Class 11

Important Formulas

Relations and Functions

Relations:

1. An ordered pair consists of two objects or elements in a given fixed order.
2. $(a_1, b_1) = (a_2, b_2) \Leftrightarrow a_1 = a_2$ and $b_1 = b_2$
3. If A and B are two non-empty sets, then $A \times B = \{(a, b) : a \in A, b \in B\}$ is called the cartesian product of A and B. If A and B are finite sets having m and n elements respectively, then $A \times B$ has mn elements.
4. $R \times R = \{(x, y) : x, y \in R\}$ is the set of all points in xy-plane.
5. $R \times R \times R = \{(x, y, z) : x, y, z \in R\}$ set of all points in three dimensional space.
6. For any three sets A, B, C, we have
   (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
   (iii) $A \times (B - C) = A \times B - A \times C$ (iv) $A \times B = B \times A \Leftrightarrow A = B$
   (v) $(A \times B) \cap (B \times A) = (A \cap B) \times (B \cap A)$ (vi) $A \times (B' \cup C')' = (A \times B) \cap (A \times C)$
   (vii) $A \times (B' \cap C')' = (A \times B) \cup (A \times C)$ (viii) $A \times B = A \times C \Rightarrow B = C$
7. Let A and B be two sets. A relation from A to B is a subset of $A \times B$.
8. If A and B are finite sets having m and n elements respectively. Then, 2^{mn} relations can be defined from A to B.
9. If R is a relation from set A to set B, then
   Domain $(R) = \{x : (x, y) \in R\}$, Range$(R) = \{y : (x, y) \in R\}$
10. A relation from a set A to itself is called a relation on A.
11. Let A, B be two sets and let R be a relation from set A to set B. Then the inverse of R, denoted by R^{-1}, is a relation from B to A and is defined by $R^{-1} = \{(b, a) : (a, b) \in R\}$.
    Clearly, $(a, b) \in R \Leftrightarrow (b, a) \in R^{-1}$
    Domain (R) = Range(R^{-1}), and Range (R) = Domain (R^{-1}).

Functions:

1. Let A and B be two non-empty sets. Then a relation f from A to B is a function, if
   (i) for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$
   (ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$.
   In other words, f is a function from A to B if each element of A appears in some ordered pair in f and no two ordered pairs in f have the same first element.
   If $(a, b) \in f$, then b is called the image of a under f.
2. A function f from a set A to a set B is a rule associating elements of set A to elements of set B such that every element in set A is associated to a unique elements in set B.
   The set A is called the domain of f and the set B is called its co-domain.
3. The range of a function f is the set of images of elements in the domain.
4. A real function has the domain and co-domain both as subsets of set R.
5. If $f : D_1 \to R$ and $g : D_2 \to R$ are two real functions and $c \in R$, then
   (i) $f \pm g : D_1 \cap D_2 \to R$ is defined as $(f \pm g)(x) = f(x) \pm g(x)$
   (ii) $fg : D_1 \cap D_2 \to R$ is defined as $(fg)(x) = f(x)\, g(x)$
   (iii) $\frac{f}{g} : D_1 \cap D_2 - \{x : g(x) = 0\} \to R$ is defined as $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$
   (iv) $cf : D_1 \cap D_2 \to R$ is defined as $(cf)(x) = c\, f(x)$.