Standard VIII

MATHEMATICS

Part - 2

Government of Kerala
Department of Education
2016

State Council of Educational Research and Training (SCERT)

KT-511-1/Maths-8(E) Vol-2

THE NATIONAL ANTHEM

Jana-gana-mana adhinayaka, jaya he
Bharatha-bhagya-vidhata.
Punjab-Sindh-Gujarat-Maratha
Dravida-Utkala-Banga
Vindhya-Himachala-Yamuna-Ganga
Uchchala-Jaladhi-taranga
Tava subha name jage,
Tava subha asisa mage,
Gahe tava jaya gatha.
Jana-gana-mangala-dayaka jaya he
Bharatha-bhagya-vidhata.
Jaya he, jaya he, jaya he,
Jaya jaya jaya, jaya he!

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give respect to my parents, teachers and all elders and treat everyone with courtesy.

I pledge my devotion to my country and my people. In their well-being and prosperity alone lies my happiness.

Prepared by:
State Council of Educational Research and Training (SCERT)
Poojappura, Thiruvananthapuram 695 012, Kerala

Website: www.scertkerala.gov.in
E-mail: scertkerala@gmail.com
Phone: 0471-2341883, Fax: 0471-2341869
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Dear children,

We have travelled much
In the world of math
Let's continue
Our explorations
Our findings
We have much more to go ahead
In Mathematics
To the ever widening world of Numbers
To the precise reasoning of Geometry
To the higher levels to Algebra
Let's continue our search...

Dr. P. A. Fathima
Director
SCERT

TEXTBOOK DEVELOPMENT

Participants

T. P. Prakashan
GHSS, Vazhakadu, Malappuram

Unnikrishnan M. V.
GHSS, Kumbala, Kasaragode

Narayanan K.
BARHSS, Bovikkanam,Kasaragode

Mohanan C.
GHSS, Angadikkal South, Chenganoor

Ubaidulla K. C.
SOHSS, Areacode, Malappuram

Vijayakumar T. K.
GHSS, Cherkkala, Kasaragode

Sreekumar
GGHSS, Karamana, Thiruvananthapuram

V. K. Balagangadharan
GMHSS, Calicut University Campus
Malappuram

Narayananunni
DIET, Palakkad

Abraham Kurian
CHSS, Pothukallu, Nilambur

Sunil Kumar V. P.
Janatha HSS, Venjaramoodu
Thiruvananthapuram

Krishnaprasad M.
PMSA HSS, Chappangadi
Malappuram

Cover

Rakesh P. Nair

Experts

Dr. E. Krishnan
Prof. (Rtd) University College, Thiruvananthapuram

Academic Co-ordinator
Sujith Kumar G.
Research Officer, SCERT

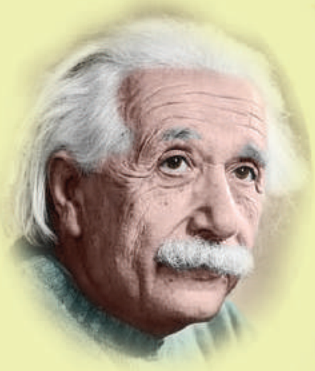
ENGLISH VERSION

Dr. E. Krishnan
Prof.(Rtd) University College,
Thiruvananthapuram

Venugopal C.
Asst. Professor,
Govt. College of Teacher Education,
Thiruvananthapuram

Academic Co-ordinator
Sujith Kumar G.
Research Officer, SCERT

State Council of Educational Research and Training (SCERT)
Vidya Bhavan, Thiruvananthapuram



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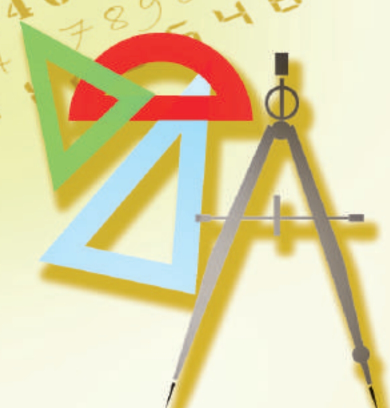


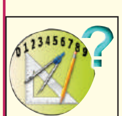
Certain icons are used in this textbook for convenience

Computer Work

Additional Problems

Project

Self Assessment



For Discussion

6

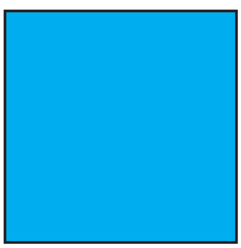
Construction of Quadrilaterals

Classification

We have seen different kinds of quadrilaterals. Let's look at their properties once more:

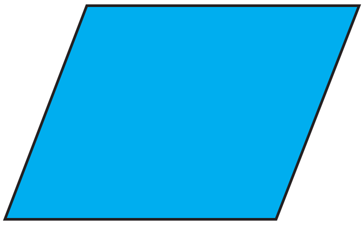
Rectangle

- Opposite sides equal
- Opposite sides parallel
- All angles right
- Diagonals equal
- Diagonals bisect each other



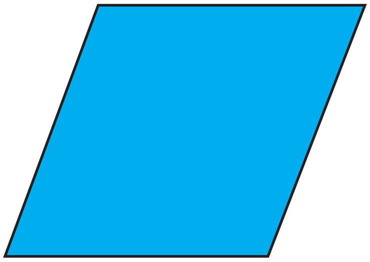
Square

- All sides equal
- Opposite sides parallel
- All angles right
- Diagonals equal
- Diagonals perpendicular bisectors of each other



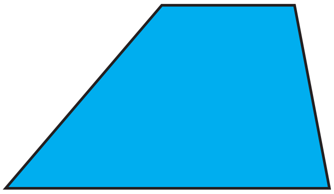
Parallelogram

- Opposite sides equal
- Opposite sides parallel
- Diagonals bisects each other
- Opposite angles equal
- Sum of angles on the same side 180°



Rhombus

- All sides equal
- Opposite sides parallel
- Diagonals perpendicular bisectors of each other
- Opposite angles equal
- Sum of angles on same side 180°



Trapezium

- Only one pair of opposite sides parallel
- Sum of angles on each of non-parallel sides 180°

Isosceles trapezium

- Only one pair of opposite sides parallel
- Non-parallel sides equal
- Diagonals equal
- Angles on each of parallel sides equal
- Sum of angles on each of non-parallel sides 180°

Squares

We have learnt to draw rectangles and squares of specified sides using set squares, in class 5. Just to refresh your memory, draw a square of side 4 centimetres.

We have seen how we can draw perpendiculars using a compass, in the lesson, **Equal Triangles**. Draw a square using this technique.

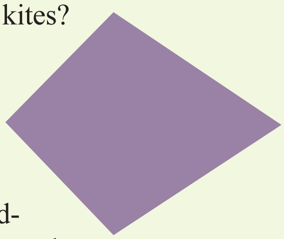
Suppose the length of a diagonal is specified, instead of the length of a side?

A Kite that doesn't fly

Have you flown kites?

What is the usual shape of a kite?

This also is a quadrilateral.

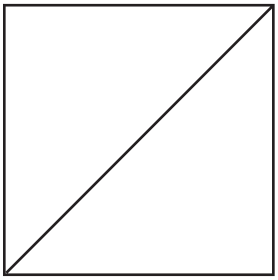


In this, pairs of adjacent sides are equal.

Such a quadrilateral is called a kite in geometry also.

For example, how do we draw a square of diagonal 5 centimetres?

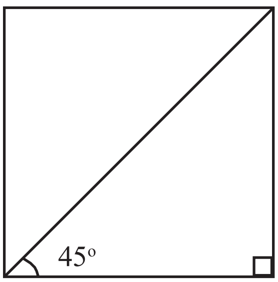
Let's make a rough sketch of a square and its diagonal.



The diagonal splits the square into two triangles. Can you say what the angles of these triangles are?

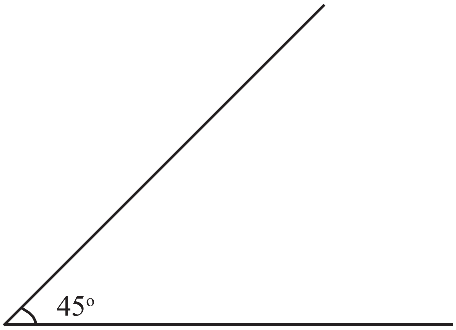
One angle in each is right; and each is an isosceles triangle.

So the other angles are 45°. (How is that?)

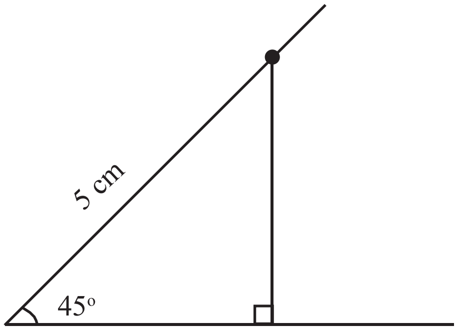


Now can't we draw a square of diagonal 5 centimetres?

First draw a horizontal line and then another line started at 45° at one end;

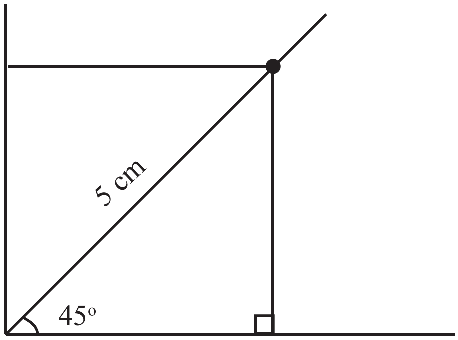


Next mark 5 centimetres on the slanted line and drop a perpendicular line from this point to the bottom line.



(We can draw the perpendicular by making a 45° angle at this point also).

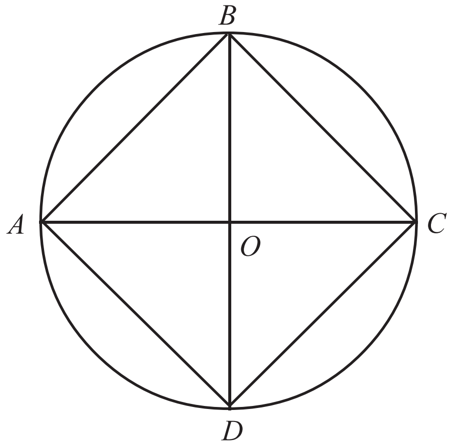
Now we can draw perpendiculars from the two corners and complete the square.



And erase the pieces jutting out to make the figure clean.

There is another way to draw a square.

Draw a circle and two perpendicular diameters. Join their ends.



The four triangles *OAB*, *OBC*, *OCD*, *ODA* are congruent.

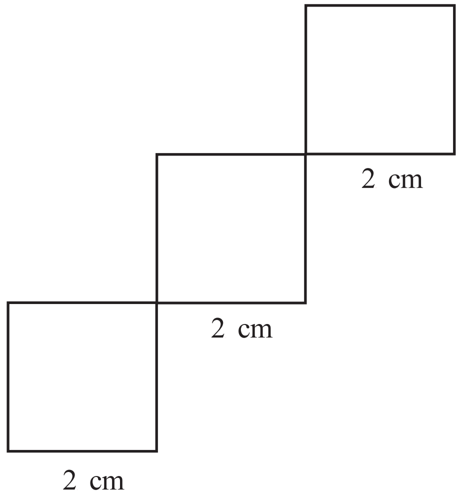
So, what can we say about the quadrilateral *ABCD*?

Doesn't this give another method to draw a square of diagonal 5 centimetres?

Draw a circle of radius 2.5 centimetres and two perpendicular diameters.

Can you draw these patterns of squares in your notebook?

(1)



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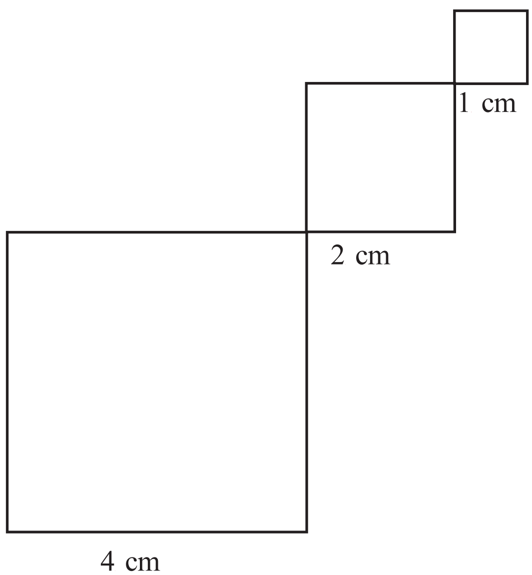

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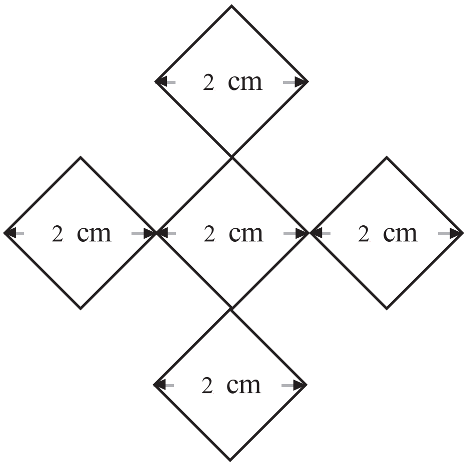


Rectangle

Can't you draw a rectangle of specified sides?

Draw a rectangle of side 8 centimetres and 5 centimetres.

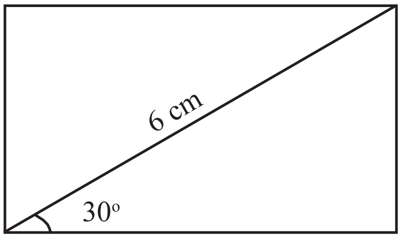
Can we draw rectangles of specified diagonals?

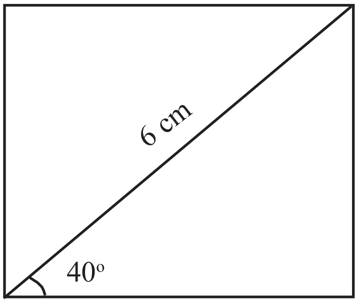
For example, how do we draw a rectangle of diagonal 6 centimetres?

We can draw a square of this diagonal as before. How do we draw a rectangle which is not a square?

Unlike a square, the angle between a diagonal and a side need not be 45° in other rectangles.

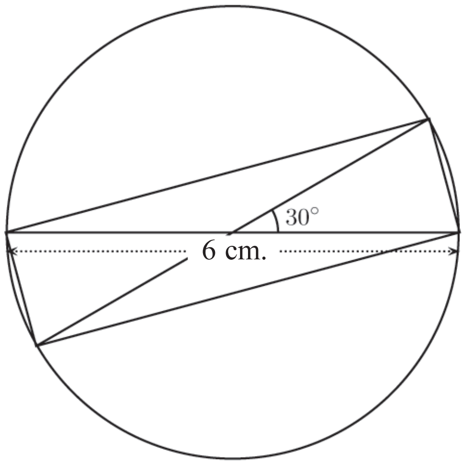
Thus we can draw many rectangles of diagonal 6 centimetres:

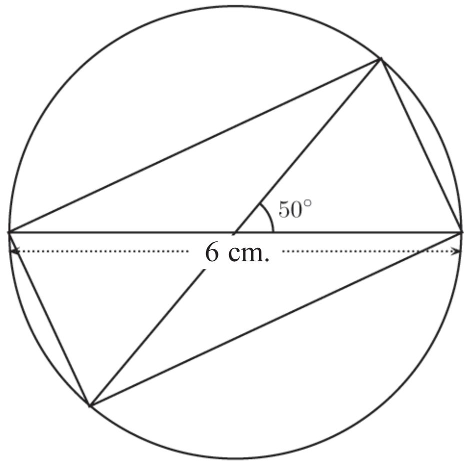




We can draw these by first drawing the angle and then perpendiculars, as we did for squares.

We can draw rectangles of specified diagonals by drawing circles also. In rectangles which are not squares, the diagonals are not perpendicular to each other; So we can draw rectangles using any pair of diameters:

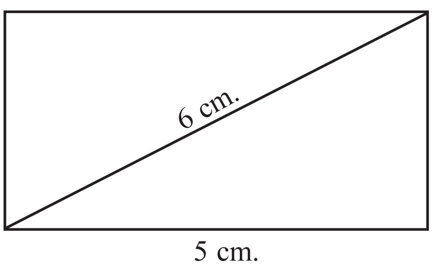




Can you draw a rectangle like this, with diagonal 5 centimetre and 40° angle between them?

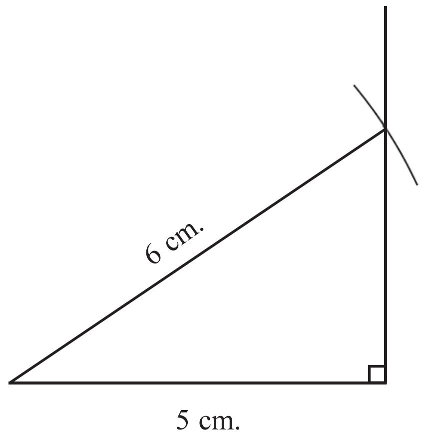
Another question : how do we draw a rectangle of one side 5 centimetres and diagonal 6 centimetres?

To get an idea of how this rectangle would look, let's draw a rough sketch without these actual lengths:



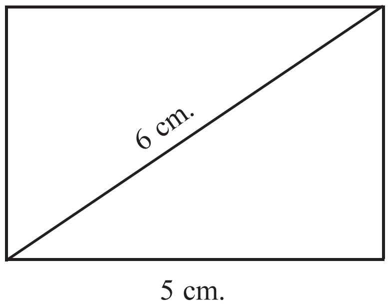
The diagonal splits the rectangle into two right triangles. How about drawing one of these triangles first?

We have to draw a right triangle of hypotenuse 6 centimetres and one of the other sides 5 centimetres:



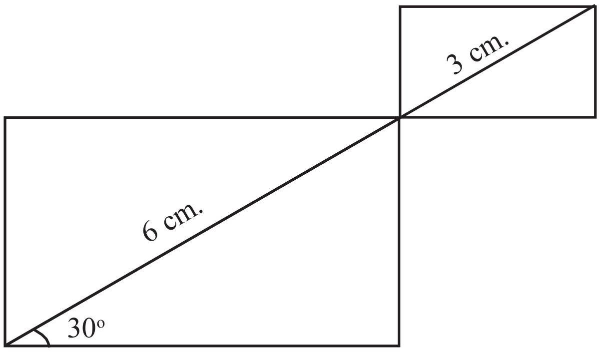
Thus we get half our rectangle.

We can draw the upper half also and complete the rectangle:



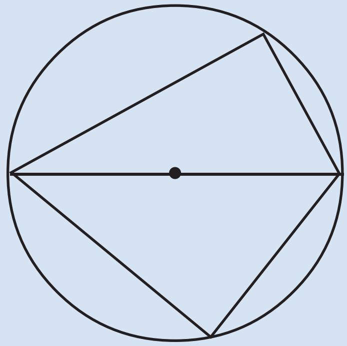
Draw the figures below in your notebook.

(1)



Rectangle in circle

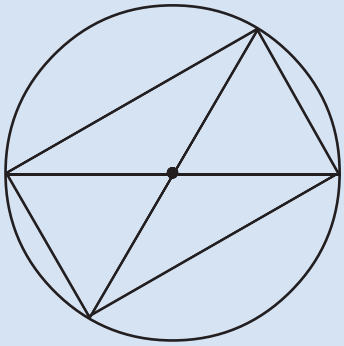
Draw a circle and a diameter. Mark a point in each half of the circle and join them with the ends of the diameter:



The quadrilateral thus get may not be a rectangle. But the angles on either side of the diameter is right. (Why?).

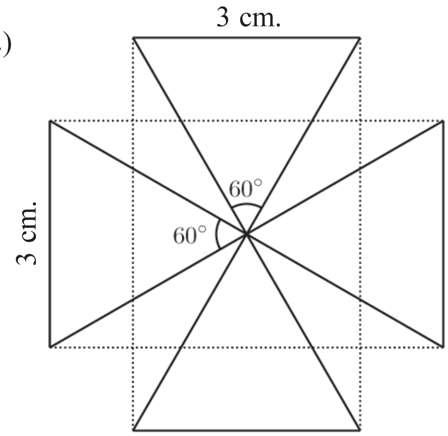
What about the other two angles?

What if the right corners are at the ends of another diameter?

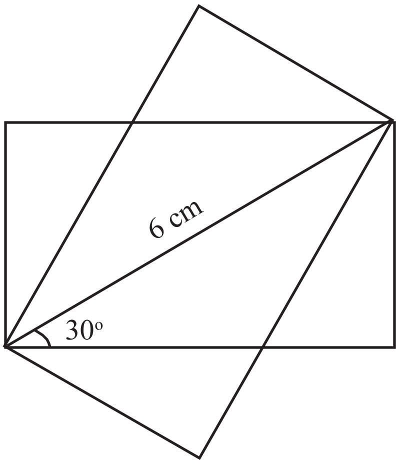


All four angles are right; that is, the quadrilateral is a rectangle.

(2)



(3)



(The rectangles must be equal)

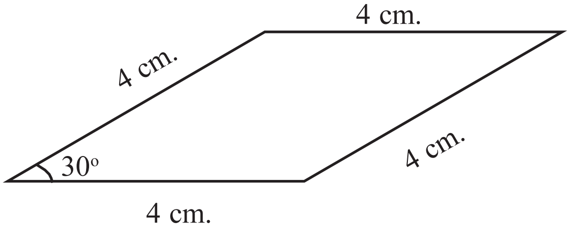
Parallelograms

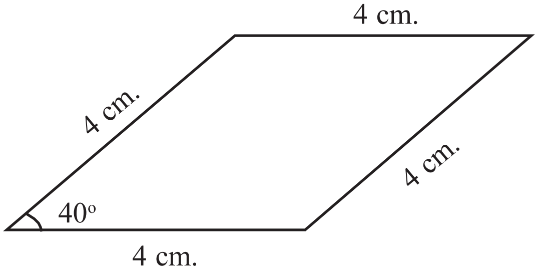
Can you draw a rhombus of sides 4 centimetres? A square is a rhombus and we can easily draw it.

What about a rhombus which is not a square?

Adjacent sides need not be perpendicular.

So we can draw with any angle between them:



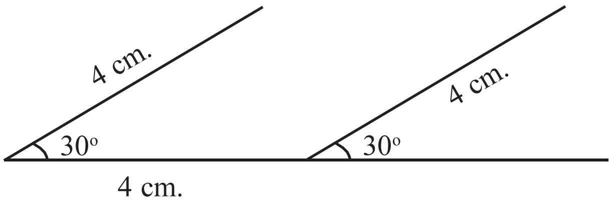


Can you draw the first figure in your notebook?

There are various ways to draw.

First draw a 4 centimetre long line and on its left end, draw another 4 centimetre long line, slanted at 30°. Draw parallels through the other ends of these lines.

Or draw a horizontal line of length 4 centimetres and draw lines of length 4 centimetre at either end, slanted at 30°.



Now we need only to join the upper ends of the slanted lines (and clean the figure by erasing the jutting pieces)

Like this, draw the other rhombus of angle 40° also.

Unlike a square, the diagonals of a rhombus may not be equal. How do we draw a rhombus with diagonals of specified lengths?

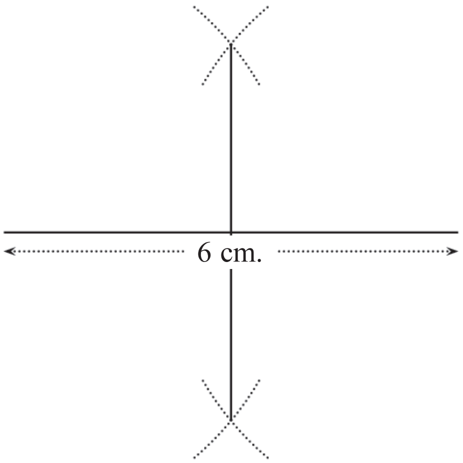
For example, suppose we want to draw a rhombus of diagonals 6 centimetres and 4 centimetres.

This is easy, once we recall that the diagonals of a rhombus are perpendicular bisectors of each other.

First draw a 6 centimetre long line and its perpendicular bisector.

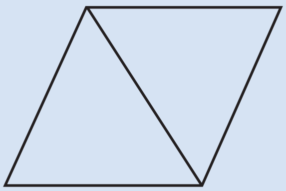
Now mark 2 centimetres on the upper and lower parts of the bisector.

We get our rhombus by joining these points with the ends of the first line.



Isosceles Triangles

A diagonal of a rhombus splits it into two isosceles triangles; and they are equal:



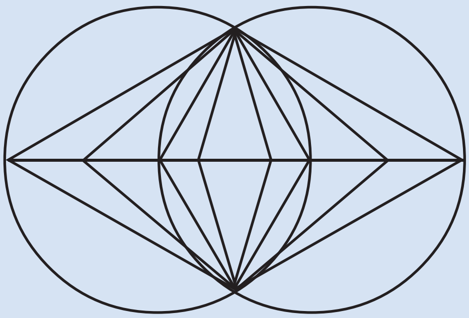
So to draw a rhombus with specified sides and diagonal, we need only to draw isosceles triangles on both sides of the diagonal.

What if the diagonal is equal to a side?

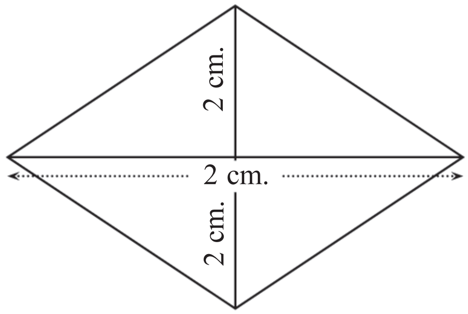
Circles and rhombus

Draw a line and two equal circles centred at its ends. Extend the line to meet the circles.

We can draw several rhombuses with a diagonal along this line.

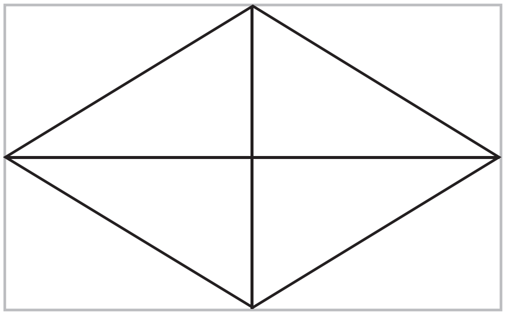


All the four rhombuses in the figure are along the same line, right?



Can we draw this in any other way?

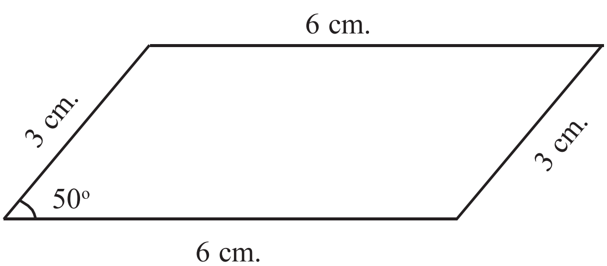
See this picture:



How do we draw a rhombus inside a rectangle?

1) Draw a rhombus of diagonals 5.5 centimetres and 3 centimetres in your notebook.
2) Draw also a rhombus of diagonals 5.5 centimetres and 3.5 centimetres.

We can draw a parallelogram, which is not a rhombus, by specifying some measurements. For example, see this figure:



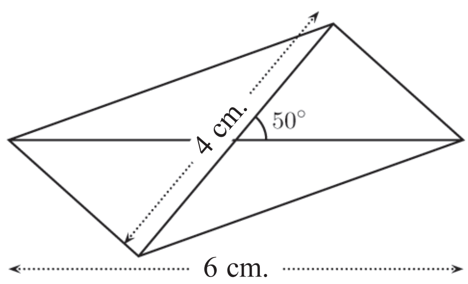
As in the case of a rhombus, we can first draw a 50° angle of sides 6 centimetres and 5 centimetres and then parallels through their ends.

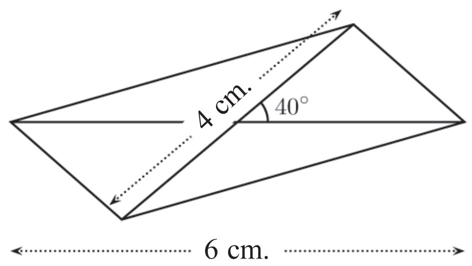
Or we can draw a 6 centimetre long line, then 3 centimetre long lines at its ends slanted at 50° and finally join the ends of the slanted lines

Try it!

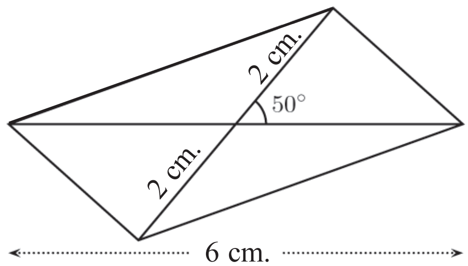
Draw also a parallelogram with sides of these lengths but slanted at 60°

In parallelograms of unequal sides also, the diagonals bisect each other; but not at right angles. So we can draw several parallelograms of same diagonals. See these figures:





We can draw these also, as we drew rhombuses. For the first, instead of drawing the perpendicular bisector of a 6 centimetre long line, we draw a line though the midpoint, slanted at 50°:

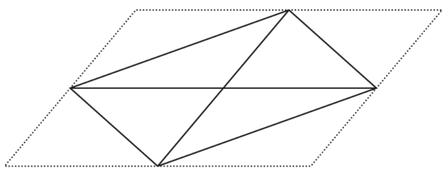


Draw the second figure like this, in your notebook

The diagonals of a parallelogram are not in general equal, and so the length of a side and a diagonal does not specify it completely. (Recall that these were sufficient to determine a rectangle.)

Another way

See this figure:



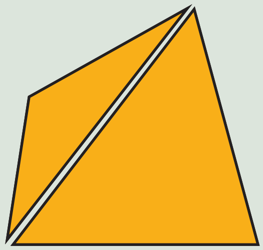
What is the relation between the sides of the outer parallelogram and the diagonals of the inner parallelogram? And what about the angles?

What is the relation between the vertices of the inner parallelogram and sides of the outer parallelogram?

Doesn't this give another method to draw a parallelogram of specified diagonals and angle between them?

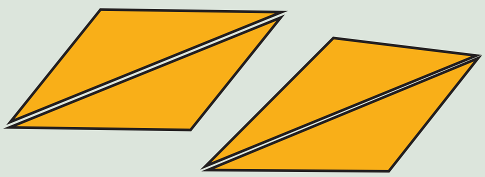
Triangles and quadrilaterals

We can split any quadrilateral into two triangles by drawing a diagonal.



Putting this in reverse, we can join any two triangles with one pair of sides equal, to make a quadrilateral.

If the whole triangles are equal, we can make a parallelogram or a kite.

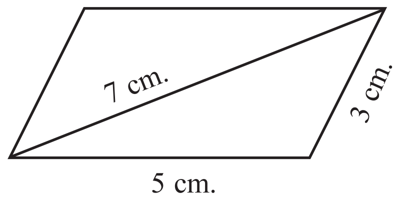


Find out the specialities of the triangles joined, to get various kinds of quadrilaterals.

Suppose the length of two sides and one diagonal are specified?

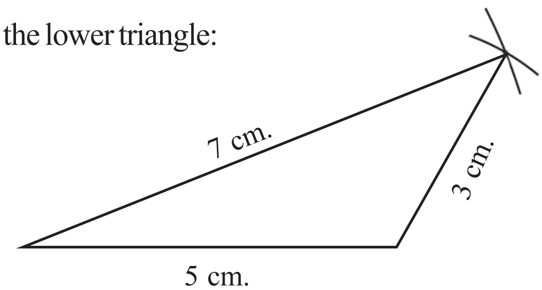
For example, how do we draw a parallelogram of sides 5 centimetres, 3 centimetres and one diagonal 7 centimetres?

Let's fist draw a rough sketch and mark these measurements:

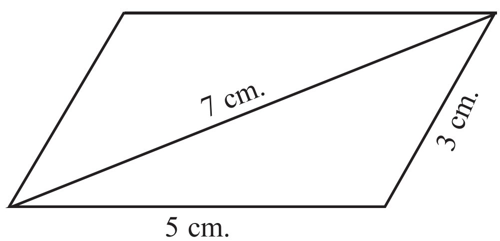


How about drawing the upper and lower triangles separately as we did for a rectangle?

First, the lower triangle:



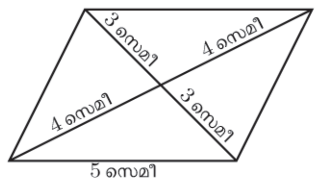
Now we can find the fourth vertex also, by drawing parallel lines or arcs:



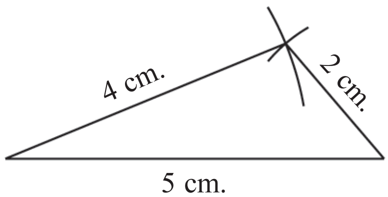
Instead of specifying two sides and a diagonal, suppose it is the other way round.

For example, how do we draw a parallelogram of one side 5 centimetres and diagonals 6 centimetres and 8 centimetres?

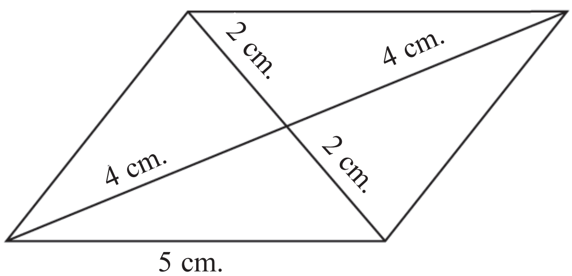
Draw a rough sketch and mark these measurements. Since the diagonals bisect each other, we can write like this:



First, let's draw the triangle formed by the bottom side and half the diagonals above it:



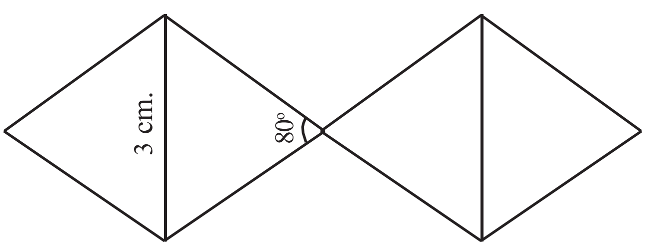
Now we can double the upper sides and complete the parallelogram.



Like this, draw a parallelogram of one side 6.5 centimetres and diagonals 8 centimetres and 7 centimetres.

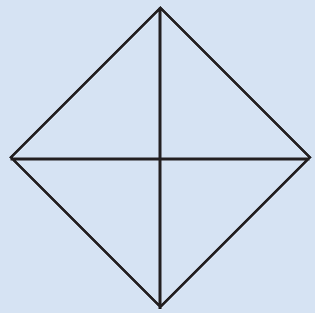
Draw these figures

1) Two equal rhombuses:

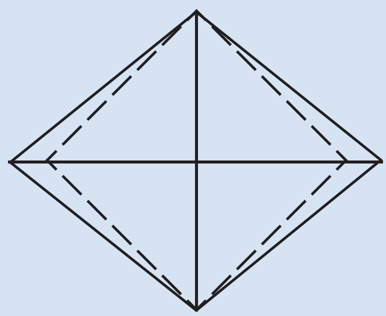


Perpendicular diagonals

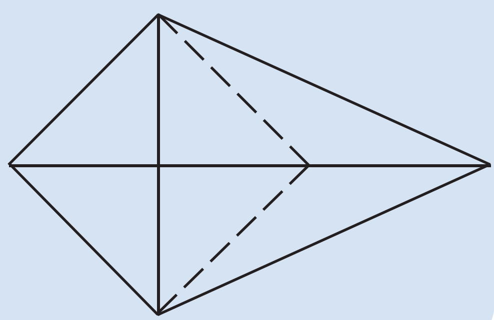
Draw two lines of equal length, bisecting each other at right angles. Joining their ends gives a square:



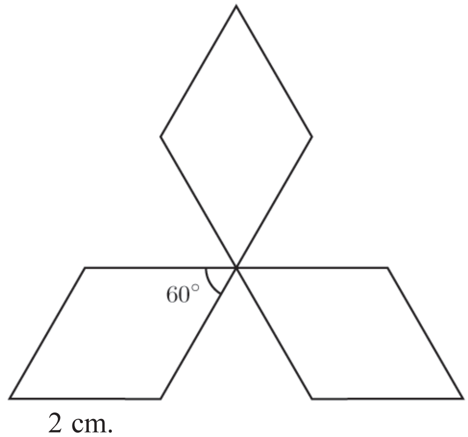
Now extend one line equally to both sides. What do we get on joining the end points?



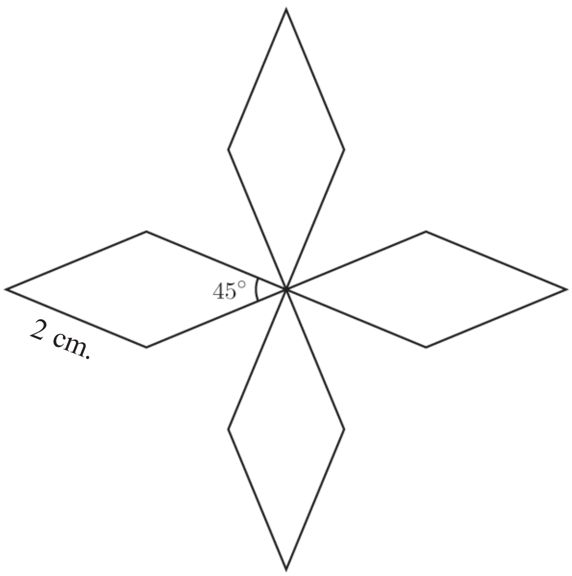
In the first figure, suppose we extend one line to just one scale. What do we get on joining the end points?



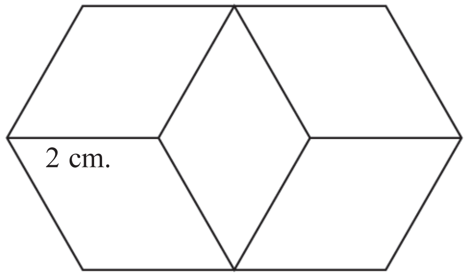
2) Three equal rhombuses:



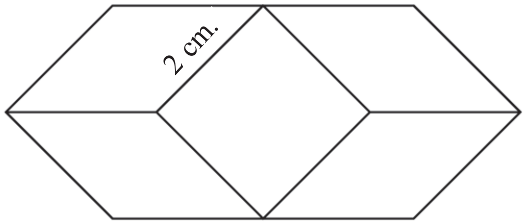
3) Four equal rhombuses:



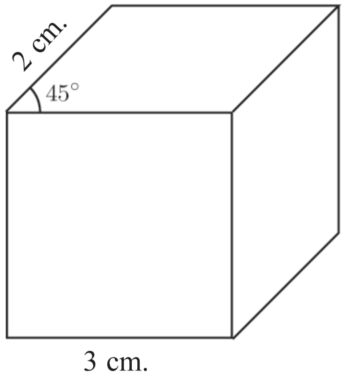
4) Five equal rhombuses:



5) Four rhombuses around a square:

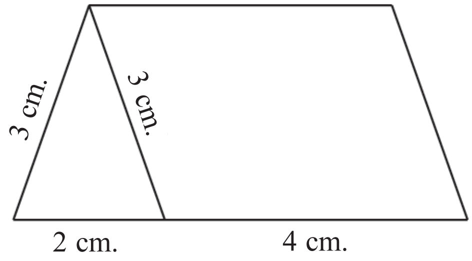


6) Parallelograms on two sides of a square:



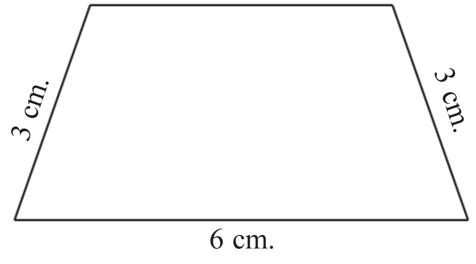
Trapeziums

The figure below shows an isosceles triangle and a parallelogram joined together:



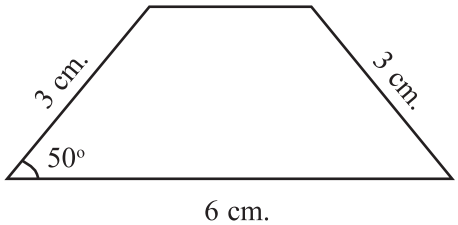
Can't you draw this?

If we erase the joining line, what do we get?



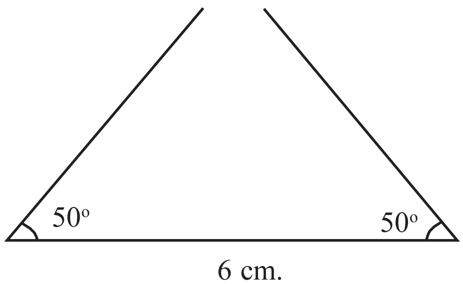
We have drawn a parallelogram of adjacent sides 6 centimetres, 8 centimetres and the angle between them 50°.

Can you draw an isosceles trapezuim of these specifications?

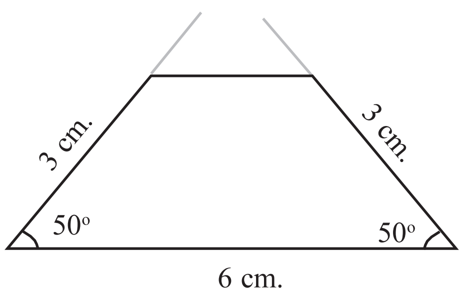


Since it is an isosceles trapezuim, the other angle on the bottom side is also 50°.

So, we can start by drawing a 6 centimetre long line and 50° angles at each end:



Marking 3 centimetres on these lines and joining the ends, we get our trapezium:

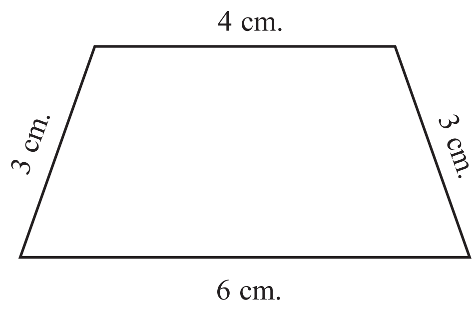


(Can you prove that the top side is parallel to the bottom side?)

Draw an isoceles trapezium with sides of these lengths, but the angle between them 60°.

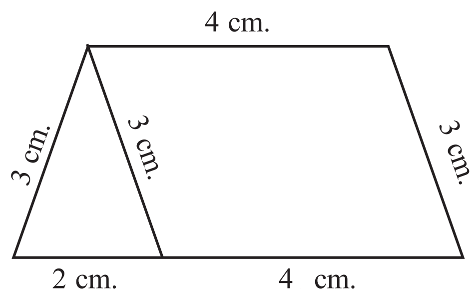
Suppose instead of the angles, the length of the fourth side is specified?

For example, how do we draw the trapezium shown below?

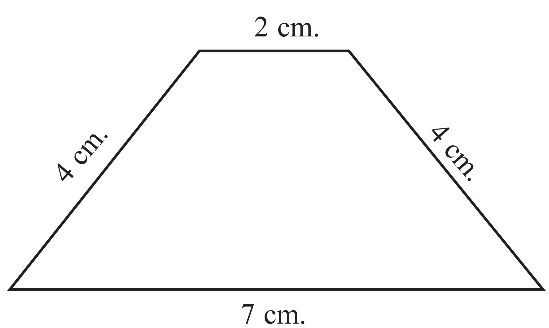


This is a figure we had already drawn.

We drew it by joining an isosceles triangle and a parallelogram:



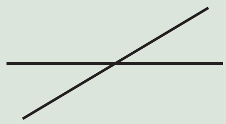
Like this, can you draw in your notebook, the trapezium shown below?



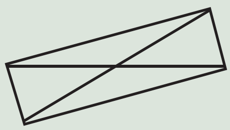
First draw a triangle and then a parallelogram.

Diagonal Changes

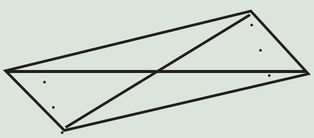
Draw two lines of equal length bisecting each other, but not at right angles;



What kind of quadrilateral do we get on joining their ends?

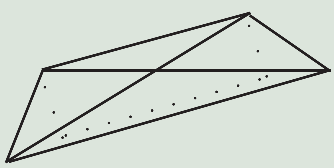


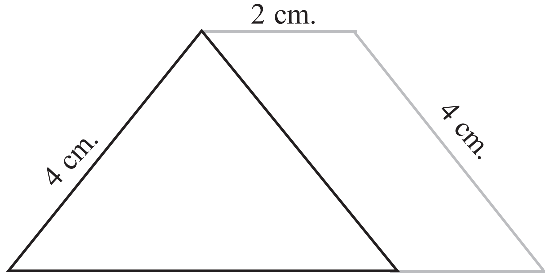
Now as before, suppose we extend one line equally to both sides and join the ends:



What kind of quadrilateral do we get?

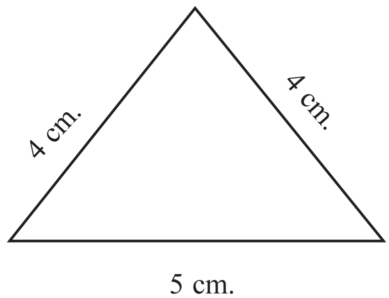
In the first figure, suppose we equally extend the horizontal line to the right and the slanted line downwards, what do we get?



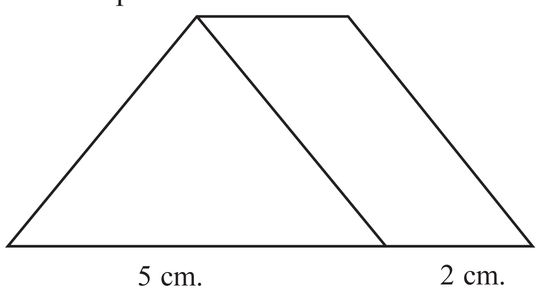


The bottom side of the triangle is 7 – 2 = 5 centimetres. What about the right side?

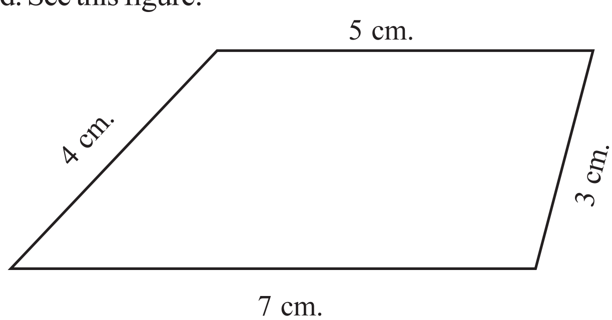
So, we draw a triangle of sides 5 centimetres, 4 centimetres, 4 centimetres:



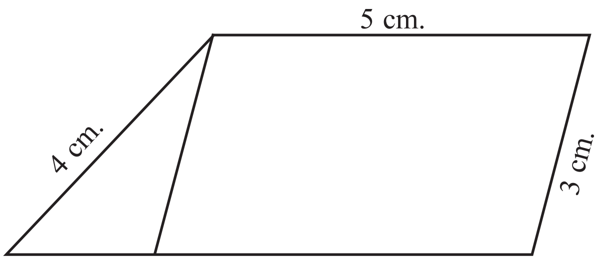
Now extending the bottom side and drawing parallels, we can draw the trapezium:



We can draw non-isosceles trapezuims also like this; all sides must be specified. See this figure:



This also we can split into a triangle and a parallelogram:

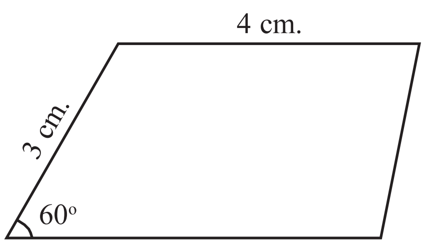


What are the lengths of the other two sides of the triangle?

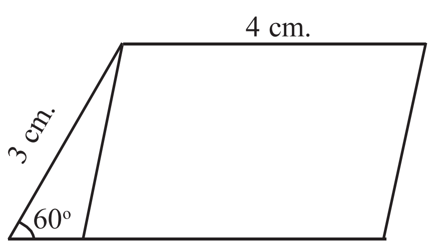
So, we first draw a triangle of sides 2 centimetres, 4 centimetres, 3 centimetres and then a parallelogram as before to make the trapezium. Try!

Instead of four sides, suppose three sides and an angle are specified.

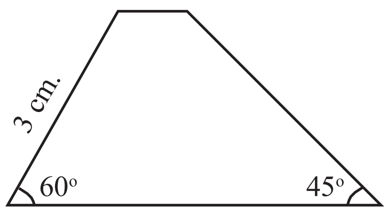
If the measurements are as in the figure alongside, it is not difficult to draw it.



As before, we can first draw a triangle and then a parallelogram:



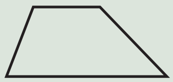
What about two sides and two angles?



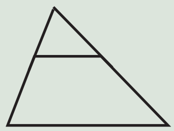
First draw a 5 centimetre long line and angles of 60° and 45° at its ends. Mark 3 centimetres on the left line and draw a line parallel to the bottom line. Try it. (We can draw the parallel to the bottom line by drawing an angle of 120° at the top end of the left lines.)

Trapezium and triangle

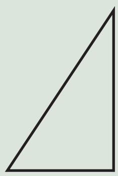
Draw a trapezium



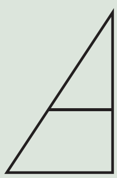
If we extend the pair of non-parallel opposite sides, they met at a point, and we get a triangle.



Now let's start with a triangle.



Draw a line inside, parallel to one of the sides.

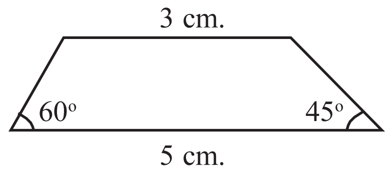


If we erase the portion of the sides above this line, we get a trapezium.

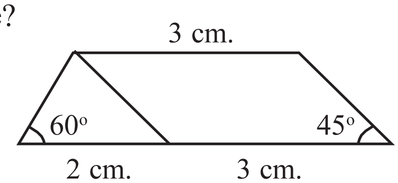
If we start with an isosceles trapezium, what kind of triangle so we get?

On the other hand, cutting an isosceles triangle as above, what kind of trapezium do we get?

How about this trapezium?



Suppose we divide this into a triangle and parallelogram as before?

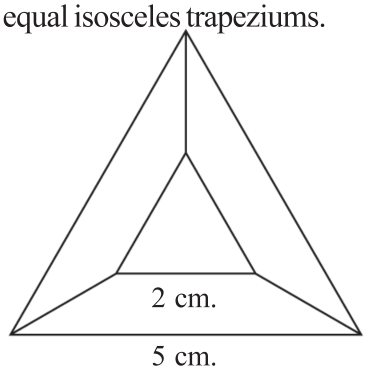


We know the bottom side of the triangle and the angle at one end of it. What about the angle at the other end?

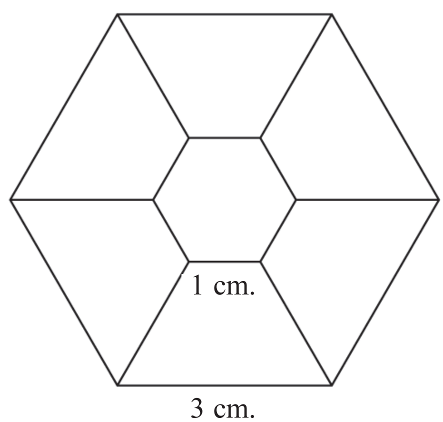
Now can't you draw the triangle and then the trapezium?

Draw the figures below:

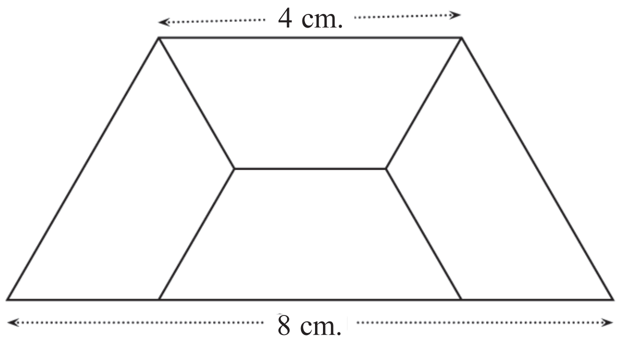
1) Three equal isosceles trapeziums.



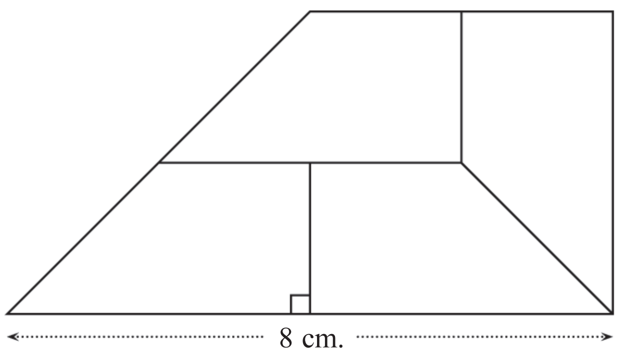
2) Six equal isosceles trapeziums.



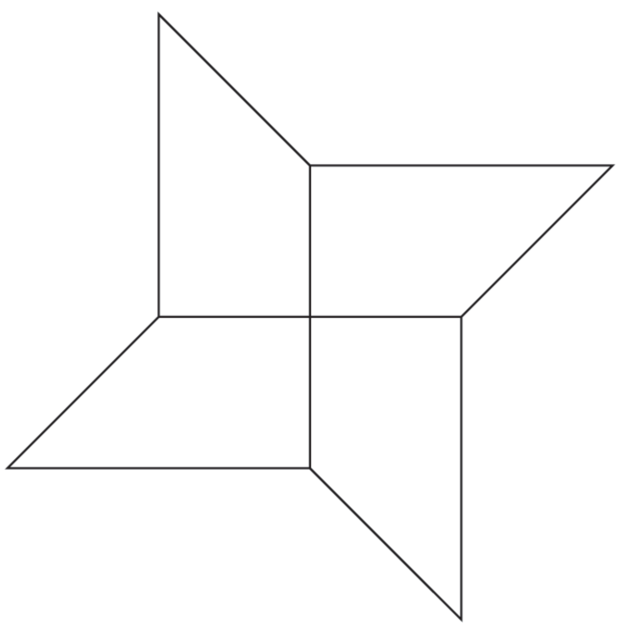
3) Four equal isosceles trapeziums:



4) Four equal trapeziums of another kind:

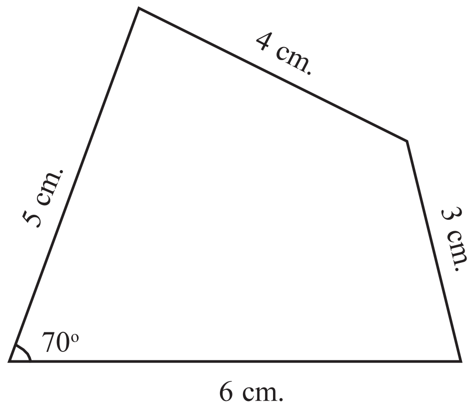


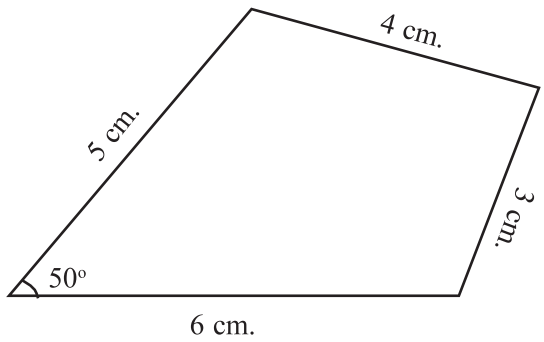
5) Another arrangement of the trapeziums in the last figure:



Quadrilaterals

Now let's draw plain quadrilaterals without any specialities. Two quadrilaterals may not be equal, even if all four sides are the same. So, we can draw different quadrilaterals of same sides. See these quadrilaterals:



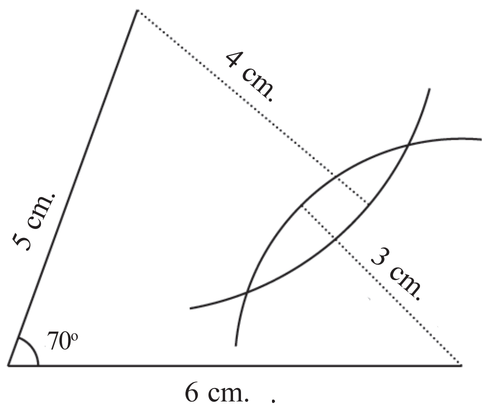


Can you draw these in your notebook?

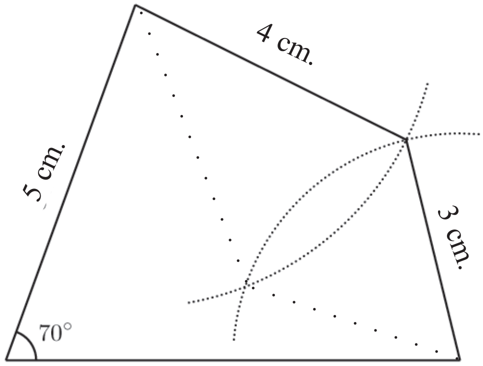
Let's see how we draw the first one. Draw a 6 centimetre long line and then a 5 centimetre line at one of its ends, slanted at 70°.

Now we have three vertices of the quadrilateral. How do we locate the fourth vertex?

It is 4 centimetres from the top vertex and 3 centimetres from the right vertex. Thus it is on the two circles centered at these vertices and of radii equal to these lengths.



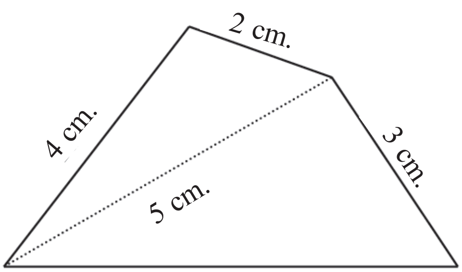
Taking one point of intersection of these circles we can draw our quadrilateral.



(And we don't take into account the depressed quadrilateral got by taking the other point)

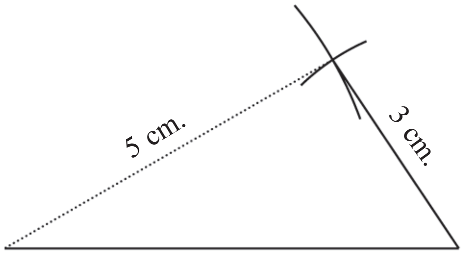
Like this, try to draw the other quadrilateral with 50° angle.

Instead of four sides and an angle, four sides and a diagonal also determines a quadrilateral.



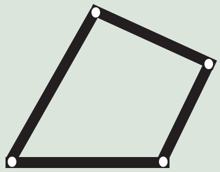
How do we draw this?

First draw the lower triangle:



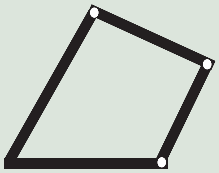
Unstable quadrilateral

Cut out these strips of plastic or thick card-board of lengths 3, 4, 5, 6 centimetres and join their ends using pins or thumbtacks to make a quadrilateral.

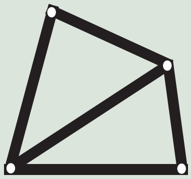


We can move the sides to get different quadrilaterals; and all of them have the same sides.

Now remove the pin from one corner and paste the ends of these sides real fast. Can you move the sides now?

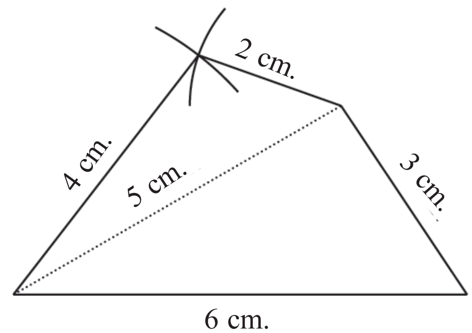


Instead of pasting two ends together, suppose we fit a cross piece like this:



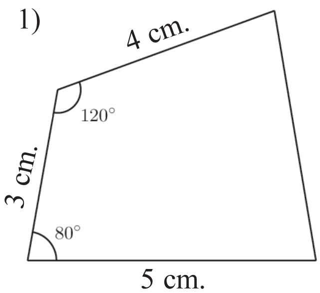
Can you move the sides now?

Draw the upper triangle also, we get the quadrilateral.

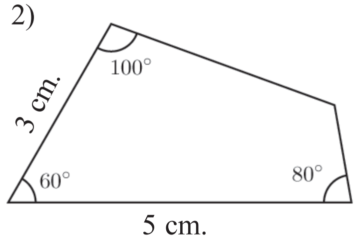


Draw the quadrilaterals shown below.

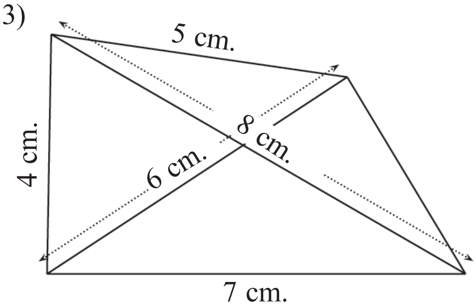
1)



2)



3)

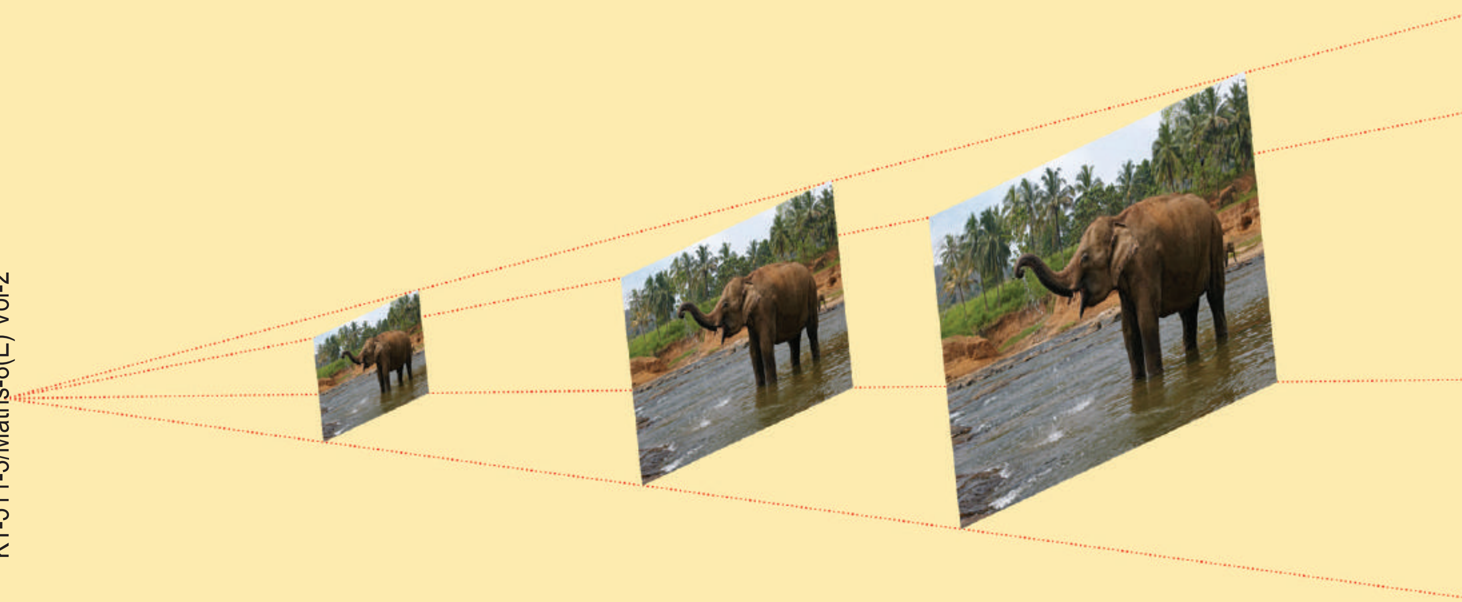


Looking back

Learning outcomes	What I can	With teacher's help	Must Improve
• Explaining various methods of drawing squares.			
• Drawing rectangles by different methods.			
• Finding the measurements needed to specify a parallelogram.			
• Drawing parallelogram according to specification.			
• Finding the measurements needed to specific trapezium			
• Explaining the methods of drawing trapezium according to specification.			
• Determining the measurements needed to specify a quadrilateral.			

7

Ratio



KT-511-3/Maths-8(E) Vol-2

Part relations

See this picture:

The line AB is divided into five equal parts.

Let's call the first three parts together as AP. In what all ways can we describe the relation between the lengths of the lines AP and BP?

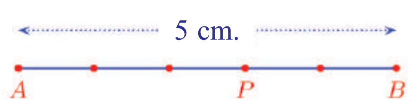
- The relations of AP and BP with AB.
  - AP is $\frac{3}{5}$ of AB
  - BP is $\frac{2}{5}$ of AB
- The relation between AP and BP.
  - BP is $\frac{2}{3}$ of AP
  - AP is $\frac{3}{2}$ of BP
- The relation between AP, BP and the natural numbers 2 and 3.
  - 2 times AP and 3 times BP are equal.
  - $\frac{1}{3}$ of AP and $\frac{1}{2}$ of BP are equal; AP is 3 times this length and BP is 2 times this length.

How do we say all these relations together?

The lengths AP, BP are in the ratio 3 : 2.

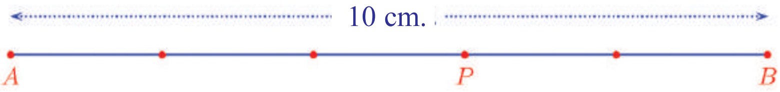
Here, the actual length of AB is not mentioned.

It is 5 centimetres in the picture below:



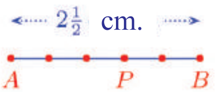
So, the length of AP is 3 centimetres and the length of BP is 2 centimetres.

Suppose we double the length of AB:



The length of AP becomes $2 \times 3 = 6$ centimetres and the length of BP becomes $2 \times 2 = 4$ centimetres. But all the relations above are still true.

What if we halve the length of AB?



Now $AP = 3 \times \frac{1}{2} = 1\frac{1}{2}$ centimeters, $BP = 2 \times \frac{1}{2}$ = 1 centimetres.

Again the old relations are not changed.

What does it mean to say that two lengths are in the ratio 3 : 5?

We can't say what the actual lengths are.

It may be 3 centimetres and 5 centimetres.

Or something like

6 centimetres, 10 centimetres

$1\frac{1}{2}$ centimetres, $2\frac{1}{2}$ centimetres

6 metres, 10 metres

It may even be that, measured by some string, the length of the first is 3 lines the string and the length of the second is 5 times the string.

In any case, we can say that the first is 3 times some non specified length and the second is 5 times this length.

Using algebra we can say in general that if we take this length as x centimetres, then the length of the first is $3x$ centimetres and the length of the second is $5x$ centimetres.

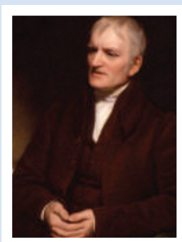
Chemical ratio

In chemistry, substances are classified as elements and compounds.

In the eighteenth century, the scientist Joseph Proust found out that in any compound, the masses of the elements in it are in a definite ratio.

For example, he found out through experiments that in copper carbonate, the mass of copper is 5.3 times that of carbon and the mass of oxygen in 4 times that of carbon. Perhaps it was the thought that such ratios could be expressed as natural numbers using very small particles, which led to the idea of atoms. Such a theory was first put forward by the nineteenth century scientist, John Dalton.

According to his theory, compounds are formed by tiny particles of the elements, called atoms. In any compound, the number of atoms of the elements in it are in a definite ratio.



We can explain ratio of other quantities also like this. For example, the capacities of two bottles are in the ratio 3 : 5 means, using some cup of unspecified size, it takes 3 cups to fill the first bottle and 5 cups to fill the second bottle.

If the capacity of the cup is denoted as x millilitres, we can say in general that the capacity of the first bottle is $3x$ milliliters and the capacity of the second cup is $5x$ milliliters.

What does it mean to say that the number of boys and girls in a class are in the ratio 3 : 5?

If the actual numbers are 30 and 50, we can think of the boys as 3 groups of 10 and the girls as 5 groups of 10.

If the actual numbers are 15 and 25, we can think of the boys and girls again as 3 and 5 groups of 5.

In any case, we can think of the boys as 3 groups and girls as 5 groups, each group with the same number of children.

If the number of children in a group is denoted by x, the number of boys is $3x$ and the number of girls is $5x$.

Elementary ratios

Water is important to sustain life. It is the substance which the human body contains the most.

The elements in water are hydrogen and oxygen. Do you know how much of each water contains? One molecule of water contains 2 atoms of hydrogen and 1 atom of Oxygen. Thus the ratio of hydrogen and oxygen in water is 2:1. The chemical formula for water is H_2O. The elements in common salt that we use are sodium (Na) and chlorine (Cl). They are contained in equal amounts. That is, they are in the ratio 1:1. The chemical formula for common salt is NaCl.

What is the general idea seen in all these examples?

If two quantities are in the ratio *a* : *b*, then there is a quantity *x* such that the first is *ax* and the second is *bx*.

Now let's look at a problem done in class 7. (The section, **Division problem of the lesson, Ratio**)

> The perimeter of a rectangle is 24 metres and its breadth and length are in the ratio 3 : 5. What is the breadth and length?

How did we do this?

We can do this in a different way, using algebra. Since breadth and length are in the ratio 3 : 5, we can take the breadth as $3x$ metres and length as $5x$ metres. To find x, we use the perimeter given in the problem.

Since breadth is $3x$ metres and length is $5x$ metres, the perimeter is

$$2(3x + 5x) = 16x \text{ metres}$$

It is said to be 24 metres. So $16x = 24$ and from this we get,

$$x = \frac{24}{16} = \frac{3}{2}$$

Now can't we calculate the breadth and length?

$$\text{Breadth is } = 3 \times \frac{3}{2} \text{ metres} = 4\frac{1}{2} \text{ metres}$$

$$\text{Length is } = 5 \times \frac{3}{2} \text{ metres} = 7\frac{1}{2} \text{ metres}$$

Let's look at another problem:

The breadth and length of a rectangle are in the ratio 4 : 7. The length is 15 centimetres more than the breadth. What are the length and breadth?

From the given ratio, we see that the breadth is $\frac{4}{11}$ of the sum of bread and length, and the length is $\frac{7}{11}$ of this sum.

So, the difference in length and breadth is $\frac{7}{11} - \frac{4}{11} = \frac{3}{11}$ of the sum. This is said to be 15 centimetres. So the sum of breadth and length is $\frac{11}{3}$ times 15; that is

$$15 \text{ metres} \times \frac{11}{3} = 55 \text{ metres}$$

Now we can calculate, the breadth and length

$$\text{Breadth is} = 55 \times \frac{7}{11} = 35 \text{ metres}$$

$$\text{Length is} = 35 - 15 = 20 \text{ metres}$$

We can do this also using algebra, as in the first problem.

We can take the breadth as $4x$ centimeters and breadth as $7x$ centimeters.

So, length is $7x - 4x = 3x$ centimeters more than the breadth. This difference is given to be 15 centimeters. So, $3x = 15$ and hence $x = 5$.

Now we can compute the breadth and length.

$$\text{Breadth is} = 4 \times 5 \text{ metres} = 20 \text{ metres}$$

$$\text{Length is} = 7 \times 5 \text{ metres} = 35 \text{ metres}$$

Sweet ratio

Do you know the elements in sugar?

Carbon, hydrogen and oxygen. One molecule of sugar contains 12 atoms of carbon, 22 atoms of hydrogen and 11 atoms of oxygen. Thus carbon, hydrogen, oxygen are in the ratio 12 : 22 : 11. The chemical formula of sugar is $C_{12}H_{22}O_{11}$.

What happens when we heat sugar? Why?

One more problem:

The breadth and length of a rectangle are in the ratio 4 : 5 and its area is 320 square metres. What are the breadth and length?

If we take the breadth as $4x$ metres and length as $5x$ metres, area is

$$4x \times 5x = 20x^2 \text{ square metres.}$$

Since we know that this is 320 square metres,

$$20x^2 = 320$$

This means, 20 times the number x^2 is 320.

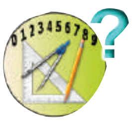
So, this number is $320 \div 20 = 16$. That is,

$$x^2 = 16$$

The number with its square 16 is 4. So, $x = 4$.

$$\text{Breadth is } 4 \times 4 \text{ metres} = 16 \text{ metres}$$

$$\text{Length is } 5 \times 4 \text{ metres} = 20 \text{ metres}$$



In this problem, if we take the breadth as 4 metres and length as 5 metres, the area is 20 square metres; the area given is 16 times this area. Why is it not right to calculate the breadth as 16 times 4 and length as 16 times 5?

1) In a regular polygon, the ratio of the inner and outer angle is 7 : 2. What is each angle? How many sides does the polygon have?

2) The number of girls and boys in a class are in the ratio 7 : 5 and there are 8 more girls than boys. How many girls and boys are there in this class?

3) Blue and yellow paints are mixed in the ratio 2 : 5 to make a new colour. 6 litres more of yellow than blue is taken. How many litres of each is mixed?

4) There are four right triangles, the ratio of perpendicular sides being 3 : 4 in each. One more fact about each is given below. Find the lengths of the sides of each triangle.

(i) The difference in the lengths of the perpendicular sides is 24 metres.

(ii) The hypotenuse is 24 metres.

(iii) The perimeter is 24 metres.

(iv) The area is 24 square metres.

Changing relations

The length of a rectangle is 6 centimetres and its breadth is 4 centimetres.

So, length and breadth are in the ratio 3 : 2.

Suppose we increase the length by 2 centimetres and enlarge the rectangle. Length and breadth are now 8 centimetres and 4 centimetres and their ratio is 2 : 1.

Here's question in reverse:

The length and breadth of a rectangle are in the ratio 3 : 2. It is enlarged by increasing the length by 2 centimetres and now length and breadth are in the ratio 5 : 8. What are the length and breadth of the original rectangle?

Since the length and breadth of the original rectangle are in the ratio 3 : 2, we take the actual length and breadth as $3x$ centimeters and $2x$ centimetres. When the length is increased by 2 centimetres, these become $3x + 2$ centimetres and $2x$ centimeters. It is said that they are in the ratio 5 : 3. How do we compute x using this fact?

One meaning of saying two quantities are in the ratio 5 : 8 is that 3 times the larger quantity and 5 times the smaller quantity are equal. In our problem, $3x + 2$ is the larger and $2x$ is the smaller. So, the relation between them is

$$3(3x + 2) = 5 \times 2x$$

This can be shortened to

$$9x + 6 = 10x$$

From this, we get $x = 6$ (How?)

To and fro

The sides of a rectangle are 33 centimetres. The sides of another rectangle are 11 centimetres and 6 centimetres. What is the ratio of their perimeters? And the ratio of their areas?
Can you find other pairs of rectangles with such a relation?

Thus the length of the original rectangle is 18 centimetres and breadth is 12 centimetres.

The length and breadth of a rectangle are in the ratio 3 : 2. Increasing the length by any amount, can we make the ratio 4 : 3? How about 5 : 3?

Another problem:

The length and breadth of a rectangle are in the ratio 3 : 2. The length is increased by half as much and the rectangle is enlarged. What is the ratio of the length and breadth of the larger rectangle?

In the original rectangle, the breadth is $\frac{2}{3}$ of the length. When the length is increased by half as much, it becomes $1\frac{1}{2}$ of the original. So, the question is how much times $\frac{2}{3}$ is $1\frac{1}{2}$.

$$1\frac{1}{2} \div \frac{2}{3} = \frac{3}{2} \times \frac{3}{2}$$
$$= \frac{9}{4}$$

Thus in the new rectangle, the length is $\frac{9}{4}$ times the breadth So, length and breadth are in the ratio 9 : 4.

We can also do this using algebra. Let's start by taking the length and breadth of the original rectangle as $3x$ centimeters and $2x$ centimeters.

Half the length is $1\frac{1}{2}x$ centimeters, adding this length becomes $4\frac{1}{2}x$ centimeters. Breadth remain $2x$ centimeters.

We may say that these are in the ratio, $4\frac{1}{2} : 2$. In terms of natural numbers, it is $9 : 4$.

Ratio and area

Two rectangles have the same perimeter. Length and breadth are in the ratio 2 : 1 in one and 3 : 2 in the other. Which has larger area?

Since perimeters are the same, sum of length and breadth are also same. Taking it as s centimeters, the lengths of the sides of the first rectangle are $\frac{1}{3}s$, $\frac{2}{3}s$ centimeters and the area is $\frac{2}{9}s^2$ square centimeters.

What is the area of the other?

$$\frac{2}{5}s \times \frac{3}{5}s = \frac{6}{25}s^2$$

Which of $\frac{2}{9}$ and $\frac{6}{25}$ is the larger?

$$\frac{2}{9} < \frac{6}{25}.$$

So, the second rectangle has larger area.

Now, what about another rectangle of the same perimeter, but the ratio of sides 1 : 2?

Which has the largest area?

Look at the difference of the sides in all these rectangles. Any connection with the areas?

1) Acid and water are mixed in the ratio 4 : 3 to make a liquid. On adding 10 more litres of Acid, the ratio changed to 3 : 1. How many litres of acid and water does the liquid contain now?

2) Two angles are in the ratio 1 : 2. On increasing the smaller angle by 6° and decreasing the larger angle by 6°, the ratio changed to 2 : 3. What were the original angles?

3) The sides of a rectangle are in the ratio 4 : 5.

i) By what fraction should the shorter side be increased to make it a square?

(ii) By what fraction should the longer side be decreased to make it a square?

4) Two quantities are in the ratio 3 : 5.

(i) If the smaller alone is made four times the original, what would the ratio be?

(ii) If the smaller is doubled and the larger is halved, what would the ratio be?

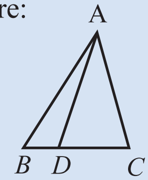
5) (i) (i) The capacities of two bottles are in the ratio 3 : 4. The smaller bottle was filled twice and the larger bottle was filled and emptied into a vessel. Twice the smaller and half the larger was emptied into another. What is the ratio of the quantities of water in the two vessels?

(ii) In the above problem, what if the capacities of the bottles are in the ratio 4 : 7?

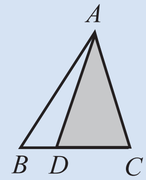
6) The breadth and length of two rectangles are in the ratio 2 : 3. In another rectangle, whose breadth is 1 centimetres less and length is 3 centimetres less than those of the first, this ratio is 3 : 4. Calculate the breadth and length of both rectangles.

Area relations

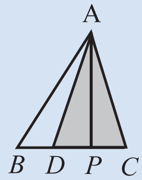
See this figure:



What is the ratio of the areas of ΔABD and ΔACD?



Draw the perpendicular from A to BC.



Taking the length of this perpendicular as h, the area of ΔABD is

$$\frac{1}{2}\ h \times BD$$

and area of ΔACD is

$$\frac{1}{2}\ h \times CD$$

So,

$$\frac{\text{Area of } \Delta ABD}{\text{Area of } \Delta ACD} = \frac{BD}{CD}$$

That is, the ratio of these areas is the same as the ratio of the lengths BD and CD.

So, how do we split a triangle into two triangles of the same area?

What if we want one part to have double the area of the other?

Three quantities

See this picture:

The line AB is divided into 11 equal parts.

AP is 2 such parts together.

PQ is 5 such parts together.

QB is 4 such parts together.

In what all ways can we describe the relations between these pieces.

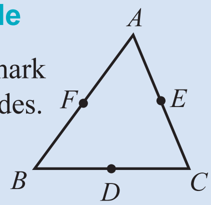
- Relations AP, PQ, QB have with AB
  - AP is $\frac{2}{11}$ of AB
  - PQ is $\frac{5}{11}$ of AB
  - QB is $\frac{4}{11}$ of AB
- Relations between AP, PQ, QB taken in pairs.
  - PQ is $\frac{5}{2}$ times AP; AP is $\frac{2}{5}$ of PQ.
  - QB is $\frac{4}{5}$ of PQ, PQ is $\frac{5}{4}$ times QB.
  - AP is $\frac{2}{4} = \frac{1}{2}$ of QB, QB is $\frac{4}{2} = 2$ times AP.
- Relations AP, PQ, QB have with the numbers 2, 5, 4.
  - 5 times AP and 2 times PQ are equal.

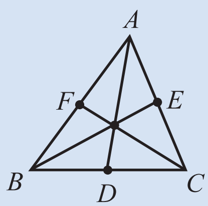
    4 times PQ and 5 times QB are equal.

    2 times AP is equal to QB.
  - $\frac{1}{2}$ of AP, $\frac{1}{5}$ of PQ and $\frac{1}{4}$ of QB are equal. AP is 2 times, PQ is 5 times and QB is 4 times this length.

Centre of a triangle

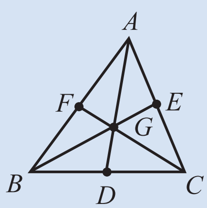
Draw a triangle and mark the midpoints of its sides.





Join each mid point to the opposite vertex.

These lines are called medians of the triangle. Don't they pass through a single point inside the triangle?



This point is called the centroid of the triangle.

The centroid divides each median in the ratio 2 : 1. Thus in our picture,

$$\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = 2$$

The centroid has another property. Cut out a cardboard figure like this. We can balance it at this point on the tip of a pencil.

Thus the centroid of a triangle is its centre of gravity.

As in the case of two quantities, here also we can combine all this by saying that AP, PQ, QB are in the ratio 2 : 5 : 4.

Thus when we say that three quantities are in the ratio 3 : 4 : 2 it means that the least is 2 times, the largest is 4 times and the medium sized 3 times some unspecified quantity.

We can put this in algebra.

If three quantities are in the ratio $a : b : c$, then there is a quantity x such that the first is ax, the second is bx and the third is cx.

Look at this problem:

The sides of a triangle are in the ratio 3 : 5 : 7 and its perimeter is 45 centimetres. What are the length of the sides?

Quantities are in the ratio 3 : 5 : 7 means, they are $\frac{3}{15}, \frac{5}{15}, \frac{7}{15}$ of the sum. In this problem, the sum of the length is the perimeter; that is, 45 centimetres. So, the length of the sides are

$$45 \text{ centimetres} \times \frac{3}{15} = 9 \text{ centimetres}$$

$$45 \text{ centimetres} \times \frac{5}{15} = 15 \text{ centimetres}$$

$$45 \text{ centimetres} \times \frac{7}{15} = 21 \text{ centimetres}$$

We can also do this using algebra. Taking the sides as $3x$, 5x and $7x$ centimeters, the perimeter is $15x$ centimetres.

Right triangles

What is the speciality of a triangle with sides 3 centimetres, 4 centimetres and 5 centimetres? Since $3^2 + 4^2 = 5^2$, it is a right triangle.

Suppose we double each side. Do we get a right triangle?

We also have $6^2 + 8^2 = 10^2$.

Thus the triangle got by doubling the sides is also right. What if we take x times each side?

$(3x)^2 + (4x)^2 = 9x^2 + 16x^2 = 25x^2 = (5x)^2$.

Thus a triangle with sides $3x$, $4x$, $5x$ is also right.

In short, any triangle whose sides are in the ratio 3 : 4 : 5 is right.

What about triangles with sides in the ratio 5 : 12 : 13?

Perimeter is given to be 45 centimetres. So we have $15x = 45$, which gives $x = 3$. Thus the lengths of the sides are $3 \times 3 = 9$ centimetres, $5 \times 3 = 15$ centimetres and $7 \times 3 = 21$ centimetres.

Can any triangle have sides in the ratio 3 : 5 : 8?

Let's look at another kind of problem:

> In triangle *ABC*, the sides *AB*, *BC* are in the ratio 2 : 3 and the sides *BC*, *CA* are in the ratio 4 : 5. What is the ratio of all three sides together?

AB, *BC* are in the ratio 2 : 3 means,

AB is $\frac{2}{3}$ of *BC*.

BC, *CA* are in the ratio 4 : 5 means,

CA is $\frac{5}{4}$ times *BC*.

So, measured by *BC*, the length of *AB* is $\frac{2}{3}$, the length of *BC* is 1 and the length of *CA* is $\frac{5}{4}$.

What if we measure by $\frac{1}{12}$ of *BC*?

All lengths get multiplied by 12.

That is, length of *AB* is $\frac{2}{3} \times 12 = 8$, the length of *BC* is 12, and the length of *CA* is $\frac{5}{4} \times 12 = 15$.

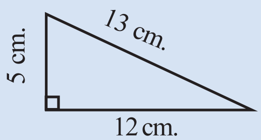
Ratio of sides 8 : 12 : 15

We can do this using algebra also.

The first ratio given means, *AB* is 2 times some length and *BC* is 3 times this length. What about the second ratio?

United triangles

A triangle with sides 5 centimetres, 12 centimetres, 13 centimetres is right:



Can we join any triangle with sides in the ratio 3 : 4 : 5 to this to make a large triangle? How many such triangles are there? What are the lengths of their sides?

Concrete mixture

To make concrete, for every sack of cement, two sacks of sand and for every sack of sand, two sacks of gravel are taken. How many sacks of gravel are needed for each sack of cement? For two sacks of sand, four sacks of gravel must be taken. Thus for each sack of cement, two sacks of sand and four sacks of gravel must be taken.

We can put it like this.

The ratio of cement to sand and sand to gravel are both 1 : 2. Writing the second ratio as 2 : 4, we can easily see that cement, sand and gravel are in the ratio 1 : 2 : 4.

BC is 4 times a length and CA is 5 times this length. The length of BC is not the same when measured by these little lengths; so these lengths are also different. Taking them as x and y centimeters,

AB = $2x$ centimetres, BC = $3x$ centimetres

BC = $4y$ centimetres, CA = 5y centimetres

Both $3x$ and $4y$ refer to the length of BC.

So, $3x = 4y$, which gives.

$$y = \frac{3}{4}x$$

Thus we find

$$CA = 5y \text{ cm.} = 5 \times \frac{3}{4}x \text{ cm.} = \frac{15}{4}x \text{ cm.}$$

and hence

$$AB = 2x \text{ cm.}$$
$$BC = 3x \text{ cm.}$$
$$CA = \frac{15}{4}x \text{ cm.}$$

This gives the ratio of AB, BC, CA as $2 : 3 : \frac{15}{4}$. In terms of natural numbers, we can write this as 8 : 12 : 15.

Another way

AB and BC are in the ratio 2 : 3 means, AB is $\frac{2}{3}$ of BC. BC and CA are in the ratio 4 : 5 means CA is $\frac{5}{4}$ times BC.

This can be put like this: measured by $\frac{1}{3}$ of BC, the length of AB is 2; and measured by $\frac{1}{4}$ of BC, the length of CA is 5. What if we measure by $\frac{1}{12}$ of BC? The length of AB would be 8 and the length of CA would be 15. And the length of BC itself is 12. Thus AB, BC, CA are in the ratio 8 : 12 : 15.

1) Johny invested 50000 rupees, Jaleel 40000 rupees and Jayan 20000 rupees to start a business together. They got 3300 rupees as profit in a month, which they divided in the ratio of their investments. How much did each get?

2. The capacities of three water tanks are in the ratio 2 : 3 : 5. The smallest of them can hold 2500 litres. How many litres can the other two hold?

3. The angles of a triangle are in the ratio 1 : 3 : 5. How much is each angle?

4. The outer angles of triangle are in the ratio 5 : 6 : 7. What are the angles?