Integers

Introduction to Integers and Their Absolute Value

Natural numbers

The counting numbers 1, 2, 3, ... are called natural numbers.

The set of natural number is denoted by the letter N.

$\therefore$ N = {1, 2, 3, ...}

1 is the smallest natural number. The set of natural numbers, N is an infinite set.

Whole numbers

The numbers 0, 1, 2, 3, ... are called whole numbers.

The set of whole numbers is denoted by the letter W.

$\therefore$ W = {0, 1, 2, 3, ...}

0 is the smallest whole number. The set of whole numbers, W is an infinite set.

Integers

We had observed that adding any two whole numbers always gives a whole number. We can examine whether this case is true for the operation ‘subtraction’. Let us consider the following examples:

$13 - 12 = 1$

$13 - 13 = 0$

$12 - 13 = ?$

We can observe that in the last case, the operation ‘subtraction’ cannot be performed in the system of whole numbers i.e., when a bigger whole number is subtracted from a smaller whole number. In order to solve such type of problems, the system of whole numbers has to be enlarged by introducing another kind of numbers called **negative integers**. These numbers are obtained by putting “−” sign before the counting numbers 1, 2, 3, … That is, negative integers are −1, −2, −3 …

The most common real life example of negative integers is the temperature of our surroundings. In winters, sometimes the temperature drops down to a negative value say −1, −3. So, in such cases negative integers are highly used.

All positive and all negative numbers including zero are called **integers** (or **directed numbers** or **signed numbers**). That is, the numbers …−3, −2, −1, 0, 1, 2, 3… are called integers. The collection or set of all integers is an infinite set and usually it is denoted by **I** or **Z.**

Convention: If there is no sign in front of a number, then we treat it as a positive number.

However, the number '0' is taken as neutral i.e., 0 is always written without any sign.

I or **Z** = {… −3, −2, −1, 0, 1, 2, 3, …}

Absolute value of an integer

The **absolute value** of an integer is its numerical value regardless of its sign. The absolute value of an integer n is denoted as $|n|$.

Therefore, |−10| = 10, |−2| = 2, |0| = 0, |7| = 7 etc.

Note: The absolute value of any integer is always non-negative.

Opposite of an integer

Numbers which are represented by points such that they are at equal distances from the origin but on the opposite sides of it are called **opposite numbers**.

Thus, the opposite of an integer is the integer with its sign reversed. The opposite of integer a is $-a$ and the opposite of integer $-b$ is $+b$ or b as a and $-a$; $-b$ and $+b$ are at equal distance from the origin but on the opposite sides.

Thus, opposite of 5 is −5, opposite of −8 is 8.

Let us discuss some examples based on these concepts.

Example 1:

Write the absolute value of 4, −19, 23 and −1.

Solution:

The absolute value of 4 = |4| = 4.

The absolute value of −19 = |−19| = 19.

The absolute value of 23 = |23| = 23.

The absolute value of −1 = |−1| = 1.

Example 2:

The absolute value of two integers are 11 and 0. What could be the possible value(s) of the those integers?

Solution:

If the absolute value of an integer is 11, then the possible values of that integer could be ±11 i.e., 11 or −11.

If the absolute value of an integer is 0, then the possible value of that integer could be 0.

Example 3:

What are the opposite of integers 51, −927 and −7?

Solution:

The opposite of 51 is −51.

The opposite of −927 is 927.

The opposite of −7 is 7.

Consider the following example.

Suppose you have five bags that contain 6 balls each. **Can you tell the total number of balls in all the bags?**

To calculate this, we have to add 6 five times, i.e. $\underbrace{6+6+6+6+6}_{5 \text{ times}}$.

Now, we know that multiplication of whole numbers is repeated addition. Therefore, instead of adding 6 five times, we can simply multiply 6 by 5, i.e., 5 × 6 = 30

Hence, the bags contain a total number of 30 balls.

Just like whole numbers, the multiplication of integers is also their repeated addition.

For example:

$$4\times(-7)=\underbrace{(-7)+(-7)+(-7)+(-7)}_{4\text{ times}}=-28$$

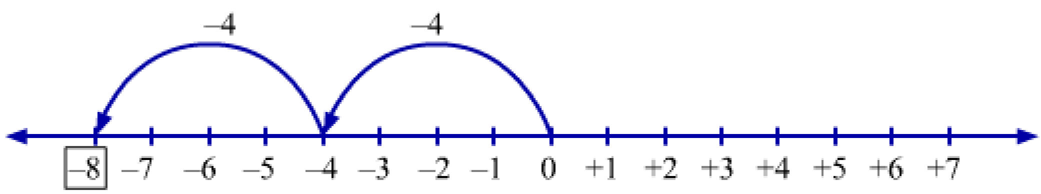
Multiplication of two numbers can also be performed on the number line.
Let us multiply (–4) with 2 on number line.

We know that multiplication is a repeated addition.

∴ (–4) × 2 = (–4) + (–4)

Now, in order to multiply (–4) with 2, jump 4 steps to the left at a time. This is continuing for 2 times.

This can be done as,



Here, the tip of the finale arrow is at –8.

∴ (–4) × 2 = (–4) + (–4) = –8

Now, let us discuss some more examples based on the above concept.

Example 1:

Find the values of the following expressions.

(i) 5 × 18

(ii) 6 × (−9)

(iii) (−42) × 12

(iv) (−25) × (−8)

(v) 6 × (−8) × 3

(vi) 2956 × 1902 × 0

(vii) (−7) × (−3) × 2 × (−5) × (−10)

(viii) (−2) × 5 × (−2) × (−20) × (−9) × (−8)

Solution:

(i) 5 × 18 = 90

(ii) 6 × (−9) = − (6 × 9) = −54 (One positive and one negative integer)

(iii) (−42) × 12 = − (42 × 12) = −504 (One positive and one negative integer)

(iv) (−25) × (−8) = (25 × 8) = 200 (Two negative integers)

(v) 6 × (−8) × 3 = − (6 × 8 × 3)

= −144 (One negative integer)

(vi) 2956 × 1902 × 0 **=** 0 (Product of any integer with zero is zero)

(vii) (−7) × (−3) × 2 × (−5) × (−10) = 7 × 3 × 2 × 5 × 10

= 2100 (Even number of negative integers)

(viii) (−2) × 5 × (−2) × (−20) × (−9) × (−8) = − (2 × 5 × 2 × 20 × 9 × 8)

= − 28800 (Odd number of negative integers)

Example 2:

Examine whether the following statements are correct or incorrect. Give reasons.

1. When −5 is multiplied *n* number of times, where *n* is even, then the sign of the product is negative.
2. The sign of the product is negative if we multiply 11 negative and 5 positive integers.
3. The product of 295 and 0 is 295.

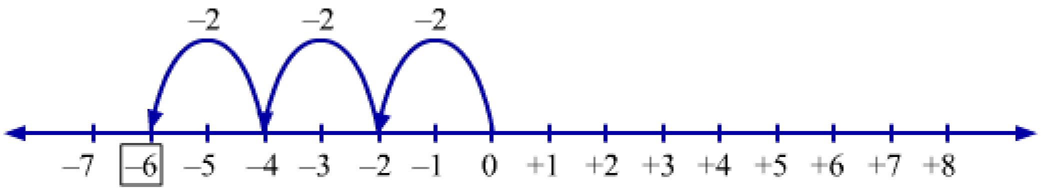
Solution:

1. False. Since *n* is even, the sign of the product should be positive.
2. True. Since we are multiplying an odd number of negative integers, the sign of the

product will be negative.
3. False. The product of any integer and zero is zero.

Example 3:
Multiply –2 with 3 on number line.

Solution:
–2 × 3 = (–2) + (–2) + (–2)
Now, in order to multiply –2 with 3, jump 2 steps to the left at a time. This is continuing for 3 times.
This can be done as,



Here, the tip of the finale arrow is at –6.
So, –2 × 3 = (–2) + (–2) + (–2) = –6

Commutative, Associative and Distributive Properties of Integers over Multiplication

Consider the integers −12 and 7. **What is the value of (−12) × 7?**

Yes, (− 12) × 7 = − 84

Now, what will you get if you interchange the positions of −12 and 7? Will the product remain the same? Let us see.

We have 7 × (−12) = −84

∴ (−12) × 7 = 7 × (−12)

Thus, we see that even when we change the positions of the integers, it makes no difference to their product. This property of integers is known as the **commutative property.** This is true for all integers. Hence, all integers are commutative under multiplication.

Thus, according to the commutative property of integers under multiplication:

If *x* and *y* are any two integers,
then *x* × *y* = *y* × *x*.

Now, consider the integers −3, −5, and −6.

Let us find **the value of the expression [(−3) × (−5)] × (−6) and (−3) × [(−5) × (−6)].**

We have, [(−3) × (−5)] × (−6) = 15 × (−6) = −90

Also, (−3) × [(−5) × (−6)] = −3 × 30 = −90

∴ [(−3) × (−5)] × (−6) = (−3) × [(−5) × (−6)]

Thus, even when we group the three integers differently, their product remains the same. This property is known as the **associative property** and it is true for all integers. Hence, all integers are associative under multiplication.

Thus, according to the associative property of integers under multiplication:

If *a*, *b,* and *c* are any three integers, then (*a* × *b*) × *c* = *a* × (*b* × *c*).

Now, let us check whether the integers −7, −9, and −16 are distributive under multiplication.

We have, (−7) × [(−9) + (−16)] = (−7) × (−25) = 175

Also, (−7) × (−9) + (−7) × (−16) = 63 + 112 = 175

∴ (−7) × [(−9) + (−16)] = (−7) × (−9) + (−7) × (−16)

This verifies the **distributive property** of multiplication for integers −7, −9, and −16. In fact, all integers are distributive under multiplication.

Thus, according to the distributive property of integers under multiplication:

If *x*, *y,* and *z* are any three integers, then *x* × (*y* + *z*) = (*x* × *y*) + (*x* × *z*).

Can we also say that the distributive property of integers over subtraction under multiplication i.e., *x* × (*y* − *z*) = (*x* × *y*) − (*x* × *z*) is also true?

Yes. This is also true for any integer *x*, *y,* and *z*. Let us now verify this with an example.

Consider the integers 5, 8, and 7.

Now, $5 \times (8 - 7) = 5 \times 1 = 5$ and $(5 \times 8) - (5 \times 7) = 40 - 35 = 5$

Thus, $5 \times (8 - 7) = (5 \times 8) - (5 \times 7)$

Thus, the distributive property of multiplication of integers over subtraction is also true.

Thus, according to the distributive property of integers over subtraction under multiplication:

If x, y, and z are any three integers, then $x \times (y - z) = (x \times y) - (x \times z)$.

Cancellation law:

If x, y, and z are any three integers such that $x \neq 0$ and $xy = xz$, then $y = z$.

Let us now look at some examples.

Example 1:

Verify the associative and distributive property for the integers 5, 19, and − 27.

Solution:

We have, $(5 \times 19) \times (-27) = 95 \times (-27) = -2565$

And, $5 \times [19 \times (-27)] = 5 \times (-513) = -2565$

$\therefore (5 \times 19) \times (-27) = 5 \times [19 \times (-27)]$

Thus, the associative property for the given integers is verified.

Next, we have to verify that $5 \times [19 + (-27)] = 5 \times 19 + 5 \times (-27)$

Now, $5 \times [19 + (-27)] = 5 \times (-8) = -40$

And, $5 \times 19 + 5 \times (-27) = 95 - 135 = -40$

$\therefore 5 \times [19 + (-27)] = 5 \times 19 + 5 \times (-27)$

Thus, the distributive property for the given integers is verified.

Example 2:

Fill in the blanks using any of the commutative and associative property of integers under multiplication.

1. **5 × __ = − 2 × __**
2. **− 20 × __ = 8 × __**
3. **(*p*×__) ×__ = __× (*r*×*s*)**
4. **(20 × − 7) × __ = __ × (__× − 9)**

Solution:

1. The commutative property holds for integers under multiplication.

   Hence, 5 × − 2 = − 2 × 5
2. The commutative property holds for integers under multiplication.

   Hence, − 20 × 8 = 8 × − 20
3. The associative property holds for integers under multiplication.

   Hence, $(p \times r) \times s = p \times (r \times s)$
4. The associative property holds for integers under multiplication.

   Hence, (20 × − 7) × − 9 = 20 × (− 7 × − 9)

Multiplicative and Additive Identities for Integers

Consider the integers 5, 7, −8, and −9. **What do we get when we multiply each of these integers with 1?**

5 × 1 = 5

7 × 1 = 7

−8 × 1 = −8

$-9 \times 1 = -9$

What do you observe?

Yes, you are right. The multiplication of an integer with 1 gives the same number again.

This is true for all integers. Thus, we can conclude that the multiplication of any integer with 1 gives the same integer again. Therefore, **1 is called the multiplicative identity of all integers.**

Thus, according to the multiplicative identity of integers:

For any integer x, $x \times 1 = x$

What happens when we add each of these integers to 0? Let us see.

$5 + 0 = 5$

$7 + 0 = 7$

$-8 + 0 = -8$

$-9 + 0 = -9$

What do you observe?

We observe here that the result of the sum of the integers with 0 is the same integer. This property of integers is known as **additive identity**. Just like multiplicative identity, ***additive identity*** **is also valid for all integers. Thus, 0 is called the additive identity of all integers.**

Thus, according to the additive identity of integers:

For any integer x, $x + 0 = x$

The additive inverse of an integer x is an integer which, when added to x, gives the sum as 0. In general, the additive inverse of an integer x is $-x$

$[x + (-x)] = 0$.

The additive inverse of an integer can be found by multiplying the integer by −1.

Let us find the additive inverses of the integers 5, 7, −8, and −9.

For this, we need to multiply each of the integers by −1.

Integer	**Additive inverse**
5	$5 \times (-1) = -5$
7	$7 \times (-1) = -7$
−8	$-8 \times (-1) = 8$
−9	$-9 \times (-1) = 9$

Let us now look at a few examples.

Example 1:

Fill in the blanks in the following expressions:

(i) 1292 × 1 = __

(ii) 7 × (−5) × __ × (−2) = −70

Solution:

(i) $1292 \times 1 = 1292$

(ii) $7 \times (-5) \times (-1) \times (-2) = -70$

Example 2:

Determine the integer whose product with (−1) is 25.

Solution:

If we multiply any integer with (−1), then we get the additive inverse of that integer.

$\therefore 25 = (-1) \times (-25)$.

Hence, the required integer is −.

Division of Integers

Suppose Manmohan wants to distribute Rs 1000 equally among his 5 grandchildren. What amount will each child receive?

To answer this question, we are required to divide 1000 by 5. The value of the expression $1000 \div 5$ is found to be 200. Thus, Manmohan gives Rs 200 to each grandchild.

We were able to answer this question easily because it involved division of whole numbers.

Sometimes, we come across the situations when we need to perform division of integers.

To understand the rules followed for dividing integers, let us look at the following video.

Properties of Division

Property 1: If a and b are two integers, then $a \div b$ might not be an integer.
Example: Let $a = 27$ and $b = 4$, then $a \div b = 27 \div 4 = 274274$ which is not an integer.

Property 2: If a is an integer and $a \neq 0$, then $a \div a = 1$.
Example: Let $a = 7$. Then $a \div a = 7 \div 7 = 77=177=1$

Property 3: If a is an integer and $a \neq 0$, then $a \div 1 = a$
Example: Let $a = -3$. Then $a \div 1 = (-3) \div 1 = -31=-3\text{-}31=\text{-}3$

Property 4: If a is an integer and $a \neq 0$, then $0 \div a = 0$
Example: Let $a = -8$. Then $0 \div a = 0 \div (-8) = 0-8=00\text{-}8=0$

Property 5: If a is a non-zero integer, then $a \div 0$ is not defined.
Example: If $a = 25$, then $a \div 0 = 25 \div 0 =$ not defined

Property 6: If a, b and c are non-zero integers, then
$(a \div b) \div c \neq a \div (b \div c)$ except when $c = 1$
Note: when $c = 1$, $(a \div b) \div c = a \div (b \div c)$

Property 7: If a, b and c are integers, such that
(i) $a > b$ and c is positive, then $(a \div c) > (b \div c)$

Example: $a = 28$, $b = 18$ and $c = 2$ then $(a \div c) = 28 \div 2 = 14$
and $(b \div c) = 18 \div 2 = 9$. So, $(a \div c) > (b \div c)$.

(ii) $a > b$ and c is negative, then $(a \div c) < (b \div c)$

Example: a = 28, b = 18 and c = −2 then $(a \div c) = 28 \div -2 = -14$ and $(b \div c) = 18 \div -2 = -9$. So, $(a \div c) < (b \div c)$.

Let us now solve some examples based on these properties.

Example 1:

Find the values of the following expressions.

(i) 26 ÷ 13

(ii) (−44) ÷ 11

(iii) 36 ÷ (−2)

(iv) $\frac{-28}{-4}$

(v) $\frac{23+(-7)}{8}$

(vi) 48 ÷ [(−6) − (+2)]

Solution:

(i) 26 ÷ 13 = 2

(ii) (−44) ÷ 11 = −4

(iii) 36 ÷ (−2) = −18

(iv) $\frac{-28}{-4} = 7$

(v) $\frac{23+(-7)}{8} = \frac{16}{8} = 2$

(vi) 48 ÷ [(−6) − (+2)] = 48 ÷ (−8) = −6

Example 2:

Fill in the boxes to satisfy the following equations.

(i) 54 ÷ $\boxed{}$ = − 6

(ii) 1 ÷ (−1) = $\boxed{}$

(iii) $\dfrac{145}{\boxed{}}$ = −145

(iv) $\dfrac{\boxed{}}{-21} = 3$

(v) $\boxed{}$ ÷ 4 = 7

Solution:

(i) 54 ÷ $\boxed{-9}$ = −6

(ii) 1 ÷ (−1) = $\boxed{-1}$

(iii) $\dfrac{145}{\boxed{-1}}$ = −145

(iv) $\dfrac{\boxed{-63}}{-21}$ = 3

(v) $\boxed{28}$ ÷ 4 = 7

Example 3:

If the cost of 6 pens is Rs 84, then what is the cost of one pen?

Solution:

Cost of 6 pens = Rs 84

∴ Cost of 1 pen = Rs $\dfrac{84}{6}$

= Rs 14

Example 4:

Isha took a test. Each question in the test carried (+4) marks for the correct answer and (−2) marks for the wrong one. Isha answered 10 questions correctly, but scored 10 marks in total. Find the number of questions that Isha answered incorrectly, if she attempted all the questions.

Solution:

Marks for one correct answer = +4

Isha attempted 10 questions correctly.

Therefore, marks which she scored for the correct answers = 10 × 4 = 40

Total marks = 10

∴ Marks which she scored for the incorrect answers = 10 − 40 = −30

Marks for 1 incorrect answer = −2

Thus, number of questions attempted incorrectly by Isha = (− 30) ÷ (− 2) = 15

Order of Performing Operations on Whole Numbers

We know how to perform addition, subtraction, multiplication and division on whole numbers. Till now, we have solved expressions involving only one mathematical operation such as 35 × 42, 50 + 14, 614 – 126, 24 ÷ 6, etc.

Now, let us learn how to solve expressions involving more than one mathematical operation.

Look at the following expression.

15 × 8 – 3

To solve this expression, one may proceed in following two manners:

15 × 8 – 3

= 120 – 3 (By solving multiplication)

= 117 (By solving subtraction)

Or

$15 \times 8 - 3$

$= 15 \times 5$ (By solving subtraction)

$= 75$ (By solving multiplication)

Now, let us take another example.

Let us solve the expression $54 \div 6 + 3$.

$54 \div 6 + 3$

$= 9 + 3$ (By solving division)

$= 12$ (By solving addition)

Or

$54 \div 6 + 3$

$= 54 \div 9$ (By solving addition)

$= 6$ (By solving division)

It can be seen that, we have obtained two different values for each of the expression. This happened because we have followed two different orders to perform the operations while solving the given expressions.

This requires the need to have a specific order to perform operations. The correct order of performing the operations in a given expression is: **Division, Multiplication, Addition, Subtraction (DMAS).**

Note: If any of the operations is not present in an expression, then we skip it and move to the next operation.

The first value of each of the above expressions is obtained by following the correct order; so it is the solution of the expression.

We also have some expressions having one operation more than once.

Consider the expression $25 + 108 \div 6 \div 2$

In such cases, we move from left to right to perform the repeated operations.

So, the given expression can be solved as follows:

25 + 108 ÷ 6 ÷ 2

= 25 + 18 ÷ 2 (By solving division on the left)

= 25 + 9 (By solving division)

= 34 (By solving addition)

Similarly, we can solve many other such expressions as well.

Use of brackets:

There are mainly three types of brackets namely, **square bracket [ ]**, **curly bracket { }** and **simple bracket ( )**. These are used to separate the terms as well as to define that which operation should be performed first in an expression.

For example, consider the following expression.

128 ÷ 8 – 4

If this expression is given as (128 ÷ 8) – 4, this means that division is to be performed first and the expression can be solved as follows:

(128 ÷ 8) – 4

= 16 – 4 (By solving division in simple bracket)

= 12 (By solving subtraction)

Also, if this expression is given as 128 ÷ (8 – 4), this means that subtraction is to be performed first and the expression can be solved as follows:

128 ÷ (8 – 4)

= 128 ÷ 4 (By solving subtraction in simple bracket)

= 32 (By solving division)

Sometimes, there may be more than one bracket and more than one operation in each bracket in an expression.

For example, consider the following expression.

(128 ÷ 8 + 12) – (4 × 2 + 3)

Here, we will solve the brackets according to the order of operations as studied above.

The expression can be solved as follows:

(128 ÷ 8 + 12) – (4 × 2 + 3)

= (16 + 12) – (4 × 2 + 3) (By solving division in simple bracket on the left)

= (16 + 12) – (8 + 3) (By solving multiplication in simple bracket on the right)

= 28 – 11 (By solving addition in both the simple brackets)

= 17 (By solving subtraction)

Similarly, we may solve two or more brackets having multiplication and division in between them.

It should be noted that when a number or a bracket is multiplied with a bracket, then we can avoid writing the sign of multiplication between them.

For example, the expressions 4 × (24 – 5) and (24 – 5) × (24 + 5) are same as 4(24 – 5) and (24 – 5) (24 + 5).

Now, what would we do if an expression has all the types of brackets in it?

For example, [25 + {(100 ÷ 4 + 12) – 7} × (2 + 3)] – 10

In such expressions, simple bracket is solved first, then curly bracket is solved and finally the square bracket is solved.

The given expression can be solved as follows:

[25 + {(100 ÷ 4 + 12) – 7} × (2 + 3)] – 10

= [25 + {(25 + 12) – 7} × (2 + 3)] – 10 (By solving division in the innermost simple bracket)

= [25 + {37 – 7} × 5] – 10 (By addition in both the simple brackets)

= [25 + 30 × 5] – 10 (By solving curly bracket)

= [25 + 150] – 10 (By solving multiplication in square bracket)

= 175 – 10 (By solving square bracket)

= 165 (By solving subtraction)

So, in this manner, we can solve expressions having more than one bracket.

Now, let us solve some more expressions to understand the concept better.

Example 1:

Find the value of the following expressions.

a) 324 ÷ 9 + 12 – 3 × 4 + 10

b) 256 ÷ 8 ÷ 16 + 12 × 3

c) 125 × 25 ÷ 5 × 4 – 50

Solution:

a) 324 ÷ 9 + 12 – 3 × 4 + 10

= 36 + 12 – 3 × 4 + 10 (By solving division)

= 36 + 12 – 12 + 10 (By solving multiplication)

= 36 + 12 + 10 – 12 (By rearranging the terms)

= 58 – 12 (By solving addition)

= 46 (By solving subtraction)

b) 256 ÷ 8 ÷ 16 + 12 × 3

= 32 ÷ 16 + 12 × 3 (By solving division on the left)

= 2 + 12 × 3 (By solving division)

= 2 + 36 (By solving multiplication)

= 38 (By solving addition)

c) 125 × 25 ÷ 5 × 4 – 50

= 125 × 5 × 4 – 50 (By solving division)

= 625 × 4 – 50 (By solving multiplication on the left)

= 2500 – 50 (By solving multiplication)

= 2450 (By solving subtraction)

Example 2:

Simplify the following expressions.

a) (112 ÷ 8 × 4) – 3(4 + 10)

b) (35 × 6 – 4) (21 ÷ 3 + 4)

c) [108 – {3 + (70 ÷ 14 –4)} + (2 + 114 ÷ 19)] ÷ 8

d) 111 + [{227 + (65 ÷ 5)} ÷ 4 – 4(238 ÷ 17)] ÷ 4

Solution:

a) (112 ÷ 8 × 4) – 3(4 + 10)

= (14 × 4) – 3(4 + 10) (By solving division in simple bracket on the left)

= 56 – 3(4 + 10) (By solving multiplication in simple bracket on the left)

= 56 – 3(14) (By solving addition in simple bracket)

= 56 – 42 (By solving multiplication)

= 14 (By solving subtraction)

b) (35 × 6 – 4) (21 ÷ 3 + 4)

= (35 × 6 – 4) (7 + 4) (By solving division in simple bracket on the right)

= (210 – 4) (7 + 4) (By solving multiplication in simple bracket on the left)

= (210 – 4)11 (By solving addition in simple bracket on the right)

= (206)11 (By solving subtraction in simple bracket)

= 2266 (By solving multiplication)

c) [108 – {3 + (70 ÷ 14 – 4)} + (2 + 114 ÷ 19)] ÷ 8

= [108 – {3 + (5 – 4)} + (2 + 6)] ÷ 8 (By solving division in simple brackets)

= [108 – {3 + (5 – 4)} + 8] ÷ 8 (By solving addition in simple bracket)

= [108 – {3 + 1} + 8] ÷ 8 (By solving subtraction in simple bracket)

= [108 – 4 + 8] ÷ 8 (By solving curly bracket)

= [108 + 8 – 4] ÷ 8 (By rearranging terms in square bracket)

= [116 – 4] ÷ 8 (By solving addition in square bracket)

= 112 ÷ 8 (By solving square bracket)

= 14 (By solving division)

d) 111 + [{227 + (65 ÷ 5)} ÷ 4 – 4(238 ÷ 17)] ÷ 4

= 111 + [{227 + 13} ÷ 4 – 4(14)] ÷ 4 (By solving division in simple brackets)

= 111 + [240 ÷ 4 – 4(14)] ÷ 4 (By solving addition in curly bracket)

= 111 + [60 – 4(14)] ÷ 4 (By solving division in square bracket)

= 111 + [60 – 56] ÷ 4 (By solving multiplication in square bracket)

= 111 + 4 ÷ 4 (By solving square bracket)

= 111 + 1 (By solving division)

= 112 (By solving addition)

Example 3:

Insert the brackets in the following expression to get the value as 2.

361 ÷ 12 + 7 – 7 ÷ 6

Solution:

To get the value of the given expression as 2, the brackets can be inserted as follows:

{361 ÷ (12 + 7) – 7} ÷ 6

It can be solved as follows:

{361 ÷ (12 + 7) – 7} ÷ 6

= {361 ÷ 19 – 7} ÷ 6 (By solving simple bracket)

= {19 – 7} ÷ 6 (By solving division in curly bracket)

= 12 ÷ 6 (By solving curly bracket)

= 2 (By solving division)