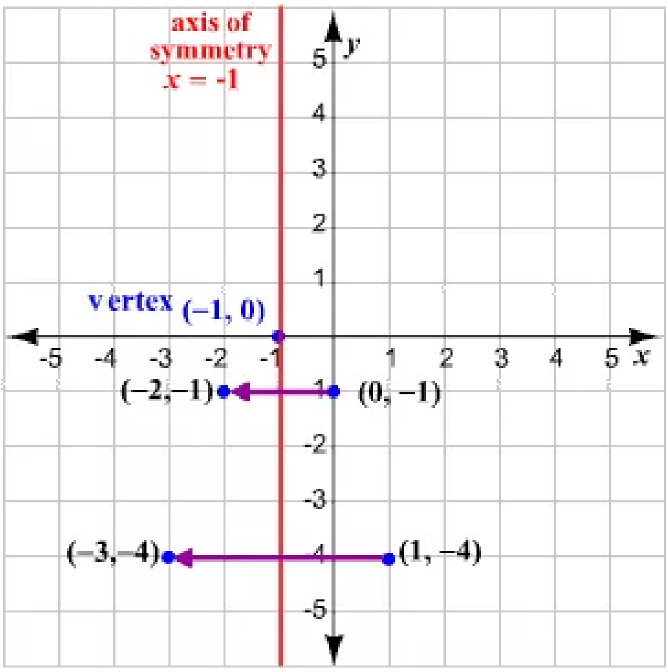
Substitute 1 for x in the function and simplify.

$$\begin{aligned} g(1) &= -(1)^2 - 2(1) - 1 \\ &= -1 - 2 - 1 \\ &= -4 \end{aligned}$$

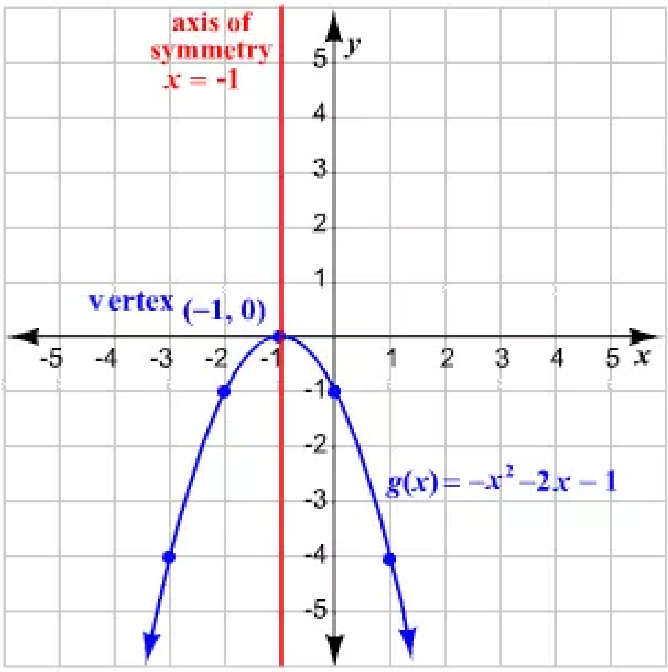
Thus, the point (1, −4) lies on the graph. Plot the point on the coordinate plane.



Reflect the point (1, –4) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 26e.

STEP 2:

The axis of symmetry is $x=-\frac{b}{2a}=-\frac{-4}{2(-6)}=-\frac{1}{3}$

Find the vertex.

Calculate the *x*-coordinate.

The vertex has *x*-coordinate= $=-\frac{b}{2a}=-\frac{-4}{2(-6)}=-\frac{1}{3}$

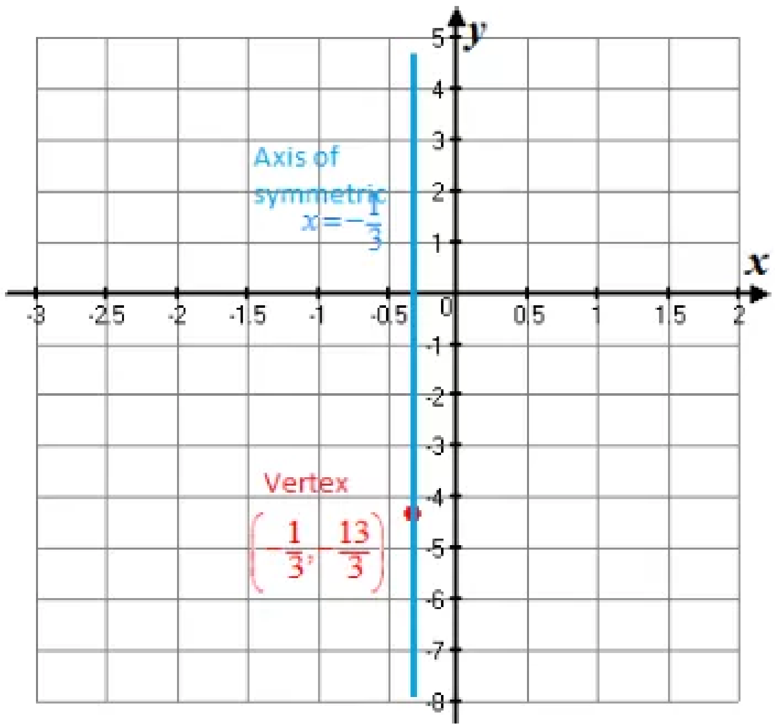
Then find the *y*-coordinate of the vertex.

$$f\left(-\frac{1}{3}\right)=-6\left(-\frac{1}{3}\right)^2-4\left(-\frac{1}{3}\right)-5=-\frac{13}{3}$$

Therefore, the vertex is $\left(-\frac{1}{3},-\frac{13}{3}\right)$.

STEP 3:

Plot the point $\left(-\frac{1}{3},-\frac{13}{3}\right)$ and draw the axis of symmetry $x=-\frac{1}{3}$.



STEP 4:

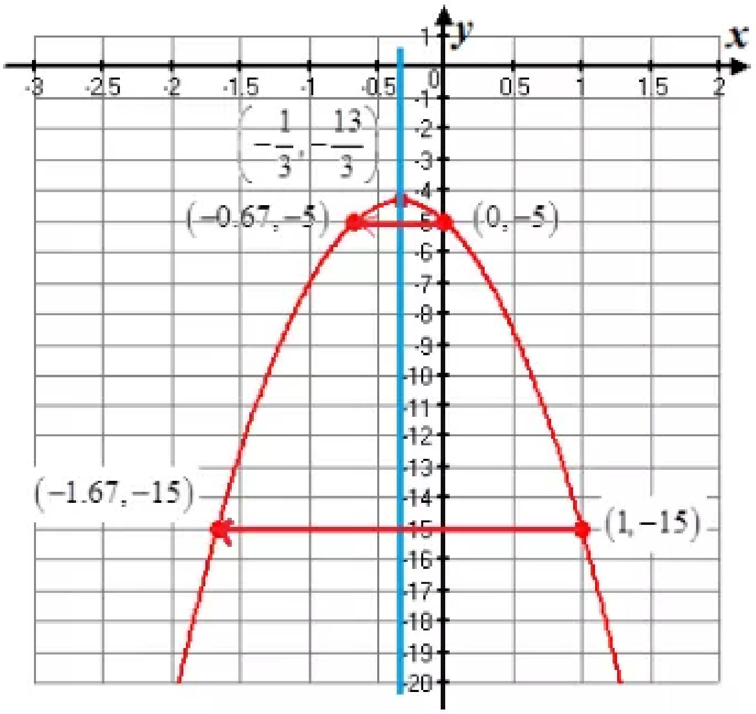
Identify the y-intercept c, which is -5. Plot the point $(0,-5)$. Then reflect this point in the axis of symmetry to plot another point, $(-0.67,-5)$.

Evaluate the function for the another value of x, such as $x=1$.

$$f(1)=-6(1)^2-4(1)-5=-15$$

Plot the point $(1,-15)$ and its reflection $(-1.67,-15)$ in the axis of symmetry.

Draw the parabola through the plotted points:



Therefore, the above graph is the required graph of the function $f(x)=-6x^2-4x-5$

Answer 27e.

STEP 2 Find the vertex. The vertex of the graph of $y=ax^2+bx+c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute $\frac{2}{3}$ for a, and -3 for b and evaluate.

$$\begin{aligned}-\frac{b}{2a} &= -\frac{-3}{2\left(\frac{2}{3}\right)} \\ &= \frac{9}{4} \\ &= 2.25\end{aligned}$$

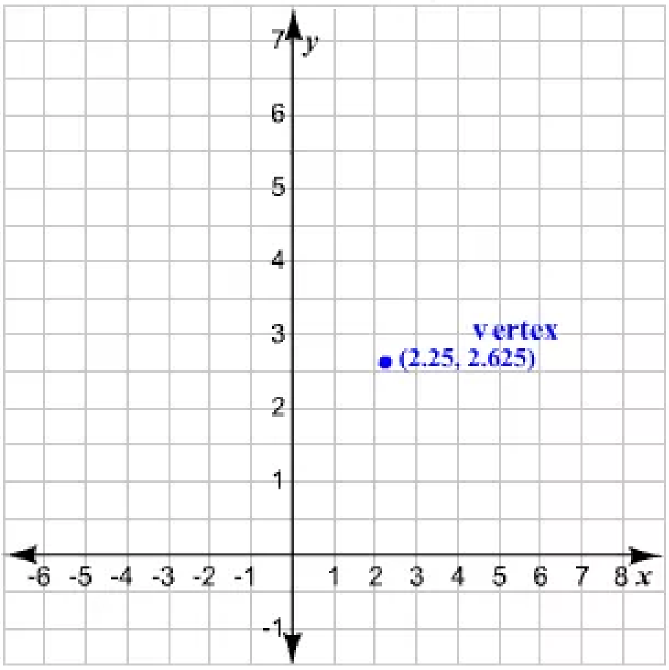
The x-coordinate of the vertex is 2.25.

Substitute 2.25 for x in the given function to find the y-coordinate.

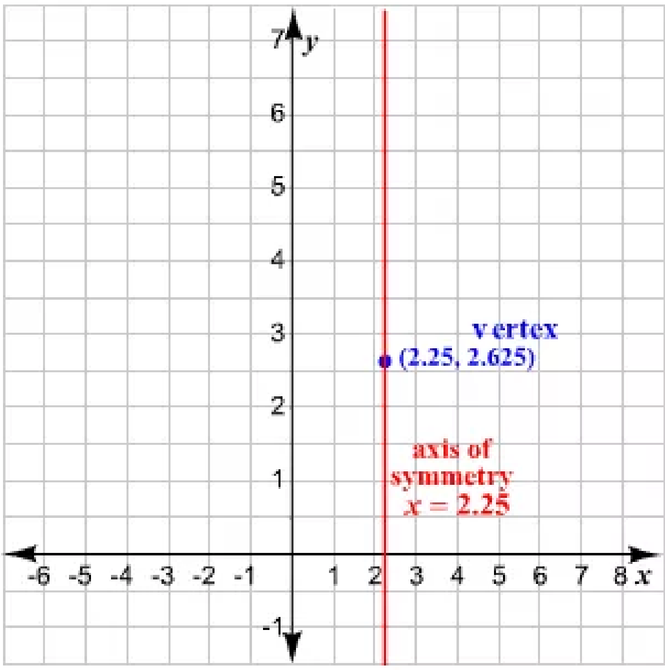
$$y = \frac{2}{3}(2.25)^2 - 3(2.25) + 6$$
$$= 3.375 - 6.75 + 6$$
$$= 2.625$$

Thus, the vertex of the graph of the given function is (2.25, 2.625).

Plot the vertex on a coordinate plane.



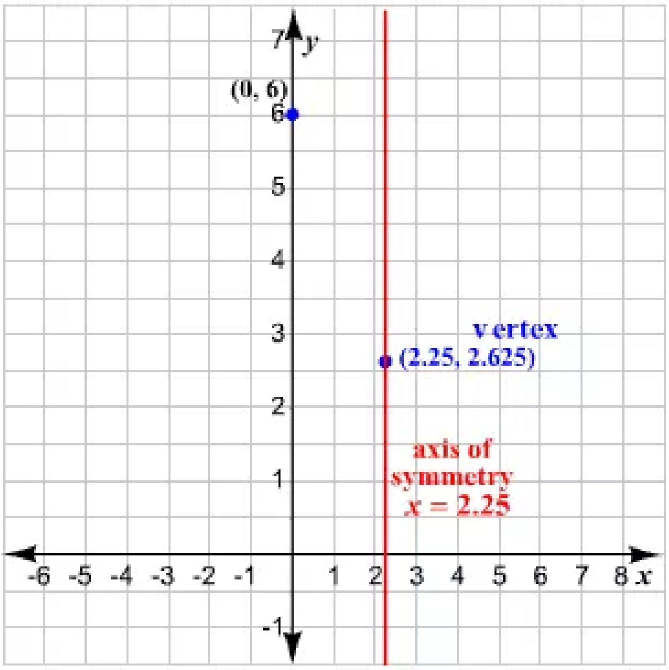
STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$. The axis of symmetry of the given function is the line $x = 2.25$. Now, draw the axis of symmetry $x = 2.25$.



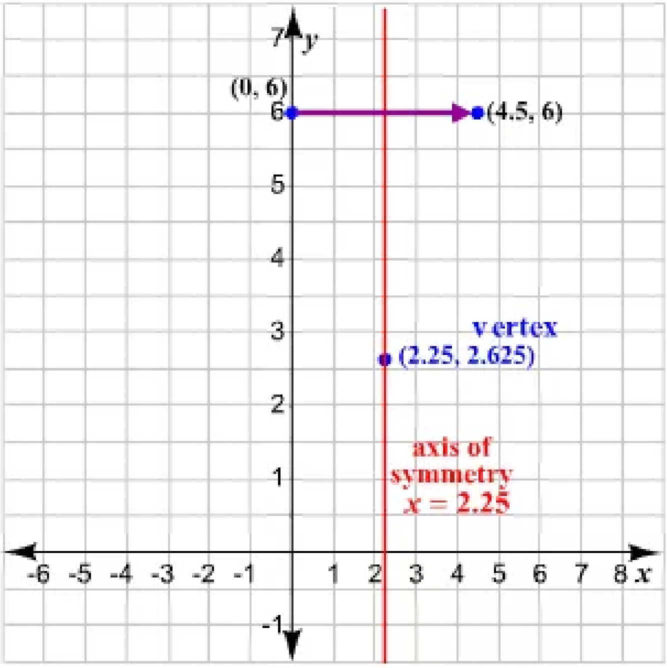
STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola.

Thus, the y-intercept of the given function is 6 and (0, 6) is on the parabola.

Plot the point (0, 6) on the same coordinate plane.



Now, reflect the point (0, 6) in the axis of symmetry to get another point.

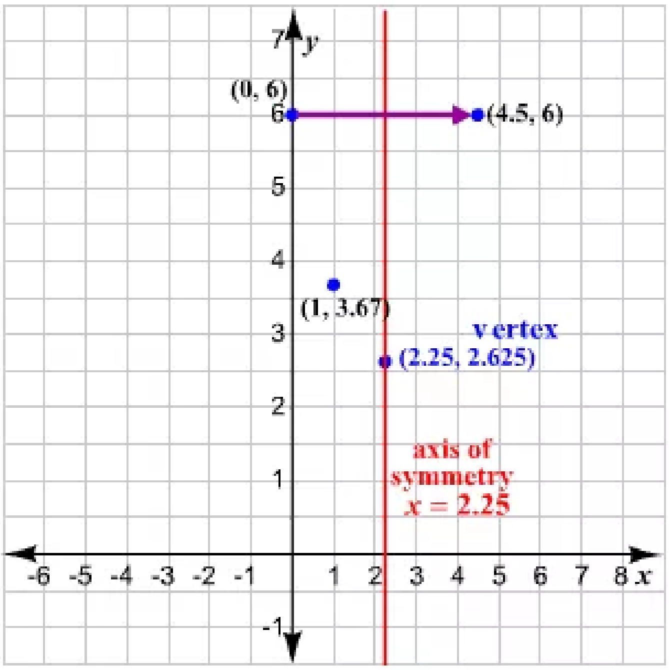


STEP 5 Evaluate the given function for another value of x, say, 1.

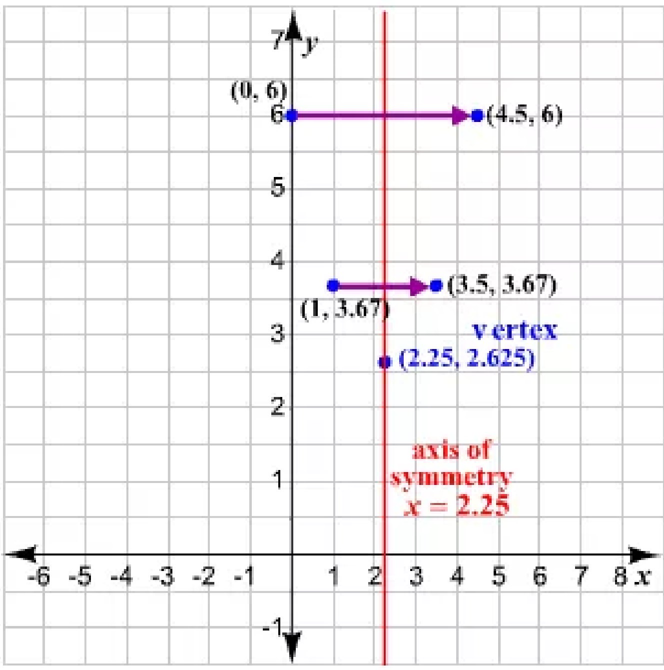
Substitute 1 for x in the function and simplify.

$$\begin{aligned} y &= \frac{2}{3}(1)^2 - 3(1) + 6 \\ &= \frac{2}{3} - 3 + 6 \\ &\approx 3.67 \end{aligned}$$

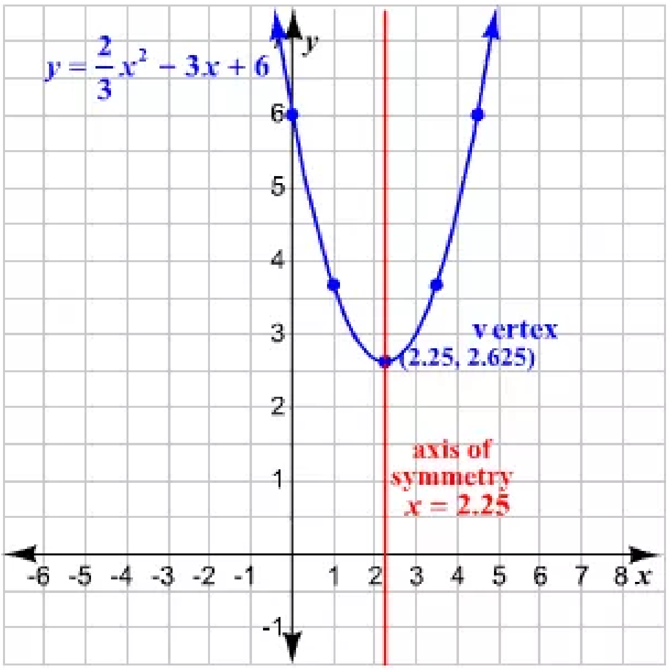
Thus, the point (1, 3.67) lies on the graph. Plot the point on the coordinate plane.



Reflect the point (1, 3.67) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 28e.

Consider the function

$$y = -\frac{3}{4}x^2 - 4x - 1$$

STEP 1:

Identify the coefficients of the function $y = -\frac{3}{4}x^2 - 4x - 1$

Comparing the function $y = -\frac{3}{4}x^2 - 4x - 1$ with $y = ax^2 + bx + c$

The coefficients are $a = -\frac{3}{4}, b = -4, c = -1$

Because $a < 0$, the parabola opens down

STEP 2:

The axis of symmetry is $x = -\frac{b}{2a} = -\frac{-4}{2 \cdot \frac{-3}{4}} = -\frac{-4}{\frac{-3}{2}} = -\frac{4 \cdot 2}{3} = -\frac{8}{3}$

Find the vertex.

Calculate the x-coordinate.

The vertex has x-coordinate $= -\frac{b}{2a} = -\frac{-4}{2 \cdot \frac{-3}{4}} = -\frac{-4}{\frac{-3}{2}} = -\frac{4 \cdot 2}{3} = -\frac{8}{3}$

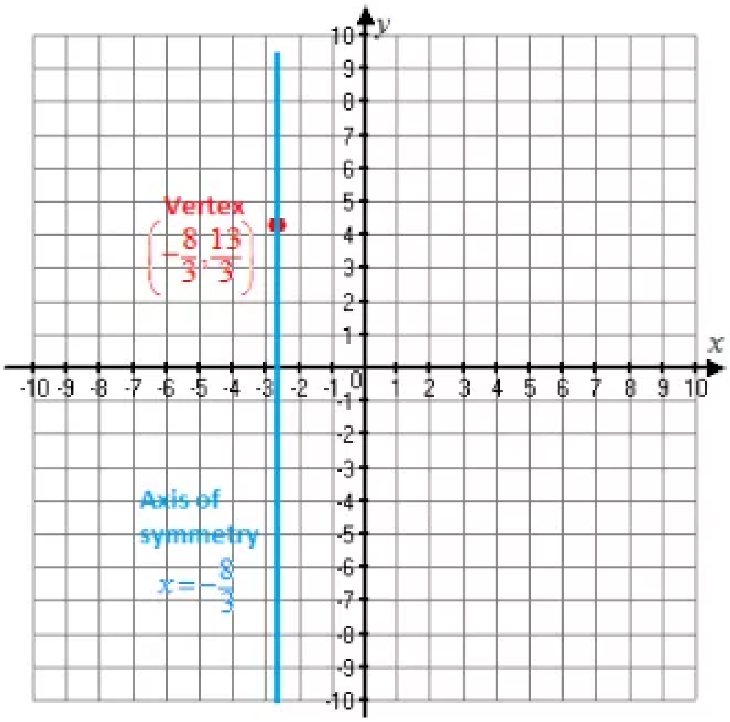
Then find the y-coordinate of the vertex.

$$y = -\frac{3}{4}\left(-\frac{8}{3}\right)^2 - 4\left(-\frac{8}{3}\right) - 1 = \frac{13}{3}$$

Therefore, the vertex is $\left(-\frac{8}{3}, \frac{13}{3}\right)$.

STEP 3:

Plot the point $\left(-\frac{8}{3},\frac{13}{3}\right)$ and draw the axis of symmetry $x=-\frac{8}{3}$.



STEP 4:

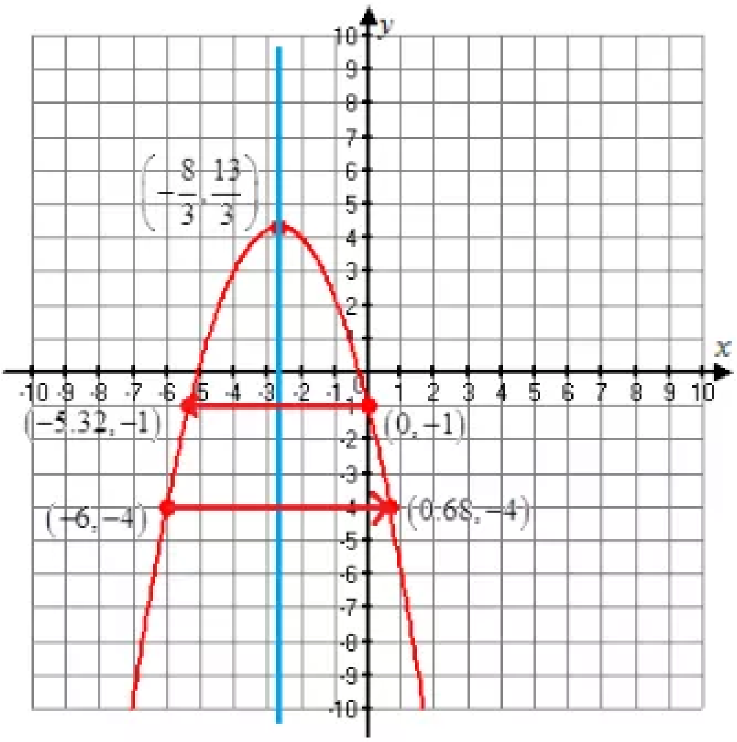
Identify the y-intercept c, which is -1. Plot the point $(0,-1)$. Then reflect this point in the axis of symmetry to plot another point, $(-5.32,-1)$.

Evaluate the function for another value of x, such as $x=-6$.

$$y=-\frac{3}{4}(-6)^2-4(-6)-1=-4$$

Plot the point $(-6,-4)$ and its reflection $(0.68,-4)$ in the axis of symmetry.

Draw the parabola through the plotted points.



Therefore, the above graph is the required graph of the function $y=-\frac{3}{4}x^2-4x-1$

Answer 29e.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y=ax^2+bx+c$. On comparing, we have a is $-\frac{3}{5}$, b is 2, and c is 2. Since $a=-\frac{3}{5}<0$, the graph opens down.

STEP 2 Find the vertex. The vertex of the graph of $y=ax^2+bx+c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute $-\frac{3}{5}$ for a, and 2 for b and evaluate.

$$-\frac{b}{2a}=-\frac{2}{2\left(-\frac{3}{5}\right)}$$
$$=\frac{5}{3}$$
$$\approx 1.67$$

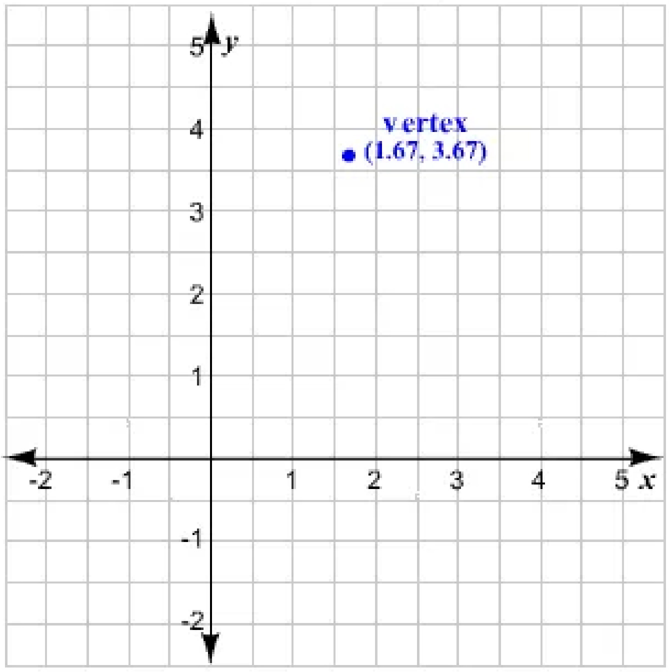
The x-coordinate of the vertex is 1.67.

Substitute 1.67 for x in the given function to find the y-coordinate.

$$g(-1.67) = -\frac{3}{5}(1.67)^2 + 2(1.67) + 2$$
$$\approx 3.67$$

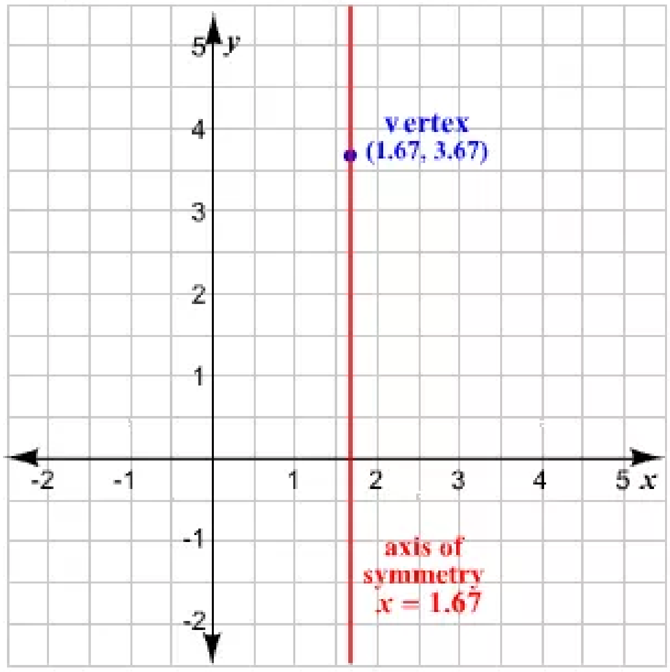
Thus, the vertex of the graph of the given function is (1.67, 3.67).

Plot the vertex on a coordinate plane.



STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$. The axis of symmetry of the given function is the line $x = 1.67$.

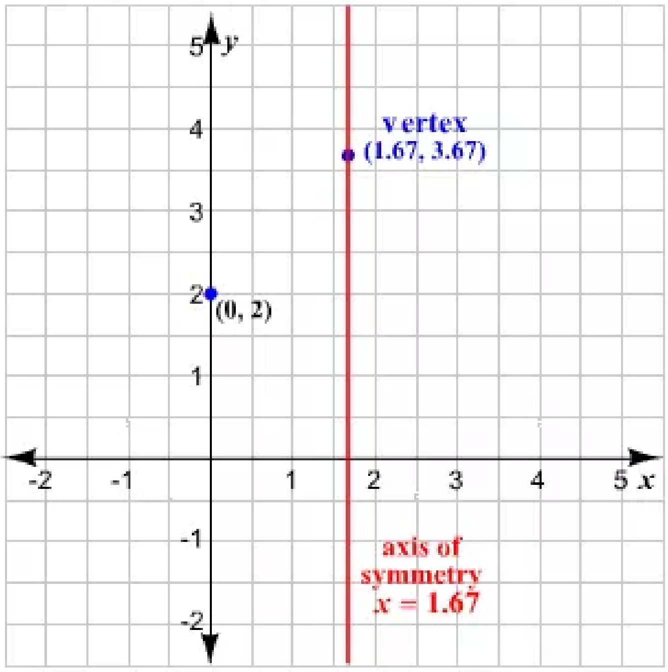
Now, draw the axis of symmetry $x = 1.67$.



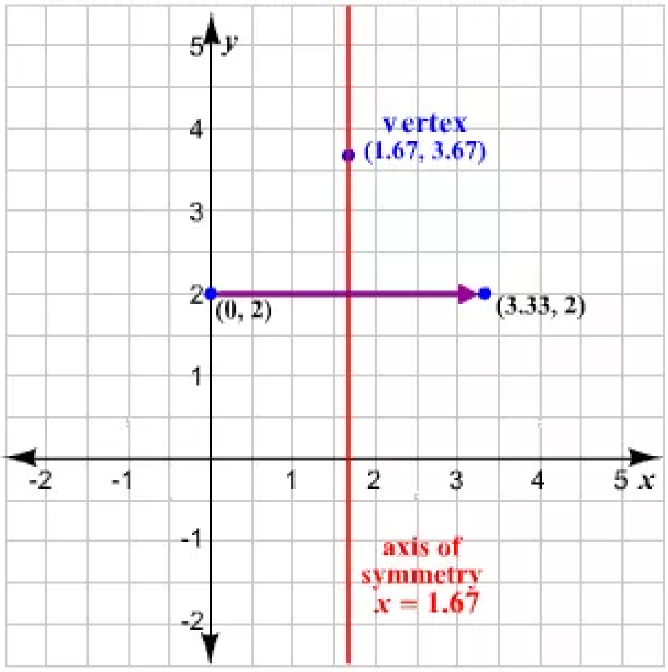
STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola.

Thus, the y-intercept of the given function is 2 and (0, 2) is on the parabola.

Plot the point (0, 2) on the same coordinate plane.



Now, reflect the point (0, 2) in the axis of symmetry to get another point.

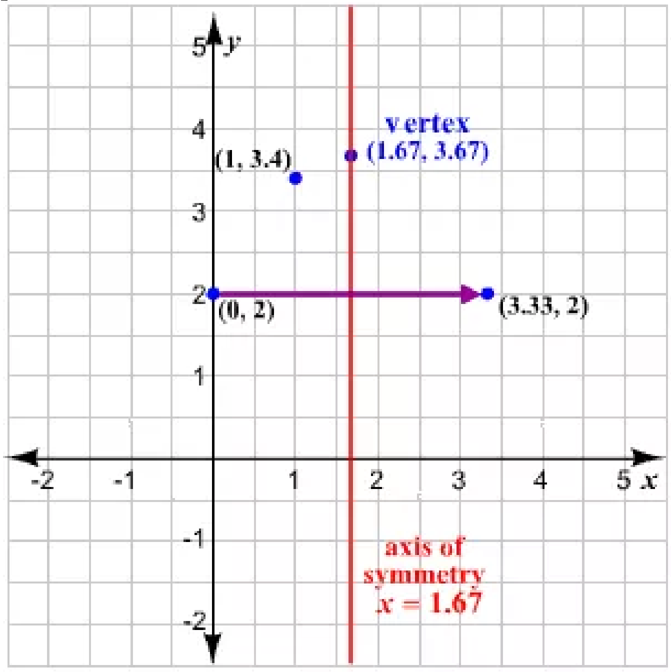


STEP 5 Evaluate the given function for another value of x, say, 1.

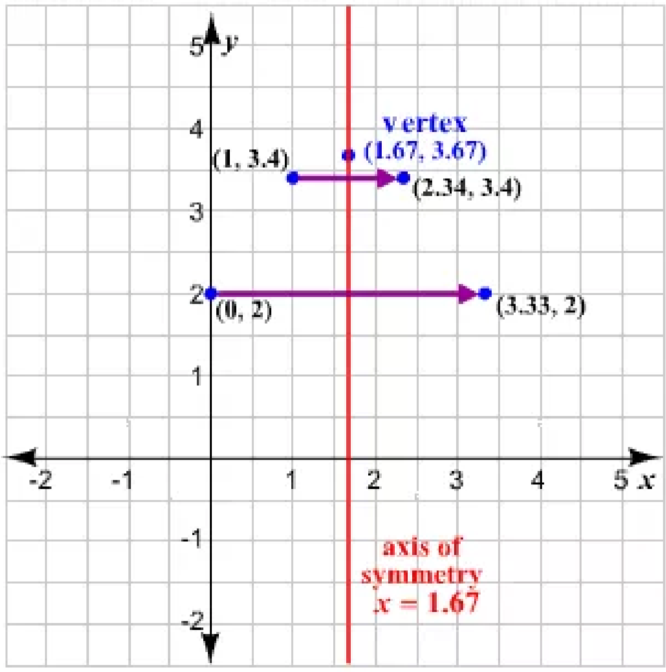
Substitute 1 for x in the function and simplify.

$$g(1) = -\frac{3}{5}(1)^2 + 2(1) + 2$$
$$= 3.4$$

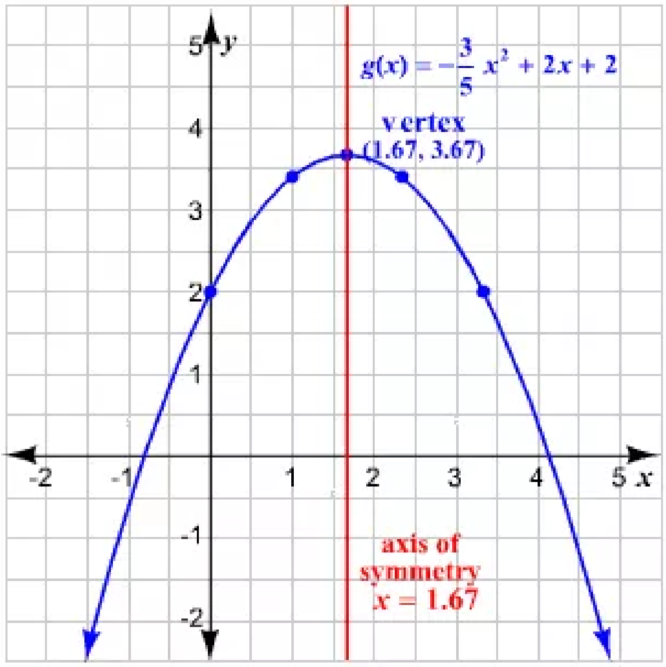
Thus, the point (1, 3.4) lies on the graph. Plot the point on the coordinate plane.



Reflect the point (1, 3.4) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 30e.

Consider the function

$f(x) = \frac{1}{2}x^2 + x - 3$

STEP 1:

Identify the coefficients of the function $f(x) = \frac{1}{2}x^2 + x - 3$

Comparing the function $f(x) = \frac{1}{2}x^2 + x - 3$ with $y = ax^2 + bx + c$

The coefficients are $a = \frac{1}{2}, b = 1, c = -3$

Because $a > 0$, the parabola opens up.

STEP 2:

The axis of symmetry is $x = -\frac{b}{2a} = -\frac{1}{2 \cdot \frac{1}{2}} = -1$

Find the vertex.
Calculate the x-coordinate.

The vertex has x-coordinate $x = -\frac{b}{2a} = -\frac{1}{2 \cdot \frac{1}{2}} = -1$

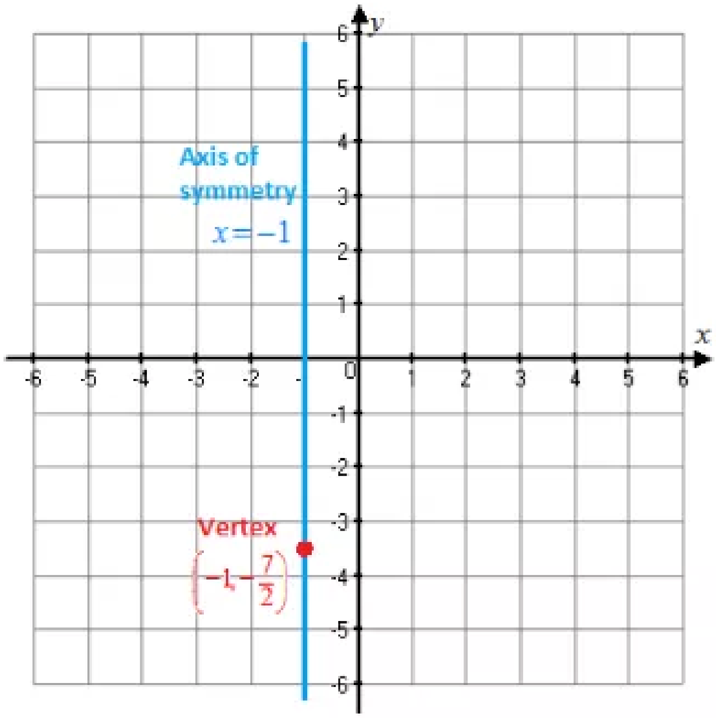
Then find the y-coordinate of the vertex.

$f(-1) = \frac{1}{2}(-1)^2 + (-1) - 3 = -\frac{7}{2}$

Therefore, the vertex is $\left(-1, -\frac{7}{2}\right)$.

STEP 3:

Plot the point $\left(-1,-\frac{7}{2}\right)$ and draw the axis of symmetry $x=-1$.



STEP 4:

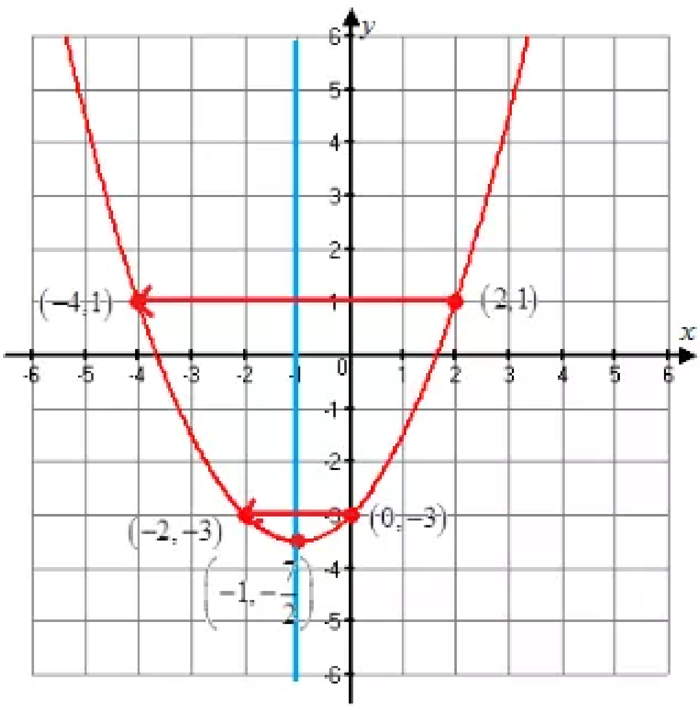
Identify the y-intercept c, which is -3. Plot the point $(0,-3)$. Then reflect this point in the axis of symmetry to plot another point, $(-2,-3)$.

Evaluate the function for another value of x, such as $x=2$.

$$f(2)=\frac{1}{2}(2)^2+(2)-3=1$$

Plot the point $(2,1)$ and its reflection $(-4,1)$ in the axis of symmetry.

Draw the parabola through the plotted points.



Therefore, the above graph is the required graph of the function $f(x)=\frac{1}{2}x^2+x-3$

Answer 31e.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y = ax^2 + bx + c$. On comparing, we have a is $\frac{8}{5}$, b is –4, and c is 5. Since $a = \frac{8}{5} > 0$, the graph opens up.

STEP 2 Find the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, substitute $\frac{8}{5}$ for a, and –4 for b and evaluate.

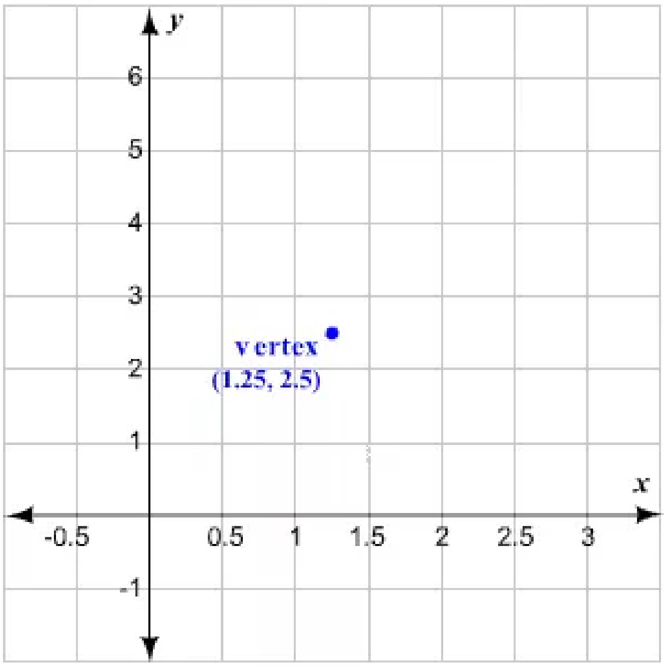
$$-\frac{b}{2a} = -\frac{-4}{2\left(\frac{8}{5}\right)} = \frac{5}{4} = 1.25$$

The x-coordinate of the vertex is 1.25.

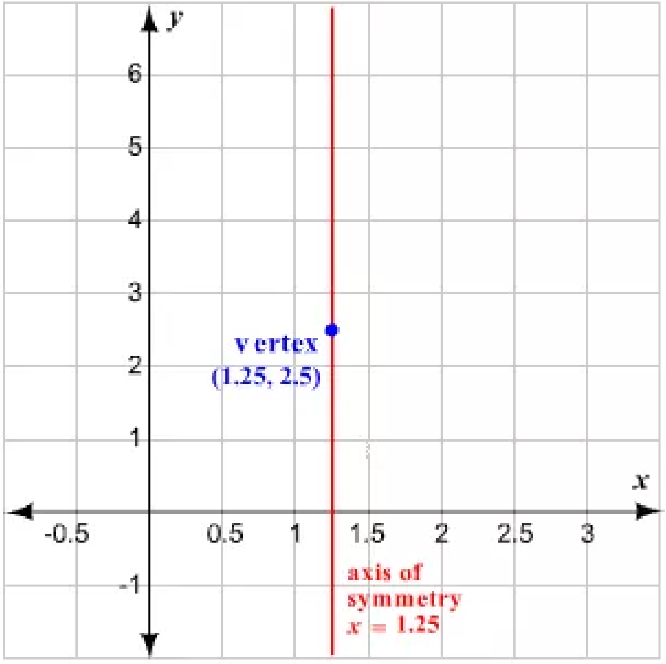
Substitute 1.25 for x in the given function to find the y-coordinate.

$$y = \frac{8}{5}(1.25)^2 - 4(1.25) + 5 = 2.5$$

Thus, the vertex of the graph of the given function is (1.25, 2.5). Plot the vertex on a coordinate plane.



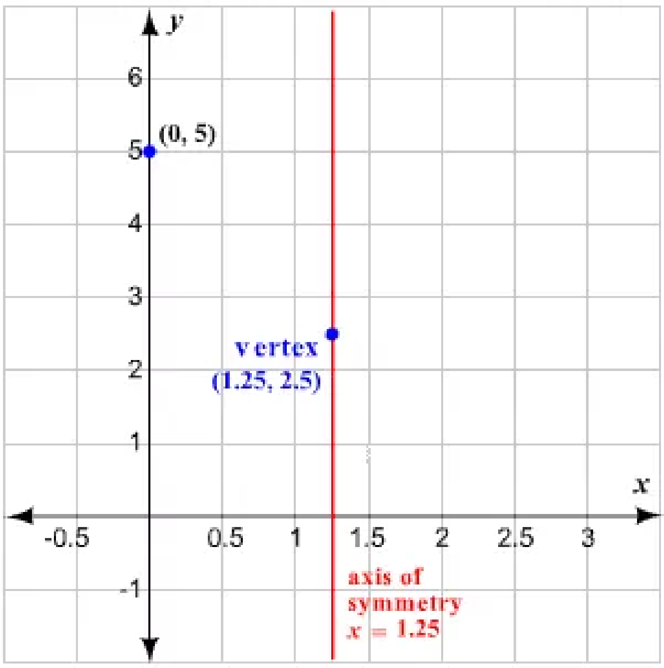
STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$. The axis of symmetry of the given function is the line $x = 1.25$. Draw the axis of symmetry.



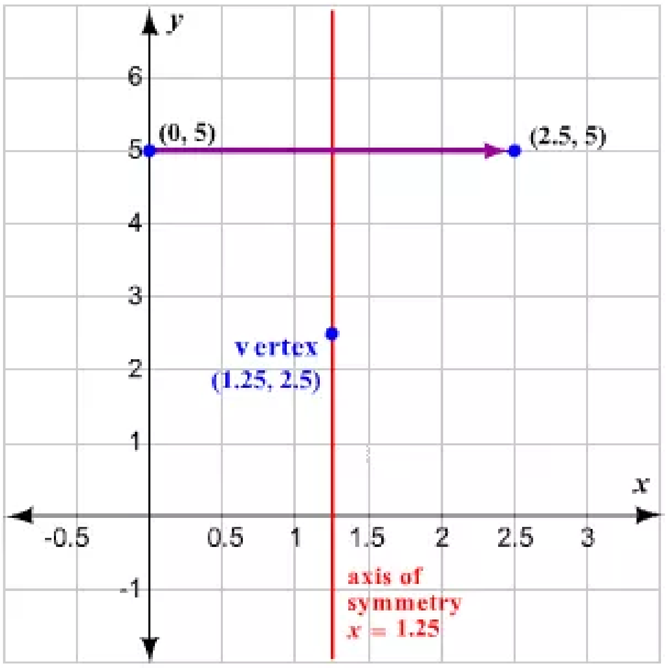
STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola.

Thus, the y-intercept of the given function is 5 and (0, 5) is on the parabola.

Plot the point (0, 5) on the same coordinate plane.



Now, reflect the point (0, 5) in the axis of symmetry to get another point.

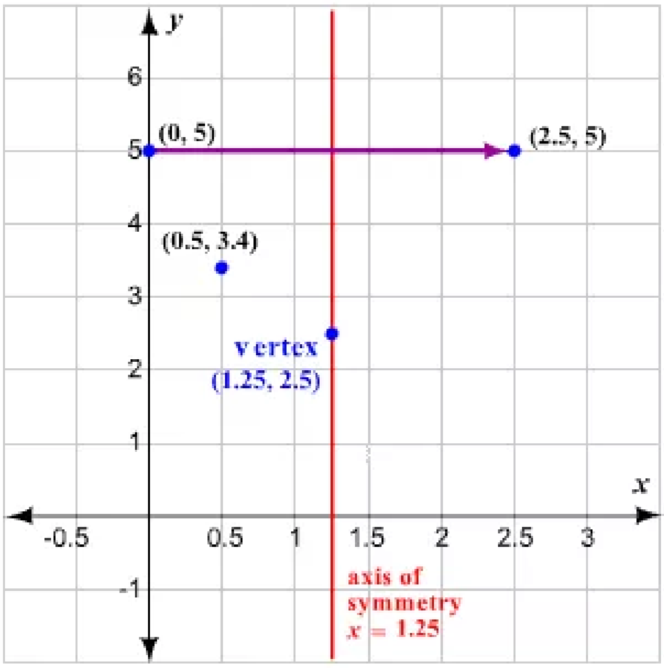


STEP 5 Evaluate the given function for another value of x, say, 0.5.

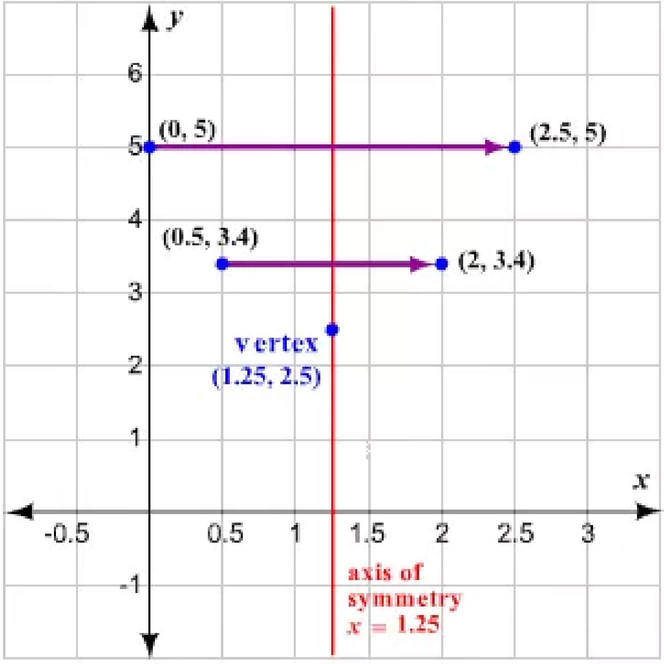
Substitute 0.5 for x in the function and simplify.

$$y = \frac{8}{5}(0.5)^2 - 4(0.5) + 5$$
$$= 3.4$$

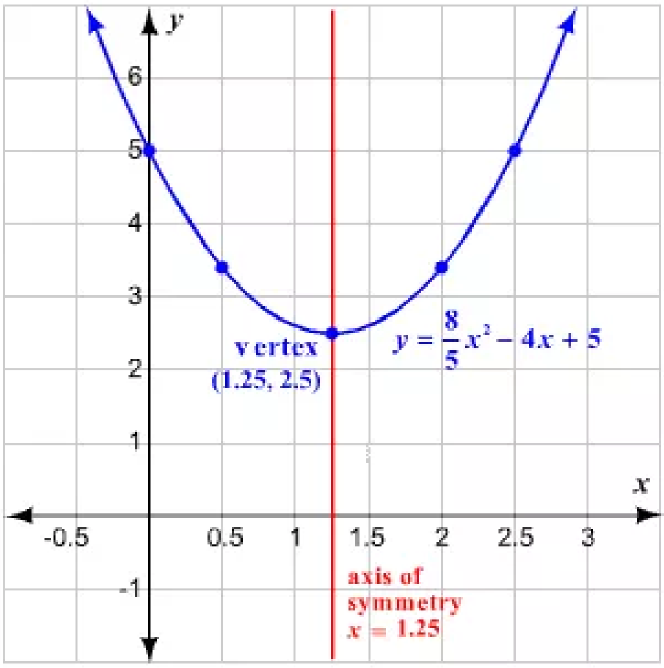
Thus, the point (0.5, 3.4) lies on the graph. Plot the point on the coordinate plane.



Reflect the point (0.5, 3.4) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 32e.

Consider the function $y=-\frac{5}{3}x^2-x-4$

STEP 1:

Identify the coefficients of the function $y=-\frac{5}{3}x^2-x-4$

Comparing the function $y=-\frac{5}{3}x^2-x-4$ with $y=ax^2+bx+c$

The coefficients are $a=-\frac{5}{3}, b=-1, c=-4$

Because $a<0$, the parabola opens down.

STEP 2:

The axis of symmetry is $x=-\frac{b}{2a}=-\frac{-1}{2\cdot\frac{-5}{3}}=-\frac{-1\cdot 3}{-10}=-\frac{3}{10}$

Find the vertex.
Calculate the x-coordinate.

The vertex has x-coordinate $=-\frac{b}{2a}=-\frac{-1}{2\cdot\frac{-5}{3}}=-\frac{-1\cdot 3}{-10}=-\frac{3}{10}$

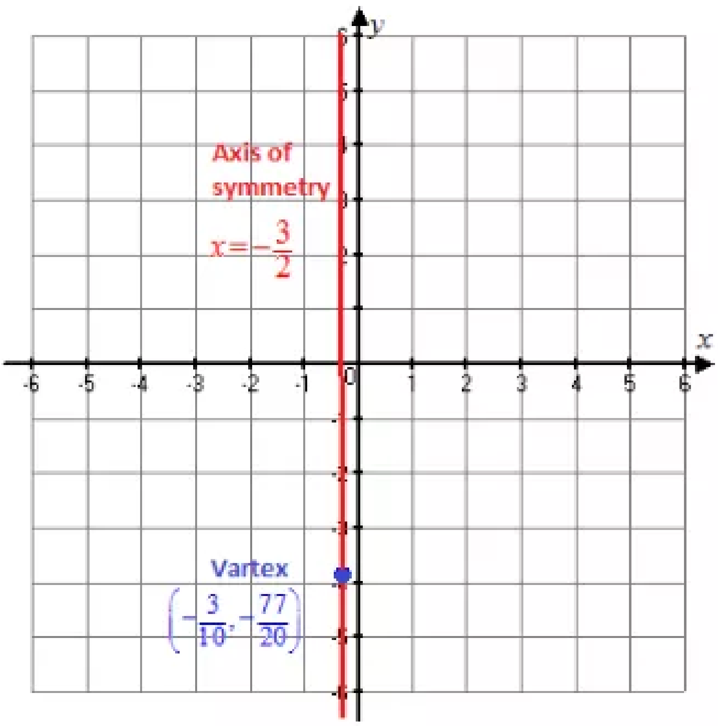
Then find the y-coordinate of the vertex.

$$y=-\frac{5}{3}\left(-\frac{3}{10}\right)^2-\left(-\frac{3}{10}\right)-4=-\frac{77}{20}$$

Therefore, the vertex is $\left(-\frac{3}{10},-\frac{77}{20}\right)$.

STEP 3:

Plot the point $\left(-\frac{3}{10},-\frac{77}{20}\right)$ and draw the axis of symmetry $x=-\frac{3}{2}$.



STEP 4:

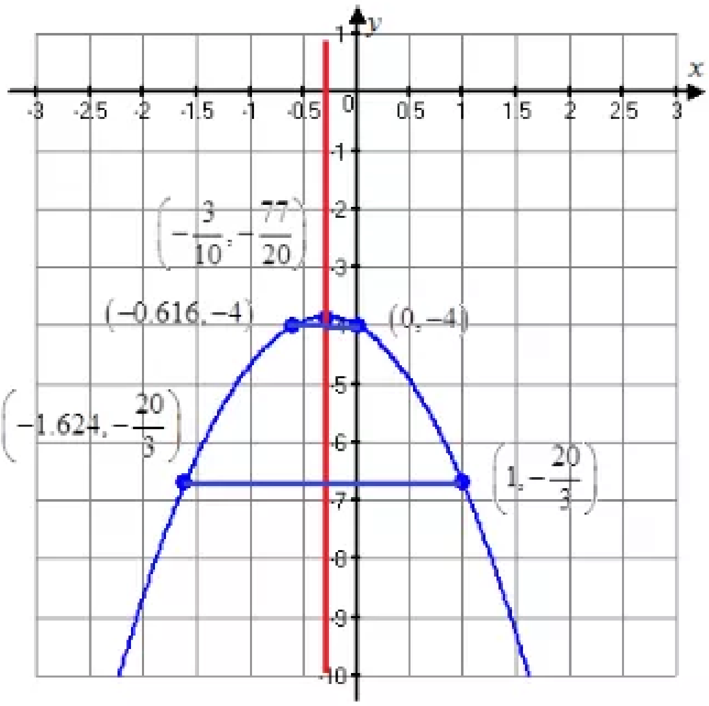
Identify the y-intercept c, which is -4. Plot the point $(0,-4)$. Then reflect this point in the axis of symmetry to plot another point, $(-0.616,-4)$.

Evaluate the function for another value of x, such as $x=1$.

$$y=-\frac{5}{3}(1)^2-(1)-4=-\frac{20}{3}$$

Plot the point $\left(1, -\frac{20}{3}\right)$ and its reflection $\left(-1.624, -\frac{20}{3}\right)$ in the axis of symmetry.

Draw the parabola through the plotted points.



Therefore, the above graph is the required graph of the function $y = -\frac{5}{3}x^2 - x - 4$

Answer 33e.

We know that for a function $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value if $a > 0$, and the maximum value if $a < 0$.

In the given function, a is -6, which is less than 0. Thus, the function has a maximum value.

Now, find the coordinates of the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has the x-coordinate as $-\frac{b}{2a}$. Find the x-coordinate by substituting -6 for a, 0 for b, and evaluate.

$$-\frac{b}{2a} = -\frac{0}{2(-6)}$$
$$= 0$$

The x-coordinate of the vertex is 0.

Substitute 0 for x in the given function to find the y-coordinate.

$$\begin{aligned} y &= -6(0)^2 - 1 \\ &= -1 \end{aligned}$$

Therefore, the maximum value of the function is -1.

Answer 34e.

Consider the function $y = 9x^2 + 7$

Comparing the given function with $y = ax^2 + bx + c$.

We get $a = 9, b = 0, c = 7$

We know that, For $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value of the function $a > 0$ and the maximum value if $a < 0$

Since $a = 9 > 0$

The function $y = 9x^2 + 7$ has a minimum value which is given by the vertex.

x-coordinate of the vertex is $-\dfrac{b}{2a}$

$= 0$

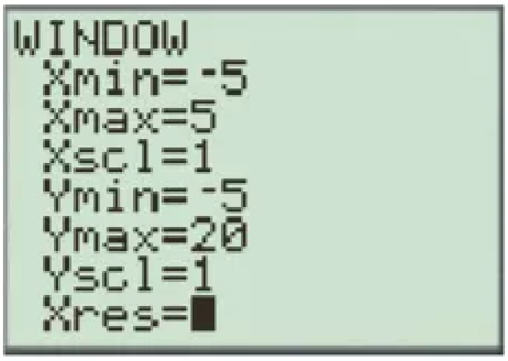
y-coordinate of the vertex is $y = 9(0^2) + 7 = 7$

Vertex is $(0, 7)$

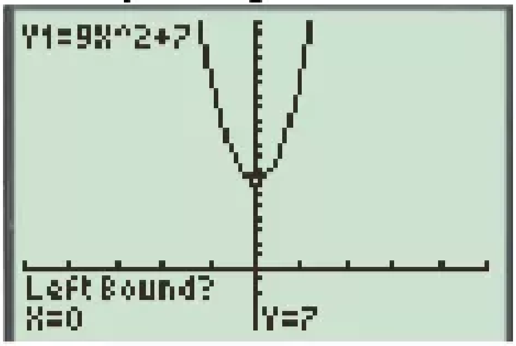
Therefore the minimum value of the given function is 7

Press Y= for the equation editor and enter the equation

Press WINDOW for the window menu.
Enter the window range given below



Press 2nd, then TRACE followed by entering 3 for minimum and graph



Therefore the minimum value of the given function is $\boxed{7}$

Answer 35e.

We know that for a function $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value if $a > 0$, and the maximum value if $a < 0$.

In the given function, a is 2, which is greater than 0. Thus, the function has a minimum value.

Now, find the coordinates of the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has the x-coordinate as $-\frac{b}{2a}$. Find the x-coordinate by substituting 2 for a, 8 for b, and evaluate.

$$-\frac{b}{2a} = -\frac{8}{2(2)}$$
$$= -2$$

The x-coordinate of the vertex is -2.

Substitute -2 for x in the given function to find the y-coordinate.

$$f(-2) = 2(-2)^2 + 8(-2) + 7$$
$$= -1$$

Therefore, the minimum value of the function is -1.

Answer 36e.

Consider the function $g(x) = -3x^2 + 18x - 5$

Comparing the given function with $y = ax^2 + bx + c$

We get $a = -3, b = 18, c = -5$

We know that, For $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value of the function $a > 0$ and the maximum value if $a < 0$

Since $a = -3 < 0$

The function $g(x) = -3x^2 + 18x - 5$ has a minimum value which is given by the vertex.

x-coordinate of the vertex is

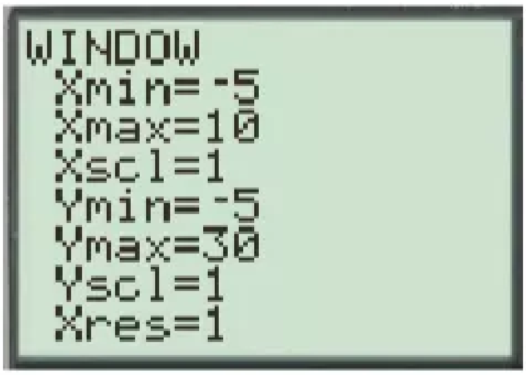
$$-\frac{b}{2a} = -\frac{18}{2(-3)}$$
$$= 3$$

y-coordinate of the vertex is $g(3) = -3(3^2) + 18(3) - 5 = 22$

Hence, the vertex is $(3, 22)$

Therefore the maximum value of the given function is 22

Press Y= for the equation editor and enter the equation



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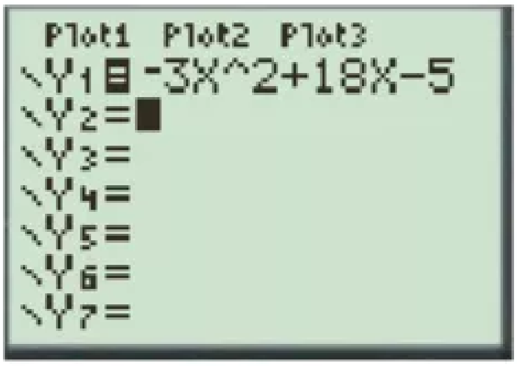
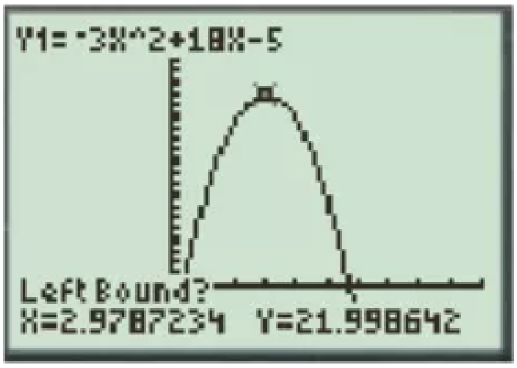


Press 2nd, then TRACE followed by entering 4 for maximum and graph



Therefore the maximum value of the given function is $\boxed{22}$

Answer 37e.

We know that for a function $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value if $a > 0$, and the maximum value if $a < 0$.

In the given function, a is $\frac{3}{2}$, which is greater than 0. Thus, the function has a minimum value.

Now, find the coordinates of the vertex. The vertex of the graph of $y = ax^2 + bx + c$ has the x-coordinate as $-\frac{b}{2a}$. Find the x-coordinate by substituting $\frac{3}{2}$ for a, 6 for b, and evaluate.

$$-\frac{b}{2a} = -\frac{6}{2\left(\frac{3}{2}\right)}$$
$$= -2$$

The x-coordinate of the vertex is -2.

Substitute −2 for x in the given function to find the y-coordinate.

$$f(-2) = \frac{3}{2}(-2)^2 + 6(-2) + 4$$
$$= -2$$

Therefore, the minimum value of the function is −2.

Answer 38e.

Consider the function $y = -\frac{1}{4}x^2 - 7x + 2$

Comparing the given function with $y = ax^2 + bx + c$

We get $a = -\frac{1}{4}, b = -7, c = 2$

We know that, For $y = ax^2 + bx + c$, the vertex's y-coordinate is the minimum value of the function $a > 0$ and the maximum value if $a < 0$

Since $a = -\frac{1}{4} < 0$

The given function has a minimum value which is given by the vertex.
x-coordinate of the vertex is

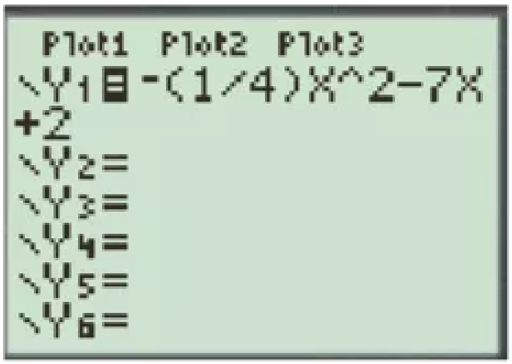
$$-\frac{b}{2a} = -\frac{(-7)}{2(\frac{-1}{4})}$$
$$= -14$$

y-coordinate of the vertex is $y = -\frac{1}{4}(-14)^2 - 7(-14) + 2 = 51$

Hence the vertex is $(-14, 51)$

Therefore the maximum value of the given function is 51

Press Y= for the equation editor and enter the equation



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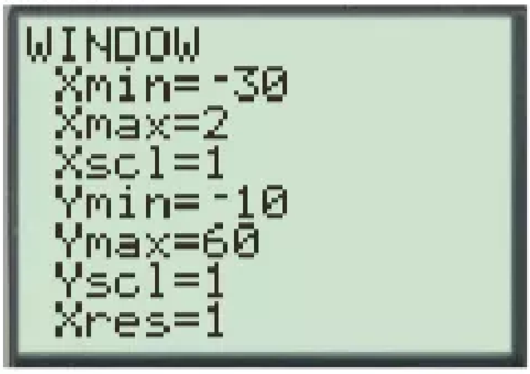
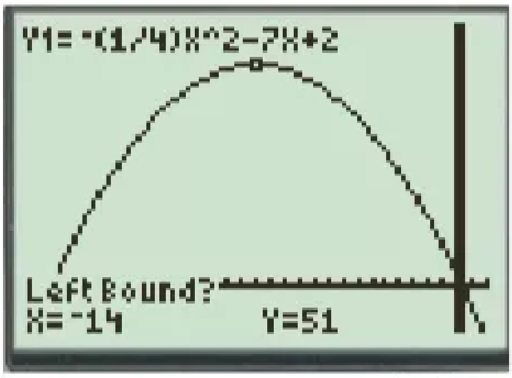


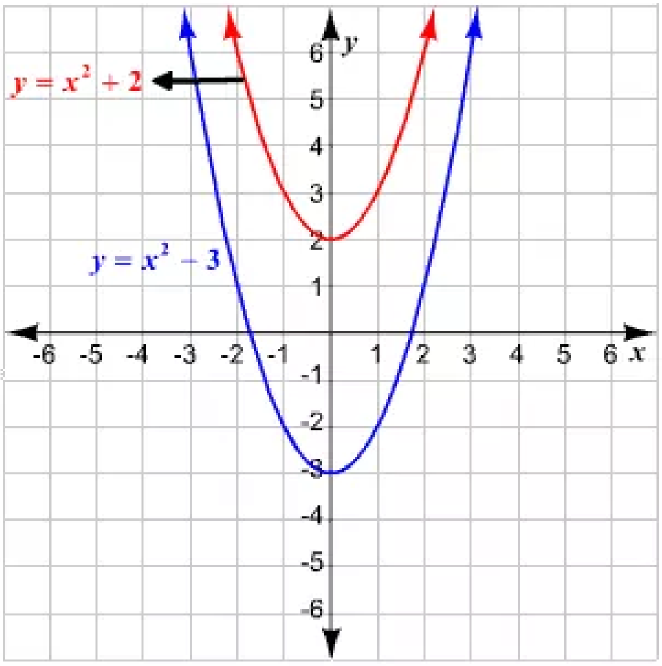
Press 2nd, then TRACE followed by entering 4 for maximum and graph



Therefore the maximum value of the given function is $\boxed{51}$

Answer 39e.

First, we have to graph the given functions on the same coordinate plane.



On comparing, we can see that both the graphs open up and have the same axis of symmetry. The graph of $y = x^2 - 3$ is wider than the graph of $y = x^2 + 2$ and its vertex moves down the y-axis.

Therefore, the correct answers are choices **A** and **D**.

Answer 40e.

Out of the given four quadratic functions, the one which has the smallest quadratic coefficient in terms of numerical value should be the one which will have widest graph. Therefore the graph of the quadratic function $y = 0.5x^2$ will be the widest graph in comparison to the graphs of the other quadratic functions.

Answer 41e.

We know that a quadratic function can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$.

Compare $y = -0.02x^2 + x + 6$ with $y = ax^2 + bx + c$.
$a = -0.02$, $b = 1$, $c = 6$

Answer 42e.

Consider $y = -0.01x^2 + 0.7x + 6$

Comparing the given function with $y = ax^2 + bx + c$

We get $a = -0.01, b = 0.7, c = 6$

Answer 43e.

We know that the axis of symmetry is $x = -\dfrac{b}{2a}$.

Substitute 4 for x.

$$4 = -\frac{b}{2a}$$

Multiply each side by $-2a$.

$$4(-2a) = -\frac{b}{2a}(-2a)$$
$$-8a = b$$

In order to find one of the quadratic functions, first evaluate b for any value of a, say, 1. Substitute 1 for a in $b = -8a$ and evaluate.

$$b = -8(1)$$
$$= -8$$

Choose any y-intercept, say, 2. The y-intercept of the function $y = ax^2 + bx + c$ is c. Thus, $c = 2$.

A quadratic function can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$.

Put the values for a, b, and c in $y = ax^2 + bx + c$.

$$y = 1x^2 - 8x + 2$$
$$y = x^2 - 8x + 2$$

Evaluate b for another value of a, say, 2 to find another quadratic function.

Replace a with 2 in $b = -8a$ and evaluate.

$$b = -8(2)$$
$$= -16$$

It is given that the functions have different y-intercepts. Choose any y-intercept other than 2, say, -1. Thus, $c = -1$.

Substitute 2 for a, -16 for b, and -1 for c in $y = ax^2 + bx + c$.

$y = 2x^2 - 16x - 1$

Now, to find the other quadratic function, substitute any value for a, say, -1 and evaluate b.

$$b = -8(-1)$$
$$= 8$$

Choose any y-intercept other than -1 and 2, say, 3. Thus, $c = 3$.

Substitute -1 for a, 8 for b, and 3 for c in $y = ax^2 + bx + c$.

$y = -x^2 + 8x + 3$

The three different quadratic functions whose graphs have the line $x = 4$ as an axis of symmetry but have different y-intercepts are

$y = x^2 - 8x + 2,$

$y = 2x^2 - 16x - 1,$

and

$y = -x^2 + 8x + 3.$

Answer 44e.

Consider the function $y = 0.5x^2 - 2x$

When we put $x = 0$ we get,

$y = 0.5(0^2) - 2(0) = 0$

$\Rightarrow$ The graph of the given function passes through $(0,0)$.

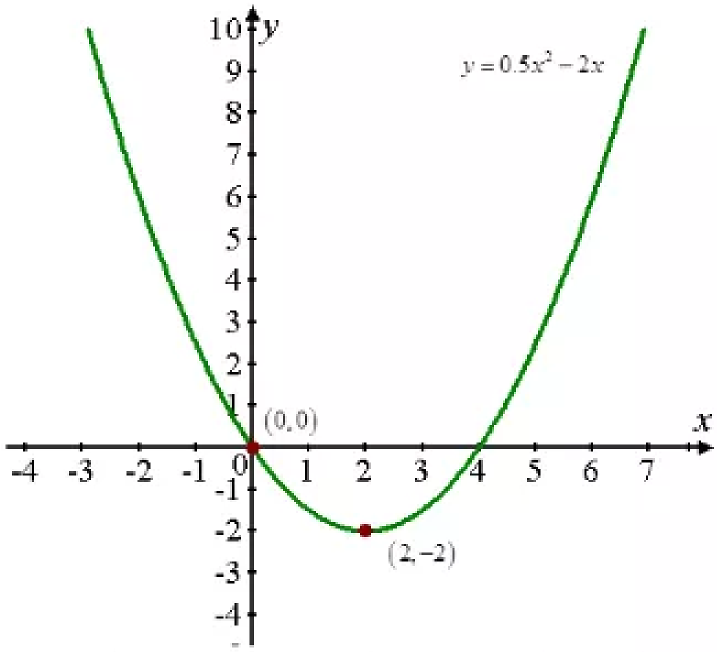
The vertex has x-coordinate

$$-\frac{b}{2a} = -\frac{-2}{2(0.5)}$$
$$= 2$$

The y-coordinate of the vertex is $y(2) = 0.5(2^2) - 2(2) = -2$

The vertex is $(2,-2)$

Sketch the graph of $y = 0.5x^2 - 2x$



Therefore the graph of the given function must be the graph in option C.

Answer 45e.

A quadratic function can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$.
Compare the given function with the standard form.
$a = 0.5$, $b = 0$, $c = 3$

The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$. In order to find the x-coordinate of the vertex, first substitute 0.5 for a, and 0 for b.

$$-\frac{b}{2a} = -\frac{0}{2(0.5)}$$
$$= 0$$

The x-coordinate of the vertex is 0.

Substitute 0 for x in the given function to find the y-coordinate.

$$y = 0.5(0)^2 + 3$$
$$= 3$$

Thus, the vertex is (0, 3).

The axis of symmetry is $x = -\frac{b}{2a}$. Since the value of b is 0, the axis of symmetry is the y-axis.

We can see that only the parabola in choice **A** has vertex at (0, 3) and the y-axis as its axis of symmetry.

Therefore, the given equation matches with choice **A**.

Answer 46e.

Consider the function $y = 0.5x^2 - 2x + 3$

Comparing the given function with $y = ax^2 + bx + c$

We get $a = 0.5, b = -2, c = 3$

Then, the axis of symmetry is $x = -\frac{b}{2a} = 2$

and the vertex has x-coordinate $-\frac{b}{2a} = 2$

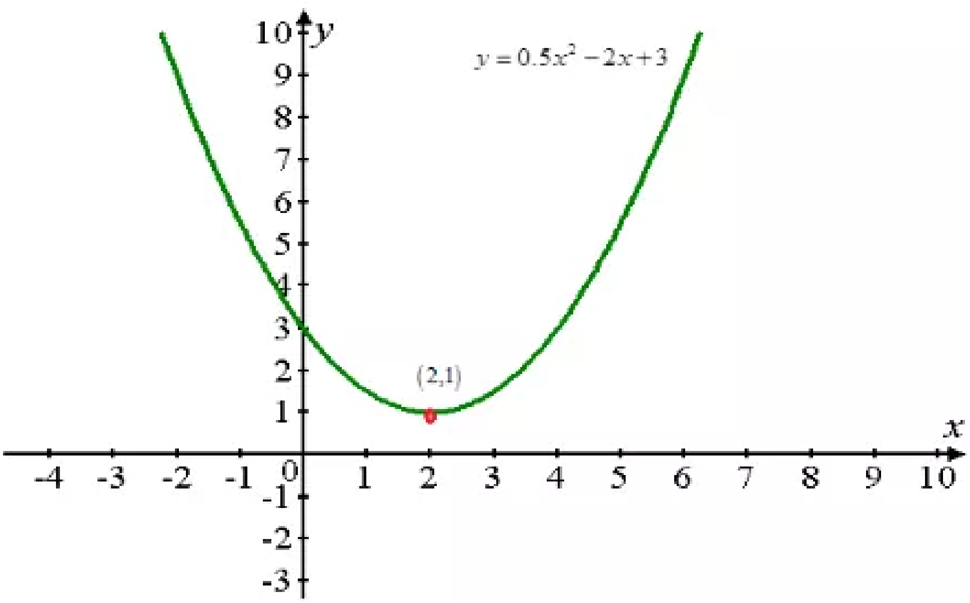
The y-coordinate of the vertex is

$$y(2) = 0.5(2^2) - 2(2) + 3$$
$$= 1$$

Hence the vertex is $(2,1)$

The graph of the given function has axis of symmetry $x = 2$ and it does not pass through the origin.

Sketch the graph of $y = 0.5x^2 - 2x + 3$



Therefore the graph of the given function must be the graph in option B.

Answer 47e.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y = ax^2 + bx + c$.

On comparing, we have a is 0.1, b is 0, and c is 2.

Since $a > 0$, the graph opens up.

STEP 2 Find the vertex.

The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\dfrac{b}{2a}$.

In order to find the x-coordinate of the vertex, substitute 0.1 for a, and 0 for b and evaluate.

$$\begin{aligned} -\frac{b}{2a} &= -\frac{0}{2(0.1)} \\ &= 0 \end{aligned}$$

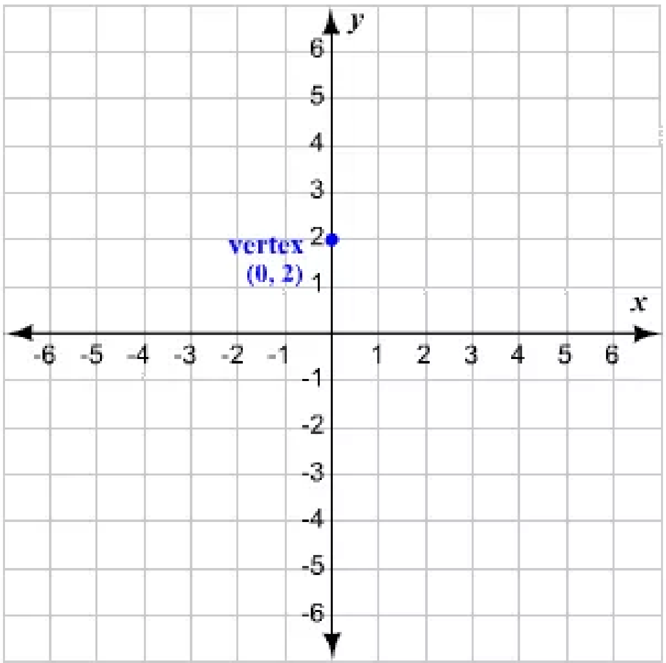
The x-coordinate of the vertex is 0.

Substitute 0 for x in the given function to find the y-coordinate.

$$\begin{aligned} f(0) &= 0.1(0)^2 + 2 \\ &= 2 \end{aligned}$$

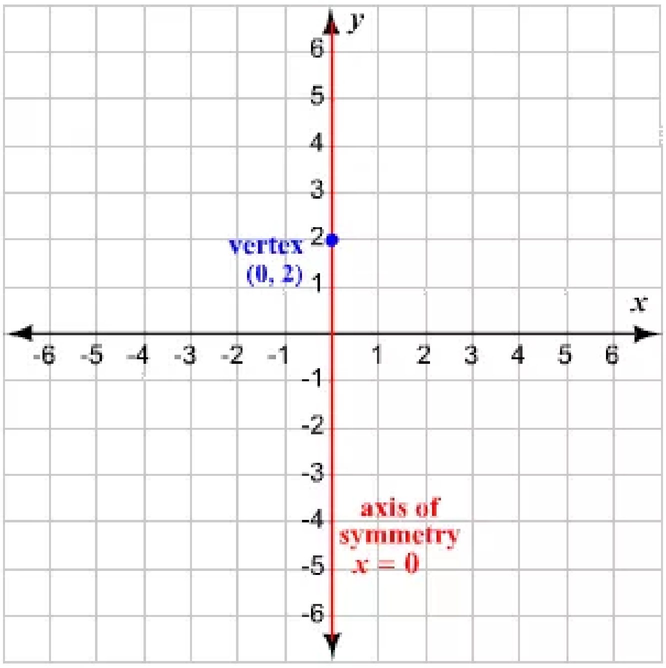
Thus, the vertex of the graph of the given function is (0, 2).

Plot the vertex on a coordinate plane.



STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$.

The axis of symmetry of the given function is the line $x = 0$ or the y-axis. Now, draw the axis of symmetry.

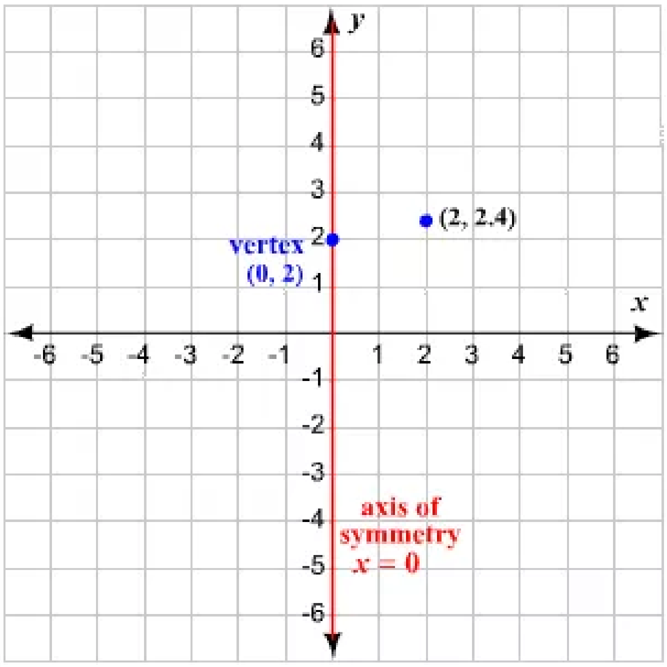


STEP 4 Evaluate the given function for another value of x, say, 2.

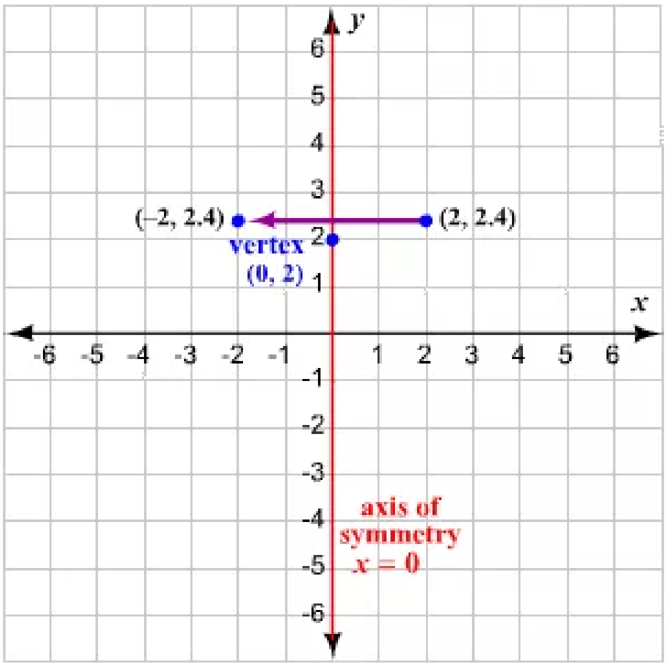
Substitute 2 for x in the function and simplify.

$$f(2) = 0.1(2)^2 + 2$$
$$= 2.4$$

Thus, the point (2, 2.4) lies on the graph. Plot the point on the coordinate plane.



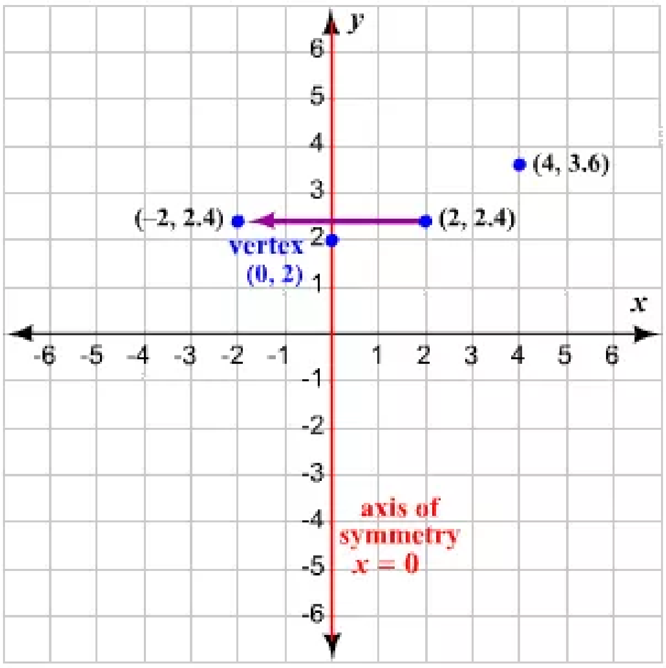
Now, reflect the point (2, 2.4) in the axis of symmetry to get another point.



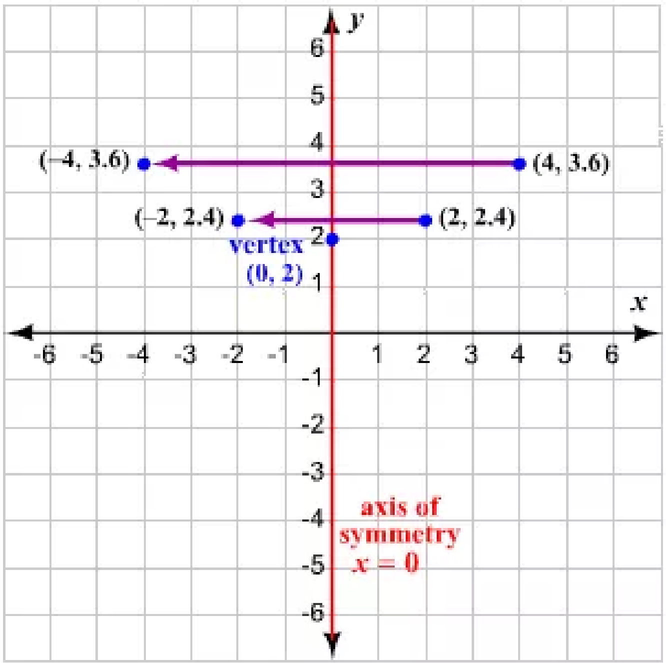
STEP 5 Substitute another value of x, say, 4 and evaluate.

$$f(4) = 0.1(4)^2 + 2$$
$$= 3.6$$

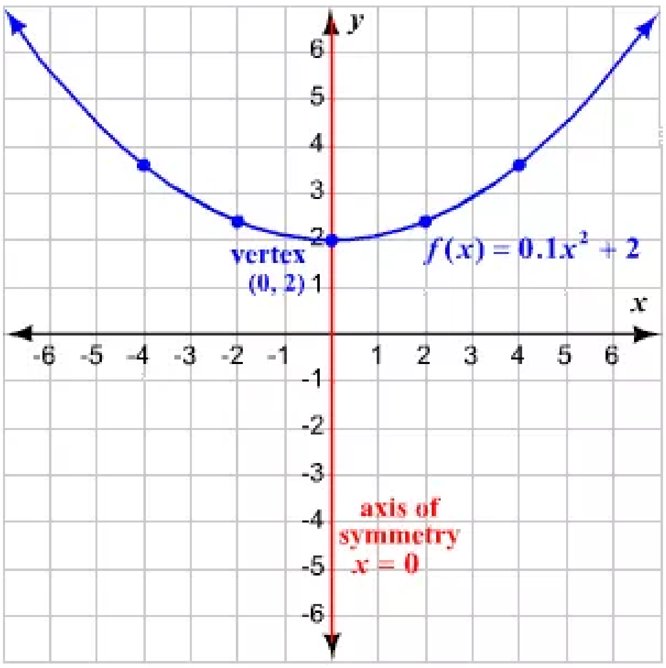
The point (4, 3.6) lies on the graph. Plot the point on the coordinate plane.



We have to plot the reflection of the point (4, 3.6) in the axis of symmetry.
Reflect the point (4, 3.6) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 48e.

Consider the function $g(x) = -0.5x^2 - 5$

Comparing the given function with $y = ax^2 + bx + c$

We get $a = -0.5, b = 0, c = -5$

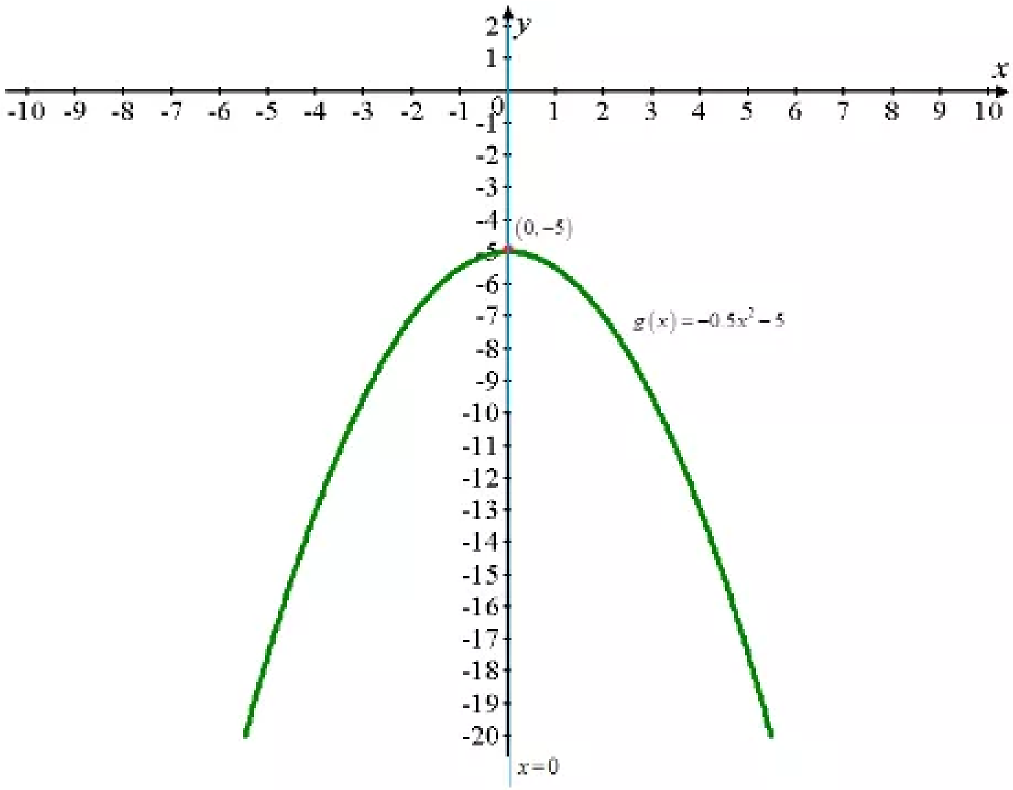
Then, the axis of symmetry is $x = -\frac{b}{2a} = 0$

and the vertex has x-coordinate $-\frac{b}{2a} = 0$

y-coordinate of the vertex is $g(0) = -0.5(0^2) - 5 = -5$

Hence, the vertex is $(0, -5)$

Sketch the graph of the function $g(x) = -0.5x^2 - 5$



Answer 49e.

STEP 1 Identify the coefficients of the function.
The given function is of the form $y = ax^2 + bx + c$.
On comparing, we have a is 0.3, b is 3, and c is -1.

Since $a > 0$, the graph opens up.

STEP 2 Find the vertex.

The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$.

In order to find the x-coordinate of the vertex, substitute 0.3 for a, and 3 for b and evaluate.

$$-\frac{b}{2a} = -\frac{3}{2(0.3)}$$
$$= -5$$

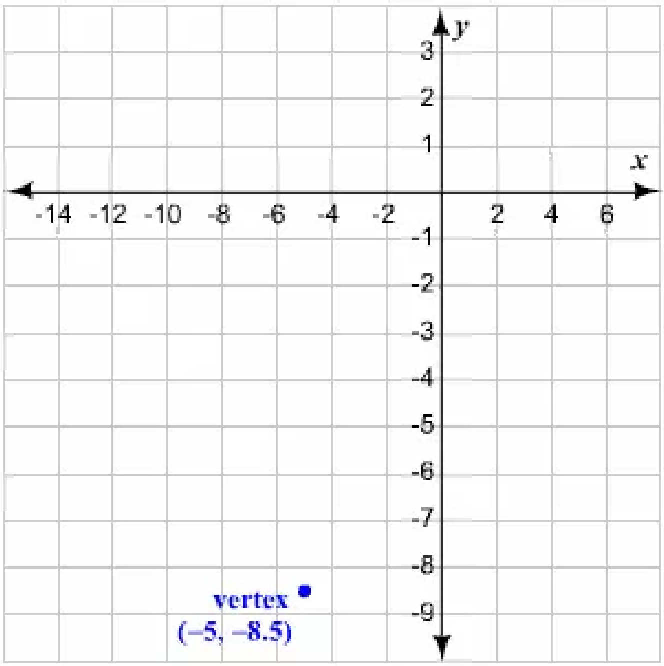
The x-coordinate of the vertex is -5.

Substitute -5 for x in the given function to find the y-coordinate.

$$y = 0.3(-5)^2 + 3(-5) - 1$$
$$= 7.5 - 15 - 1$$
$$= -8.5$$

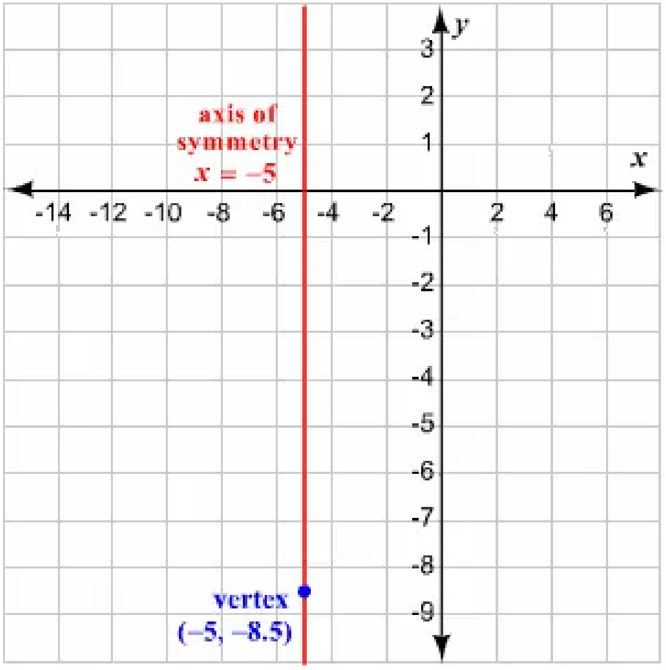
Thus, the vertex of the graph of the given function is $(-5, -8.5)$.

Plot the vertex on a coordinate plane.

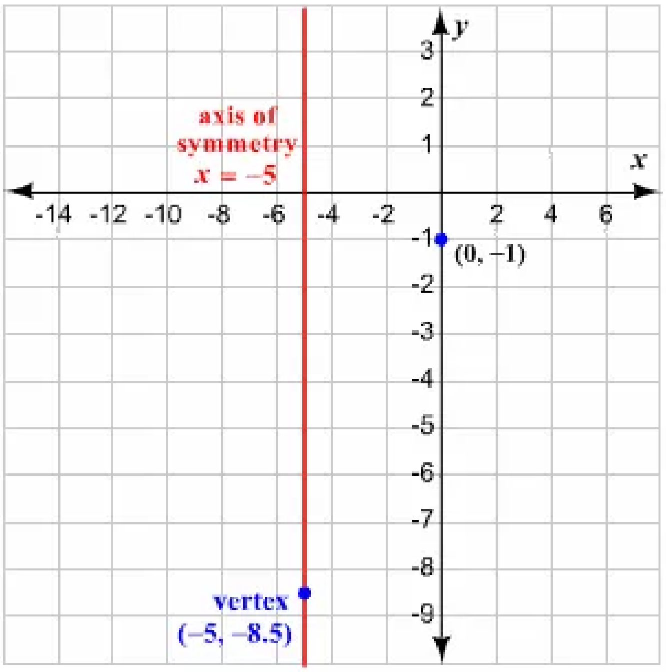


STEP 3 We know that the axis of symmetry is $x = -\dfrac{b}{2a}$.

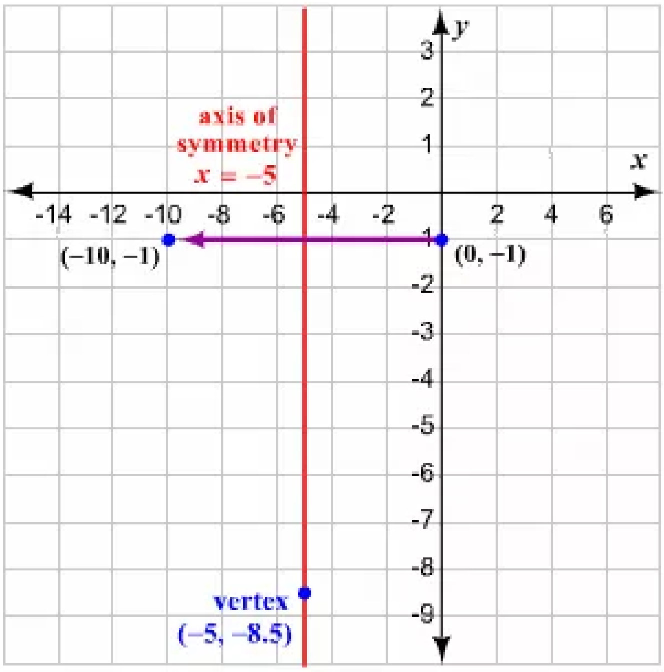
The axis of symmetry of the given function is the line $x = -5$.
Now, draw the axis of symmetry $x = -5$.



STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola.
Thus, the y-intercept of the given function is -1 and $(0, -1)$ is on the parabola.
Plot the point $(0, -1)$ on the same coordinate plane.



Now, reflect the point (0, −1) in the axis of symmetry to get another point.

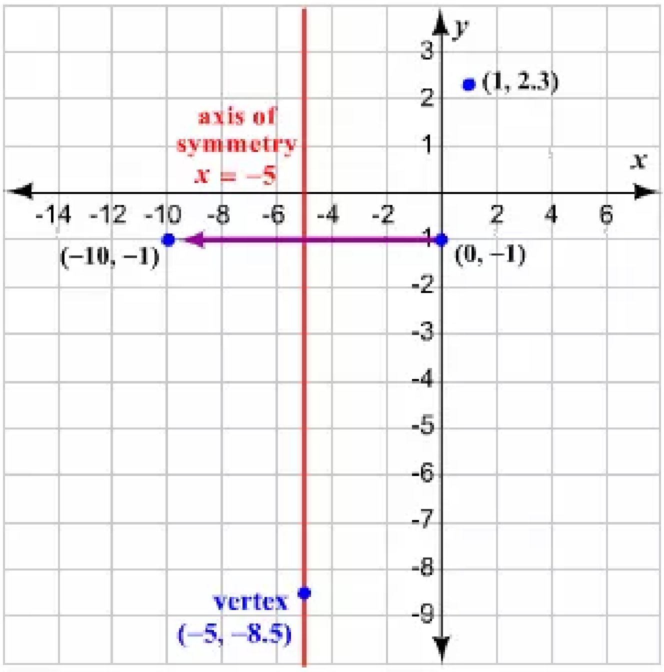


STEP 5 Evaluate the given function for another value of x, say, 1.

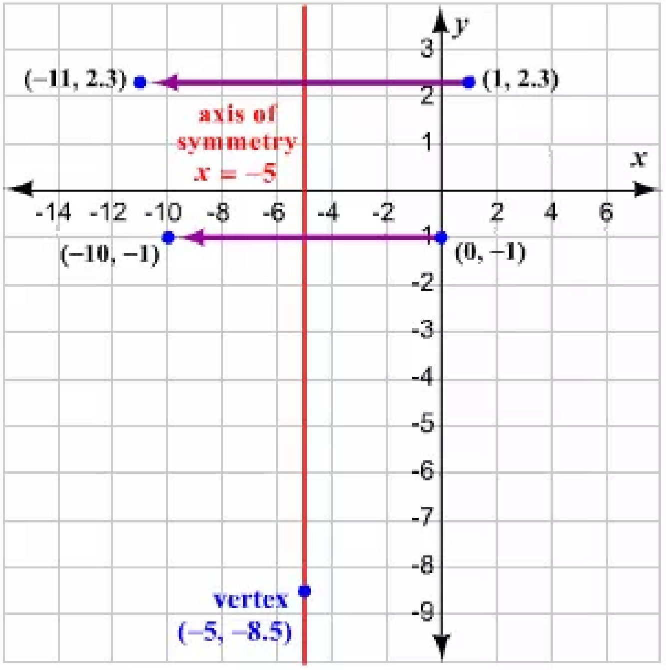
Substitute 1 for x in the function and simplify.

$$\begin{aligned} y &= 0.3(1)^2 + 3(1) - 1 \\ &= 0.3 + 3 - 1 \\ &= 2.3 \end{aligned}$$

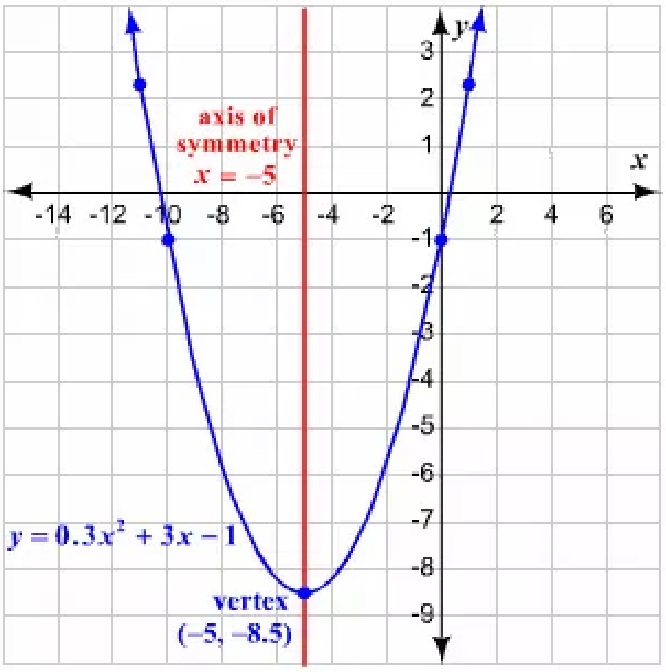
Thus, the point (1, 2.3) lies on the graph. Plot the point on the coordinate plane.



We have to plot the reflection of the point (1, 2.3) in the axis of symmetry.
Reflect the point (1, 2.3) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 50e.

Consider the function $y = 0.25x^2 - 1.5x + 3$

Comparing the given function with $y = ax^2 + bx + c$.

We get $a = 0.25, b = -1.5, c = 3$

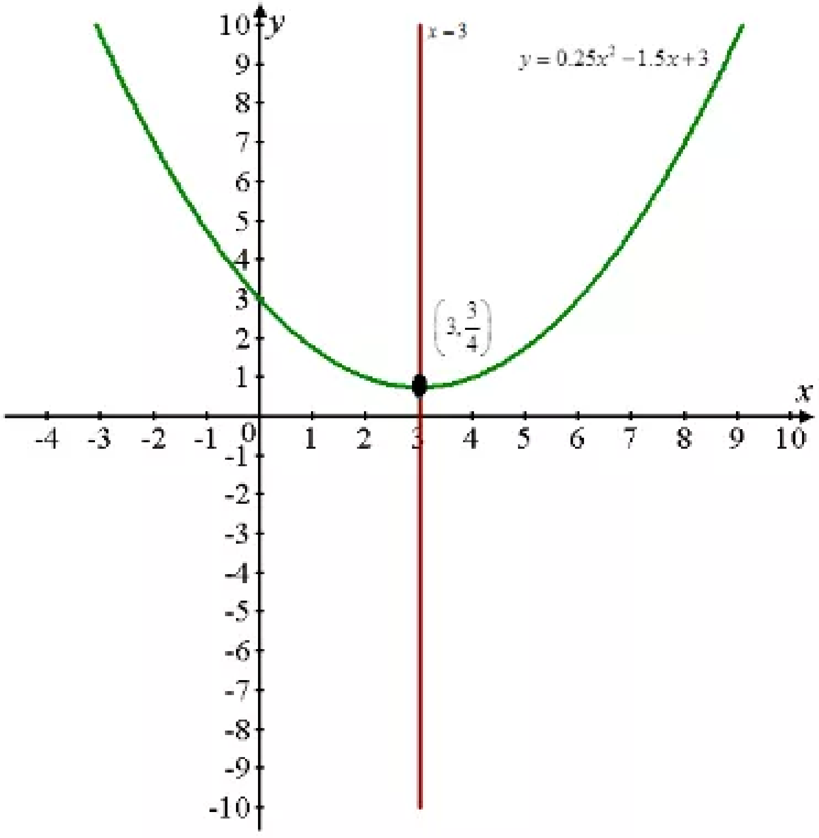
Then, the axis of symmetry is $x = -\frac{b}{2a} = 3$

and the vertex has x-coordinate $-\frac{b}{2a} = 3$

The y-coordinate of the vertex is $y = 0.25(3^2) - 1.5(3) + 3 = \frac{3}{4}$

Hence the vertex is $\left(3, \frac{3}{4}\right)$

Sketch the graph of the function $y = 0.25x^2 - 1.5x + 3$



Answer 51e.

STEP 1 Identify the coefficients of the function.

The given function is of the form $y = ax^2 + bx + c$.

On comparing, we have a is 4.2, b is 6, and c is -1.

Since $a = 4.2 > 0$, the graph opens up.

STEP 2 Find the vertex.

The vertex of the graph of $y = ax^2 + bx + c$ has x-coordinate $-\frac{b}{2a}$.

In order to find the x-coordinate of the vertex, substitute 4.2 for a, and 6 for b and evaluate.

$$-\frac{b}{2a} = -\frac{6}{2(4.2)}$$
$$\approx -0.71$$

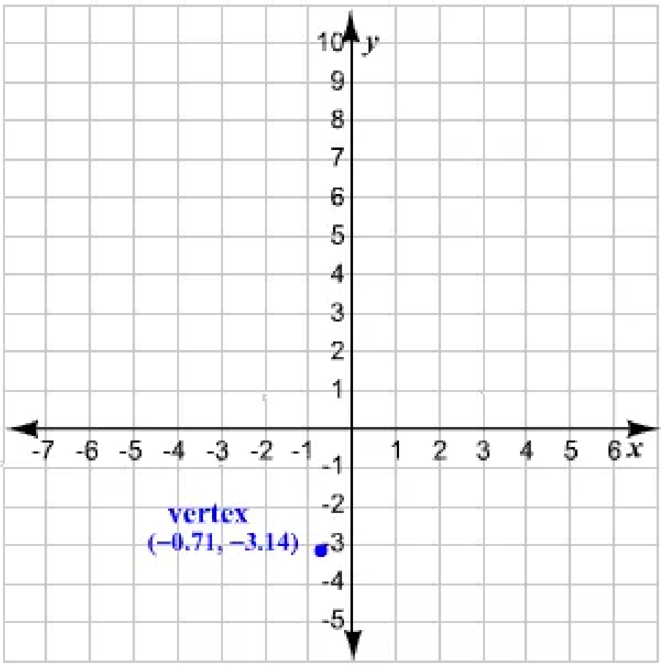
The x-coordinate of the vertex is -0.71.

Substitute -0.71 for x in the given function to find the y-coordinate.

$$f(-0.71) = 4.2(-0.71)^2 + 6(-0.71) - 1$$
$$\approx -3.14$$

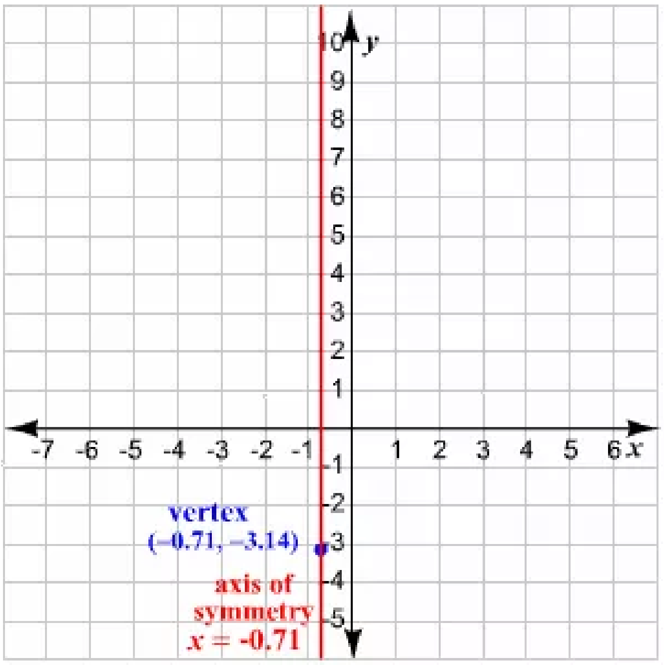
Thus, the vertex of the graph of the given function is $(-0.71, -3.14)$.

Plot the vertex on a coordinate plane.

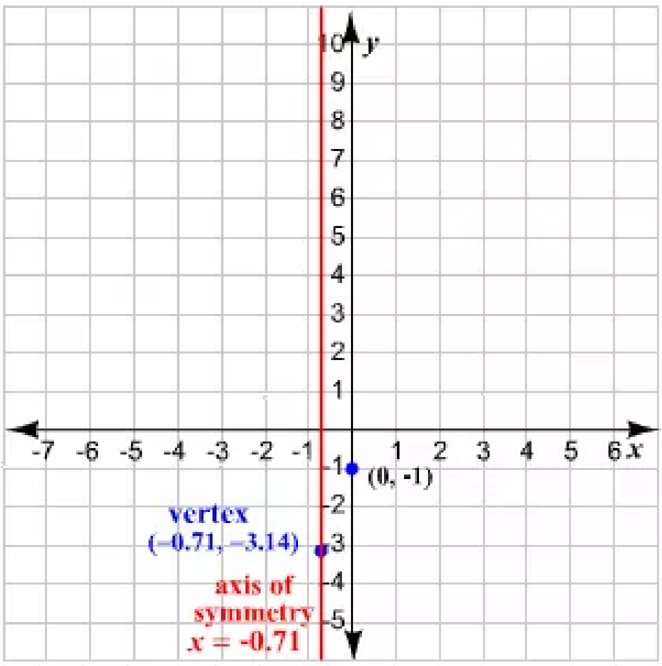


STEP 3 We know that the axis of symmetry is $x = -\frac{b}{2a}$.

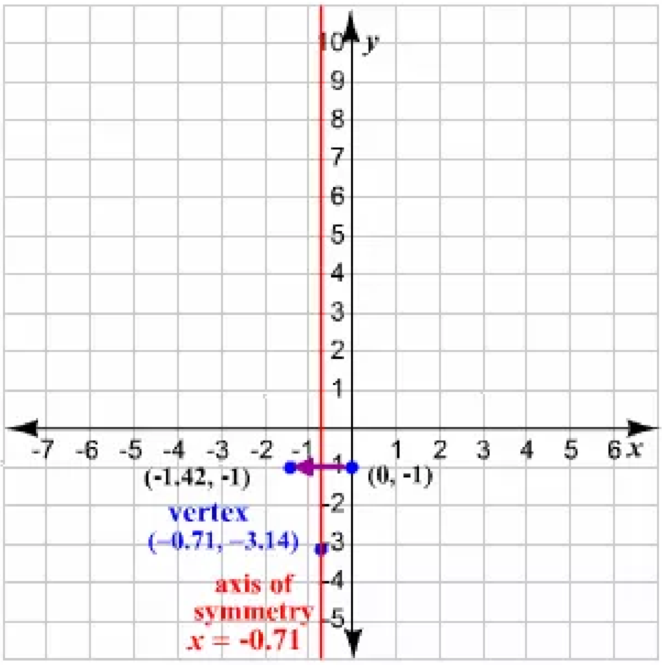
The axis of symmetry of the given function is the line $x = -0.71$.
Now, draw the axis of symmetry $x = -0.71$.



STEP 4 The y-intercept of $y = ax^2 + bx + c$ is c and the point $(0, c)$ is on the parabola.
Thus, the y-intercept of the given function is -1 and $(0, -1)$ is on the parabola.
Plot the point $(0, -1)$ on the same coordinate plane.



Now, reflect the point (0, −1) in the axis of symmetry to get another point.

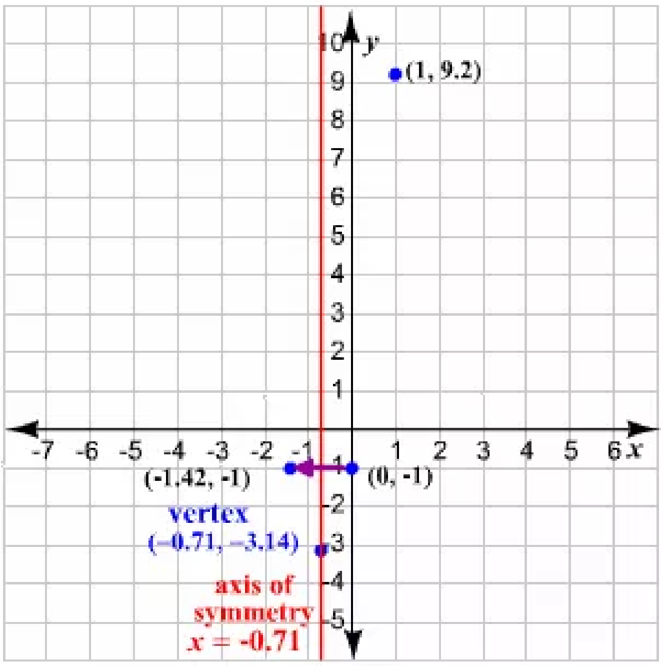


STEP 5 Evaluate the given function for another value of x, say, 1.

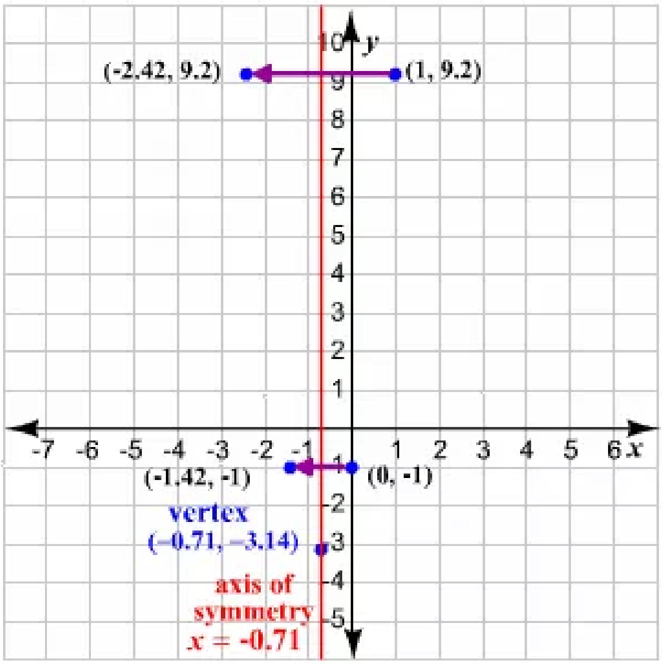
Substitute 1 for x in the function and simplify.

$$f(1) = 4.2(1)^2 + 6(1) - 1$$
$$= 9.2$$

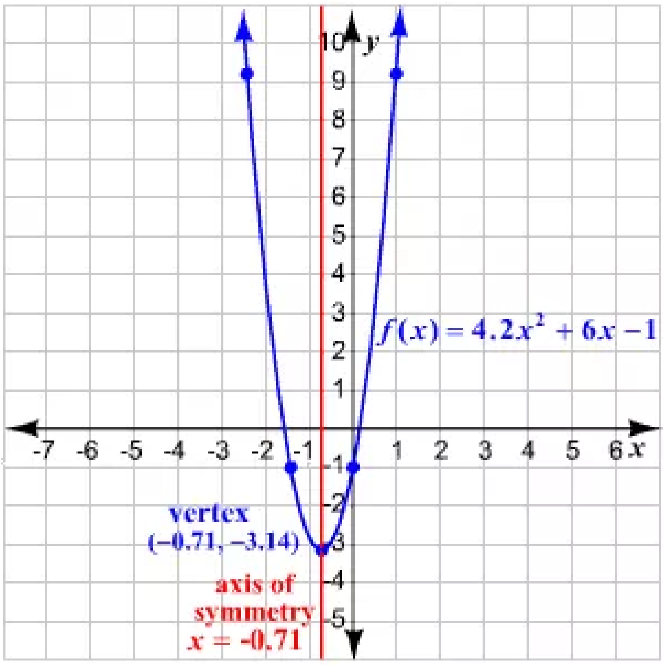
Thus, the point (1, 9.2) lies on the graph. Plot the point on the coordinate plane.



We have to plot the reflection of the point (1, 9.2) in the axis of symmetry.
Reflect the point (1, 9.2) in the axis of symmetry.



STEP 6 Draw a smooth curve through the plotted points.



Answer 52e.

Consider the function $g(x)=1.75x^2-2.5$

Comparing the given function with $y=ax^2+bx+c$.

We get $a=1.75, b=0, c=-2.5$

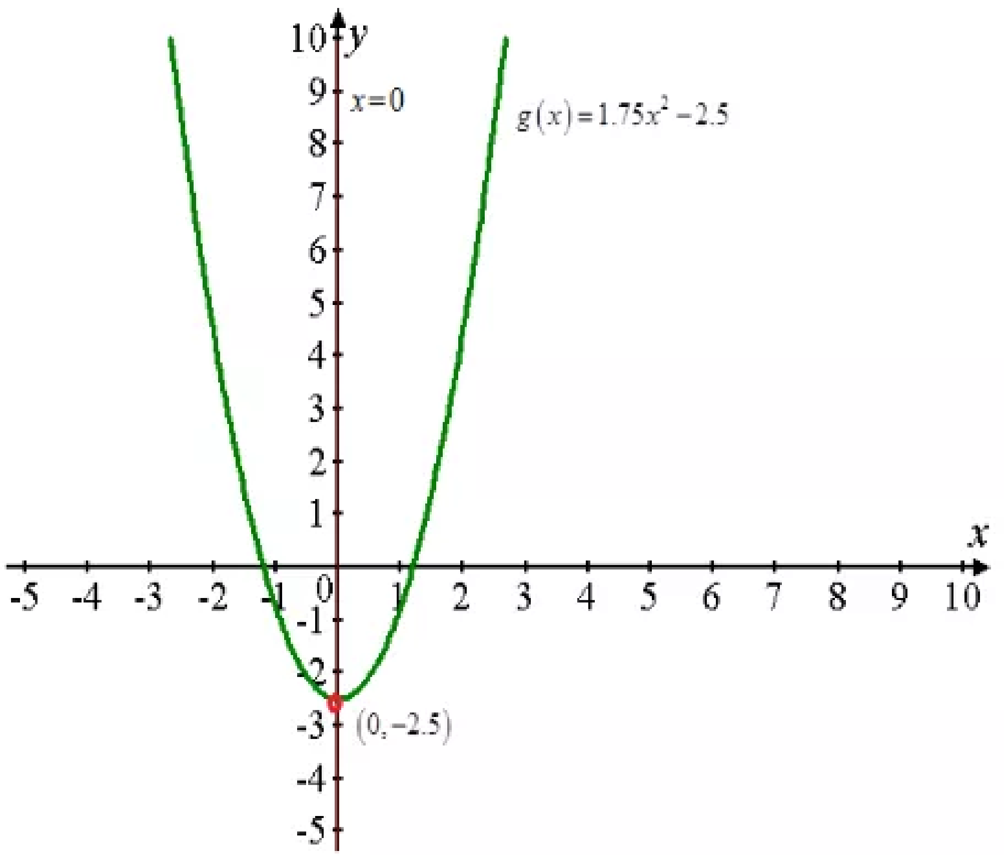
Then, the axis of symmetry is $x=-\frac{b}{2a}=0$

and the vertex has x-coordinate $-\frac{b}{2a}=0$

The y-coordinate of the vertex is $g(0)=1.75(0^2)-2.5=-2.5$

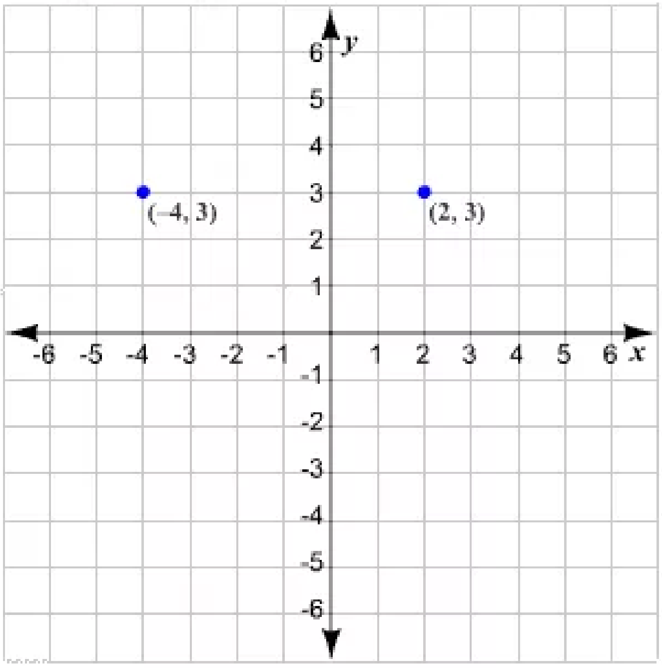
Therefore, the vertex is $(0,-2.5)$

Sketch the graph of the function $g(x)=1.75x^2-2.5$ is shown as below



Answer 53e.

First, plot the given points on a coordinate plane.



We know that the axis of symmetry lie half way between the two x-coordinates.
From the given points, we can see that the y-coordinates are the same. Thus, the points are reflections of each other. The midpoint of the x-coordinates is -1.
Thus, the line $x = -1$ lie half way between the two x-coordinates -4 and 2. Therefore, the equation of the axis of symmetry is
$x = -1$.

Answer 54e.

Consider the function $y = ax^2 + bx + c$

The axis of symmetry is $x = -\dfrac{b}{2a}$

Since the x-coordinate of the vertex is $-\dfrac{b}{2a}$

Show that the y-coordinate of the vertex is $-\dfrac{b^2}{4a} + C$

Substitute $x = -\dfrac{b}{2a}$ in $y = ax^2 + bx + c$ then we get

$$y = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c$$

$$= a\left(\frac{b^2}{4a^2}\right) - \frac{b^2}{2a} + c$$

$$= \frac{b^2}{4a} - \frac{b^2}{2a} + c$$

$$= -\frac{b^2}{4a} + C$$

Therefore, y-coordinate of the vertex is $\boxed{y = -\dfrac{b^2}{4a} + C}$

Answer 55e.

We have the equation $R(x) = (1 + 0.05x)(4000 - 80x)$.

Apply the distributive property.
$R(x) = 4000 - 80x + 200x - 4x^2$

Combine the like terms.
$R(x) = 4000 + 120x - 4x^2$

Now, we have to find the coordinates $(x, R(x))$ of the vertex.

The x-coordinate of the vertex is $x = -\frac{b}{2a}$.
Substitute 120 for b, and −4 for a, and evaluate.

$$\begin{aligned} x &= -\frac{120}{2(-4)} \\ &= \frac{-120}{-8} \\ &= 15 \end{aligned}$$

Evaluate $R(15)$. For this, replace x with 15 in $R(x) = 4000 + 120x - 4x^2$ and evaluate.

$$\begin{aligned} R(15) &= 4000 + 120(15) - 4(15)^2 \\ &= 4000 + 1800 - 900 \\ &= 4900 \end{aligned}$$

The vertex is at (15, 4900). Therefore, raise the price by 15×0.05 or \$0.75 to increase the revenue to \$4900 per day.

Answer 56e.

Consider the electronics store sells about 70 of a new model of digital camera per month at a price of \$320 each.
For each \$20 decrease in price, about 5 more cameras per month are sold.
Obtain a function that models the above situation and use the function to maximize the monthly revenue.

Let \$$20x$ be the decrease in price, then $5x$ is the increase in sales.
Let $T(x)$ denote the total revenue per month. Then,

$T(x) = (320 - 20x)(70 + 5x)$

$\Rightarrow T(x) = -100x^2 + 200x + 22400$

Comparing the given function with $y = ax^2 + bx + c$.
We have $a = -100, b = 200, c = 22400$

Since $a<0$, then the given function attains a maximum vale which is given by the vertex.

x-coordinate of the vertex is $-\frac{b}{2a}=1$

$\Rightarrow$ y-coordinate of the vertex is $T(1)=-100\left(1^2\right)+200(1)+22400=22500$

$\Rightarrow$ vertex is $(1,22500)$

Therefore the maximum value of the given function is 22500

That is, by reducing price by $\$20$, total revenue can be increased up to $\boxed{\$22500}$.

Answer 57e.

The height h above the road of a cable at its lowest point is the y-coordinate of the vertex of the parabola.

First, we have to find the x-coordinate.

The x-coordinate of the vertex is $x = -\frac{b}{2a}$. Substitute $-\frac{7}{15}$ for b, and $\frac{1}{9000}$ for a and evaluate.

$$x = -\frac{-\frac{7}{15}}{2\left(\frac{1}{9000}\right)}$$
$$= 2100$$

Replace x with 2100 to find the y-coordinate.

$$y = \frac{1}{9000}(2100)^2 - \frac{7}{15}(2100) + 500$$
$$= 10$$

Therefore, the height h above the road of a cable at its lowest point is 10 feet.

Answer 58e.

Consider the function $y=-0.2x^2+1.3x$

Find if $y=3$ for any real x.

$\Rightarrow \quad 3=-0.2x^2+1.3x$

$\Rightarrow \quad 0.2x^2-1.3x+3=0$

The above equation is of the form $ax^2+bx+c=0$
The descriminant of the quadratic equation is b^2-4ac
Let D denote the descriminant of the above quadratic equation.

$$D=(-1.3)^2-4(0.2)(3)$$
$$=1.69-2.4$$
$$=-0.71<0$$

The given quadratic equation has no real root
$y\neq 3$ for any $x\in\mathbb{R}$
If the fence 3 feet high the mouse cannot be jump over a fence.

Answer 59e.

a. Let x be the number of \$1 decreases in price and $P(x)$ be the theater's weekly profit.
It is given that the price per ticket is \$20. Thus, the expression for the price is $20-x$.
Since about 10 more tickets per week are sold for each \$1 decrease in price, the expression for the sales is $150+10x$.

The verbal model that represents the given situation is

Profit (dollars)	=	price (dollars)	·	sales	−	expenses (dollars)
⇓		⇓		⇓		⇓
$P(x)$	=	$(20-x)$	·	$(150+10x)$	−	1500 .

Apply the distributive property.
$P(x)=3000+200x-150x-10x^2-1500$

Combine the like terms.
$P(x)=-10x^2+50x+1500$

Therefore, the quadratic function is
$P(x)=-10x^2+50x+1500$.

b. We have the quadratic function $P(x) = -10x^2 + 50x + 1500$.

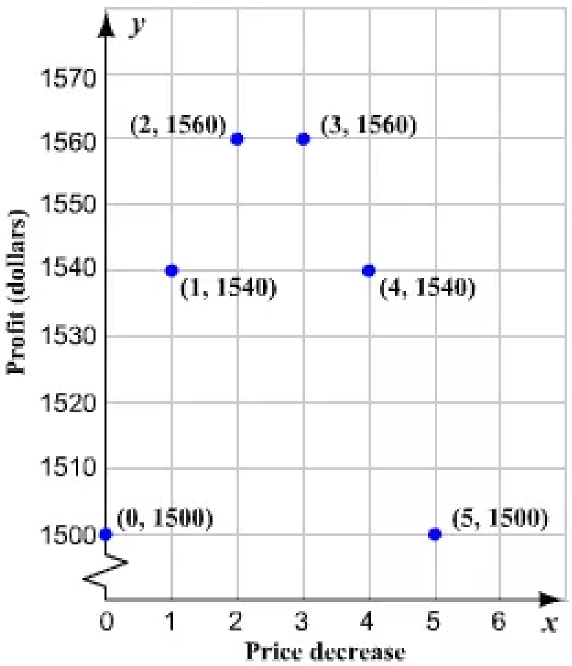
In order to make a table of values, first substitute 0 for x in the quadratic function and evaluate.

$$\begin{aligned} P(0) &= -10(0)^2 + 50(0) + 1500 \\ &= 1500 \end{aligned}$$

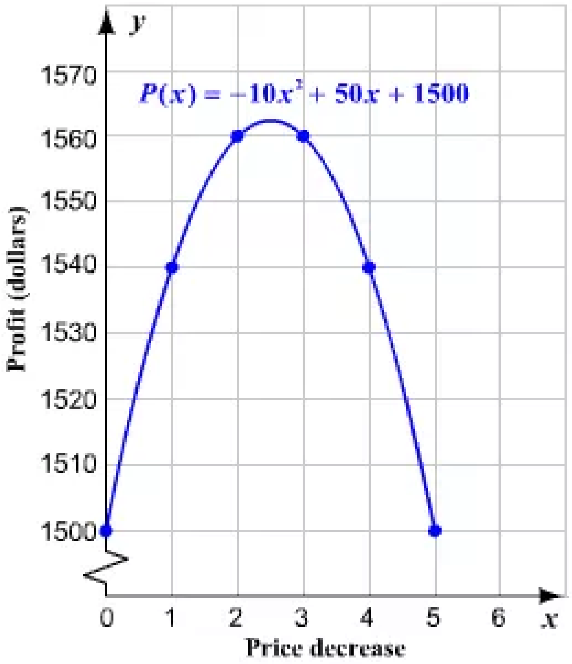
Choose some more values for x and find the corresponding values of $P(x)$.

x	$P(x)$
0	1500
1	1540
2	1560
3	1560
4	1540
5	1500

c. Plot the points from the table on a coordinate plane.



Draw a smooth curve through the plotted points.



Now, we have to find the coordinates $(x, P(x))$ of the vertex.

The x-coordinate of the vertex is $x = -\frac{b}{2a}$. Substitute 120 for b, and −4 for a and evaluate.

$$\begin{aligned} x &= -\frac{50}{2(-10)} \\ &= 2.5 \end{aligned}$$

Evaluate $P(2.5)$. For this, replace x with 2.5 in $P(x) = -10x^2 + 50x + 1500$ and evaluate.

$$\begin{aligned} P(2.5) &= -10(2.5)^2 + 50(2.5) + 1500 \\ &= 1562.5 \end{aligned}$$

The vertex is at (2.5, 1562.5). Therefore, reduce the price by \$2.50 to increase profits to \$1562.50 per week.

Answer 60e.

Consider the path of the golf ball hit an angle of 45^0 and with a speed of 100 feet per second can be modeled by $y = -\frac{g}{10000}x^2 + x$

Where x is the ball's horizontal position (in feet), y is the corresponding height (in feet), and g is the acceleration due to gravity (in feet per second squared).

(a) From the above information, we clearly see that,

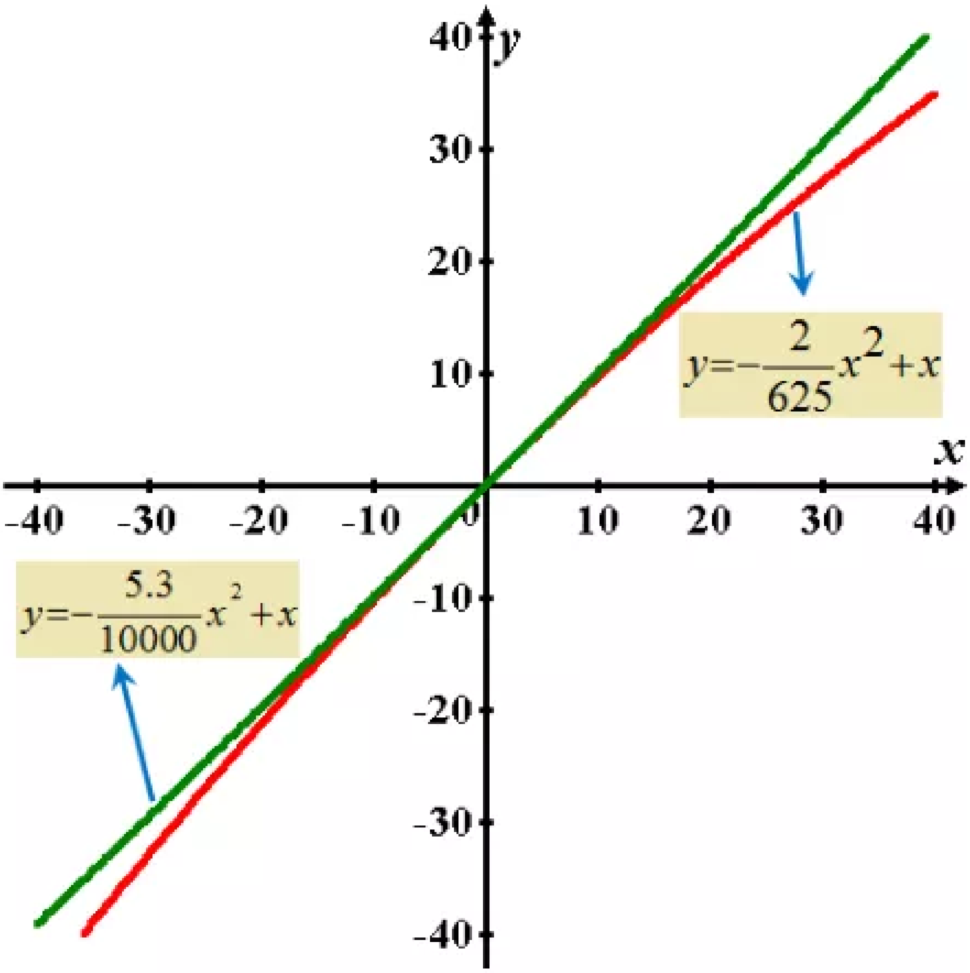
The function for the path of a golf ball hit on earth is

$$y = -\frac{32}{10000}x^2 + x$$

$$y = -\frac{2}{625}x^2 + x$$

The function for the path of a golf ball hit on moon is $\boxed{y = -\frac{5.3}{10000}x^2 + x}$

(b) The following diagram contains the graphs of the functions $y = -\frac{2}{625}x^2 + x$ and $y = -\frac{5.3}{10000}x^2 + x$.



In the above diagram, the graph of $y = -\frac{2}{625}x^2 + x$ is represented by red curve and the graph of $y = -\frac{5.3}{10000}x^2 + x$ is represented by green curve.

Plug $y = 0$ in $y = -\frac{2}{625}x^2 + x$. We get,

$$\frac{2}{625}x^2 = x \Rightarrow x = \frac{625}{2}$$
$$= 312.5$$

Therefore the ball hit on earth travels $\boxed{312.5}$ feet.

Put $y=0$ in $y=-\frac{5.3}{10000}x^2+x$. We get,

$$\frac{5.3}{10000}x^2=x \quad \Rightarrow \quad x=\frac{10000}{5.3}\approx 1886.8$$

Therefore the ball hit on moon travels $\boxed{1886.8}$ feet.

(c) Let d_1 and d_2 denote the distance traveled by the golf ball on earth and moon respectively.

Then, $d_1 : d_2$

$=312.5:1886.8$

$\approx 1:6$

Therefore a ball hit on moon travels around 6 times longer distance than a ball hit on earth.

As we clearly see if the acceleration due to gravity g decreases, then the distance traveled by the ball increases.

Answer 61e.

First, we have to write an equation for the length of the rope.

$P = l + 2w \qquad (1)$

We know that l is some fraction of P. Let x be some value between 0 and 1. Thus, $l = xP$.

Now, write w in terms of P. For this, substitute xP for l in equation (1).

$P = xP + 2w$

Subtract xP from each side.

$P - xP = xP + 2w - xP$

$P - xP = 2w$

Factor out P from the left side.

$(1-x)P = 2w$

Divide each side by 2 to isolate w.

$$\frac{(1-x)P}{2} = \frac{2w}{2}$$

$$\frac{(1-x)P}{2} = w$$

We have to write an equation for the area of the rectangular swimming section.

The area of a rectangle is the product of its length and width. Thus,

$$A = \left(\frac{1-x}{2}\right)P \times xP.$$

Apply the distributive property.

$$A = \left(-\frac{P^2}{2}\right)x^2 + \frac{P^2}{2}x$$

An equation for the area of the rectangular swimming section is

$$A = \left(-\frac{P^2}{2}\right)x^2 + \frac{P^2}{2}x.$$

The above equation is in the form $y = ax^2 + bx + c$, where a is $-\frac{P^2}{2}$, b is $\frac{P^2}{2}$, and c is 0. The value of a is less than 0 and so the graph opens down. Since the parabola opens down, the y-coordinate of the vertex gives the maximum area.

First, we need to find the x-coordinate.

The x-coordinate of the vertex is $x = -\frac{b}{2a}$. Substitute $-\frac{P^2}{2}$ for a, and $\frac{P^2}{2}$ for b and evaluate.

$$x = -\frac{\frac{P^2}{2}}{2\left(-\frac{P^2}{2}\right)}$$

$$= \frac{1}{2}$$

Substitute $\frac{1}{2}$ for x in the equation for the area to find the y-coordinate.

$$A = \left(-\frac{P^2}{2}\right)\left(\frac{1}{2}\right)^2 + \frac{P^2}{2}\left(\frac{1}{2}\right)$$

Simplify.

$$A = \frac{P^2}{8}$$

Therefore, the swimming section can have a maximum area of $\frac{P^2}{8}$ ft ^{2}.

Answer 62e.

Consider the equation $x-3=0$

Solve the equation $x-3=0$

$x-3=0$	Write original equation
$(x-3)+3=0+3$	Add 3 on both sides
$x+(-3+3)=3$	Combine like terms
$x+0=3$	Simplify

The solution is $\boxed{x=3}$

Check: check $x=3$ in the original equation

$$x-3=3-3$$
$$=0$$

Answer 63e.

Subtract 4 from both sides of the given equation.

$$3x+4-4 = 0-4$$
$$3x = -4$$

Divide both the sides by 3.

$$\frac{3x}{3} = \frac{-4}{3}$$
$$x = -\frac{4}{3}$$

The solution is $-\frac{4}{3}$.

Answer 64e.

Consider the equation $-9x+7=-4x-5$

Solve the above equation

$-9x+7=-4x-5$	Write original equation
$(-9x+7)+(4x-7)=(-4x-5)+(4x-7)$	Add both sides $4x-7$ on both sides
$(-9x+4x)+(7-7)=(-4x+4x)+(-5-7)$	Combine like terms
$-5x+0=0-12$	Simplify
$-5x=-12$	Divide -5 on both sides
$x=\frac{12}{5}$	

The solution is $\boxed{x=\frac{12}{5}}$.

Check: check $x=\frac{12}{5}$ in the original equation

$$-9x+7=-4x-5$$

$$-9\left(\frac{12}{5}\right)+7=-4\left(\frac{12}{5}\right)-5$$

$$\frac{-108+35}{5}=\frac{-48-25}{5}$$

$$-\frac{73}{5}=-\frac{73}{5}$$

Answer 65e.

Add 2 to each side of the equation.

$$5x-2+2=-2x+12+2$$

$$5x=-2x+14$$

Add $2x$ to each side.

$$5x+2x=-2x+14+2x$$

$$7x=14$$

Divide each side by 7.

$$\frac{7x}{7}=\frac{14}{7}$$

$$x=2$$

The solution is 2.

Answer 66e.

Consider the equation $0.7x+3=0.2x-2$

Solve the above equation

$0.7x+3=0.2x-2$ Write original equation

$(0.7x+3)+(-0.2x-3)=(0.2x-2)+(-0.2x-3)$ Add both sides $-0.2x-3$ on both sides

$(0.7x-0.2x)+(3-3)=(0.2x-0.2x)+(-2-3)$ Combine like terms

$0.5x+0=0-5$ Simplify

$0.5x=-5$ Divide 0.5 on both sides

$x=-\frac{5}{0.5}$

The solution is $\boxed{x=-10}$

Check: check $x=-10$ in the original equation

$0.7x+3=0.2x-2$

$0.7(-10)+3=0.2(-10)-2$

$-7+3=-2-2$

$-4=-4$

Answer 67e.

Add 0.5 to each side of the given equation.

$0.4x+0.5x=-0.5x-5+0.5x$

$0.9x=-5$

Divide each side by 0.9.

$\frac{0.9x}{0.9}=\frac{-5}{0.9}$

$x=-\frac{50}{9}$

The solution is $-\frac{50}{9}$.

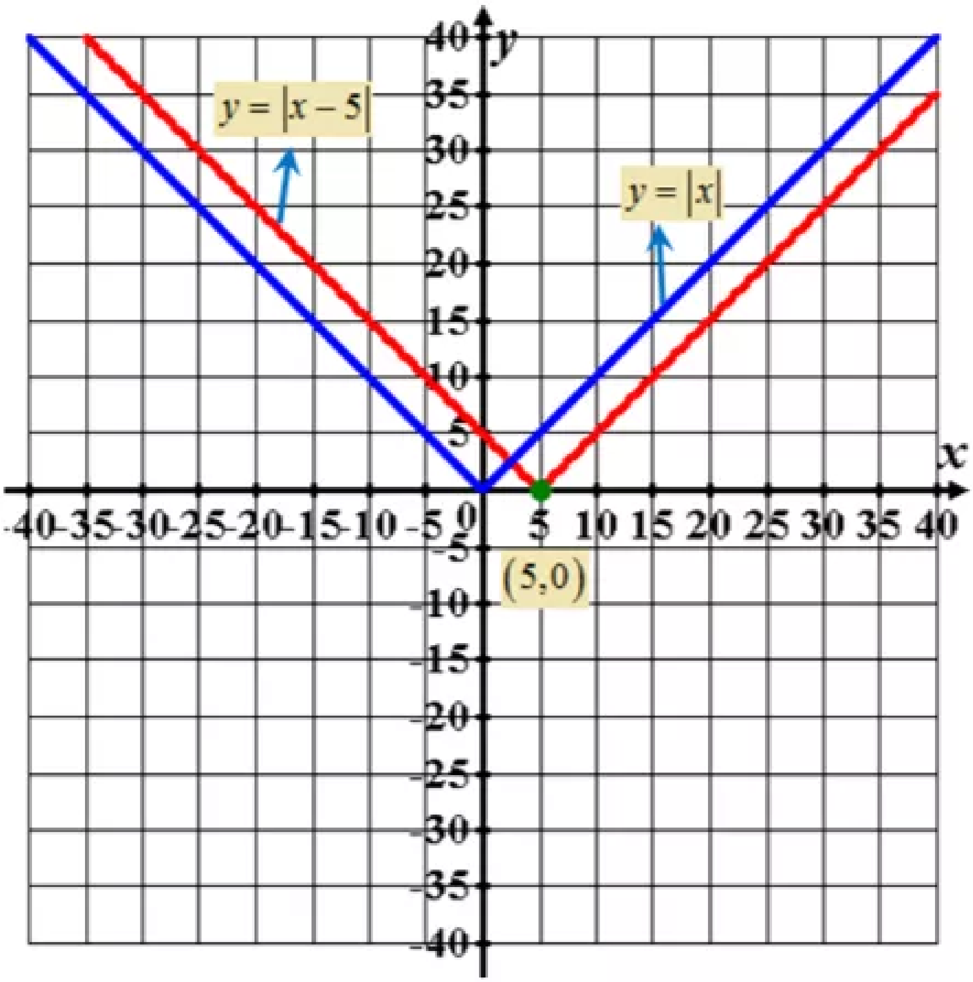
Answer 68e.

Consider the function $y=|x-5|$

The graph of $y=|x-5|$ is the graph of $y=|x|$ translated 5 units horizontally.

The vertex of $y=|x-5|$ is $(5,0)$.

The following diagram contains the graph of the function $y = |x - 5|$.

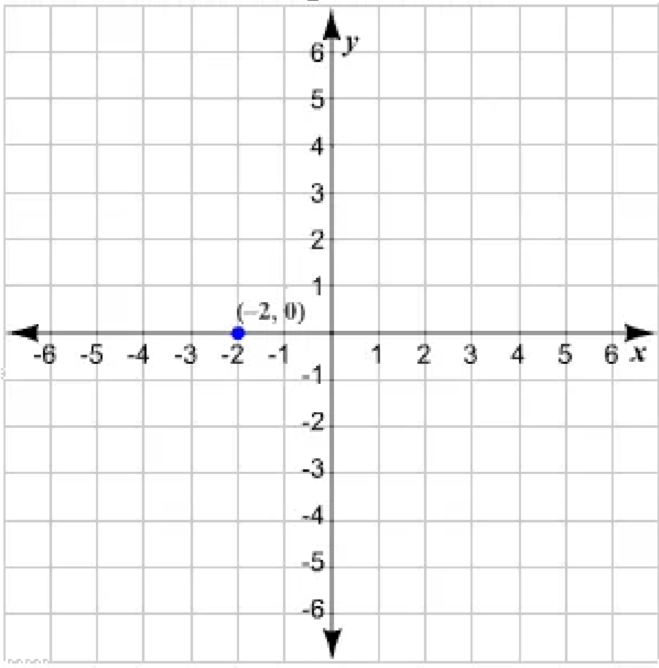


Answer 69e.

STEP 1 The given function is of the form $y = |x - h| + k$, where (h, k) is the vertex of the function.

We get the value of h as -2 and of k as 0. Thus, the vertex is $(-2, 0)$.

Plot the vertex of the given function.



STEP 2 Use symmetry to find two more points.

Substitute any value, say, −3 for y in the given function.

$$-3 = -|x + 2|$$

Divide each side by −1.

$$\frac{-3}{-1} = \frac{-|x+2|}{-1}$$

$$3 = |x + 2|$$

We get $x + 2 = 3$ and $x + 2 = -3$.

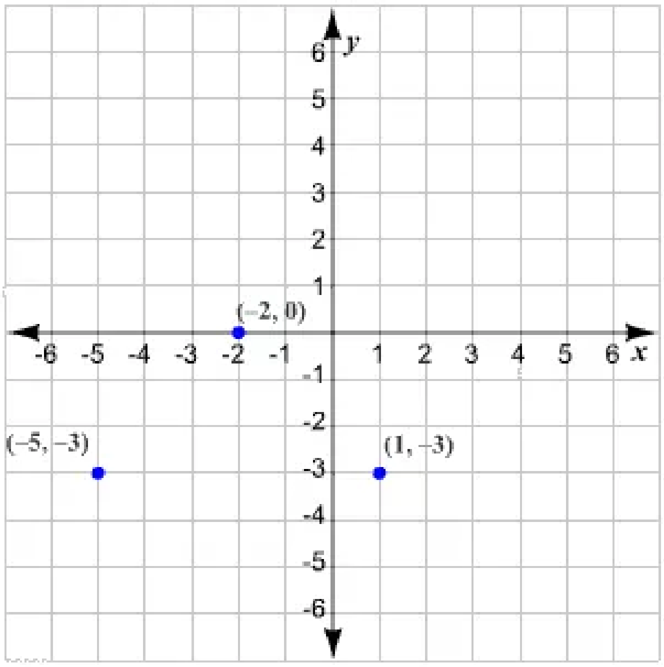
Subtract 2 from both the sides of the two equations.

$x + 2 - 2 = 3 - 2$ and $x + 2 - 2 = -3 - 2$

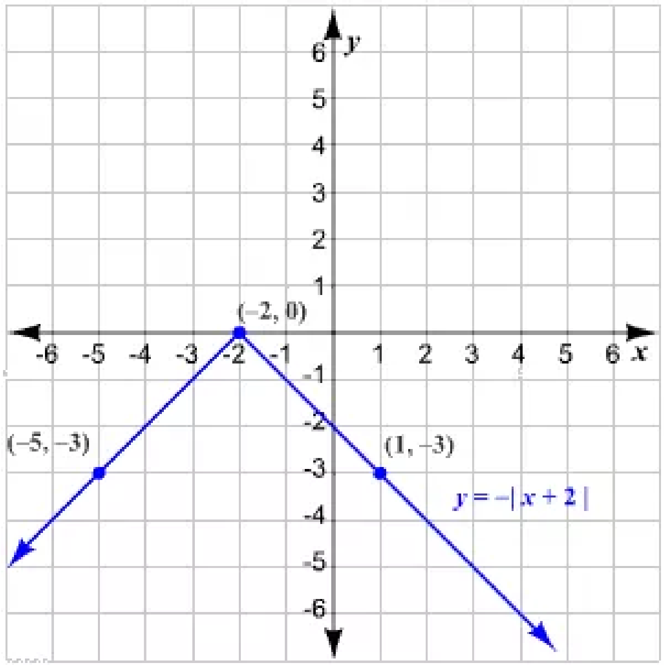
$x = 1$ and $x = -5$

The two points are (1, −3) and (−5, −3).

Plot these points on the graph.



STEP 3 Connect the points using straight lines to obtain a V-shaped graph.



Answer 70e.

Consider the function $y = 3|x-1|$

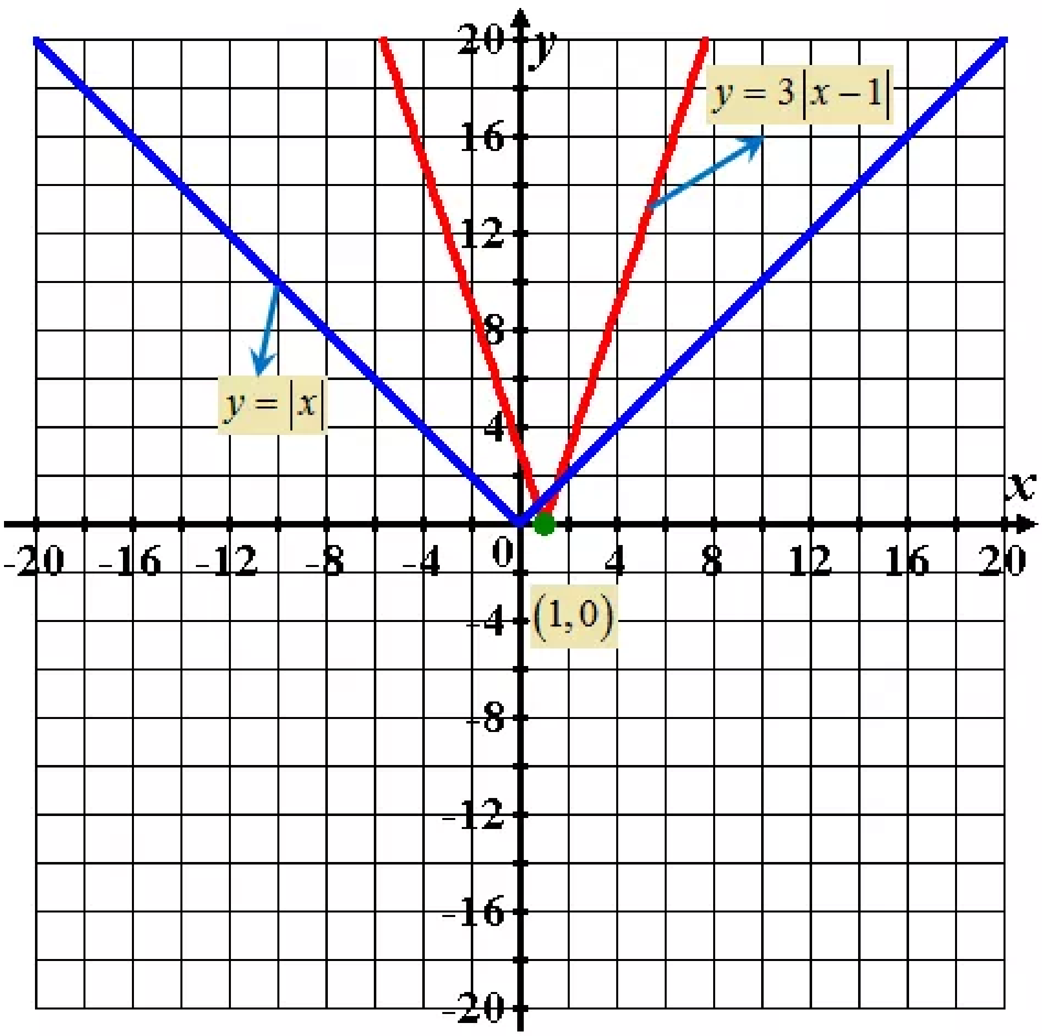
$|x-1|$ shifts each of the *x* term in $|x|$ to the right by 1 unit.

$3|x-1|$ raises each point on $|x-1|$ at 3 times.

So, each point on $|x-1|$ is raised vertically three times.

Put the observations together, $3|x-1|$ is a horizontal shift of 1 unit and vertical shift of three times that of $|x|$

The graph of $|x|$ in blue color and $3|x-1|$ in red color confirms the horizontal and vertical shifts as

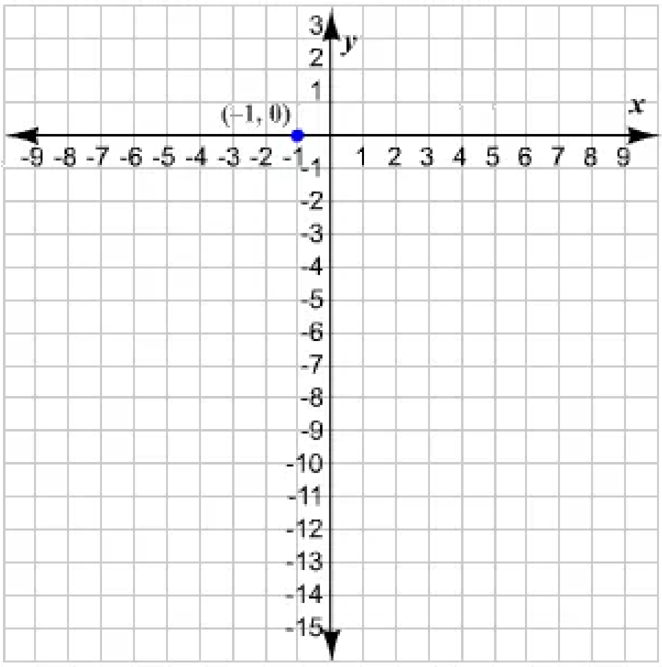


Answer 71e.

STEP 1 The given function is of the form $y = a|x - h| + k$, where (h, k) is the vertex of the function.

We get the value of h as -1 and of k as 0. Thus, the vertex of the given function is $(-1, 0)$.

Plot the vertex of the given function.



STEP 2 Use symmetry to find two more points.
Substitute any value, say, -12 for y in the given function.
$-12 = -4|x + 1|$

Divide each side by -4.

$$\frac{-12}{-4} = \frac{-4|x+1|}{-4}$$

$$3 = |x+1|$$

We get $x + 1 = 3$ and $x + 1 = -3$.

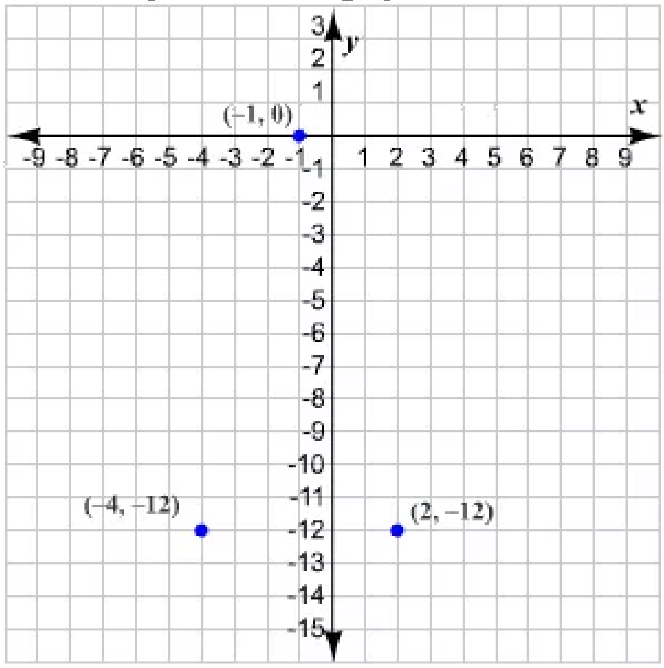
Subtract 1 from both the sides of the two equations.

$x + 1 - 1 = 3 - 1$ and $x + 1 - 1 = -3 - 1$

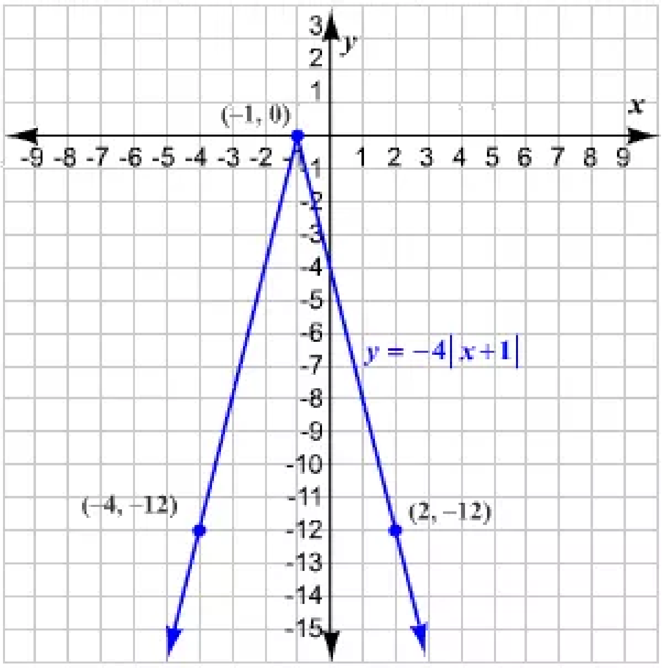
$x = 2$ and $x = -4$

The two points are $(2, -12)$ and $(-4, -12)$.

Plot these points on the graph.



STEP 3 Connect the points using straight lines to obtain a V-shaped graph.



Answer 72e.

Consider the function $f(x)=2|x-3|+6$

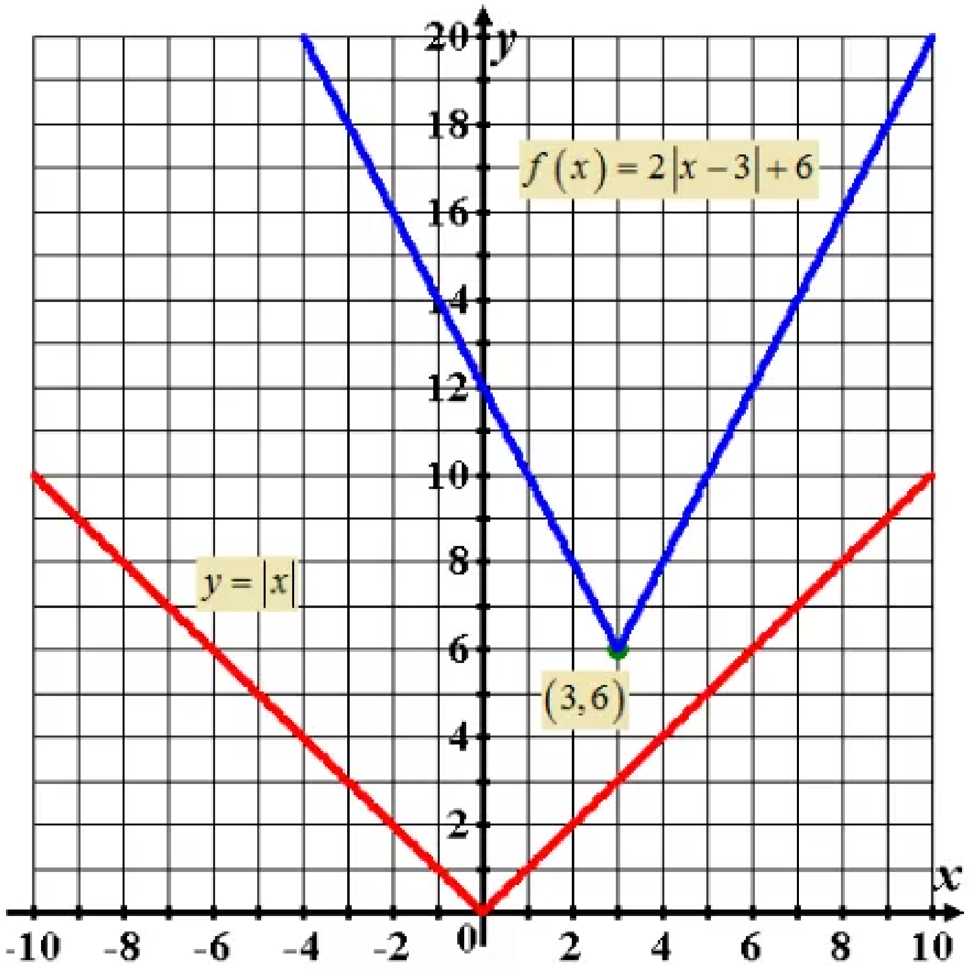
$|x-3|$ shifts each of the x term in $|x|$ to the right by 3 unit.

$2|x-3|$ raises each point on $|x-3|$ at 2 times.

So, each point on $|x-3|$ is raised vertically two times.

The graph of $y=|x|$ moves 6 units vertically upwards.

The following diagram contains the graph of the function $f(x)=2|x-3|+6$

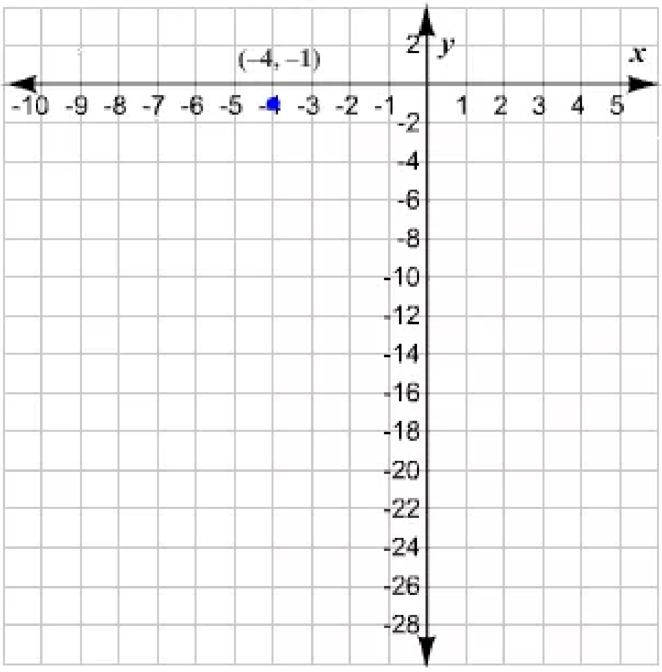


Answer 73e.

STEP 1 The given function is of the form $y=a|x-h|+k$, where (h, k) is the vertex of the function.

We get the value of h as −4 and of k as −1. Thus, the vertex of the given function is (−4, −1).

Plot the vertex of the given function.



STEP 2 Use symmetry to find two more points.
Substitute any value, say, −26 for y in the given function.

$-26 = -5\,|x+4| - 1$

Add 1 to each side.

$$-26 + 1 = -5|x+4| - 1 + 1$$

$$-25 = -5|x+4|$$

Divide each side by −5.

$$\frac{-25}{-5} = \frac{-5|x+4|}{-5}$$

$$5 = |x+4|$$

We get $x + 4 = 5$ and $x + 4 = -5$.

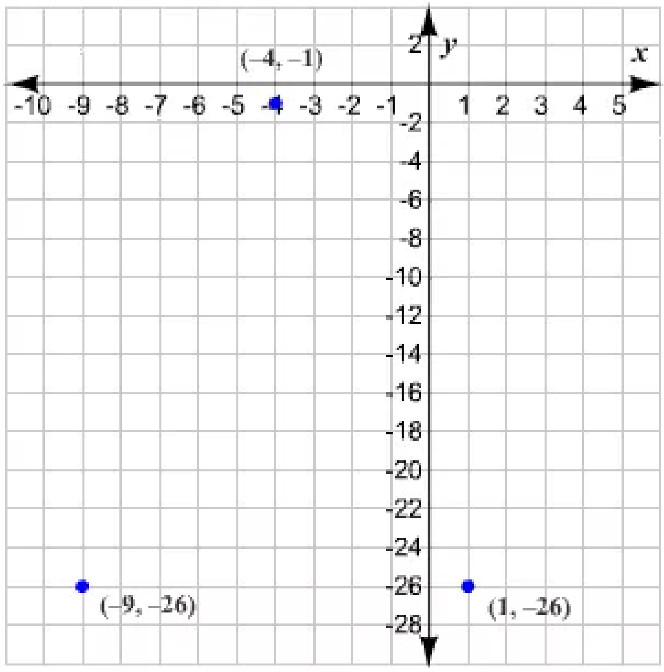
Subtract 4 from both the sides of the two equations.

$x + 4 - 4 = 5 - 4$ and $x + 4 - 4 = -5 - 4$

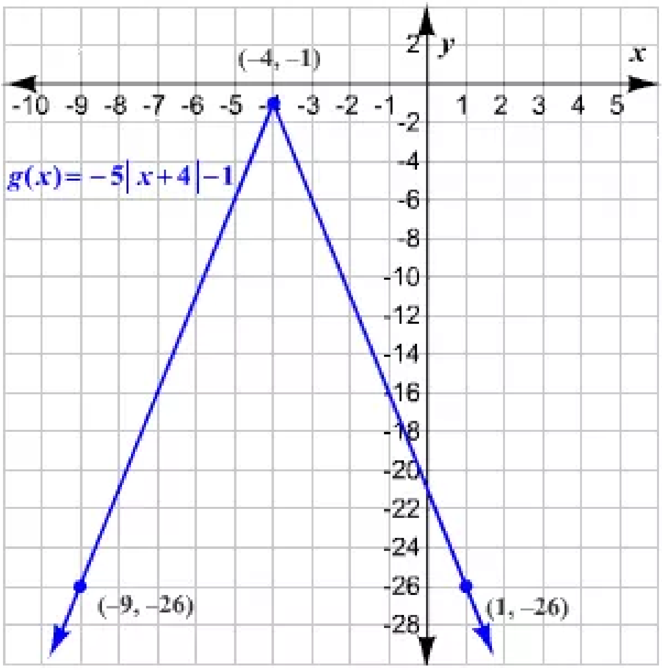
$x = 1$ and $x = -9$

The two points are (1, −26) and (−9, −26).

Plot these points on the graph.



STEP 3 Connect the points using straight lines to obtain a V-shaped graph.



Answer 74e.

From the given information, it is clear that we have traveled a distance of 270 miles in 5 hours.

Therefore, the average speed is $\frac{270}{5} = 54$ miles per hour