HOTS (Higher Order Thinking Skills)

Que 1. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to from an embankment. Find the height of the embankment.

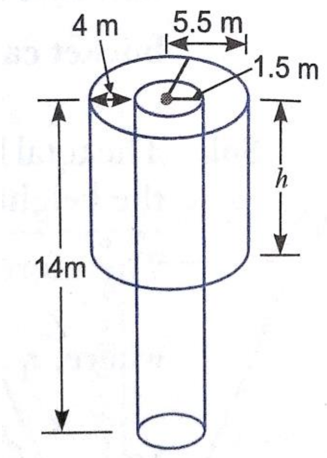

Fig. 13.40

Sol. Here, radius of the well $= \frac{3}{5} = 1.5\ m$

Depth of the well = 14 m
Width of the embankment = 4 m
∴ Radius of the embankment = 1.5 + 4 = 5.5 m
Let h be the height of the embankment.
∴ Volume of the embankment
= Volume of the well (Cylinder)

$\Rightarrow \pi[(5.5)^2 - (1.5)^2] \times h = \pi(1.5)^2 \times 14$

$\Rightarrow (30.25 - 2.25) \times h = 2.25 \times 14$

$\Rightarrow h = \frac{2.25 \times 14}{28} = 1.125\ m$

Que 2. A hollow cone is cut by a plane parallel to the base and upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line segments into which the cone's altitude is divided by the plane.

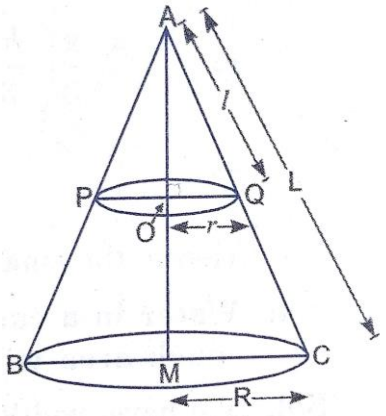

Fig. 13.41

Sol. In Fig. 13.41, the smaller cone APQ has been cut off through the plane PQ||BC. Let r and R be the radii of the smaller and larger cone and I and L their slant heights respectively.

Here, in the adjoining figure

OQ = r, MC = R, AQ = I, AC = L.

Now, $\Delta AOQ \sim \Delta AMC$

$$\Rightarrow \quad \frac{OQ}{MC} = \frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{r}{R} = \frac{I}{L} \quad \Rightarrow \quad r = \frac{RI}{L} \qquad \ldots(i)$$

Since, curved surface area of the remainder = $\frac{8}{9}$ of the curved surface area of the whole cone, therefore, we get,

CSA of smaller cone = $\frac{1}{9}$ of the CSA of the whole cone

$$\therefore \quad \pi rl = \frac{1}{9}\pi RL$$

$$\Rightarrow \quad \pi\left(\frac{RI}{L}\right) I = \frac{1}{9}(\pi RL) \qquad \text{[Using (i)]}$$

$$\Rightarrow \quad I^2 = \frac{L^2}{9} \quad \Rightarrow \quad \frac{I}{L} = \frac{1}{3}$$

Now again in similar triangles, AOQ and AMC, we have

$$\frac{AO}{AM} = \frac{AQ}{AC} \quad \Rightarrow \quad \frac{AO}{AM} = \frac{I}{L} = \frac{1}{3} \quad \Rightarrow \quad AO = \frac{AM}{3}$$

$$\Rightarrow \quad OM = AM - OA = AM - \frac{AM}{3} = \frac{2}{3}AM$$

$$\therefore \quad \frac{AO}{OM} = \frac{AM/3}{2AM/3} = \frac{1}{2}$$

Hence, the required ratio of their heights = 1: 2

Que 3. The height of the cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$ of the given cone, at what height above the base is the section made?

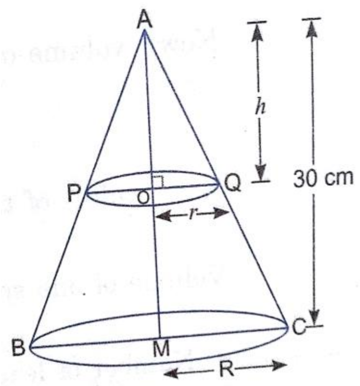

Fig. 13.42

Sol. Let the small cone APQ be cut off at the top by the plane PQ||BC.
Let r and h be the radius and height of the smaller cone, respectively and also let the radius of larger cone = R.

Now, $\Delta AOQ \sim \Delta AMC$

$\Rightarrow \quad \frac{AO}{AM} = \frac{OQ}{MC}$

$\Rightarrow \quad \frac{h}{30} = \frac{r}{R} \qquad \therefore \; r = \frac{hR}{30} \qquad \ldots$(i)

Since, it is given that volume of the smaller cone = $\frac{1}{27}$ (Volume of larger cone)

$$\frac{1}{3}\pi r^2 h = \frac{1}{27}\left(\frac{1}{3}\pi R^2 . 30\right)$$

$\Rightarrow \quad \frac{\pi}{3}\left(\frac{hR}{30}\right)^2 h = \frac{1}{27} \times \frac{1}{3}\pi R^2 . 30 \qquad$ [From (i)]

$\Rightarrow \quad h^3 = \frac{30\times30\times30}{27} = 1000 \qquad \therefore \; h = 10\, cm$

Hence, the smaller cone has been cut off at a height of (30 – 10) cm = 20 cm from the base.

Que 4. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

Sol. We have, width of the canal = 6 m

Depth of the canal = 1.5 m

Now, length of water flowing per hour = 10 km

∴ Length of water flowing in half hour = 5 km = 5,000 m

∴ Volume of water flow in 30 minutes = 1.5 × 6 × 5,000 = 45,000 m^3

Here, standing water needed is 8 cm = 0.08 m

∴ Area irrigated in 30 minutes = $\frac{Volume}{Height} = \frac{45,000}{0.08}$

$= 562500 \text{ m}^2 \qquad [1\ hectare = 10000\ m^2]$

$= 56.25$ hectares

Que 5. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots in the vessel.

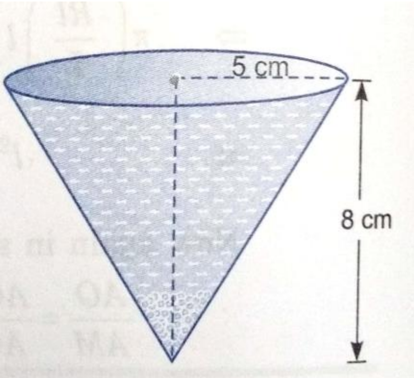

Fig. 13.43

Sol. We have,

Height of conical vessel = h = 8 cm

and its radius = r = 5 cm

Now, volume of cone = Volume of water in the cone

$$= \frac{1}{2}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 8 = \frac{4{,}400}{21} \text{cm}^3$$

Now, Volume of water flows out = Volume of lead shots

$$= \frac{1}{4} \times Volume\ of\ water\ in\ the\ cone = \frac{1}{4} \times \frac{4{,}400}{21} = \frac{1{,}100}{21} cm^3$$

Now, radius of the lead shots = 0.5 cm = $\frac{5}{10} cm = \frac{1}{2} cm$

Volume of one spherical lead shot = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{11}{21} \text{cm}^3$

∴ Number of lead shots dropped in the vessel = $\dfrac{Volume\ of\ water\ flows\ out}{Volume\ of\ one\ lead\ shot}$

$$= \frac{\frac{1{,}100}{21}}{\frac{11}{21}} = \frac{1{,}100}{21} \times \frac{21}{11} = 100$$