# JEE (Main)-2025 (Online) Session-2 Memory Based Question with & Solutions (Physics, Chemistry and Mathematics) 7th April 2025 (Shift-2)

Time: 3 hrs. M.M.: 300

### **IMPORTANT INSTRUCTIONS:**

- **(1)** The test is of 3 hours duration.
- **(2)** This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.
- (3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.
- **(4)** Section A : Attempt all questions.
- (5) Section B: Attempt all questions.
- (6) Section A (01 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.
- (7) Section B (21 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

# **MEMORY BASED QUESTIONS JEE-MAIN EXAMINATION - APRIL, 2025**

(Held On Monday 7th April, 2025)

# TIME: 3:00 PM to 6:00 PM

## **PHYSICS**

### SECTION-A

1. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as reason (R)

**Assertion** (A): Refractive index of glass is more than air

**Reason (R):** Optical density of a medium is directly related to its mass density.

In the light of the above statements, choose the correct answer from the options given below

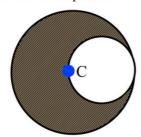
- (1) Both (A) and (R) are true and (R) is correct explanation of (A)
- (2) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)
- (3) (A) is true but (R) is false
- (4) (A) is false but (R) is true

Ans. (3)

**Sol.** Assertion  $\rightarrow$  True

Reason → False

2. The figure shows a circular portion of radius  $\frac{R}{2}$  removed from a disc of mass m and radius R. The moment of inertia about an axis passing through the centre C of the disc and perpendicular to the plane is



- $(1)\frac{11}{64}mR^2$
- $(2) \frac{mR^2}{16}$
- $(3) \frac{mR^2}{2}$
- $(4)\frac{13}{32}mR^2$

Ans. (4

Sol.  $I = \frac{MR^2}{2} + \frac{\left(-\frac{M}{4}\right)\left(\frac{R}{2}\right)^2}{2} + \left(-\frac{M}{4}\right)\left(\frac{R}{2}\right)^2$ 

$$= \frac{mR^{2}}{2} - \frac{mR^{2}}{32} - \frac{mR^{2}}{16}$$

$$\frac{16mR^{2} - mR^{2} - 2mR^{2}}{32}$$

$$= \frac{13}{32}mR^{2}$$

- 3. Which of the following quantity has same dimensions as  $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ ?
  - (1) Resistance
- (2) Voltage
- (3) Capacitance
- (4) Inductance

Ans. (1)

**Sol.** Ampere's law  $B\ell: \mu_0 i$ 

Gauss' law 
$$\frac{q}{\epsilon_0} = EA$$

$$\sqrt{\frac{\mu_0}{\epsilon_0}} = \left(\frac{1}{\sqrt{\mu_0 \epsilon_0}}\right) \mu_0 = C \times \frac{B}{ni}$$

$$= \frac{C(EL)}{CNi} = \frac{V}{Ni} = R$$

4. Give below are two statements. One is labelled as Statement-I and the other is labelled as Statement-II.

Statement-I: A magnetic monopole does not exist.

Statement-II: Magnetic lines are continuous and form closed loops.

In the light of the above statements, choose the correct answer from the options given below:-

- (1) Statement-I is correct, statement-II is correct and statement-II is correct explanation of statement-I.
- (2) Statement-I is correct, statement-II is correct and statement-II is not the correct explanation of statement-I.
- (3) Statement-I is correct and statement-II is incorrect.
- (4) Statement-I is incorrect and statement-II is correct.

Ans. (1)

**Sol.**  $\oint \mathbf{B} \cdot d\mathbf{A} = 0$ 

- **5.** Potential energy is not defined for which of the force:-
  - (1) Gravitational force (2) Restoring force
  - (3) Friction
- (4) Electrostatic force

Ans. (3)

- **Sol.** Frictional force is Non conservative force
- 6. The equation of a wave travelling on a string is  $y = \sin \left[ 20\pi x + 10\pi t \right]$  where x and t are distances and time in SI units, the minimum distance between two points having zero oscillating speed is:-
  - (1) 20 cm
- (2) 5.0 cm
- (3) 2.5 cm
- (4) 10 cm

Ans. (2)

- **Sol.**  $y = \sin [20\pi x + 10\pi t]$ 
  - $\omega = 10\pi$

 $k = 20\pi$ 

$$\Delta \phi = \frac{2\pi}{\lambda} \Delta x$$

 $\pi = 20\pi \ \Delta x$ 

$$\Delta x = \frac{1}{20} m = 5 cm$$

- 7. For a given physical expression  $\sqrt{\frac{2I}{C\epsilon_0}}$  which of the unit matches:-
  - (1) N/C
- (2) m/s C
- (3) Amp./m
- (4) N m/C

Ans. (1)

**Sol.**  $I = \frac{W}{m^2}$ 

C = m/s

$$\epsilon_0 = \frac{C^2}{Nm^2}$$

$$\sqrt{\frac{2I}{\epsilon_0 C}} = \sqrt{\frac{\frac{W / m^2}{C^2}}{\frac{C^2}{Nm^2} \times m / s}} = \frac{N}{C}$$

- **8.** Match the following:-
  - (a) Power
- (P)  $[ML^{-3}T^{0}]$
- (b) Mass density
- (Q)  $[ML^2T^{-3}]$
- (c) Angular Impulse
- $(R) [ML^2T^{-1}]$
- (S)  $[ML^3T^{-2}]$
- (1) (a)-(Q), (b)-(P), (c)-(R)
- (2) (a)-(Q), (b)-(P), (c)-(S)
- (3) (a)-(S), (b)-(P), (c)-(R)
- (4)(a)-(P),(b)-(Q),(c)-(R)

Ans. (1)

- **Sol.** (a) Power =  $[ML^2T^{-3}]$ 
  - (b) Mass density =  $[ML^{-3}T^{0}]$
  - (c) Angular Impulse =  $[ML^2T^{-1}]$
- 9. A helicopter is moving 2 km above the ground horizontally with a speed of 360 km/hr. A packet is dropped from helicopter, find distance between the points where the packet was released to the point where packet lands on the ground
  - $(1)\sqrt{2}$  km
- (2)  $2\sqrt{2} \text{ km}$
- (3) 2 km
- $(4)\frac{1}{\sqrt{2}}$  km

Ans. (2)

Sol. 
$$R = \sqrt{\frac{2h}{g}} \times u$$

$$=350\times\frac{5}{18}\times\sqrt{\frac{2\times2000}{10}}$$

- $=100\times20=2000$ m
- = 2 km

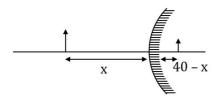
Distance between the points =  $2\sqrt{2}$  km

- 10. A convex mirror is used to produce an image with magnification of  $\frac{1}{4}$ . If the distance between object and its image is 40 cm, the focal length of the mirror is:-
  - (1) 12.7 cm
- (2) 15 cm
- (3) 10 cm
- (4) 10.7 cm

Ans.

(4)

Sol.



$$m = -\left(\frac{40 - x}{-x}\right) = \frac{1}{4}$$

$$160 - 4x = x$$

$$x = 32$$

$$\frac{1}{8} - \frac{1}{32} = \frac{1}{f}$$

$$f = \frac{32}{3}$$
cm

- 11. An object with mass 500 g moves along x-axis with speed  $V = 4\sqrt{x} m/s$  then force acting on the object is :-
  - (1) 5 N
- (2) 6 N
- (3) 8 N
- (4) 4 N

(4) Ans.

 $v = 4\sqrt{x}$ Sol.

$$a = v \frac{dv}{dx}$$

$$=4\sqrt{x}\,\frac{2}{\sqrt{x}}=8$$

$$F = 4N$$

- 12. A capillary tube is put in water which is having tube radius of 0.1 mm. Find capillary rise in the tube, If angle of contact =  $30^{\circ}$ , surface tension of  $\rho_{\text{water}} = \frac{1 \text{gram}}{\text{cm}^3}, g =$ liquid dyne/cm,  $9.8 \text{ m/sec}^2$ 
  - (1) 21.35 cm
- (2) 12.37 cm
- (3) 15.35 cm
- (4) 11.37 cm

Ans. **(2)** 

 $h = \frac{2T\cos\theta}{\text{ogr}} = \frac{2 \times 7 \times \frac{\sqrt{3}}{2}}{1 \times 980 \times 0.01} = \frac{1.732}{0.14} = 12.37 \text{cm}$ 

- 13. Flux through a plane parallel to x-z plane is 6 SI units. Find area of plane if electric field in the region is  $\vec{E} = (\hat{i} + 4\hat{j} + \hat{k}) 10^3 \text{N/C}$ .
  - (1)  $2.5 \times 10^{-2} \text{ m}^2$  (2)  $2 \times 10^{-3} \text{ m}^2$
  - (3)  $2.5 \times 10^{-3} \text{ m}^2$  (4)  $1.5 \times 10^{-3} \text{ m}^2$

Ans. (4)

Sol.  $\phi = \vec{E} \cdot \vec{A}$ 

$$6 = \left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}\right) \cdot \left(A\hat{\mathbf{j}}\right) \times 10^3$$

$$6 = 4A \times 10^3$$

$$\Rightarrow$$
 A = 1.5×10<sup>-3</sup> m<sup>2</sup>

14. Assertion: Aeroplane is made of metal to prevent from lighting strike.

> Reason: Electric field in cavity inside conductor at equilibrium remain zero.

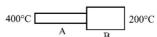
- (1) Both Assertion and Reason are correct
- (2) Assertion is correct but Reason incorrect
- (3) Assertion is incorrect and Reason is correct
- (4) Both are incorrect.

Ans. **(1)** 

Assertion → Correct Sol.

Reason → Correct

If  $\frac{L_A}{L_B} = 2$ ,  $\frac{K_A}{K_B} = 2$ ,  $\frac{R_A}{R_B} = \frac{1}{2}$ , where L = length of 15. Rod, K = thermal conductivity, R = Radius of cross-section of Rod.



Find temperature of Junction.

- $(1) 280^{\circ}C$
- (2) 300°C
- (3) 240°C
- (4) 320°C

(3) Ans.

 $R_{A} = \frac{L_{A}}{K_{A} \pi R_{A}^{2}}$ Sol.

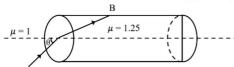
$$R_{\rm B} = \frac{L_{\rm B}}{K_{\rm B}\pi R_{\rm B}^2}$$

$$\frac{R_{A}}{R_{B}} = \frac{L_{A}}{L_{B}} \times \frac{K_{B}}{K_{A}} \times \frac{R_{B}^{2}}{R_{A}^{2}} = 2 \times \frac{1}{2} \times 4 = 4$$

Drop across  $A = \frac{4}{5} \times 200 = 160$ °C

T = 400 - 160 = 240°C

16. The maximum value of  $\theta$  (as shown in figure) for which total internal reflection can happen at point B is



- $(1) \sin^{-1}\left(\frac{3}{4}\right)$
- (2)  $\cot^{-1}\left(\frac{3}{4}\right)$
- $(3) \cos^{-1}\left(\frac{3}{4}\right)$
- (4)  $\tan^{-1}\left(\frac{4}{3}\right)$

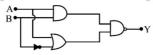
- Ans.
- $\sin\theta_{\rm C} = \frac{1}{\mu} = \frac{1}{5/4}$ Sol.

$$\theta_{\rm C} = 53^{\circ}$$

$$1\sin\theta = \frac{5}{4} \times \sin i$$

$$\sin \theta = \frac{5}{4} \times \sin 37^{\circ} = \frac{5}{4} \times \frac{3}{5} = \frac{3}{4}$$

17. Find input A and B for which output Y is zero



- (1) A = 1, B = 1
  - (2) A = 0, B = 0
- (3) A = 0, B = 1
- (4) A = 1, B = 0

- Ans. **(1)**
- Sol.  $A \rightarrow 1$ 
  - $B \rightarrow 1$
- 18. Distance between plates of capacitor is 0.5 mm, potential difference between plates is 5 volt. Now a dipole is placed between plates of capacitor, having its dipole moment parallel to electric field. Magnitude of one charge of dipole is  $2 \mu C$ and distance between charges is 0.5 µm. Now dipole is tilted at an angle  $\theta$ = 30°, then torque acting on the dipole is
  - $(1) 5 \times 10^{-8} \text{ Nm}$
- (2)  $5 \times 10^{-9} \,\mathrm{Nm}$
- (3)  $5 \times 10^{-10} \,\mathrm{Nm}$
- (4) None of these

- Ans.
- $E = \frac{5 \times 10}{0.5 \times 10^{-3}} = 10^4 \text{ V/m}$ Sol.

$$\tau = PE \sin 30^{\circ} = (2 \times 10^{-6} \times 0.5 \times 10^{-6}) \times 10^{4} \times \frac{1}{2}$$

$$\tau = 5 \times 10^{-9} \, \text{Nm}$$

- A photon of wavelength  $\lambda_1$  is incident on a photo-sensitive surface of threshold wavelength  $\lambda_0$  . The de-Broglie wavelength of fastest photoelectron is:-
  - $(1) \lambda_1 \lambda_0$
- $(2)\,\sqrt{\lambda_1^2-\lambda_0^2}$

$$(3)\sqrt{\frac{h}{2mc}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_0}\right)} \qquad (4)\sqrt{\frac{h}{2mc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_0}\right)}}$$

Ans. (4)

**Sol.** 
$$KE_{max} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_0} = hc \left( \frac{1}{\lambda_i} - \frac{1}{\lambda_0} \right)$$

$$\lambda = \frac{h}{\sqrt{2mKE}} = \frac{h}{\sqrt{2m_e hc \left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}$$

$$\lambda = \sqrt{\frac{h}{2m_e c \left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}$$

### **SECTION - B**

1. An inductor of reactance  $50\Omega$ , a capacitor of reactance 50  $\Omega$  and a resistor of resistance 100 $\Omega$ are connected in series with an AC source 10V, 50 Hz. Average power dissipated by the circuit is W.

Ans. **(1)** 

Sol.  $P = VI\cos\Delta\phi$ 

$$Z = \sqrt{100^2} = 100\Omega$$

$$I_{rms} = \frac{10}{100} = \frac{1}{10} A$$

$$\tan \Delta \phi = \frac{X_C - X_L}{R} = 0$$

$$P_{avg} = 1W$$

### **SECTION-A**

- 1. Which of the following is correct order of acidic character of oxides of vanadium?
  - (1)  $V_2O_5 > VO_2 > V_2O_3$
  - (2)  $V_2O_3 > VO_2 > V_2O_5$
  - (3)  $V_2O_5 > V_2O_3 > VO_2$
  - (4)  $VO_2 > V_2O_3 > V_2O_5$

Ans. (1)

**Sol.** For oxides of same element

**As.** O.S. increases, acidic character increases So acidic character order  $V_2O_5 > VO_2 > V_2O_3$ 

- 2. Which of the following is the correct Hybridisation of XeF<sub>4</sub>?
  - (1) sp<sup>3</sup>d
- (2)  $sp^{3}$
- $(3) sp^3d^2$
- $(4) sp^3d^3$

Ans. (3)

Sol.  $F \searrow_{Xe} F$ 

Steric Number = B.P. + 1.P.

$$=4+2=6$$

No. of hybrid orbitals formed = 6Hybridization =  $sp^3d^2$ 

Tryondization—sp d

- 3. Consider the following species
  - (i) <u>S</u>O<sub>2</sub>
- (ii)  $\underline{N}_3^-$
- (iii) NO<sub>2</sub>

Find the hybridization of underlined atom.

- (1) (i) sp<sup>2</sup> (ii) sp<sup>2</sup> (iii) sp<sup>2</sup>
- (2) (i) sp<sup>2</sup> (ii) sp (iii) sp<sup>2</sup>
- (3) (i) sp<sup>3</sup> (ii) sp (iii) sp<sup>2</sup>
- (4) (i) sp (ii) sp<sup>2</sup> (iii) sp<sup>3</sup>

Ans. (2)

- **Sol.** (i)  $SO^2 \rightarrow sp^2$ 
  - (ii)  $\underline{N}_3^- \rightarrow sp$
  - (iii)  $\underline{NO}_2^- \rightarrow sp^2$

4. Match the following

|     | Complex                                                  | Primary<br>Valency |   | Secondary<br>Valency |
|-----|----------------------------------------------------------|--------------------|---|----------------------|
| (A) | [Co(en) <sub>2</sub> Cl <sub>2</sub> ]Cl                 | (I)                | 3 | 6                    |
| (B) | [Pt(NH <sub>3</sub> ) <sub>2</sub> Cl(NO <sub>2</sub> )] | (II)               | 3 | 4                    |
| (C) | [Co(SCN) <sub>4</sub> ] <sup>-1</sup>                    | (III)              | 2 | 6                    |
| (D) | [Mg(EDTA)] <sup>2</sup>                                  | (IV)               | 2 | 4                    |

- (1) A-I, B-II, C-IV, D-III
- (2) A-III, B-IV, C-II, D-I
- (3) A-III, B-IV, C-I, D-II
- (4) A-I, B-IV, C-II, D-III

Ans. (4)

Sol.

(A)  $\lceil \text{Co(en)}_2 \text{Cl}_2 \rceil \text{Cl} \Rightarrow \text{Primary valency} = 3$ 

Sec. Valency 
$$= 6$$

(B)  $\left[ Pt(NH_3)_2 Cl(NO_2) \right] \Rightarrow Primary valency = 2$ 

Sec. Valency = 4

(C)  $\left[ \text{CO(SCN)}_4 \right]^{-1} \Rightarrow \text{Primary valency} = 3$ 

Sec. Valency = 4

(D)  $\left[ Mg(EDTA) \right]^{2^{-}} \Rightarrow Primary valency = 2$ 

Sec. Valency = 6

- **5.** Among the given order, the incorrect order of atomic radii is
  - $(1) r_{Rb} < r_{Cs}$
- $(2) r_{Mg} < r_{Al}$
- (3)  $r_{Cl} < r_{Br}$
- (4)  $r_{K} < r_{Rb}$

Ans. (2)

- **Sol.** (1)  $r_{Rb} < r_{Cs}$  correct
  - (2)  $r_{Mg} \le r_{Al}$  incorrect
  - (3)  $r_{Cl} < r_{Br}$  correct
  - (4)  $r_K < r_{Rb}$  correct

- If  $P_A^o = 350$  torr and  $P_B^o = 750$  torr and the two 6. volatile liquids (A) and (B) form an ideal solution. X<sub>A</sub> and X<sub>B</sub> are the respective k mole fraction of (A) and (B) in solution and YA and YB are the respective mole fraction of (A) and (B) in the vapour phase. Which one of the following relation is correct?
  - $(1)\frac{Y_A}{Y_B} = \frac{X_A}{X_B}$

$$(2)\frac{Y_A}{Y_B} < \frac{X_A}{X_B}$$

- $(3)\frac{Y_{A}}{Y_{B}} > \frac{X_{A}}{X_{B}}$   $(4)\frac{Y_{A}}{Y_{B}} = \frac{X_{B}}{X_{A}}$

Ans.

**Sol.** 
$$P_A = P_A^o X_A = P_T \times Y_A \dots (1)$$

$$P_{\rm B} = P_{\rm B}^{\rm o} X_{\rm B} = P_{\rm T} \times Y_{\rm B} \qquad \dots (2)$$

Dividing eq. (1) by (2)

$$\frac{P_A^o X_A}{P_B^o X_B} = \frac{Y_A}{Y_B}$$

$$\frac{350 \times X_{A}}{750 \times X_{B}} = \frac{Y_{A}}{Y_{B}}$$

$$\frac{X_{_A}}{X_{_B}} > \frac{Y_{_A}}{Y_{_B}}$$

7. Match List-I with List-II and select the correct option.

| List-I (Solution) |                                   | List-II (Properties) |                                 |
|-------------------|-----------------------------------|----------------------|---------------------------------|
| (A)               | Benzene + Toluene                 | (P)                  | Show +ve deviation              |
| (B)               | Aniline +<br>CH <sub>3</sub> COOH | (Q)                  | $\Delta v_{ m mix} = 0$         |
| (C)               | Water + ethanol                   | (R)                  | $\Delta H_{mix} = -ve$          |
| (D)               | Acetone + CHCl <sub>3</sub>       | (S)                  | Form minimum boiling Azeotrope. |

- (1)  $A \rightarrow Q$ ;  $B \rightarrow R$ ;  $C \rightarrow P$ ,S;  $D \rightarrow R$
- (2) A  $\rightarrow$  S; B  $\rightarrow$  O; C  $\rightarrow$  P,S; D  $\rightarrow$  R
- (3)  $A \rightarrow Q$ ;  $B \rightarrow P$ , S;  $C \rightarrow R$ ;  $D \rightarrow P$
- (4)  $A \rightarrow P$ ;  $B \rightarrow S$ ;  $C \rightarrow P$ ;  $D \rightarrow R$

**(1)** Ans.

Sol.

- (A) Benzene + Toleune  $\Rightarrow$  Ideal solution ( $\Delta V_{mix} = 0$ )
- (B) Aniline + CH<sub>3</sub>COOH ⇒ Negative deviation

 $(\Delta H_{mix} = negative)$ 

- (C) Water + ethanol  $\Rightarrow$  Positive deviation
  - (Form minimum boiling Azeotrope.)
- (D) Acetone + CHCl<sub>3</sub> ⇒ Negative deviation  $(\Delta H_{mix} = negative)$

List-I mentions thermodynamic process of List-II mention property

| List-I |            | List-II |                |  |
|--------|------------|---------|----------------|--|
| (A)    | Isothermal | (I)     | q = 0          |  |
| (B)    | Adiabatic  | (II)    | $\Delta T = 0$ |  |
| (C)    | Isobaric   | (III)   | $\Delta V = 0$ |  |
| (D)    | Isochoric  | (IV)    | $\Delta P = 0$ |  |

- (1)  $A \rightarrow II$ ;  $B \rightarrow I$ ;  $C \rightarrow IV$ ;  $D \rightarrow III$
- (2)  $A \rightarrow I$ ;  $B \rightarrow II$ ;  $C \rightarrow III$ ;  $D \rightarrow IV$
- (3) A  $\rightarrow$  I; B  $\rightarrow$  IV; C  $\rightarrow$  III; D  $\rightarrow$  II
- (4)  $A \rightarrow II$ ;  $B \rightarrow I$ ;  $C \rightarrow III$ ;  $D \rightarrow IV$

Ans. **(1)** 

- Sol. (A) Isothermal  $\Rightarrow \Delta T = 0$ 
  - (B) Adiabatic  $\Rightarrow$  q = 0
  - (C) Isobaric  $\Rightarrow \Delta P = 0$
  - (D) Isochoric  $\Rightarrow \Delta V = 0$
- In electrolysis of aqueous solution of AgNO<sub>3</sub>, 9. Cu(NO<sub>3</sub>)<sub>2</sub>, Hg(NO<sub>3</sub>)<sub>2</sub> and Au(NO<sub>3</sub>)<sub>3</sub> is carried out, then correct order of deposition of metal cathode give

| Metal ion            | SRP(V) |
|----------------------|--------|
| $Ag^+/Ag$            | 0.79   |
| Cu <sup>2+</sup> /Cu | 0.34   |
| $Hg^{2+}/Hg$         | 0.85   |
| $Au^{3+}/Au$         | 1.4    |
| 775 1 == 1           | 0.00   |

- (1) Au > Hg > Ag > Cu
- (2) Au > Hg > Cu > Ag
- (3) Au > Ag > Hg > Cu
- (4) Cu > Ag > Hg > Au

Ans. **(1)** 

Sol.

| SRP(V) |  |
|--------|--|
| 0.79   |  |
| 0.34   |  |
| 0.85   |  |
| 1.4    |  |
|        |  |

Higher the SRP higher the tendency to get reduced on cathode.

10. How many of the following coordination compounds having same coordination number and paramagnetic in nature?

$$\begin{split} &K_{3}[Fe(CN)_{6}],\,K_{4}[Fe(CN)_{6}][Cu(NH_{3})_{4}]SO_{4},\\ &K_{2}Hgl_{4},\,[Ni(H_{2}O)_{6}]^{2^{+}} \end{split}$$

- (1) 2
- (2) 3
- (3) 1
- (4) 4

Ans. (1)

Sol.

C.No.

 $K_3[Fe(CN)_6]$ 

6

 $Fe^{+3}:3d^5$ 

Paramagnetic

 $K_4[Fe(CN)_6]$ 

6

 $Fe^{+2}:3d^{6}$ 

Diamagnetic

 $[Cu(NH_3)_4]SO_4$ 

4

 $Cu^{+2}:3d^9$ 

Square planar

Paramagnetic

 $K_2[HgI_4]$ 

4

 $Hg^{+2}:5d^{10}$ 

tetrahedral

diamagnetic

 $[Ni(H_2O)_6]^{+2}$ 

6

 $Ni^{+2}:3d^{8}$ 

Paramagnetic

- 11. Hydration energy of K<sup>+</sup> is -x kJ/mol and of Cl<sup>-</sup> is -y kJ/mol and lattice energy of KCl is -Z kJ/mol then what is the heat of dissolution of KCl?
  - (1) Z (x + y)
- (2) Z (x + y)
- (3) Z + x + y
- (4) Z + (x + y)

Ans. (1)

Sol

$$K_{(g)}^{+} + Cl_{(g)}^{-} \rightarrow KCl_{(s)} : L.E. = -Z \text{ kJ/mol}$$
 
$$\Delta H_{Sol.} = -x$$
 
$$kJ/mol$$
 
$$\Delta H_{dissolution} = -x$$
 
$$\Delta H_{dissolution} = Z + (-x - y)$$

=Z-(x+y)

12. The correct order to basic strength of the following molecules is

$$\underbrace{ \bigcirc ^{\mathrm{NH_2}}_{\mathrm{MeO}} \underbrace{ \bigcirc ^{\mathrm{NH_2}}_{\mathrm{O_2N}} \underbrace{ \bigcirc ^{\mathrm{NH_2}}_{\mathrm{CH_3NH_2}}}_{\mathrm{CH_3NH_2}}$$

- (1) A > B > C > D
- (2) B > C > D > A
- (3) D > B > A > C
- (4) B > A > C > D

Ans. (3)

Sol. Basic strength

$$CH_3NH_2 > \bigcirc OMe > OMe > NH_2$$

**13.** Match the following List-I with List-II:

|      | List-I (Reactions)                                          | List-I | Į.       |
|------|-------------------------------------------------------------|--------|----------|
|      |                                                             | (Reac  |          |
|      |                                                             |        | erature) |
| (A)  | Cl OH                                                       | (I)    | Slight   |
|      |                                                             |        | Warming  |
|      |                                                             |        |          |
|      |                                                             |        |          |
| (B)  | Cl OH                                                       | (II)   | 368 K    |
|      |                                                             |        |          |
|      |                                                             |        |          |
|      |                                                             |        |          |
|      | $NO_2$ $NO_2$                                               |        |          |
| ( G) |                                                             | (***)  |          |
| (C)  | Cl OH                                                       | (III)  | 443 K    |
|      | NO <sub>2</sub> NO <sub>2</sub>                             |        |          |
|      |                                                             |        |          |
|      |                                                             |        |          |
|      | NO <sub>2</sub> NO <sub>2</sub>                             |        |          |
|      |                                                             |        |          |
| (D)  | Cl OH                                                       | (IV)   | 623 K    |
|      | NO <sub>2</sub> NO <sub>2</sub> NO <sub>2</sub>             |        |          |
|      | $  $ $  $ $  $ $  $ $  $ $  $ $ $                           |        |          |
|      |                                                             |        |          |
|      | $\begin{array}{ccc} & & &   \\ & NO_2 & & NO_2 \end{array}$ |        |          |
|      | 1,02                                                        |        |          |
|      |                                                             |        |          |

Choose the correct answer from the options given below:

- (1) A-II, B-III, C-I, D-IV
- (2) A-IV, B-III, C-II, D-I
- (3) A-I, B-II, C-III, D-IV
- (4) A-II, B-IV, C-III, D-I

Ans. (2)

Sol.

|     | (Reactions)                                   |  |  |
|-----|-----------------------------------------------|--|--|
| (A) | Cl OH 623 K                                   |  |  |
| (B) | Cl OH  443 K  NO <sub>2</sub> NO <sub>2</sub> |  |  |
| (C) | $OH$ $NO_2$ $NO_2$ $NO_2$ $NO_2$              |  |  |
| (D) | $NO_2$ Slight $NO_2$ NO2 NO2 NO2              |  |  |

14. Assertion: Cl Cl has more dipole moment
than Br Br
Br

than 
$$\operatorname{Br}$$
  $\operatorname{Br}$ 

Choose the correct option.

- (1) Both Assertion and Reason are correct, Reason is correct explanation of Assertion.
- (2) Both Assertion and Reason are correct, Reason is not correct explanation of Assertion.
- (3) Assertion is correct, Reason is incorrect.
- (4) Assertion is incorrect, Reason is correct.

Ans. (3)

Sol. Dipole moment  $\begin{array}{c|cccc} Cl & Cl & Br & Br \\ \hline \end{array}$ 

Due to more E.N. of Cl as compare to Br:

Br 
$$\rightarrow$$
 Br  $\rightarrow$  B

15. Which of the following compounds molecular formula C<sub>6</sub>H<sub>12</sub>O. Positively to 2,4-DNP test and Tollen's regent test

$$(1) \ \mathbf{H_3C} - \mathbf{C} - \mathbf{CH_2} - \mathbf{C} - \mathbf{CH_3} \\ \parallel \quad \parallel \quad \parallel \\ \mathbf{O} \quad \quad \mathbf{H}$$

$$\begin{array}{c} \text{(2) H--C--CH}_2\text{--CH}_2\text{--CH}_2\text{--CH}_3\\ \parallel & \text{O} \end{array}$$

$$(4) CH_{3} - C - C - CH_{3}$$

$$CH_{3} - CH_{3}$$

Ans. (2)

Sol. (1) 
$$H_3C - C - CH_2 - C - CH_3$$

Positive 2,4-DNP

Negative Tollen's test

$$\begin{array}{c} \text{(2) H--C--CH}_2\text{---CH}_2\text{---CH}_2\text{---CH}_2\\ \text{ }\\ \end{array}$$

Positive 2,4-DNP

Positive Tollen's test

Positive 2,4-DNP

Negative Tollen's test

Positive 2,4-DNP

Negative Tollen's test

### **SECTION-B**

**16.** Find the number of sp<sup>2</sup> carbons in the final product 's'.

 $ace to phenone \xrightarrow{\ ^{PCl_3}\ } X \xrightarrow{\ ^{3NaNH2}\ } Y \xrightarrow{\ ^{Acidification}\ } Z \xrightarrow{\ ^{HBO}\ } S$ 

Ans. (7)

Sol.  $Ph-C-CH_3$   $\xrightarrow{PCl_3}$   $Ph-C-CH_3$   $\xrightarrow{PCl_3}$   $Ph-C-CH_3$   $\xrightarrow{Ph-C=CH}$   $\xrightarrow{Ph-C=CH}$   $\xrightarrow{Ph-C=C}$   $\xrightarrow{Ph-C=CH}$   $\xrightarrow{Ph-C=CH}$   $\xrightarrow{Ph-C=CH}$ 

17. Consider the following molecule

$$\begin{array}{cccc} CH_3 & \stackrel{CH_3}{=} \\ & & \stackrel{C}{=} \\ H_3C-CH=CH-C-CH=CH-C-CH=CH-CH_3 \\ & \stackrel{E}{=} & \stackrel{C}{\to} \\ & \stackrel{E}{\to} & H \end{array}$$

(ii)H2O2/OH

Number of optically active molecule(s) formed after complete reductive ozonolysis of above compound is

Ans. (0)

$$\begin{array}{cccc} & CH_3 & & CH_3 \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\$$

Sol.

$$\begin{array}{c} & & & & \\ & & & & \\ & & & & \\ & & & \\ \text{C-CH=O} + \text{O=CH-C-CH=O+} \\ & & & \\ & & \text{O.I.} & & \\ & & & \\ & & & \\ & & & \\ & & & \\ \end{array}$$

$$\begin{array}{c} CH_3\\ \hline \\ \hline \\ O=CH-C-CH=O+O=CH-CH_3\\ \hline \\ O.I.\\ \end{array}$$
 O.I.

reductive ozonolysis

**18.** Consider the following oxides

V<sub>2</sub>O<sub>5</sub>, Cr<sub>2</sub>O<sub>3</sub>, Mn<sub>2</sub>O<sub>7</sub>, V<sub>2</sub>O<sub>3</sub>

Number of oxides which are acidic is x.

Consider the following complex compound

 $[Co(NH_2CH_2CH_2NH_2)_3]_2 (SO_4)_3$ 

The primary valency of complex is y

The value of x + y is

Ans. (4)

**Sol.** Acidic oxide  $\Rightarrow$  Mn<sub>2</sub>O<sub>7</sub>

Amphoteric oxide  $\Rightarrow$  V<sub>2</sub>O<sub>5</sub>, Cr<sub>2</sub>O<sub>3</sub>

Basic oxide  $\Rightarrow$  V<sub>2</sub>O<sub>3</sub>

x = no. of acidic oxide = 1

 $[\text{Co}(\text{NH}_2\text{CH}_2\text{CH}_2\text{NH}_2)_3]_2 (\text{SO}_4)_3 \Rightarrow \text{CO}^{3+}$ 

Primary valency = oxidation state of CMA = 3

y = 3

x + y = 1 + 3 = 4

**19.** Total number of ions having 3d<sup>5</sup> electronic configuration.

Ans. (3)

**Sol.**  $Mn^{+2}$ ,  $Fe^{+3}$ ,  $Cr^{+}$ 

**20.** Consider the reaction given below:

$$C_4H_{10} + \frac{13}{2}O_2 \rightarrow 4CO_2 + 5H_2O.$$

If 174 g of butane reacts with 320 g of O<sub>2</sub>. Find the volume of H<sub>2</sub>O formed in ml. (given density of H<sub>2</sub>O is 1 g/ml)

Ans. (138)

**Sol.** 
$$C_4H_{10_{(g)}} + \frac{13}{2}O_{2_{(g)}} \longrightarrow 4CO_{2_{(g)}} + 5H_2O(\ell)$$

$$n_{C_4H_{10}} = \frac{174}{58} = 3$$
  $n_{O_2} = \frac{320}{32} = 10$ 

Stoichiometric ratio

$$\frac{3}{1}$$
  $\frac{10}{\frac{13}{2}} = \frac{20}{13} = 1.53$ 

L.R.

$$\frac{13}{2}$$
 mol O<sub>2</sub> produces moles of H<sub>2</sub>O = 5

1 mol O<sub>2</sub> produces moles of H<sub>2</sub>O = 
$$\frac{5}{13/2} = \frac{10}{13}$$

10 mol O<sub>2</sub> produces moles of H<sub>2</sub>O = 
$$\frac{10}{13} \times 10 = \frac{100}{13}$$

Mass of H<sub>2</sub>O produced = 
$$\frac{100}{13} \times 18 = 138.46 \approx 138$$

Volume of 
$$H_2O$$
 produced =  $138 \times 1$  ml

$$= 138 \, ml$$

# **MATHEMATICS**

1. If x|x-3|+3|x-2|+1=0, then the number of real solution is

**(1)** Ans.

Sol. 
$$x|x-3|+3|x-2|+1=0$$

In 
$$(-\infty, 2)$$

$$-x^2 + 3x - 3(x-2) + 1 = 0$$

$$-x^2 + 7 = 0$$

$$x = \pm \sqrt{7}$$
 but  $x = -\sqrt{7}$  only

$$-x(x-3)+3(x-2)+1=0$$

$$-x^2 + 3x + 3x - 5 = 0$$

$$x^2 - 6x + 5 = 0$$

$$x = 1,5$$

Hence no solution in [2, 3]

In 
$$(3, \infty)$$

$$x(x-3)+3(x-2)+1=0$$

$$x^2 - 5 = 0$$

$$x = \pm \sqrt{5}$$

Hence no solution in  $(3, \infty)$ 

 $Re\left(\frac{2z+i}{z+i}\right) + Re\left(\frac{2\bar{z}-i}{\bar{z}-i}\right) = 2$  is a circle of radius r and centre (a, b), then  $\frac{15ab}{r^2}$  is equal to 2.

Ans.

Sol. Since 
$$Re(z) + Re(\overline{z}) = 2Re(z)$$

hence 
$$2\operatorname{Re}\left(\frac{2z+i}{z+i}\right) = 2$$

$$\frac{2x^2 + (2y+1)(y+1)}{x^2 + (y+1)^2} = 1$$

$$2x^{2} + 2y^{2} + 3y + 1 = x^{2} + 2y + 1 + y^{2}$$
  
 $x^{2} + y^{2} + y = 0$ 

$$x^2 + y^2 + y = 0$$

centre 
$$\left(0, -\frac{1}{2}\right)$$

$$\frac{2z+i}{z+i}$$

$$= \frac{2x+i(2y+1)}{x+i(y+1)}$$

$$= \frac{2x^2+(2y+1)(y+1)}{x^2+(y+1)^2}$$

$$\frac{15ab}{r^2} = 0$$

3. Let  $\vec{a}$  and  $\vec{b}$  be the vectors of the same magnitude such that  $\frac{|\vec{a}+\vec{b}|+|\vec{a}-\vec{b}|}{|\vec{a}+\vec{b}|-|\vec{a}-\vec{b}|} = \sqrt{2}+1$ ,

then the value of  $\frac{\left|\vec{a} + \vec{b}\right|^2}{\left|\vec{a}\right|^2}$  is equal to

**Ans.** 
$$2 + \sqrt{2}$$

**Sol.** 
$$\frac{\left|\vec{a} + \vec{b}\right| + \left|\vec{a} - \vec{b}\right|}{\left|\vec{a} + \vec{b}\right| - \left|\vec{a} - \vec{b}\right|} = \frac{\sqrt{2} + 1}{1}$$

Using componendo and dividendo

$$\Rightarrow \frac{2\left|\vec{a} + \vec{b}\right|}{2\left|\vec{a} - \vec{b}\right|} = \frac{\sqrt{2} + 2}{\sqrt{2}}$$

$$\Rightarrow \left| \vec{a} + \vec{b} \right| = \left| 1 + \sqrt{2} \right| \left| \vec{a} - \vec{b} \right|$$

$$\Rightarrow \frac{\left|\vec{a} + \vec{b}\right|^2}{\left|\vec{a} - \vec{b}\right|^2} = \left(1 + \sqrt{2}\right)^2$$

$$=3+2\sqrt{2}$$

$$\Rightarrow \frac{a^2 + a^2 + 2a^2 \cos \theta}{a^2 + a^2 - 2a^2 \cos \theta} = 3 + 2\sqrt{2}$$

$$\Rightarrow \frac{4}{4\cos\theta} = \frac{4+2\sqrt{2}}{2+2\sqrt{2}}$$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\left|\vec{a} + \vec{b}\right|^2}{\left|\vec{a}\right|^2} = \frac{a^2 + a^2 + 2a^2 \frac{1}{\sqrt{2}}}{a^2}$$
$$= 2 + \sqrt{2}$$

**4.** Let 
$$f(x) = \frac{5-x}{x^2-3x+2}$$
, if range of  $f(x)$  is  $(-\infty, \alpha] \cup [\beta, \infty)$ . Then  $\alpha^2 + \beta^2$  equals to

Ans. (194)

Sol. 
$$y = \frac{5-x}{x^2 - 3x + 2}$$
  
 $\Rightarrow x^2y - 3xy + 2y - 5 + x = 0$   
 $\Rightarrow x^2y + x(1 - 3y) + 2y - 5 = 0$   
solution must satisfy  $y \neq 0$  and  $D \ge 0$   
 $D \ge 0 \Rightarrow (1 - 3y)^2 - 4y(2y - 5) \ge 0$   
 $\Rightarrow 1 + 9y^2 - 6y - 8y^2 + 20y \ge 0$   
 $\Rightarrow y^2 + 14y + 1 \ge 0$   
solution for  $y^2 + 14y + 1 = 0$  are  $\frac{-14 \pm \sqrt{192}}{2} = \frac{-14 \pm 8\sqrt{3}}{2}$   
 $= -7 \pm 4\sqrt{3}$ 

So, 
$$y \in \left(-\infty - 7 - 4\sqrt{3}\right] \cup \left[-7 + 4\sqrt{3}, \infty\right)$$

So, 
$$\alpha^2 + \beta^2 = (-7 - 4\sqrt{3})^2 + (-7 + 4\sqrt{3})^2 = 194$$
.

5. Let 
$$f(x)$$
 be a polynomial of degree 4 such that it has  $x = 4.5$  as extremas. If  $\lim_{x \to 0} \frac{f(x)}{x^2} = 5$ , then  $f(2) = 6$ 

Ans.

Sol. 
$$\lim_{x \to 0} \frac{f(x)}{x^2} = 5$$

$$\lim_{x \to 0} \frac{f(x)}{x^2} = \lim_{x \to 0} \frac{ax^4 + bx^3 + cx^2 + dx + e}{x^2}$$

$$\Rightarrow c = 5 \text{ and } d = e = 0$$

$$f(x) = ax^4 + bx^3 + 5x^2$$
  
$$f'(x) = 4ax^3 + 3bx^2 + 10x = 0$$

$$f'(x) = 4ax^3 + 3bx^2 + 10x = 0$$
  

$$\Rightarrow x(4ax^2 + 3bx + 10) = 0$$

$$\Rightarrow x \left( \frac{x^2}{2} - \frac{9x}{2} + 10 \right) = 0$$

$$\Rightarrow a = \frac{1}{8} \text{ and } b = \frac{-3}{2}$$

So, 
$$f(2) = \frac{1}{8}(2)^4 - \frac{3}{2}(2)^3 + 5(2)^2$$

$$=2-12+20=10$$

6. Let p denote no. of  $\Delta's$  possible from the vertices of n sided regular polygon & q denote no. of quadrilateral possible from the same. If p+q=126, then eccentricity of  $\frac{x^2}{16} + \frac{y^2}{n} = 1$  is

Ans. 
$$\frac{1}{\sqrt{2}}$$

$${}^{n}C_{3} + {}^{n}C_{4} = 126$$

$$\Rightarrow {}^{n+1}C_{4} = 126 \Rightarrow {}^{9}C_{4} \Rightarrow n+1=9 \Rightarrow n=8$$
So, the ellipse is  $\frac{x^{2}}{16} + \frac{y^{2}}{8} = 1$ 

$$e = \sqrt{1 - \frac{8}{16}} = \frac{1}{\sqrt{2}}$$

- 7. A box contains 19 unbiased coins and 1 biased coin with both faces head. If a coin is randomly chosen out of this box and tossed, if the head appears, then the probability that the unbiased coin was selected
  - $(1)^{\frac{1}{6}}$
- $(2) \frac{1}{3}$
- $(3)\frac{1}{5}$
- $(4)\frac{19}{21}$

Ans. (4)

Sol. Required probability = 
$$\frac{\frac{19}{20} \times \frac{1}{2}}{\frac{19}{20} \times \frac{1}{2} + \frac{1}{20} \times 1} = \frac{19}{21}$$

8. If  $f(\theta) = \frac{\tan(\tan \theta) - \tan(\sin \theta)}{\tan \theta - \sin \theta}$  is continuous at  $\theta = 0$ , then the value of  $f(\theta)$  at  $\theta = 0$  is equal to

Ans. (1

Sol. 
$$f(0) = \lim_{\theta \to 0} f(\theta) = \lim_{\theta \to 0} \frac{\tan(\tan \theta) - \tan(\sin \theta)}{\tan \theta - \sin \theta}$$
$$= \lim_{\theta \to 0} \frac{\sin(\tan \theta - \sin \theta)}{\cos(\tan \theta) \cdot \cos(\sin \theta)} = 1$$

9. Let a random variable X take values 0, 1, 2, 3 with P(X = 0) = P(X = 1) = p and P(X = 2) = P(X = 3) and  $E(X^2) = 2E(X)$ , then the value of 8p - 1 is equal to

Ans. (2

So, 8p-1=2

Sol. 
$$2p + 2q = 1$$
  
 $p + q = \frac{1}{2}$   
 $E(x^2) = \sum_{i=0}^{3} x_i^2 P(x_i) = 0 \cdot p + 1 \cdot p + 4 \cdot q + 9 \cdot q = p + 13q$   
 $E(x) = \sum_{i=0}^{3} x_i P(x_i) = 0 \cdot p + 1 \cdot p + 2q + 3q = p + 5q$   
 $\Rightarrow p + 13q = 2(p + 5q)$   
 $\Rightarrow p = 3q$   
So,  $q = \frac{1}{8} & p = \frac{3}{8}$ 

10. Let 
$$a_n$$
 be the  $n^{th}$  term of an A. P. If  $S_n=a_1+a_2+a_3+\cdots+a_n=700$ ,  $a_6=7$  and  $S_7=7$ , then  $a_n$  is equal to

Ans. (64

**Sol.** 
$$S_n = 700 \Rightarrow \frac{n}{2} [2a + (n-1)d] = 700$$
 ...(i)  $a_6 = 7 \Rightarrow a + 5d = 7$  ...(ii)  $S_7 = 7 \Rightarrow \frac{7}{2} [2a + 6d] = 7$ 

$$\Rightarrow 2a + 6d = 2 \qquad \dots(iii)$$

Solving (ii) & (iii) 
$$a = -8, d = 3$$

$$\frac{n}{2} \left[ -16 + 3n - 3 \right] = 700$$

$$\Rightarrow 3n^2 - 19n - 1400 = 0$$
$$\Rightarrow (3n + 56)(n - 25) = 0$$

$$\therefore n = 25$$
$$\therefore a_{25} = a + 24d$$

$$=-8+24\times3$$

$$=-8+72$$

11. Let 
$$y = y(x)$$
 and  $(1 + x^2)y' - 2xy = (x^4 + 2x^2 + 1)\cos x$ . If  $y(0) = 1$ , then  $\int_{-3}^{3} y(x) dx$  is equal to (1) 10 (2) 15 (3) 20 (4) 24

Ans. (4)

Sol. 
$$(1+x^2)y'-2xy = (x^2+1)^2 \cos x$$
  
 $y'-\frac{2x}{1+x^2}y = (1+x^2)\cos x$ 

$$P = \frac{-2x}{1+x^2}, Q = (1+x^2)\cos x$$

$$1 + x^{2} = 0$$

$$I.F. = e^{-\int \frac{2x}{1+x^{2}} dx} = e^{-\ln(1+x^{2})} = \frac{1}{1+x^{2}}$$

$$y \times \frac{1}{1+x^2} = \int \cos x dx$$

$$\frac{y}{1+x^2} = \sin x + c \Longrightarrow y = (1+x^2)(\sin x + c)$$

$$c=1$$
$$y = (1+x^2)(\sin x + 1)$$

$$I = \int_{-3}^{3} \left\{ \left(1 + x^{2}\right) \sin x + \left(1 + x^{2}\right) \right\} dx$$

$$=2\int_{0}^{3} (1+x^{2}) dx = 2\left(\frac{x^{3}}{3} + x\right)\Big|_{0}^{3}$$

$$=2(9+3)=24$$

12. The sum of series 
$${}^2C_1 \cdot (1 \times 2) + {}^3C_2 \cdot (2 \times 3) + {}^4C_3 \cdot (3 \times 4) + \dots + {}^{19}C_{18} \cdot (18 \times 19)$$
 is  $S$ , then  $\frac{S}{295}$  is equal to

(4) 114

Ans. (4

Sol. 
$$S = 2 \cdot (1 \times 2) + 3(2 \times 3) + 4(3 \times 9) + \dots + 19 (18 \times 19)$$
  
 $S = \sum_{r=1}^{18} r(r+1)^2 \Rightarrow S = \sum_{r=1}^{18} r(r+1)(r+2-1)$   
 $\Rightarrow S = \sum_{r=1}^{18} r(r+1)(r+2) - \sum_{r=1}^{18} r(r+1)$   
 $\Rightarrow S = \frac{1}{4} \sum_{r=1}^{18} \left[ r(r+1)(r+2)(r+3-(r-1)) \right] - \frac{1}{3} \sum_{r=1}^{18} r(r+1)(r+2-(r-1))$   
 $= \frac{1}{4} \times 18 \times 19 \times 20 \times 21 - \frac{1}{3} \times 18 \times 19 \times 20$   
 $S = 18 \times 19 \times 20 \left[ \frac{21}{4} - \frac{1}{3} \right]$   
 $S = 18 \times 19 \times 20 \times \frac{59}{12} = 6 \times 19 \times 295$   
 $\frac{S}{295} = 6 \times 19 = 114$ 

13. Let the length of a latus rectum of an ellipse 
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
 be 10. If its eccentricity is the minimum value of the function  $f(t) = t^2 + t + \frac{11}{12}$ ,  $t \in \mathbb{R}$ , then  $a^2 + b^2$  is equal to

Ans. (126

Sol. 
$$\frac{2b^2}{a} = 10 \Rightarrow b^2 = 5a$$

$$e = f(t)\Big|_{\min} = t^2 + t + \frac{11}{12}\Big|_{t = \frac{-1}{2}} = \frac{1}{4} - \frac{1}{2} + \frac{11}{12}$$

$$= \frac{3 - 6 + 11}{12} = \frac{8}{12}$$

$$\Rightarrow \sqrt{1 - \frac{b^2}{a^2}} = \frac{2}{3}$$

$$\Rightarrow 1 - \frac{b^2}{a^2} = \frac{4}{9} \Rightarrow \frac{b^2}{a^2} = \frac{5}{9} \Rightarrow b^2 = \frac{5a^2}{9}$$

$$\Rightarrow 5a = \frac{5a^2}{9} \Rightarrow a = 9 \text{ ; So, } b^2 = 45$$

Hence,  $a^2 + b^2 = 81 + 45 = 126$ .

14. Let 
$$\alpha$$
 and  $\beta$  be roots of the equation  $\left[ (t+2)^{\frac{1}{7}} - 1 \right] x^2 + \left[ (t+2)^{\frac{1}{6}} - 1 \right] x + \left( (t+2)^{\frac{1}{21}} - 1 \right) = 0$ .

If  $\lim_{t\to -1} \alpha = a$  and  $\lim_{t\to -1} \beta = b$ , then  $72(a+b)^2$  is equal to

(1)36

(4)98

Ans.

Sol. 
$$a+b = \lim_{x \to -1} -\frac{\left[\left(t+2\right)^{\frac{1}{6}}-1\right]}{\left[\left(t+2\right)^{\frac{1}{7}}-1\right]}$$

$$= \lim_{h \to 0} - \frac{\left[ (1+h)^{\frac{1}{6}} - 1 \right]}{\left[ (1+h)^{\frac{1}{7}} - 1 \right]}$$

$$=\lim_{h\to 0} -\frac{\left[\frac{h}{6}\right]}{\frac{h}{2}} = \frac{-7}{6}$$

So, 
$$72(a+b)^2 = 72 \times \frac{49}{36} = 98$$

If the equation of the line passing through the point  $\left(0, -\frac{1}{2}, 0\right)$  and Perpendicular to the lines 15.

$$\vec{r} = \lambda(\hat{\imath} + a\hat{\jmath} + b\hat{k})$$
 and  $\vec{r} = (\hat{\imath} - \hat{\jmath} - 6\hat{k}) + \mu(-b\hat{\imath} + a\hat{\jmath} + 5\hat{k})$  is  $\frac{x-1}{-a} = \frac{y+4}{d} = \frac{z-c}{-4}$ ,

then a + b + c + d is equal to

(4)Ans.

Sol. 
$$\vec{\ell} = (1, a, b) \times (-b, a, 5)$$
  
=  $(5a - ab, -b^2 - 5, a + ab)$ 

$$\Rightarrow \frac{5a-ab}{a} = \frac{-b^2-5}{d} = \frac{a+ab}{4}$$

$$\Rightarrow \frac{b-5}{1} = \frac{-b^2-5}{d} = \frac{a+ab}{-4}$$

So, 
$$20 - 4b = a + ab \Rightarrow 20 = a + ab + 4b$$

$$\Rightarrow \frac{0-1}{-a} = \frac{-\frac{1}{2}+4}{d} = \frac{0-c}{-4} \Rightarrow \frac{1}{a} = \frac{7}{2d} = \frac{c}{4}$$

...(iii)

...(i)

So, 
$$d = \frac{7}{2}a$$

So, 
$$d = \frac{7}{2}a$$
  $b-5 = \frac{-b^2 - 5}{7a} \Rightarrow 7ab = 35a - 2b^2 - 10$ 

$$\Rightarrow a = \frac{20 - 4b}{1 + b}$$

$$\Rightarrow b-5 = \frac{-b^2 - 5}{\frac{7a}{2}} \Rightarrow b-5 = \frac{-2 b^2 + 5}{7a}$$

$$\Rightarrow 7a = -2\frac{(b^2 + 5)}{b - 5} = \frac{2b^2 + 10}{5 - b}$$

On solving b = 3, a = 2, d = 7, c = 2

$$\Rightarrow$$
 a + b + c + d = 14

**16.** Let 
$$A = \{(\alpha, \beta) \in \mathbb{R} \times \mathbb{R} : |\alpha - 1| \le 4 \text{ and } |\beta - 5| \le 6\}$$
 and

$$B = \{(\alpha, \beta) \in \mathbb{R} \times \mathbb{R} : 16(\alpha - 2)^2 + 9(\beta - 6)^2 \le 144 \}, \text{ then }$$

(1) Neither 
$$A \subset B$$
 nor  $B \subset A$ 

$$(2) A \subset B$$

$$(3) A \cup B = \{(x, y) : -4 \le x \le 4, -1 \le y \le 1\}$$

$$(4) B \subset A$$

Ans. (2)

**Sol.** For  $\alpha$ 

$$-4 \le \alpha - 1 \le 4$$

$$-3 \le \alpha \le 5$$

For B

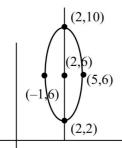
$$|\beta - 5| \le 6$$

$$\Rightarrow -6 \le \beta - 5 \le 6$$

$$-1 \le \beta \le 11$$

$$16(\alpha-2)^2+9(\beta-6)^2 \le 144$$

$$\Rightarrow \frac{\left(\alpha - 2\right)^2}{9} + \frac{\left(\beta - 6\right)^2}{16} \le 1$$

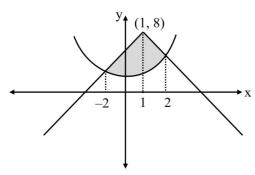


We can understand from the figure that  $B \subset A$ 

- 17. If the area of the region  $\{(x,y): 1+x^2 \le y \le \min\{x+7,11-3x\}\}$  is A, then 3A is equal to
  - (1)46
- (2)47
- (3)49
- (4)50

Ans. (4)

Sol.



$$x + 7 = 11 - 3x$$

$$\Rightarrow$$
 x = 1

and 
$$1 + x^2 = x + 7$$

$$\Rightarrow$$
  $x^2 - x - 6 = 0$ 

$$\Rightarrow$$
 (x + 5) (x - 2) = 0

$$x = -5, 2$$

Area = 
$$\int_{-2}^{1} (7 - x^2 + x) dx + \int_{1}^{2} (11 - 3x - x^2 - 1) dx = \frac{50}{3}$$

18. The number of solutions of the equation  $\cos 2\theta \cos \frac{\theta}{2} + \cos \frac{5\theta}{2} = 2 \cos^3 \frac{5\theta}{2}$  in  $\left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]$  is

Ans. (7

**Sol.** 
$$\cos 2\theta \cos \left(\frac{\theta}{2}\right) + \cos \frac{5\theta}{2} = 2\cos^3 = \frac{5\theta}{2}$$

$$\Rightarrow \frac{1}{2} \left( 2\cos 2\theta \cos \frac{\theta}{2} \right) + \cos \frac{5\theta}{2} = \frac{1}{2} \left( \cos \frac{15\theta}{2} + 3\cos \frac{5\theta}{2} \right)$$

On solving,

$$\Rightarrow \cos \frac{3\theta}{2} = \cos \frac{15\theta}{2}$$

$$\Rightarrow \frac{3\theta}{2} = 2n\pi \pm \frac{15\theta}{2}$$

$$\Rightarrow \cos\frac{15\theta}{2} - \cos\frac{3\theta}{2} = 0$$

$$\Rightarrow 2\sin 3\theta \cdot \sin \frac{9\theta}{2} = 0$$

$$3\theta = n\pi$$
 and  $\frac{9\theta}{2} = m\pi$ 

$$\Rightarrow \theta = \frac{n\pi}{3} \quad \text{and} \quad \theta = \frac{2m\pi}{9}$$

$$\Rightarrow -\frac{\pi}{3}, \frac{\pi}{3}, 0$$
 and  $-\frac{4\pi}{9}, -\frac{2\pi}{9}, \frac{4\pi}{9}, \frac{2\pi}{9}$