Chapter 7 Powers Roots and Radicals

Ex 7.4

Answer 1e.

A common logarithm is denoted by $\log_{10}$ or simply log, and it is known as a logarithm with base 10.

Thus, the given statement can be completed as
A logarithm with base 10 is called a **common logarithm**.

Answer 1gp.

The given equation is of the form $\log_b y = x$. We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 3, y is 81, and x is 4.
Thus,
$3^4 = 81$.

The given equation can be written in the exponential form as $3^4 = 81$.

Answer 1mr.

a. The three points on the given graph are (3, 1), (5, 2), and (11, 3).
We know that by translating the graph of the parent function $y = \log_b x$, we get logarithmic function of the form $y = \log_b(x - h) + k$.

Substitute 3 for x, 1 for y, and 3 for b in $y = \log_b(x - h) + k$ to get the first equation.
$1 = \log_3(3 - h) + k$ Equation 1

Now, substitute 5 for x, 2 for y, and 3 for b in $y = \log_b(x - h) + k$ to get the second equation.
$2 = \log_3(5 - h) + k$ Equation 2

Subtract Equation 1 from Equation 2.

$$\begin{aligned} 2 &= \log_3(5-h) + k \\ 1 &= \log_3(3-h) + k \\ \hline 1 &= \log_3(5-h) - \log_3(3-h) \end{aligned}$$

Use the quotient property.

$$1 = \log_3 \frac{(5-h)}{(3-h)}$$

Simplify.

$$3^1 = \frac{5-h}{3-h}$$

$$3 = \frac{5-h}{3-h}$$

Solve for h.

$$9 - 3h = 5 - h$$

$$4 = 2h$$

$$2 = h$$

Substitute 2 for h in Equation 1.

$$1 = \log_3(3-2) + k$$

Solve for k.

$$1 = \log_3 1 + k$$

$$1 = k$$

Substitute the values for h, k and b in $y = \log_b(x-h) + k$.

$$y = \log_3(x-2) + 1$$

Thus, the equation of the graph is $y = \log_3(x-2) + 1$.

b. Switch the variables x and y.

$$x = \log_3(y-2) + 1$$

Subtract 1 from both the sides.

$$x - 1 = \log_3(y-2) + 1 - 1$$

$$x - 1 = \log_3(y-2)$$

Solve for y.

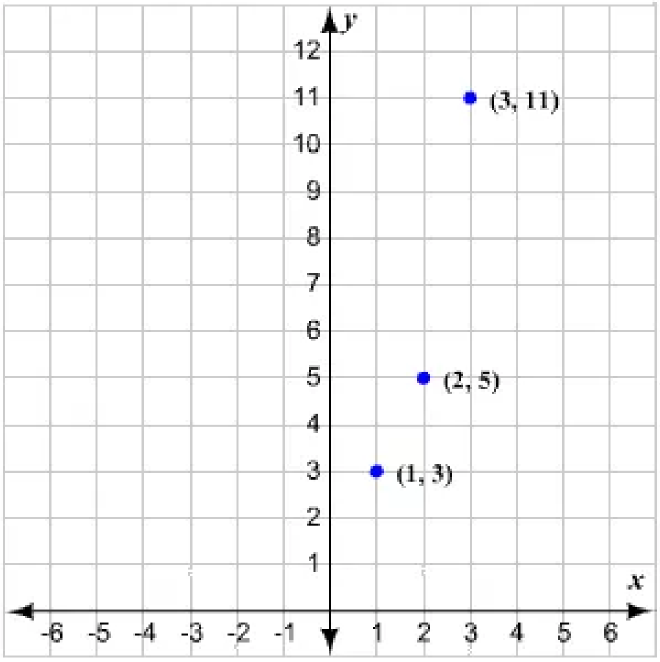
$$3^{x-1} = y - 2$$

$$3^{x-1} + 2 = y$$

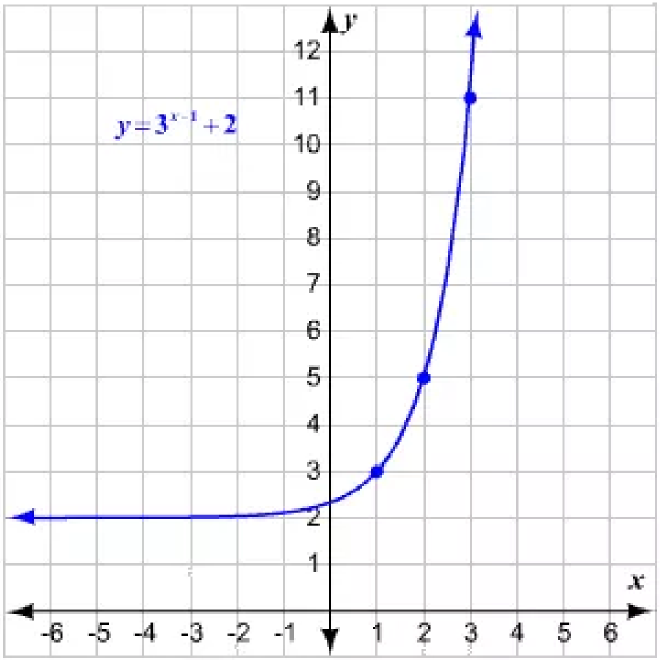
Thus, the inverse of the function is $y = 3^{x-1} + 2$.

Since the points on the given graph is (3, 1), (5, 2), and (11, 3), the corresponding points on its inverse function will be (1, 3), (2, 5), and (3, 11).

Plot the points on a coordinate plane.



Draw a curve that starts just to the right of the *y*-axis and moves up through the plotted points.



Answer 2e.

By the definition of a logarithm, it follows that the logarithmic function $g(x)=\log_b b^x = x$ is the inverse of the exponential function $f(x)=b^x$. This means that: $g(f(x))=\log_b b^x = x$ and $f(g(x))=b^{\log_b x}=x$

Describing the relationship between $y=5^x$ and $\log_5 x$.

Thus $y=5^x$ is the inverse function of $\log_5 x$.

Answer 2gp.

This definition tells you that the equations $\log_b y = x$ and $b^x = y$ are equivalent. The first is in logarithmic form and the second is in exponents form.

Rewriting the equations in exponential form

$\log_3 81 = 4$

$\log_3 3^4 = 4$ $\quad\left[\text{Because; } 81 = 3^4\right]$

$3^4 = 81$ $\quad\left[\text{Because; } \log_b y = x \text{ and } b^x = y\right]$

Therefore the exponential form is $\boxed{3^4 = 81}$.

Answer 2mr.

It is given that a piece of paper is folded in two regions and the area becomes half. This process is repeated. As long as the number of fold increases the area decreases.

(a)

For the first folding the regions become two and the area become half to maintain the total area of 1. Therefore if we fold the paper second times then the number of regions becomes 4 with equal area. Thus the area of each region will be $\frac{1}{4}$.

Similarly for the third fold of the paper the numbers of regions increase to 8 and the area of each region become $\frac{1}{8}$ and for the fourth fold the numbers of regions increase to 16 and the area become $\frac{1}{16}$.

Therefore the complete table is:

Fold number	0	1	2	3	4
Number of regions	1	2	$\boxed{4}$	$\boxed{8}$	$\boxed{16}$
Fractional area of each regions	1	$\frac{1}{2}$	$\boxed{\frac{1}{4}}$	$\boxed{\frac{1}{8}}$	$\boxed{\frac{1}{16}}$

(b)

As long as the number of folding increases, the numbers of regions increase with the power of 2 and the area of each region decrease with the power of $\frac{1}{2}$.

Therefore the number of regions $R(n)$ is:

$$\boxed{R(n)=2^n}$$

And the area of each fractional regions $A(n)$ after n fold is:

$$\boxed{A(n)=\frac{1}{2^n}}$$

Answer 3e.

The given equation is of the form $\log_b y = x$. We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 4, y is 16, and x is 2.
Thus,
$4^2 = 16$.

The given equation can be written in the exponential form as $4^2 = 16$.

Answer 3gp.

The given equation is of the form $\log_b y = x$. We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 14, y is 1, and x is 0.
Thus,
$14^0 = 1$.

The given equation can be written in the exponential form as $14^0 = 1$.

Answer 3mr.

We know that if a real life quantity increases by a fixed percent each year, the amount y of the quantity after t years can be modeled by $y = a(1 + r)^t$.

Let us assume that the value of r is 3%.
Since the amount of one item increases by r%, substitute 0.03 for r, 2 for t, and 100 for y in the model.
$100 = a(1 + 0.03)^2$

Solve for a.

$$\begin{aligned} 100 &= a(1.03)^2 \\ 100 &= a(1.0609) \\ 94.26 &= a \end{aligned}$$

In order to find the function, substitute for a, and r in $y = a(1 + r)^t$.
$y = 94.26(1 + 0.03)^t$

Simplify.
$y = 94.26(1.03)^t$

Thus, the function is $y = 94.26(1.03)^t$.

We know that if a real life quantity decreases by a fixed percent each year, the amount y of the quantity after t years can be modeled by $y = a(1 - r)^t$.

Since the amount of another item decreases by r%, substitute 0.03 for r, 2 for t, a_1 for a, and 100 for y in the model.
$100 = a_1(1 - 0.03)^2$

Solve for a_1.

$$\begin{aligned} 100 &= a_1(0.97)^2 \\ 100 &= a_1(0.9409) \\ 106.281 &= a_1 \end{aligned}$$

In order to find the function, substitute for a_1, and r in $y = a(1 - r)^t$.
$y = 106.281(1 - 0.03)^t$

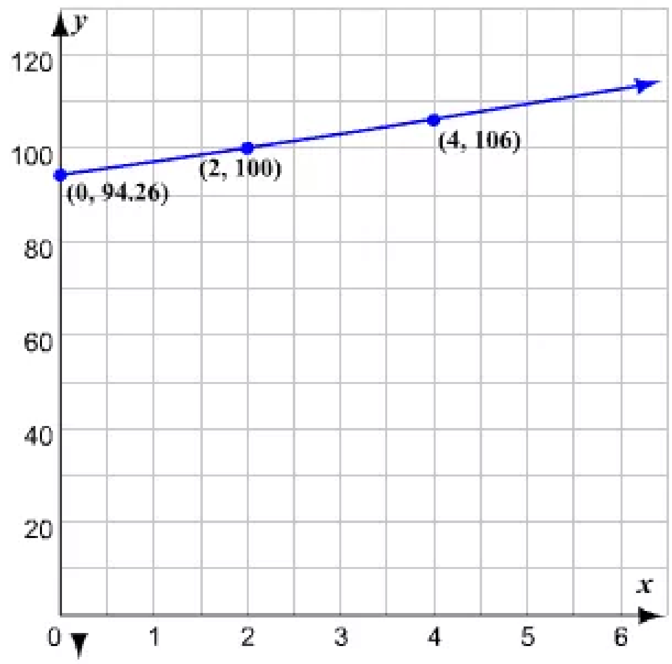
Simplify.
$y = 106.281(0.97)^t$

Thus, the function is $y = 106.281(0.97)^t$.

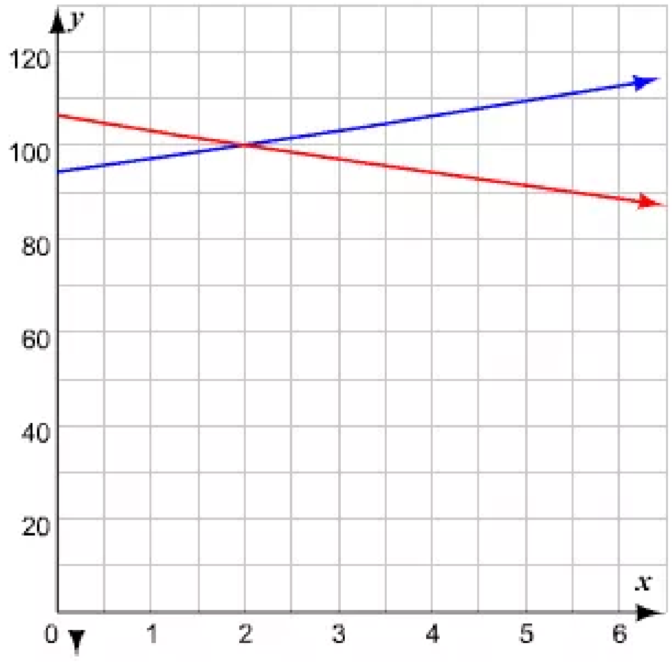
Make a table of values to graph $y = 94.26(1.03)^t$.

t	0	2	4
y	94.26	100	106

Plot the points and connect them using a straight line.



Similarly, graph $y = 106.281(0.97)^t$. on the same coordinate plane.



Answer 4e.

This definition tells you that the equations $\log_b y = x$ and $b^x = y$ are equivalent. The first is in logarithmic form and the second is in exponents form.

Rewriting the equations in exponential form

$\log_7 343 = 3$

$\log_7 7^3 = 3$ $\quad$ [Because; $343 = 7^3$]

$7^3 = 343$ $\quad$ [Because; $\log_b y = x$ and $b^x = y$]

Therefore the exponential form is $\boxed{7^3 = 343}$.

Answer 4gp.

This definition tells you that the equations $\log_b y = x$ and $b^x = y$ are equivalent. The first is in logarithmic form and the second is in exponents form.

Rewriting the equations in exponential form

$\log_{1/2} 32 = -5$

$\log_{1/2} \left(\frac{1}{2}\right)^{-5} = -5$ $\quad$ [Because; $32 = \left(\frac{1}{2}\right)^{-5}$]

$\left(\frac{1}{2}\right)^{-5} = 32$ $\quad$ [Because; $\log_b y = x$ and $b^x = y$]

Therefore the exponential form is $\boxed{\left(\frac{1}{2}\right)^{-5} = 32}$.

Answer 4mr.

Continuously Compounded Interest

When interest is compounded continuously, the amount A is an account after *t* years is given by the formula

$A = Pe^{rt}$

Where P is the principal and *r* is the annual interest rate expressed as a decimal.

We are starting with 2000 dollars and earning interest for two years.

Our interest rate is $4\% = 0.04$ so we have,

$A = Pe^{rt}$	
$= 2000e^{0.04(2)}$	**Substitute**
$= 2000e^{0.08}$	
$= 2000(1.0833)$	**Apply exponent**
$= 2166.60$	**Multiply**

So you have made \$166.60 after two years.

To find out how many years it will take to reach or exceed 2250 set with $A \geq 2250$.

$2250 \geq 2000e^{0.04t}$	**Substitute**
$\frac{2250}{2000} \geq e^{0.04t}$	**Divide both sides by 2000**
$1.125 \geq e^{0.04t}$	**Divide**
$\ln 1.125 \geq \ln e^{0.04t}$	**Take natural log**
$\ln 1.125 \geq 0.04t$	$\ln e^{x} = x$
$\frac{\ln 1.125}{0.04} \geq t$	**Divide both sides by 0.04**
$2.9446 \approx t$	**Use a calculator**

Here, $t = 2.9446$, but we are looking for how many full years it will be so we will have to wait 3 years.

Answer 5e.

The given equation is of the form $\log_b y = x$. We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 6, y is $\frac{1}{36}$, and x is -2.

Thus,

$6^{-2} = \frac{1}{36}$.

The given equation can be written in the exponential form as $6^{-2} = \frac{1}{36}$.

Answer 5gp.

In order to evaluate the logarithm, we need to find a number that gives 32 when 2 is raised to that number.

The number 2 to a power of 5 gives 32.
$2^5 = 32$

We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 2, y is 32, and x is 5. Thus,
$\log_2 32 = 5$.

Answer 5mr.

a. The amount A in an account after t years for a continuously compounded interest is given by the formula $A = Pe^{rt}$ where P is the principal and r is the annual interest rate expressed as a decimal.

In the first case, the principal is \$1500, annual interest expressed in decimal is 0.02, and the time is 3 years.
Substitute 1500 for P, 0.02 for r, and 3 for t in the formula
$A = 1500e^{0.02(3)}$

Now, evaluate using a calculator.

$$\begin{aligned} A &= 1500e^{0.06} \\ &\approx 1592.75 \end{aligned}$$

The balance after 3 years is about \$1592.75.

Now, subtract the principal amount from the balance at the end of 3 years.
$1592.75 - 1500 = 92.75$

Thus, the amount of interest earned in 3 years is about \$92.75.

In the second case, the principal is \$2000, annual interest expressed in decimal is 0.03, and the time is 5 years.
Substitute 2000 for P, 0.03 for r, and 5 for t in the formula
$A = 2000e^{0.03(5)}$

Now, evaluate using a calculator.

$$A = 2000e^{0.15}$$
$$\approx 2323.67$$

The balance after 5 years is about $2323.67.

Now, subtract the principal amount from the balance at the end of 5 years.

$2323.67 - 2000 = 323.67$

Thus, the amount of interest earned in 5 years is about $323.67.

b. We know that the interest earned by the three-year CD is about $92.75 and that by the five-year CD is about $323.67.

Subtract interest earned by the three-year CD is about $92.75 from that of the five-year CD.

$323.67 - 92.75 = 230.92$

c. In the first case we are depositing $1500 for 3 years. After 3 years we get an interest of $92.73. Thus, the benefit is that we are getting the invested amount after 3 years and the drawback is that the interest rate is 2% and the amount of interest earned will be only $92.7.

In the second case we are depositing $2000 for 5 years. After 5 years we get an interest of $323.67. Thus, the benefit is that the interest rate is 3% and the amount of interest earned will be $323.67 and the drawback is that we are getting the invested amount only after 5 years.

Answer 6e.

This definition tells you that the equations $\log_b y = x$ and $b^x = y$ are equivalent. The first is in logarithmic form and the second is in exponents form.

Rewriting the equations in exponential form

$\log_{64} 1 = 0$

$\log_{64} 64^0 = 0$ [Because; $64^0 = 1$]

$64^0 = 1$ [Because; $\log_b y = x$ and $b^x = y$]

Therefore the exponential form is $\boxed{64^0 = 1}$.

Answer 6gp.

Evaluating the logarithm

$\log_{27} 3$

$= \log_{27} 27^{1/3}$ $\quad \left[\text{Because; } 27^{1/3} = 3\right]$

$= 1/3$ $\quad \left[\text{Use the formula } \log_b b^x = x\right]$

Therefore the answer is $\boxed{1/3}$.

Answer 6mr.

It is given that the amount of tritium (in milligram) left after t years is:

$$y = 10e^{-0.0564t}$$

We need to calculate the amount of tritium left after 10 years.

Substituting t by 10 in $y = 10e^{-0.0564t}$, we have

$$y = 10e^{-0.0564 \times 10}$$
$$= 10 \times e^{-0.564}$$
$$= 10 \times 0.5689$$
$$= 5.689$$
$$y \approx 5.69$$

Therefore the amount of tritium left after 10 years is $\boxed{5.69 \text{ milligrams}}$.

Answer 7e.

The given equation is of the form $\log_b y = x$. We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 2, y is $\frac{1}{8}$, and x is -3.

Thus,

$\log_2 \frac{1}{8} = -3$.

Therefore, the error is that -3 and $\frac{1}{8}$ have been interchanged.

Answer 7gp.

First, press [LOG] key. Enter the number 12. Now, press [ )] key and then [ENTER] key to display the result.

The display will be 1.079181246. This result might vary slightly depending on the calculator you use.

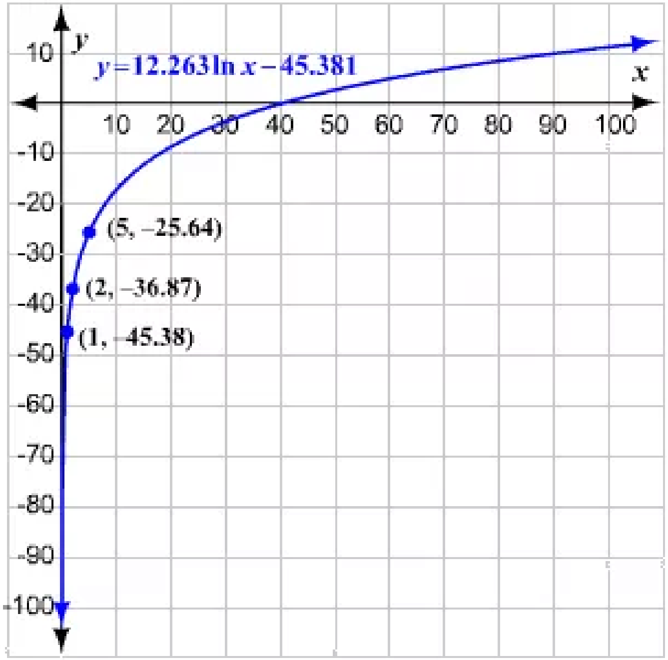
Therefore,
$\log 12 \approx 1.079181246$.

Answer 7mr.

a. Choose some points for x and find the corresponding y-values.

x	1	2	5
y	−45.38	−36.87	−25.64

Now, plot the points on a coordinate plane and connect them using a smooth curve.



b. Substitute 1000 for x in the given model.
$y = 12.263 \ln 1000 - 45.381$

Use a calculator to evaluate.
$y = 39$.

Thus, after drilling 1000 well you would collect about 39 billion barrels of oils.

c. Substitute 50 for y in the given model.

$50 = 12.263 \ln x - 45.381$

Add 45.381 to both the sides.

$$50 + 45.381 = 12.263 \ln x - 45.381 + 45.381$$
$$95.38 = 12.263 \ln x$$

Divide both the sides by 12.263.

$$\frac{95.38}{12.263} = \frac{12.263 \ln x}{12.263}$$
$$7.77 \approx \ln x$$

Exponentiate both the sides.

$$e^{7.78} = e^{\ln x}$$
$$2392.3 = x$$

Thus, to collect 50 billion barrels of oil you have to drill about 2392 wells.

Answer 8e.

Evaluating the logarithm

$\log_{15} 15$

$= \log_{15} 15^1$ [Because; $15^1 = 15$]

$= 1$ [Use the formula $\log_b b^x = x$]

Therefore the answer is $\boxed{1}$.

Answer 8gp.

Using the calculator to evaluate the logarithm

$\ln 0.75$

$= -0.287682072$ [Use calculator]

Therefore the answer is $\boxed{-0.287682072}$.

Answer 9e.

In order to evaluate the logarithm without using a calculator, we need to find a number that gives 49 when 7 is raised to that number.

The number 7 to a power of 2 gives 49.

$7^2 = 49$

We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 7, y is 49, and x is 2. Thus,
$\log_7 49 = 2$.

Answer 9gp.

Substitute 150 for d in the given model.

$$s = 93 \log d + 65$$
$$s = 93 \log 150 + 65$$

Use a calculator to evaluate.

$$s \approx 93(2.176) + 65$$
$$= 267.368$$

The wind speed near the tornado's center was about 267 miles per hour.

Answer 10e.

Evaluating the logarithm

$\log_6 216$

$= \log_6 6^3$ $\quad$ [Because; $6^3 = 216$]

$= 3$ $\quad$ [Use the formula $\log_b b^x = x$]

Therefore the answer is $\boxed{3}$.

Answer 10gp.

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b b^x = x$ is the inverse of the exponential function $f(x) = b^x$. This means that: $g(f(x)) = \log_b b^x = x$ and $f(g(x)) = b^{\log_b x} = x$

Simplifying the expression

$8^{\log_8 x}$

$= x$ $\quad$ [Use the formula $b^{\log_b x} = x$]

Therefore the answer is $\boxed{x}$.

Answer 11e.

In order to evaluate the logarithm without using a calculator, we need to find a number that gives 64 when 2 is raised to that number

The number 2 to a power of 6 gives 64.
$2^6 = 64$

We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 2, y is 64, and x is 6. Thus,
$\log_2 64 = 6$.

Answer 11gp.

We know that by an inverse property of logarithms, $\log_b b^x = x$.

In the given expression, the value of b is 7 and that of x is $-3x$.

Thus,
$\log_7 7^{-3x} = -3x$.

Answer 12e.

Evaluating the logarithm
$\log_9 1$

$= \log_9 9^0$ $\quad$ $\left[\text{Because; } 9^0 = 1\right]$

$= 0$ $\quad$ $\left[\text{Use the formula } \log_b b^x = x\right]$

Therefore the answer is $\boxed{0}$.

Answer 12gp.

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b b^x = x$ is the inverse of the exponential function $f(x) = b^x$. This means that: $g(f(x)) = \log_b b^x = x$ and $f(g(x)) = b^{\log_b x} = x$

Simplifying the expression

$\log_2 64^x$

$= \log_2 \left(2^6\right)^x$ [Express 62 as a power with base 2]

$= \log_2 2^{6x}$ [Power of a power property]

$= 6x$ [Use the formula $b^{\log_b x} = x$]

Therefore the answer is $\boxed{6x}$.

Answer 13e.

In order to evaluate the logarithm without using a calculator, we need to find a number that gives 8 when $\frac{1}{2}$ is raised to that number.

The number $\frac{1}{2}$ to a power of -3 gives 8.

$\left(\frac{1}{2}\right)^{-3} = 8$

We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is $\frac{1}{2}$, y is 8, and x is -3. Thus,

$\log_{\frac{1}{2}} 8 = -3$.

Answer 13gp.

We know that $e^{\ln x} = x$. In the given expression, the value of x is 20.

Thus,

$e^{\ln 20} = 20$.

Answer 14e.

Evaluating the logarithm

$\log_3 \frac{1}{27}$

$= \log_3 \frac{1}{3^3}$ $\left[\text{Because; } \frac{1}{3^3} = \frac{1}{27}\right]$

$= \log_3 3^{-3}$ $\left[\text{Because; } \frac{1}{3^3} = 3^{-3}\right]$

$= -3$ $\left[\text{Use the formula } \log_b b^x = x\right]$

Therefore the answer is $\boxed{-3}$.

Answer 14gp.

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b b^x = x$ is the inverse of the exponential function $f(x) = b^x$. This means that: $g(f(x)) = \log_b b^x = x$ and $f(g(x)) = b^{\log_b x} = x$

Finding the inverse of the function

$y = 4^x$

$y = \log_4 x$ [From the definition of logaritham]

Therefore the answer is $\boxed{\log_4 x}$.

Answer 15e.

In order to evaluate the logarithm without using a calculator, we need to find a number that gives $\frac{1}{4}$ when 16 is raised to that number.

We know that the number 16 to a power of $-\frac{1}{2}$ gives $\frac{1}{4}$.

$(16)^{-\frac{1}{2}} = \frac{1}{4}$

We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 16, y is $\frac{1}{4}$, and x is $-\frac{1}{2}$. Thus,

$$\log_{16} \frac{1}{4} = -\frac{1}{2}.$$

Answer 15gp.

Switch the variables x and y.

$x = \ln (y - 5)$

Rewrite the expression in exponential form.

Thus,

$e^x = y - 5.$

Add 5 to both the sides.

$$e^x + 5 = y - 5 + 5$$

$$e^x + 5 = y$$

An equation for the inverse of the function is

$y = e^x + 5.$

Answer 16e.

Evaluating the logarithm

$\log_{1/4} 16$

$= \log_{1/4} 4^2$ $\quad \left[\text{Because; } 4^2 = 16\right]$

$= \log_{1/4} \left(\frac{1}{4}\right)^{-2}$ $\quad \left[\text{Because; } 4^2 = \left(\frac{1}{4}\right)^{-2}\right]$

$= -2$ $\quad \left[\text{Use the formula } \log_b b^x = x\right]$

Therefore the answer is $\boxed{-2}$.

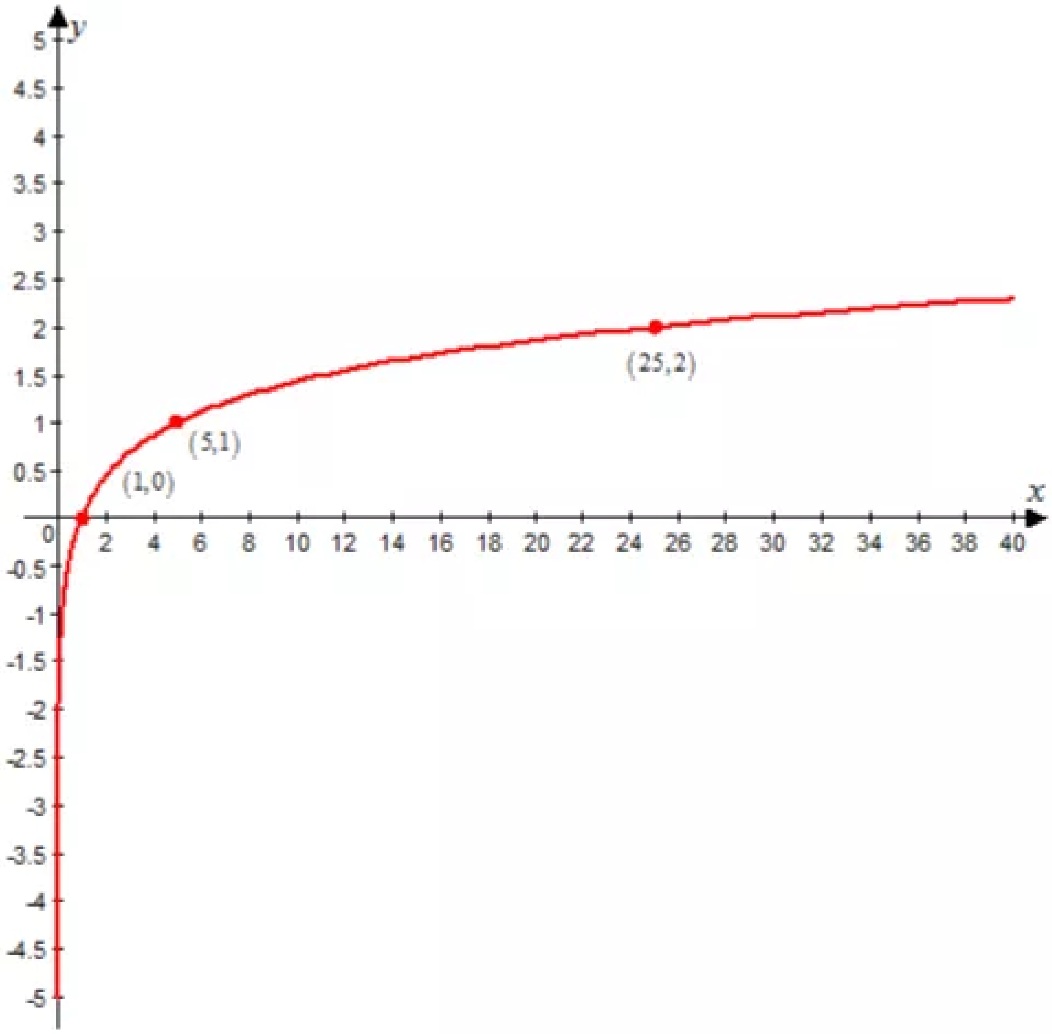
Answer 16gp.

Graphing logarithmic function:

$y = \log_5 x$

Plot several convenient points, such as $(1,0)$, $(5,1)$ and $(25,2)$. The y-axis is a vertical asymptote.

From left to right, draw a curve that starts just to the right of the y-axis and moves up through the plotted points, as shown below.



The domain is $x > 0$, and the range is all real numbers.

Answer 17e.

In order to evaluate the logarithm without using a calculator, we need to find a number that gives 512 when 8 is raised to that number.

We know that the number 8 to a power of 3 gives 512.
$8^3 = 512$

We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation, b is 8, y is 512, and x is 3. Thus,
$\log_8 512 = 3$.

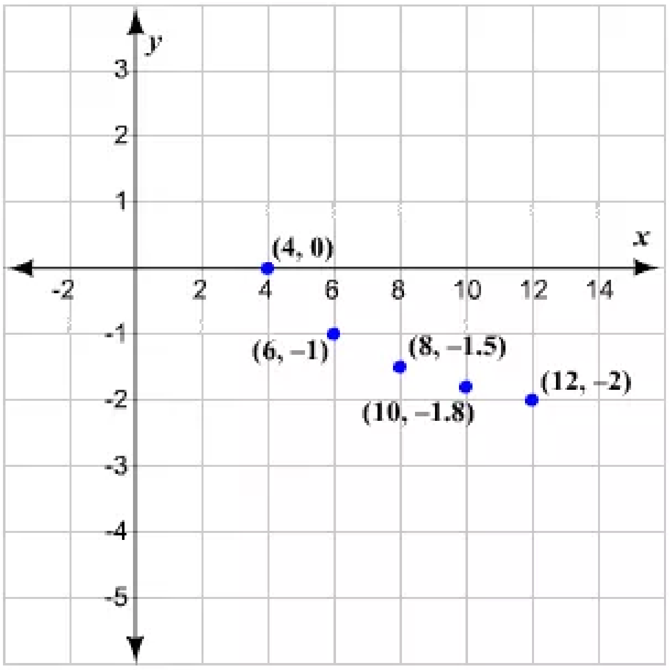
Answer 17gp.

Choose some values for x and find the corresponding y-values to graph the function. Organize the results in a table.

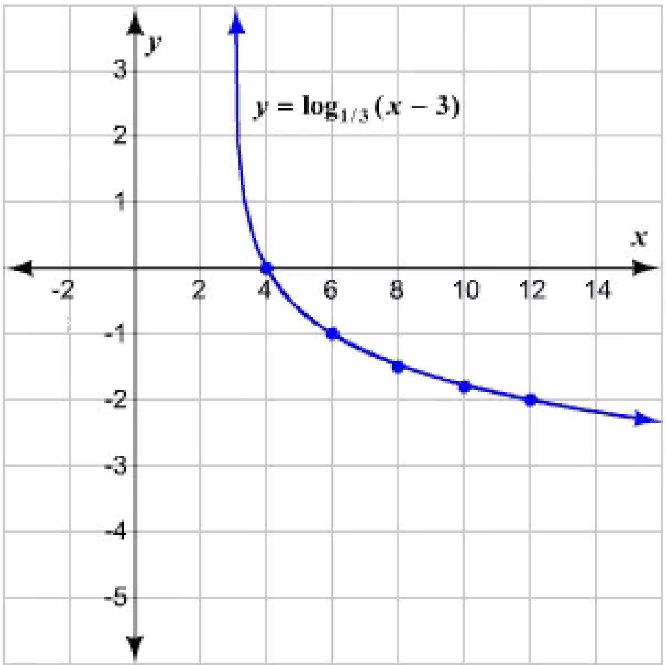
x	4	6	8	10	12
y	0	−1	−1.5	−1.8	−2

The points are (4, 0), (6, −1), (8, −1.5), (10, −1.8), and (12, −2).

Plot the points on a coordinate plane.



Now, from left to right, draw a curve that starts just to the right of the y-axis and moves up through the plotted points as shown below.



The set of input values is the domain and the set of output values is the range. The domain consists of all nonnegative values of x and the range consists of all values of y.

Thus, the domain of the given function is $x > 0$ and the range of the given function is all real numbers.

Answer 18e.

Evaluating the logarithm

$\log_5 625$

$= \log_5 5^4$ $\quad$ [Because; $5^4 = 625$]

$= 4$ $\quad$ [Use the formula $\log_b b^x = x$]

Therefore the answer is $\boxed{4}$.

Answer 18gp.

Graphing logarithmic function:

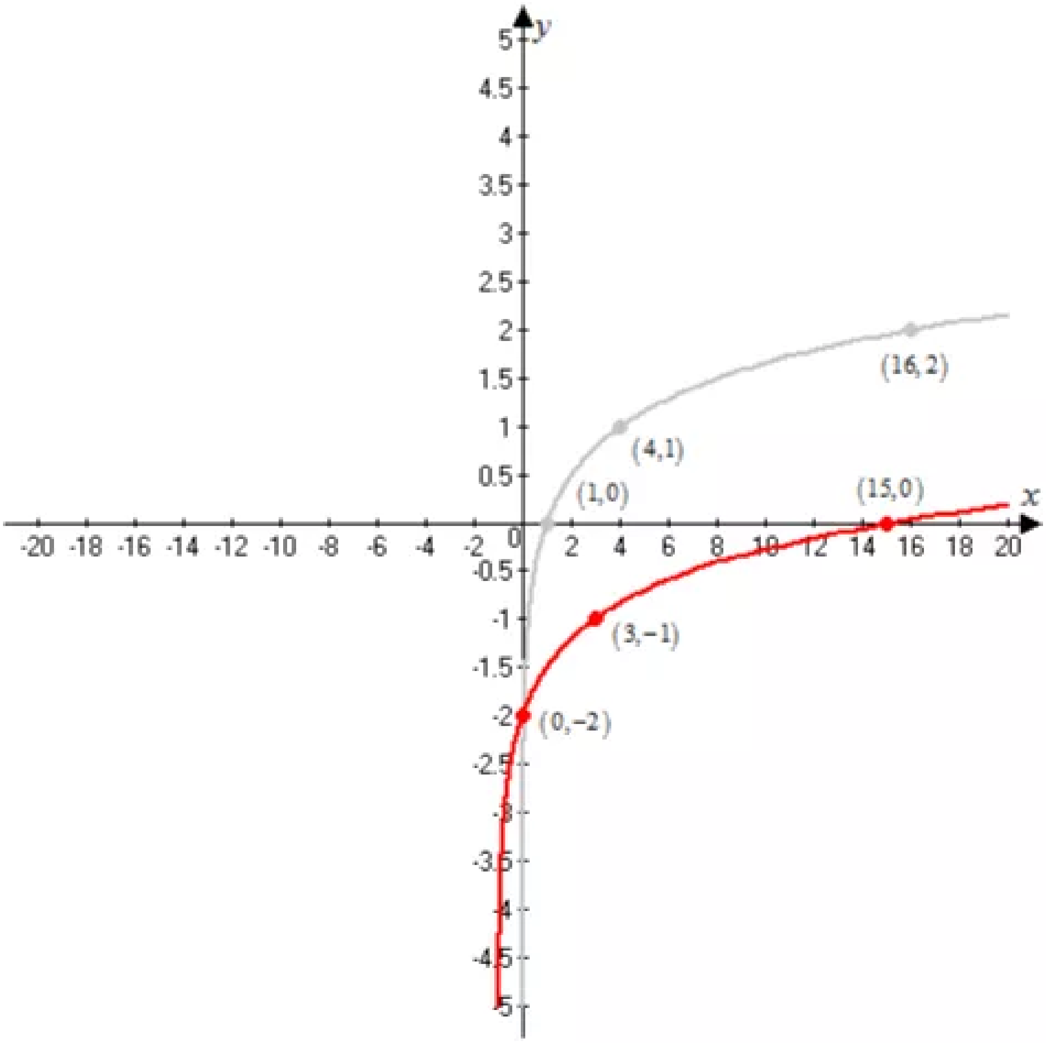
$y = \log_4(x+1) - 2$

Step 1

Sketch the graph of the parent function $y = \log_4 x$, which passes through $(1,0)$, $(4,1)$ and $(16,2)$.

Step 2

Translate the parent graph right 4 unit and up 2 units. The translated graph passes through $(0,-2)$, $(3,-1)$ and $(15,0)$.



The graph's asymptote is $x = -2$. The domain is $x > -2$, and the range is all real numbers.

Answer 19e.

In order to evaluate the logarithm without using a calculator, we need to find a number that gives 121 when 11 is raised to that number.

We know that the number 11 to a power of 2 gives 121.
$11^2 = 121$

We know that $\log_b y$ is defined as $\log_b y = x$ if and only if $b^x = y$, where b and y are positive numbers and $b \neq 0$.

In the given equation b is 11, y is 121, and x is 2. Thus,
$\log_{11} 121 = 2$.

Answer 20e.

Using the calculator to evaluate the logarithm
$\log 14$

$= 1.146128036$ [Use calculator]

Therefore the answer is $\boxed{1.146128036}$.

Answer 21e.

First, press $\boxed{\text{LN}}$ key. Enter the number 6. Now, press the $\boxed{)}$ key and then the $\boxed{\text{ENTER}}$ key to display the result.

The display will be 1.791759469. This result might vary slightly depending on the calculator you use.

Therefore,
$\ln 6 \approx 1.791759469$.

Answer 22e.

Using the calculator to evaluate the logarithm
$\ln 0.43$

$= -0.84397007$ [Use calculator]

Therefore the answer is $\boxed{-0.84397007}$.

Answer 23e.

First, press $\boxed{\text{LOG}}$ key. Enter the number 6.213. Now, press the $\boxed{)}$ key and then the $\boxed{\text{ENTER}}$ key to display the result.

The display will be 0.7933013536. This result might vary slightly depending on the calculator you use.

Therefore,
$\log 6.213 \approx 0.7933013536$.

Answer 24e.

Using the calculator to evaluate the logarithm

$\log 27$

$= 1.431363764$ [Use calculator]

Therefore the answer is $\boxed{1.431363764}$.

Answer 25e.

First, press $\boxed{\text{LN}}$ key. Enter the number 5.38. Now, press the $\boxed{)}$ key and then the $\boxed{\text{ENTER}}$ key to display the result.

The display will be 1.682688374. This result might vary slightly depending on the calculator you use.

Therefore,
$\ln 5.38 \approx 1.682688374$.

Answer 26e.

Using the calculator to evaluate the logarithm

$\log 0.746$

$= -0.127261172$ [Use calculator]

Therefore the answer is $\boxed{-0.127261172}$.

Answer 27e.

First, press $\boxed{\text{LN}}$ key. Enter the number 110. Now, press the $\boxed{)}$ key and then the $\boxed{\text{ENTER}}$ key to display the result.

The display will be 4.700480366. This result might vary slightly depending on the calculator you use.

Therefore,
$\ln 110 \approx 4.700480366$.

Answer 28e.

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b b^x = x$ is the inverse of the exponential function $f(x) = b^x$. This means that: $g(f(x)) = \log_b b^x = x$ and $f(g(x)) = b^{\log_b x} = x$

Simplifying the expression
$7^{\log_7 x}$

$= x$ $\qquad$ $\left[\text{Use the formula } b^{\log_b x} = x\right]$

Therefore the answer is $\boxed{x}$.

Answer 29e.

We know that by an inverse property of logarithms, $\log_b b^x = x$.

In the given expression, the value of b is 5.

Thus,
$\log_5 5^x = x$.

Answer 30e.

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b b^x = x$ is the inverse of the exponential function $f(x) = b^x$. This means that: $g(f(x)) = \log_b b^x = x$ and $f(g(x)) = b^{\log_b x} = x$

Simplifying the expression
$30^{\log_{30} 4}$

$= 4$ $\qquad$ $\left[\text{Use the formula } b^{\log_b x} = x\right]$

Therefore the answer is $\boxed{4}$.

Answer 31e.

We know that by an inverse property of logarithms, $b^{\log_b x} = x$.

Compare the given expression with $b^{\log_b x} = x$. We get $b = 10$ and $x = 8$.

Thus,
$10^{\log 8} = 8$.

Answer 32e.

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b b^x = x$ is the inverse of the exponential function $f(x) = b^x$. This means that: $g(f(x)) = \log_b b^x = x$ and $f(g(x)) = b^{\log_b x} = x$

Simplifying the expression
$\log_6 36^x$

$= \log_6 (6^2)^x$ [Express 36 as a power with base 6]

$= \log_6 6^{2x}$ [Power of a power property]

$= 2x$ [Use the formula $b^{\log_b x} = x$]

Therefore the answer is $\boxed{2x}$.

Answer 33e.

Rewrite 81 as a power of 3.
$\log_3 81^x = \log_3 (3^4)^x$

Use the power of a power property.
$\log_3 (3^4)^x = \log_3 3^{4x}$

We know that by an inverse property of logarithms, $b^{\log_b x} = x$.

In the given expression, b is 3 and x is $4x$. Thus,
$\log_3 3^{4x} = 4x$.

The given expression simplifies to $4x$.

Answer 34e.

By the definition of a logarithm, it follows that the logarithmic function $g(x)=\log_b b^x = x$ is the inverse of the exponential function $f(x)=b^x$. This means that: $g(f(x))=\log_b b^x = x$ and $f(g(x))=b^{\log_b x}=x$

Simplifying the expression

$\log_5 125^x$

$=\log_5 (5^3)^x$ [Express 125 as a power with base 3]

$=\log_5 5^{3x}$ [Power of a power property]

$=3x$ [Use the formula $b^{\log_b x}=x$]

Therefore the answer is $\boxed{3x}$.

Answer 35e.

Rewrite 32 as 2^5.

$\log_2 32^x = \log_2 (2^5)^x$

Use the power of a power property.

$\log_2 (2^5)^x = \log_2 2^{5x}$

We know that by an inverse property of logarithms, $b^{\log_b x} = x$.

In the given expression, b is 2 and x is $5x$. Thus,

$\log_2 2^{5x} = 5x$.

The given expression can be simplified as $5x$.

Answer 36e.

By the definition of a logarithm, it follows that the logarithmic function $g(x)=\log_b b^x = x$ is the inverse of the exponential function $f(x)=b^x$. This means that: $g(f(x))=\log_b b^x = x$ and $f(g(x))=b^{\log_b x}=x$

Expression is equivalent to $\log 100^x$.

$\log 100^x$

$= \log\left(10^2\right)^x$ [Express 100 as a power with base 10]

$= \log 10^{2x}$ [Power of a power property]

$= 2x$ [Use the formula $b^{\log_b x} = x$]

Therefore the answer is option $\boxed{B. 2x}$.

Answer 37e.

Switch the variables x and y.
$x = \log_8 y$

We know that $\log_b p$ can be defined as $\log_b p = q$ if and only if $b^q = p$, where b and p are positive numbers and $b \neq 0$.

Since the value of b is 8, $y = 8^x$. An equation for the inverse of the function is $y = 8^x$.

Answer 38e.

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b b^x = x$ is the inverse of the exponential function $f(x) = b^x$. This means that: $g(f(x)) = \log_b b^x = x$ and $f(g(x)) = b^{\log_b x} = x$

Finding the inverse of the function

$y = 7^x$

$y = \log_7 x$ [From the definition of logaritham]

Therefore the answer is $\boxed{\log_7 x}$.

Answer 39e.

Switch the variables x and y.
$x = (0.4)^y$

We know that $\log_b p$ can be defined as $\log_b p = q$ if and only if $b^q = p$, where b and p are positive numbers and $b \neq 0$.

Since the value of b is 0.4, $y = \log_{0.4} x$. An equation for the inverse of the function is $y = \log_{0.4} x$.

Answer 40e.

By the definition of a logarithm, it follows that the logarithmic function $g(x)=\log_b b^x = x$ is the inverse of the exponential function $f(x)=b^x$. This means that: $g(f(x))=\log_b b^x = x$ and $f(g(x))=b^{\log_b x} = x$

Finding the inverse of the function

$y=\log_{1/2} x$

$y=(1/2)^x$ [From the definition of logaritham]

Therefore the answer is $\boxed{(1/2)^x}$.

Answer 41e.

Switch the variables x and y.

$x = e^{y+2}$

We know that $\log_b p$ can be defined as $\log_b p = q$ if and only if $b^q = p$, where b and p are positive numbers and $b \neq 0$.

Thus,

$\ln x = y + 2$.

Subtract 2 from each side.

$\ln x - 2 = y + 2 - 2$

$\ln x - 2 = y$

An equation for the inverse of the function is

$y = \ln x - 2$.

Answer 42e.

By the definition of a logarithm, it follows that the logarithmic function $g(x)=\log_b b^x = x$ is the inverse of the exponential function $f(x)=b^x$. This means that: $g(f(x))=\log_b b^x = x$ and $f(g(x))=b^{\log_b x} = x$

Finding the inverse of the function

$$y = 2^x - 3$$

$$y + 3 = 2^x \quad \text{[Add 3 to each side]}$$

$$\log_2(y+3) = x \quad \text{[Definition of logritham]}$$

$$\log_2(x+3) = y \quad \text{[Switch } x \text{ and } y\text{]}$$

$$y = \log_2(x+3)$$

Therefore the answer is $\boxed{y = \log_2(x+3)}$.

Answer 43e.

Switch the variables x and y.
$x = \ln(y + 1)$

Rewrite the expression in exponential form.
Thus,
$e^x = y + 1$.

Subtract 1 from each side.

$$e^x - 1 = y + 1 - 1$$

$$e^x - 1 = y$$

An equation for the inverse of the function is
$y = e^x - 1$.

Answer 44e.

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b b^x = x$ is the inverse of the exponential function $f(x) = b^x$. This means that: $g(f(x)) = \log_b b^x = x$ and $f(g(x)) = b^{\log_b x} = x$

Finding the inverse of the function

$$y = 6 + \log x$$

$$x = 6 + \log y \quad \text{[Switch } x \text{ and } y\text{]}$$

$$x - 6 = \log y \quad \text{[Subtract 6 from each side]}$$

$$10^{x-6} = y \quad \text{[Definition of logrithm]}$$

$$y = 10^{x-6}$$

Therefore the answer is $\boxed{10^{x-6}}$.

Answer 45e.

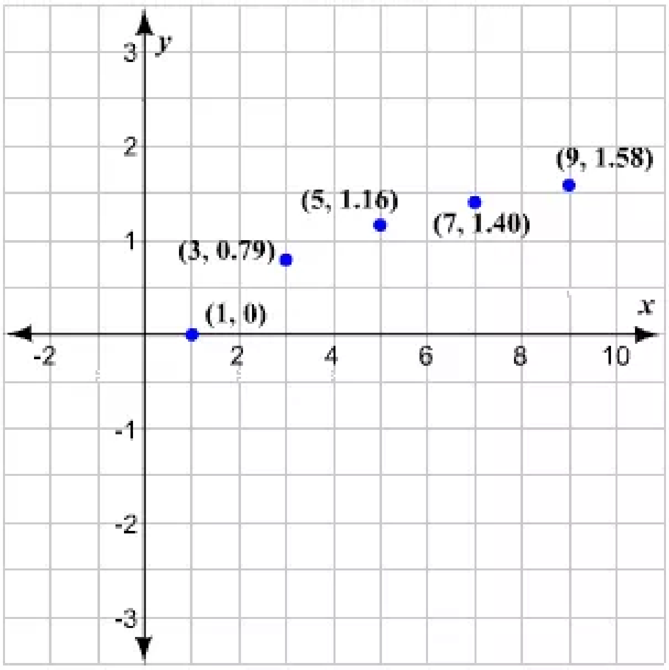
In order to graph the function, choose some values for x and find the corresponding y-values.

Organize the results in a table.

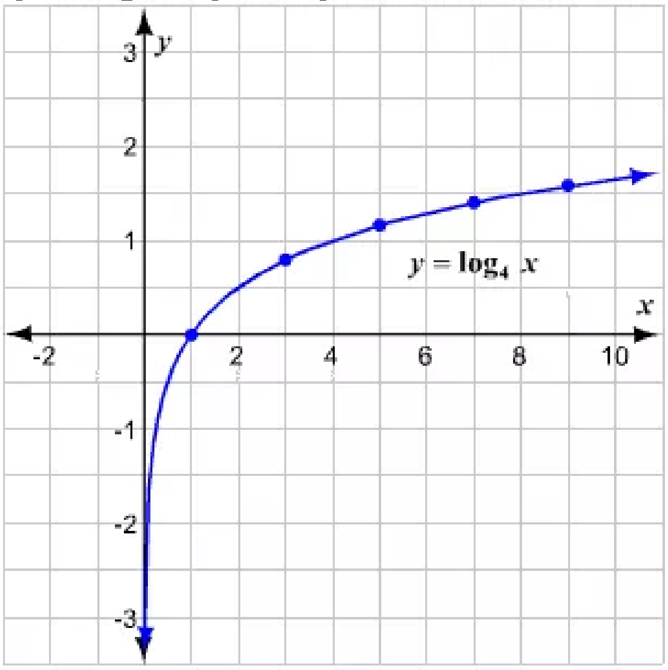
x	1	3	5	7	9
y	0	0.79	1.16	1.40	1.58

The points are (1, 0), (3, 0.79), (5, 1.16), (7, 1.40), and (9, 1.58).

Plot the points on a coordinate plane.



Now, from left to right, draw a curve that starts just to the right of the y-axis and moves up through the plotted points as shown below.



The set of input values is the domain and the set of output values is the range. The domain consists of all nonnegative values of x and the range consists of all the values of y.

Thus, the domain of the given function is $x > 0$ and the range of the given function is all real numbers.

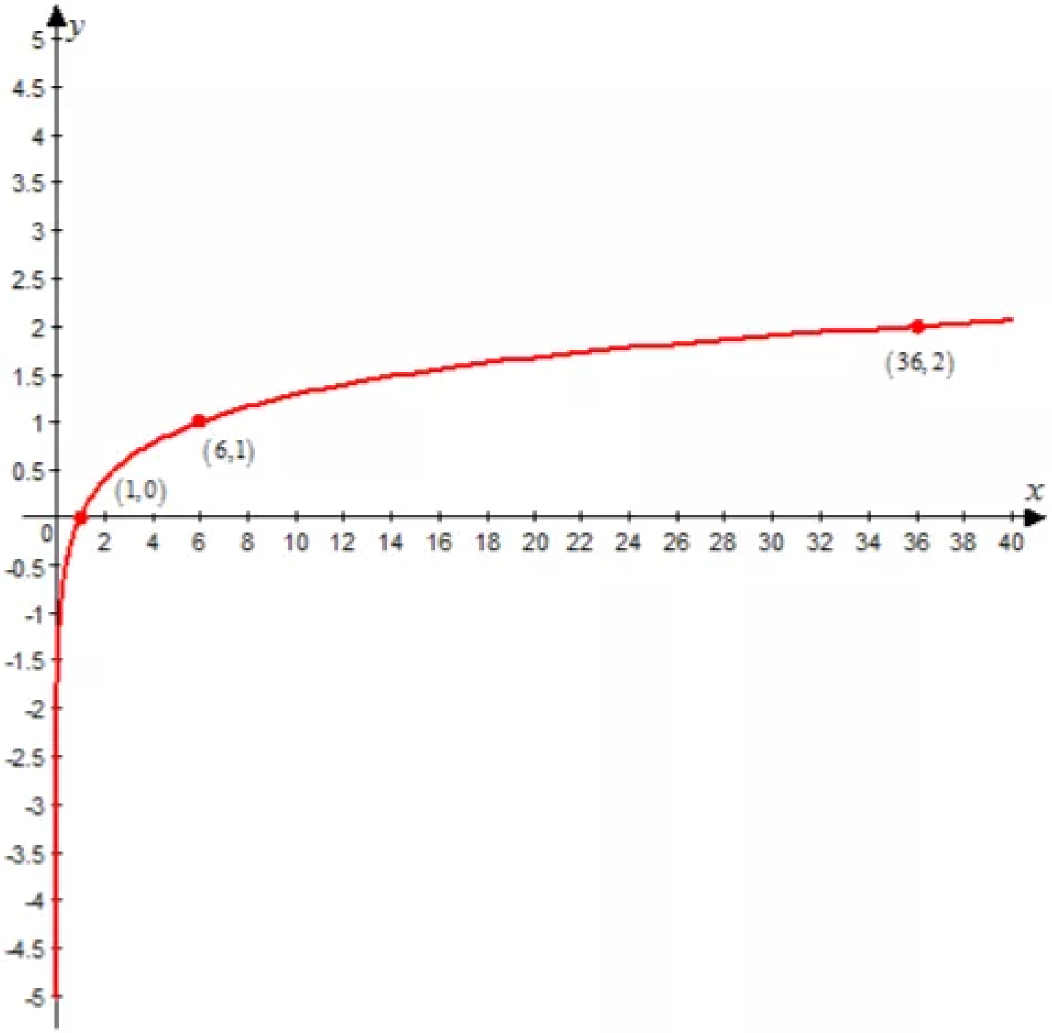
Answer 46e.

Graphing logarithmic function:

$y = \log_6 x$

Plot several convenient points, such as $(1,0)$, $(6,1)$ and $(36,2)$. The y-axis is a vertical asymptote.

From left to right, draw a curve that starts just to the right of the y-axis and moves up through the plotted points, as shown below.



The domain is $x > 0$, and the range is all real numbers.

Answer 47e.

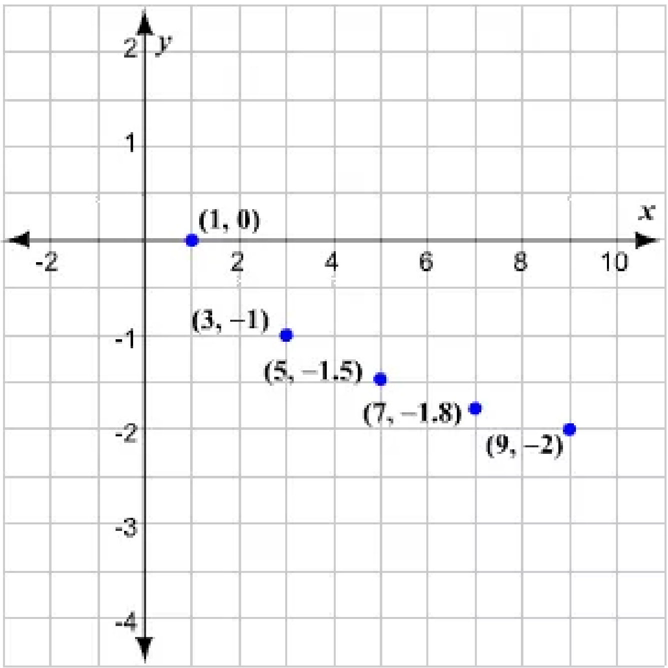
In order to graph the function, choose some values for x and find the corresponding y-values.

Organize the results in a table.

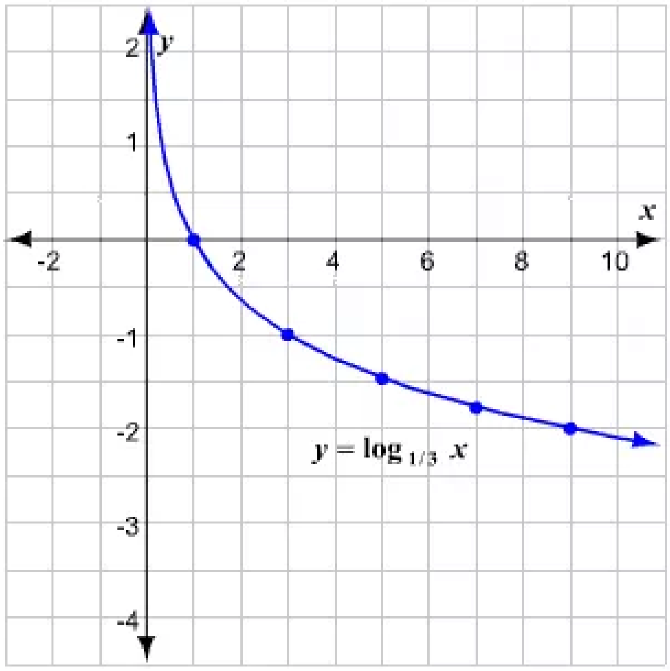
x	1	3	5	7	9
y	0	−1	−1.5	−1.8	−2

The points are (1, 0), (3, −1), (5, −1.5), (7, −1.8), and (9, −2).

Plot the points on a coordinate plane.



Now, from left to right, draw a curve that starts just to the right of the y-axis and moves up through the plotted points as shown below.



The set of input values is the domain and the set of output values is the range. The domain consists of all nonnegative values of x and the range consists of all the values of y.

Thus, the domain of the given function is $x > 0$ and the range of the given function is all real numbers.

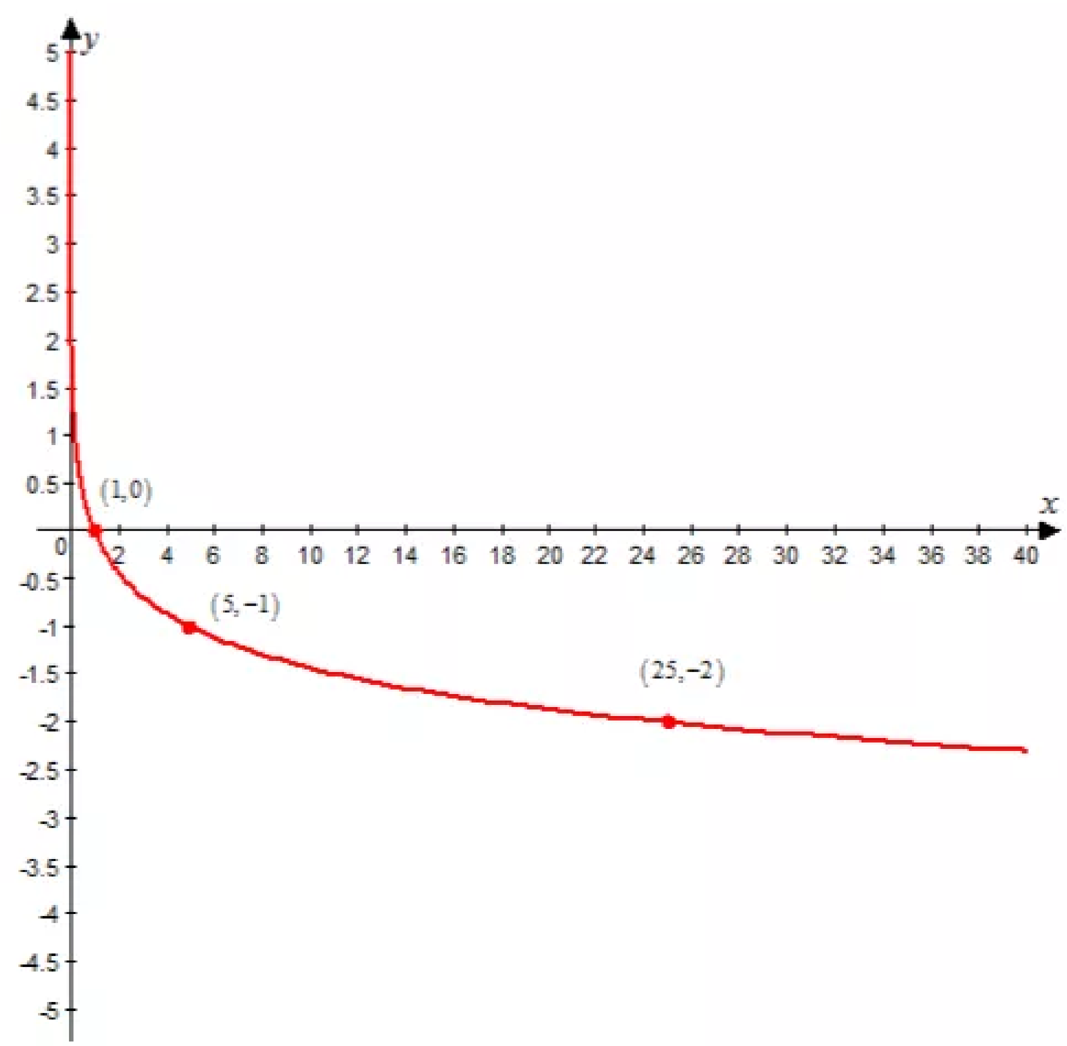
Answer 48e.

Graphing logarithmic function:

$y = \log_{1/5} x$

Plot several convenient points, such as $(1,0)$, $(5,-1)$ and $(25,-2)$. The y-axis is a vertical asymptote.

From left to right, draw a curve that starts just to the right of the y-axis and moves up through the plotted points, as shown below.



The domain is $x > 0$, and the range is all real numbers.

Answer 49e.

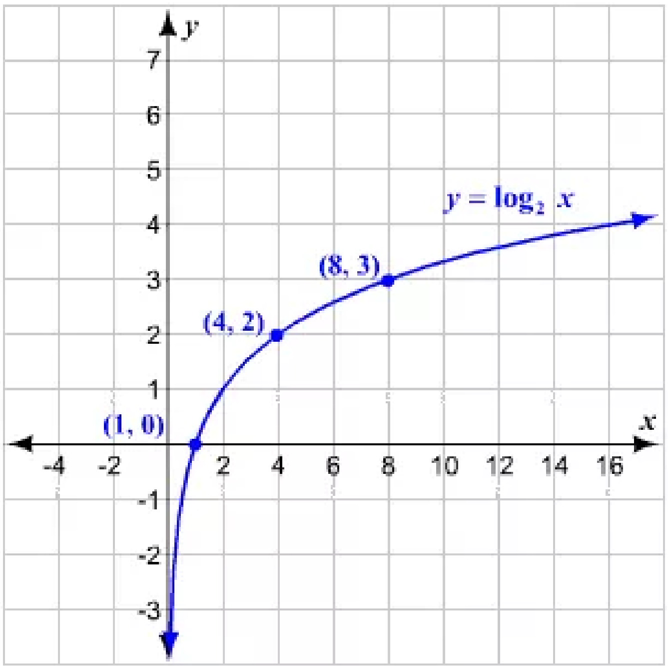
In order to graph the given function, first graph the parent function $y = \log_2 x$ on a coordinate plane.

Choose some values for x and find the corresponding y-values.
Organize the results in a table.

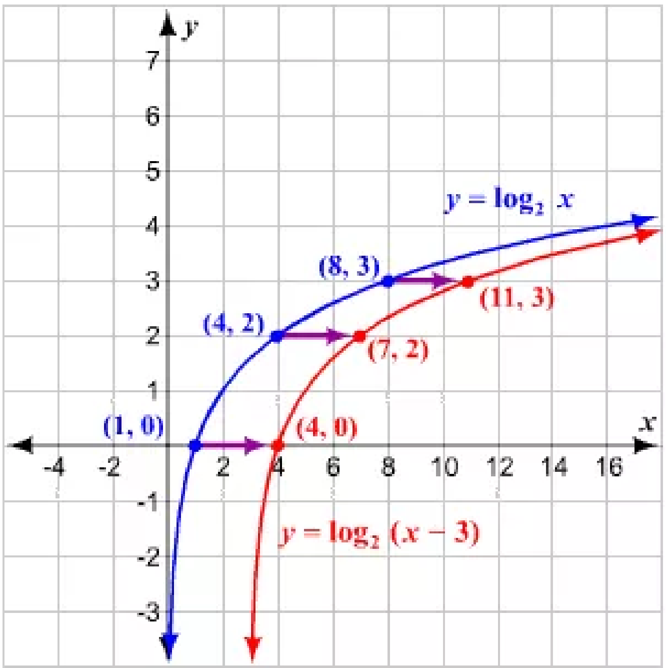
x	1	4	8
y	0	2	3

The points are (1, 0), (4, 2), and (8, 3).

Plot the points on a coordinate plane and draw a curve that starts just to the right of the y-axis and moves up through the plotted points as shown below.



Now, translate the graph of $y = \log_2 x$ right by 3 units. The resulting graph passes through the points (4, 0), (7, 2), and (11, 3).

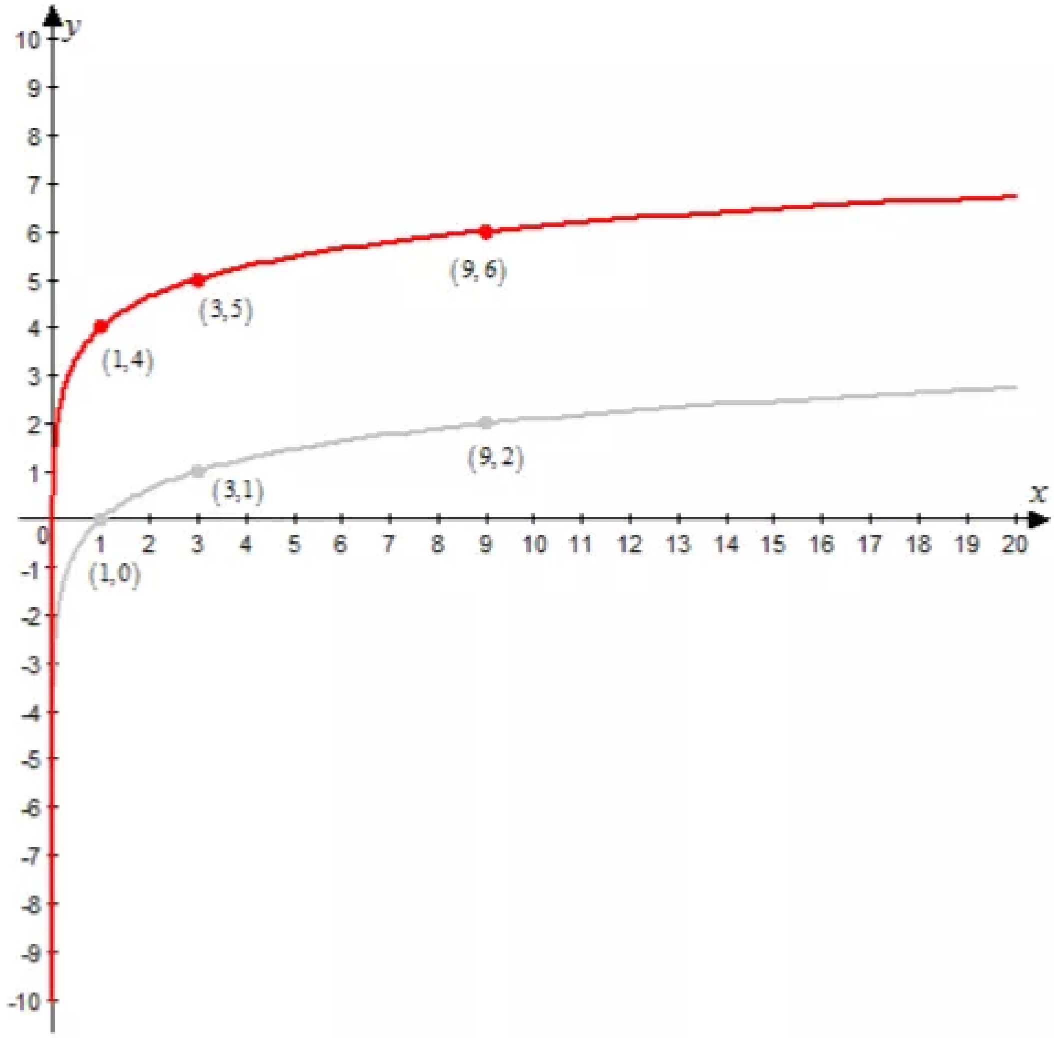


The set of input values is the domain and the set of output values is the range.

Thus, the domain of the given function is $x > 3$ and the range of the given function is all real numbers.

Answer 50e.

Graphing logarithmic function:
$y = \log_3 x + 4$
Step 1
Sketch the graph of the parent function $y = \log_3 x$, which passes through $(1,0)$, $(3,1)$ and $(9,2)$.
Step 2
Translate the parent graph up 4 units. The translated graph passes through $(1,4)$, $(3,5)$ and $(9,6)$.



The domain is $x > 0$, and the range is all real numbers.

Answer 51e.

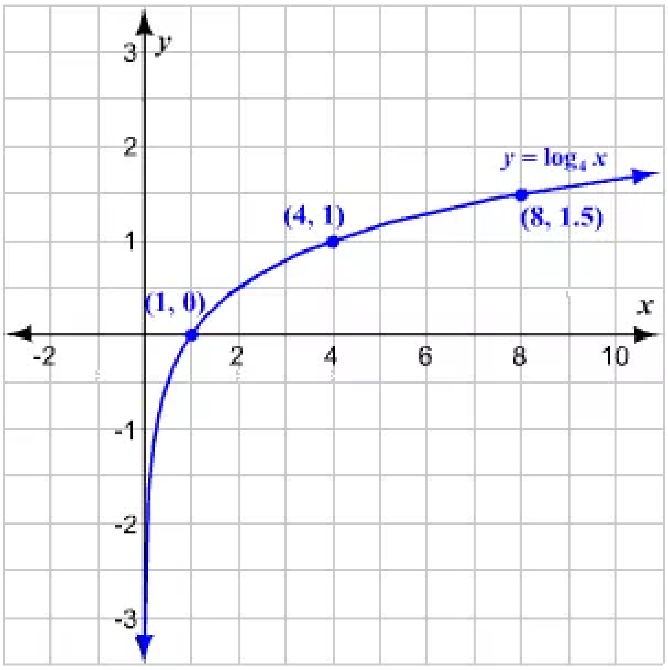
In order to graph the given function, first graph the parent function $y = \log_4 x$ on a coordinate plane.

Choose some values for x and find the corresponding y-values.
Organize the results in a table.

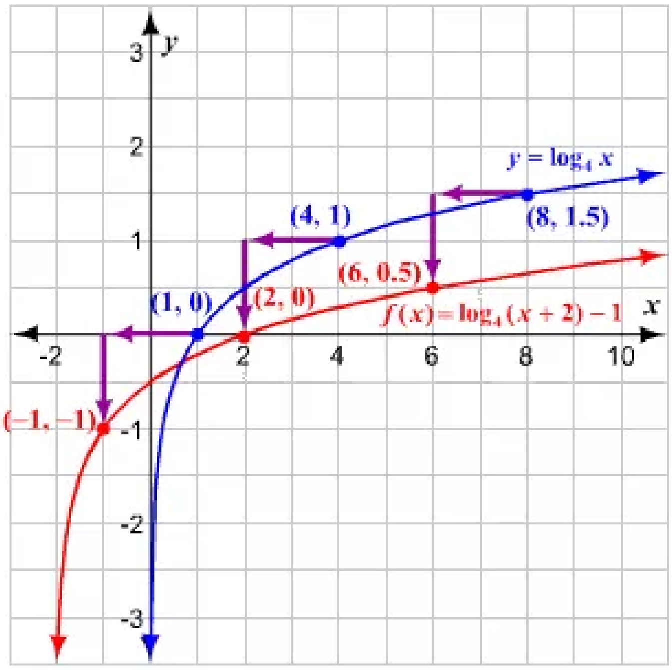
x	1	4	8
y	0	1	1.5

The points are (1, 0), (4, 1), and (8, 1.5).

Plot the points on a coordinate plane and draw a curve that starts just to the right of the y-axis and moves up through the plotted points as shown below.



Now, translate the graph of $y = \log_4 x$ to the left by 2 units and down by 1 unit. The resulting graph passes through the points (−1, −1), (2, 0), and (6, 0.5).



The set of input values is the domain and the set of output values is the range.

Thus, the domain of the given function is $x > -2$ and the range of the given function is all real numbers.

Answer 52e.

Graphing logarithmic function:

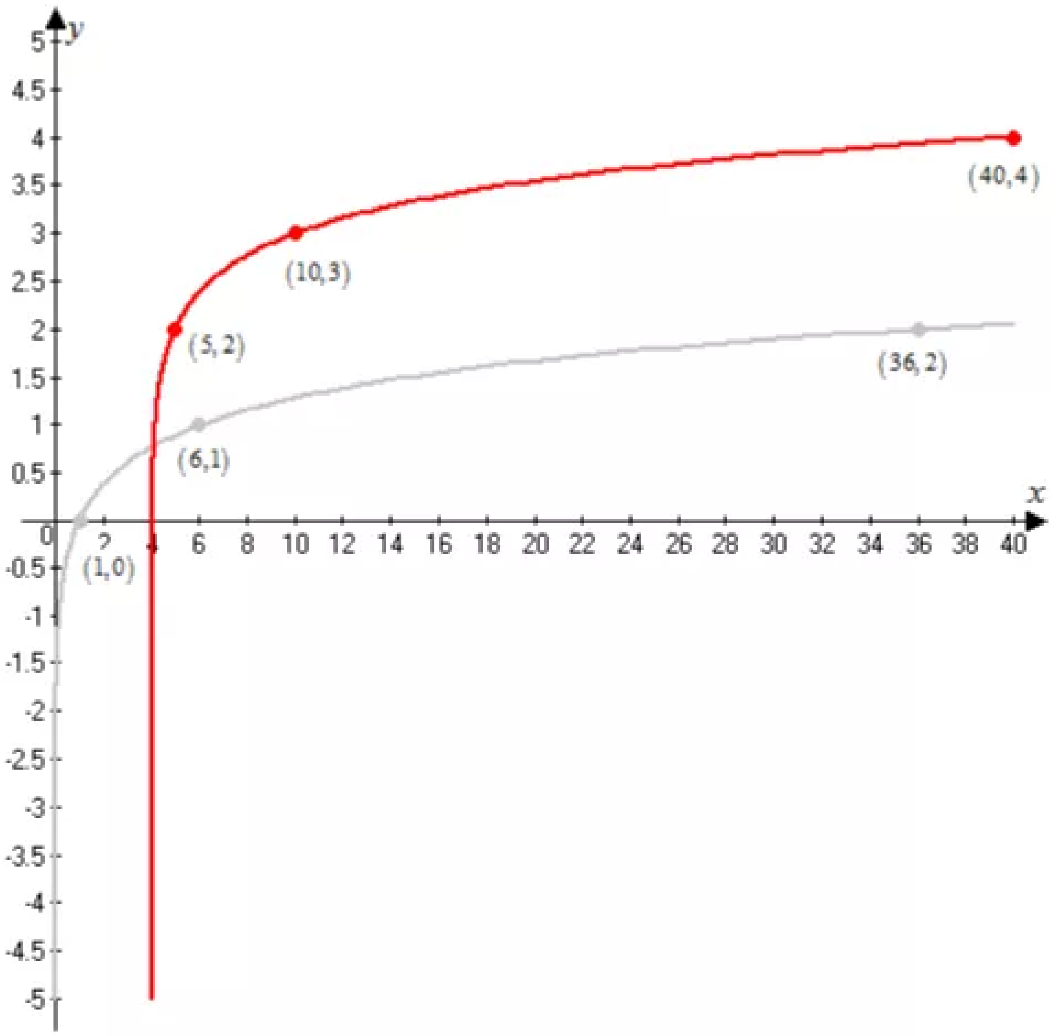
$y = \log_6(x-4)+2$

Step 1

Sketch the graph of the parent function $y = \log_6 x$, which passes through $(1,0)$, $(6,1)$ and $(36,2)$.

Step 2

Translate the parent graph right 4 unit and up 2 units. The translated graph passes through $(5,2)$, $(10,3)$ and $(40,4)$.



The graph's asymptote is $x = 4$. The domain is $x > 4$, and the range is all real numbers.

Answer 53e.

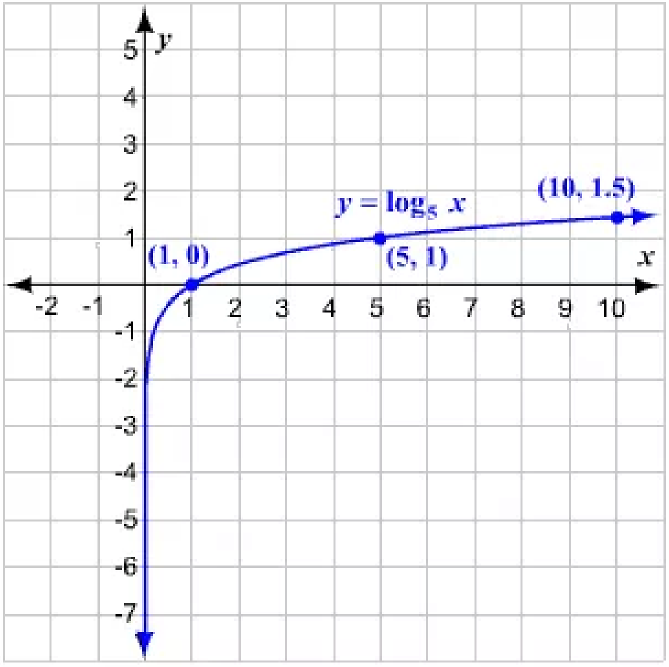
In order to graph the given function, first graph the parent function $y = \log_5 x$ on a coordinate plane.

Choose some values for x and find the corresponding y-values.
Organize the results in a table.

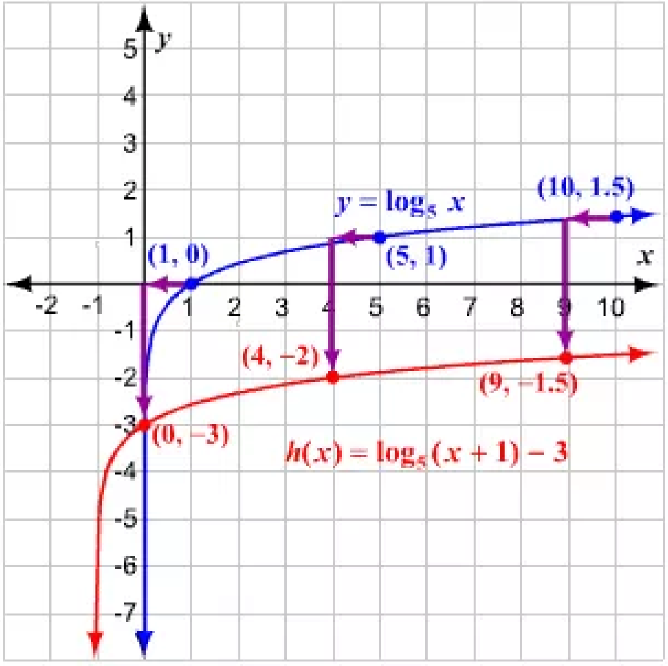
x	1	5	10
y	0	1	1.5

The points are (1, 0), (5, 1), and (10, 1.5).

Plot the points on a coordinate plane and draw a curve that starts just to the right of the y-axis and moves up through the plotted points as shown below.



Now, translate the graph of $y = \log_5 x$ left by 1 unit and down by 3 units. The resulting graph passes through the points (0, −3), (4, −2), and (9, −1.5).



The set of input values is the domain and the set of output values is the range.

Thus, the domain of the given function is $x > -1$ and the range of the given function is all real numbers.

Answer 54e.

Evaluating the logarithm

$\log_{27} 9$

$= \log_{27} 27^{2/3}$ [Express 9 as a power with base 27]

$= 2/3$ [Use the formula $\log_b b^x = x$]

Therefore the answer is $\boxed{2/3}$

Answer 55e.

Rewrite 8 and 32 as the powers of 2.

$\log_8 32 = \log_{2^3} 2^5$

Now, rewrite 2^5 as $(2^3)^{\frac{5}{3}}$.

$\log_{2^3} 2^5 = \log_{2^3} (2^3)^{\frac{5}{3}}$

Use the inverse property of logarithms.
$\log_b b^x = x$

In the equation, b is 2^3, and x is $\frac{5}{3}$. Thus,

$\log_{2^3} (2^3)^{\frac{5}{3}} = \frac{5}{3}$.

Therefore, $\log_8 32 = \frac{5}{3}$.

Answer 56e.

Evaluating the logarithm

$\log_{125} 625$

$= \log_{125} 125^{4/3}$ [Express 625 as a power with base 125]

$= 4/3$ [Use the formula $\log_b b^x = x$]

Therefore the answer is $\boxed{4/3}$

Answer 57e.

Rewrite 4 and 128 as the powers of 2.

$\log_4 128 = \log_{2^2} 2^7$

Now, rewrite 2^7 as $(2^2)^{\frac{7}{2}}$.

$\log_{2^2} 2^7 = \log_{2^2} (2^2)^{\frac{7}{2}}$

Use the inverse property of logarithms.
$\log_b b^x = x$

In the equation, b is 2^2 and x is $\frac{7}{2}$. Thus,

$\log_{2^2} (2^2)^{\frac{7}{2}} = \frac{7}{2}$.

Therefore, $\log_4 128 = \frac{7}{2}$.

Answer 58e.

The altitude h (in meters) above sea level is related to the air pressure P (in pascals) by the function:

$$h = -8005 \ln \frac{P}{101,300}$$

Substitute 57,000 for P to find the altitude above sea level.

$h = -8005 \ln \frac{57,000}{101,300}$	Substitute
$h = -8005 \ln(0.563)$	Divide
$h = -8005(-575)$	Use a calculator
$h = 4,603$	Multiply

Thus the altitude above sea level is 4,603 meters when the air pressure is 57,000 pascals.

Answer 59e.

Substitute $10^{-2.3}$ for $[H^+]$ in $pH = -\log[H^+]$.
$pH = -\log 10^{-2.3}$

Use the inverse property of logarithms.
$\log_b b^x = x$

In $-\log 10^{-2.3}$, b is 10 and x is -2.3. Thus,
$-\log 10^{-2.3} = -(-2.3)$.

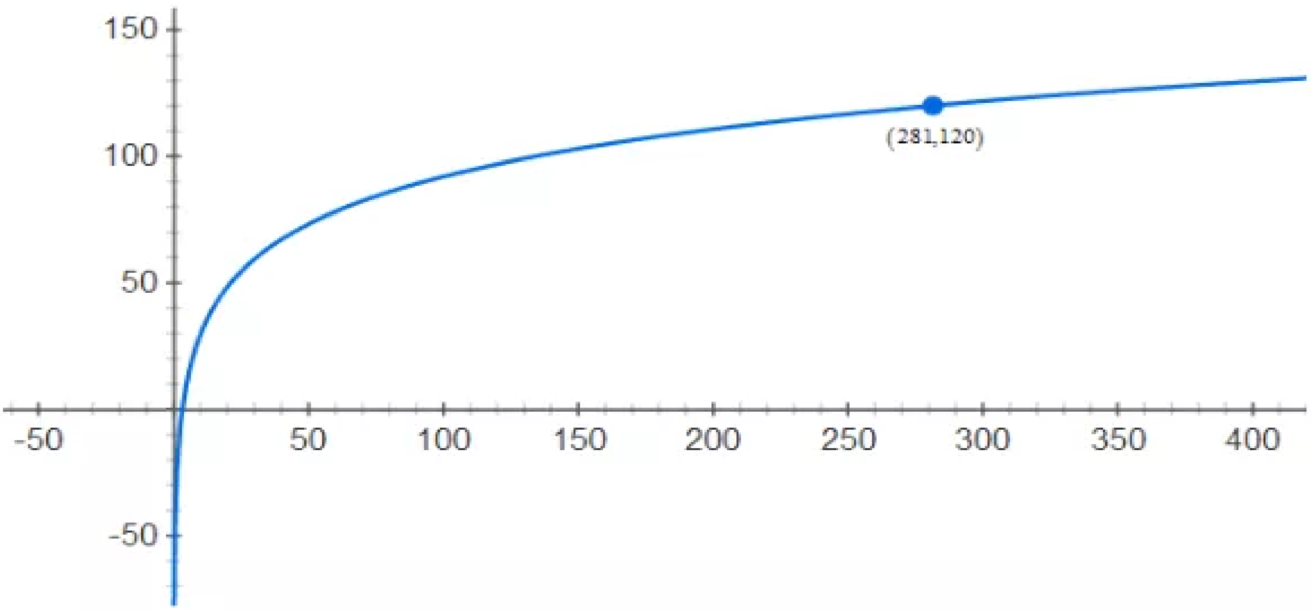
Simplify.
$-(-2.3) = 2.3$

The pH value of lemon juice is 2.3.

Answer 60e.

Biologists have found that an alligator's length l (in inches) and weight w (in pounds) are related by the function
$l = 27.1 \ln w - 32.8$

Graphing the function $l = 27.1 \ln w - 32.8$



By using the graph:
The weight of an alligator is 281 pounds that is 10 feet (120 inches) long.

Answer 61e.

a. Substitute 2.5×10^{24} for E in $M = 0.29(\ln E) - 9.9$.

$$M = 0.29\left[\ln\left(2.5 \times 10^{24}\right)\right] - 9.9$$

Use a calculator to evaluate.

$$M \approx 6.39$$

The energy magnitude of the earthquake was about 6.39.

b. Add 9.9 to each side of the given function.

$$M + 9.9 = 0.29(\ln E) - 9.9 + 9.9$$

$$M + 9.9 = 0.29(\ln E)$$

Divide each side by 0.29.

$$\frac{M + 9.9}{0.29} = \frac{0.29(\ln E)}{0.29}$$

$$\frac{M + 9.9}{0.29} = \ln E$$

Now, write in exponential form.

$$E = e^{\frac{M+9.9}{0.29}}$$

Thus, an equation for the inverse of the function is $E = e^{\frac{M+9.9}{0.29}}$.

The inverse represents the amount of energy released as a function of the energy magnitude.

Answer 62e.

Consider the function is:

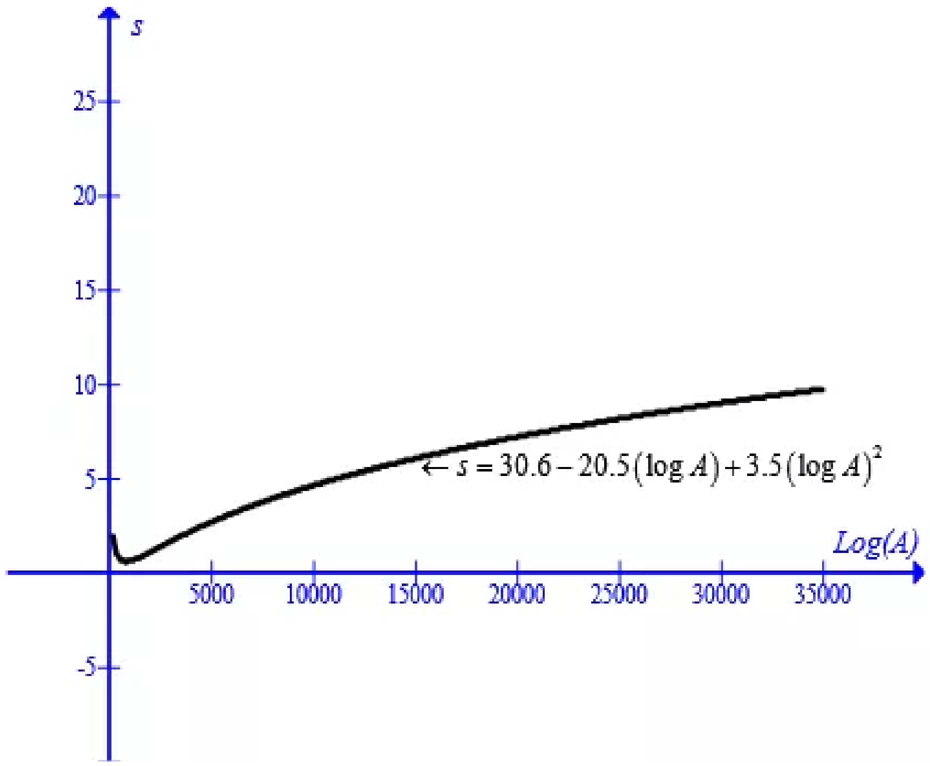
$$s = 30.6 - 20.5(\log A) + 3.5(\log A)^2$$

where s is the number of species of fish and A is the area of the pool or lake.

(a)

We need to graph the function in the range $200 \le A \le 35,000$.

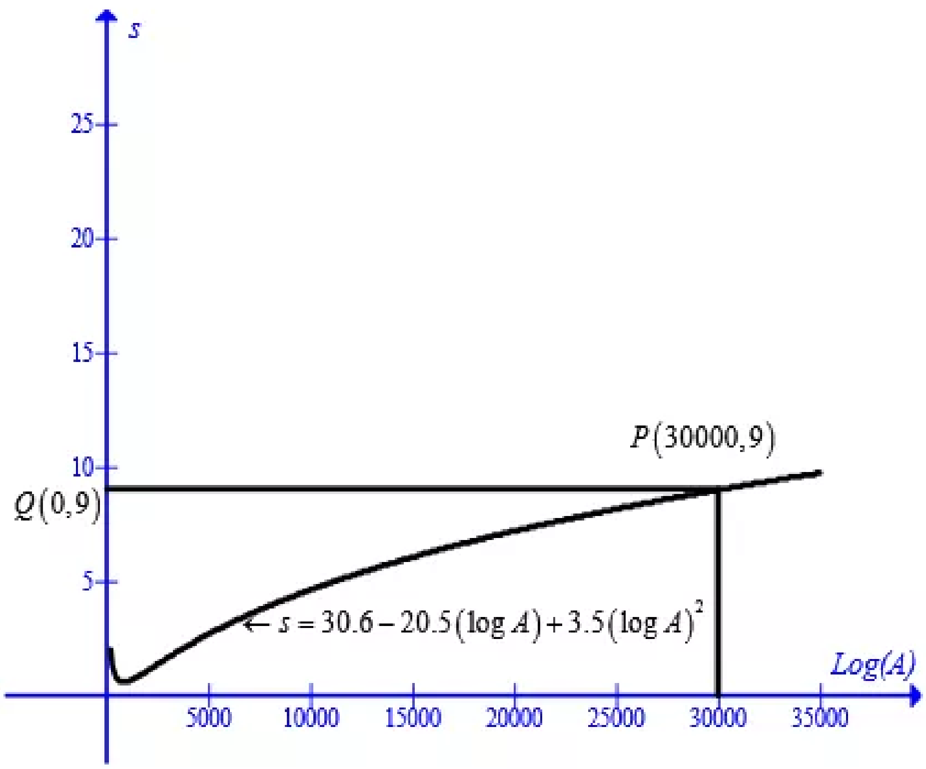
The graph of the function $s = 30.6 - 20.5(\log A) + 3.5(\log A)^2$ is shown below:



(b)

We need to estimate the number of fish species in a lake with an area of 30,000 square meters.

We will draw a line $\log(A) = 30000$ which will be parallel to y - axis and intersect the function at point P. Now from the point P, we will draw a line that is parallel to x - axis and extends it so that it cuts the y - axis at Q as shown in the graph.

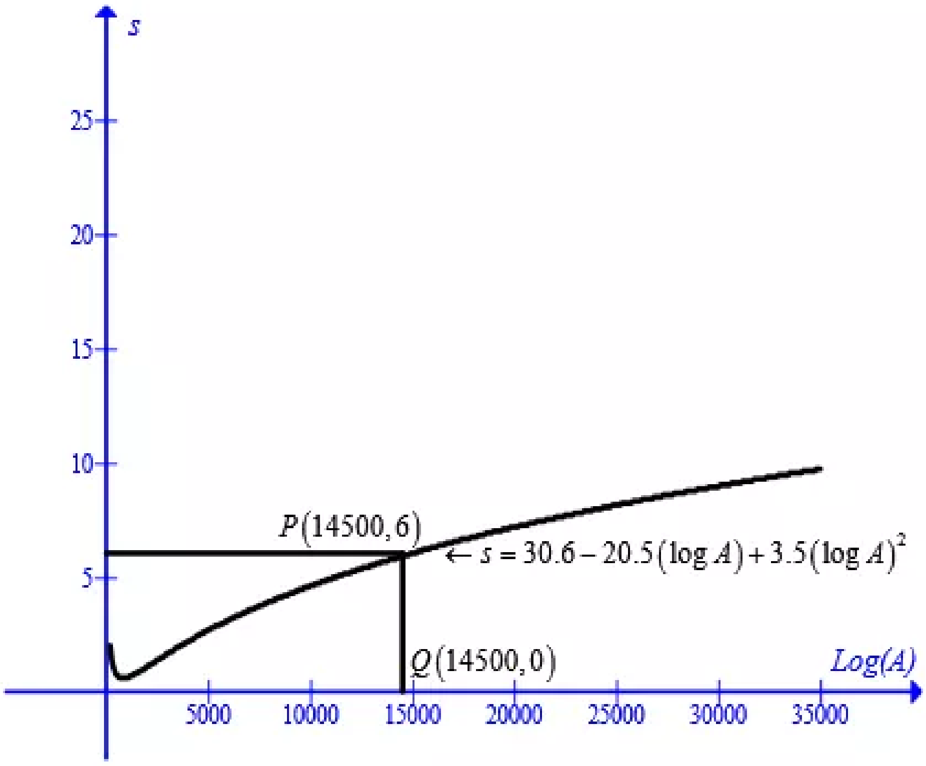


From the graph we can verify that the number of fish species in the lake or pool with an area of 30,000 square meters is $\boxed{9}$.

(c)

We need to estimate the area of a lake that contains 6 species of fish.

We will draw a line $s = 6$ which will be parallel to x - axis and intersect the function at point P. Now from the point P, we will draw a line that is parallel to y - axis and extends it so that it cuts the x - axis at Q as shown in the graph.



From the graph we can verify that the area of a lake that contains 6 species of fish is **14,500 square meter**.

(d)

As the area of a pool or lake increases the number of species is increases which indicate that a wide will contain more species.

Answer 63e.

Subtract 0.159 from each side of the given function.

$$s - 0.159 = 0.159 + 0.118(\log d) - 0.159$$

$$s - 0.159 = 0.118(\log d)$$

Divide each side by 0.118.

$$\frac{s - 0.159}{0.118} = \frac{0.118(\log d)}{0.118}$$

$$\frac{s - 0.159}{0.118} = \log d$$

Write in exponential form.

$$d = 10^{\frac{s-0.159}{0.118}}$$

An equation for the inverse of the function is $d = 10^{\frac{s-0.159}{0.118}}$.

In order to estimate the average diameter of the sand particles on a beach with a slope of 0.2, first substitute 0.2 for s.

$$d = 10^{\frac{0.2 - 0.159}{0.118}}$$

Use a calculator to evaluate.

$$d \approx 2.2$$

The average diameter of the sand particles on a beach with a slope of 0.2 will be about 2.2 mm.

Answer 64e.

Evaluating the expression

$$2^3 \cdot 2^5$$

$$= 2^{3+5} \qquad \left[\text{Because } m^a \cdot m^b = m^{a+b}\right]$$

$$= 2^8 \qquad [\text{Add}]$$

Therefore the answer is $\boxed{2^8}$

Answer 65e.

The property of the power of a power states that for any real number a, and integers m and n,

$(a^m)^n = a^{mn}$.

Apply the property and simplify.

$$\left(5^{-3}\right)^2 = 5^{-3(2)}$$

$$= 5^{-6}$$

Now, rewrite the expression with positive exponents using the rule $a^{-m} = \frac{1}{a^m}$, $a \neq 0$.

$$5^{-6} = \frac{1}{5^6}$$

Simplify.

$$\frac{1}{5^6} = \frac{1}{15,625}$$

The given expression evaluates to $\frac{1}{15.625}$.

Answer 66e.

Evaluating the expression

$8^1 \cdot 8^3 \cdot 8^{-5}$

$= 8^{1+3-5}$ $\quad$ [Because $m^a \cdot m^b \cdot m^c = m^{a+b+c}$]

$= 8^{-1}$ $\quad$ [Simplify]

$= \frac{1}{8}$ $\quad$ $\left[\text{Because } m^{-a} = \frac{1}{8}\right]$

Therefore the answer is $\boxed{\frac{1}{8}}$

Answer 67e.

In the given expression, a quotient is raised to a power.

The property of the power of a quotient states that for any real numbers a and b, and integers m and n,

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0.$$

Apply the property by raising both the numerator and the denominator to the power.

$$\left(\frac{5}{3}\right)^{-3} = \frac{(5)^{-3}}{(3)^{-3}}$$

Now, rewrite the expression with positive exponents using the rule $a^{-m} = \frac{1}{a^m}, a \neq 0.$

$$\frac{(5)^{-3}}{(3)^{-3}} = \frac{3^3}{5^3}$$

Simplify.

$$\frac{3^3}{5^3} = \frac{27}{125}$$

The given expression evaluates to $\frac{27}{125}$.

Answer 68e.

Evaluating the expression

$$\frac{10^6}{10^4}$$

$= 10^6 \cdot 10^{-4}$ $\left[\text{Because } \frac{1}{m^a} = m^{-a}\right]$

$= 10^{6-4}$ $\left[\text{Because } m^a \cdot m^b = m^{a+b}\right]$

$= 10^2$ [Simplify]

$= 100$

Therefore the answer is $\boxed{100}$

Answer 69e.

The property of the power of a power states that for any real number a, and integers m and n,

$(a^m)^n = a^{mn}$.

Apply the property and simplify.

$$\begin{aligned}\left(6^{-2}\right)^{-1} &= 6^{-2(-1)} \\ &= 6^2 \\ &= 36\end{aligned}$$

The given expression evaluates to 36.

Answer 70e.

Evaluating the expression

$$\frac{4^2}{4^5}$$

$= 4^2 \cdot 4^{-5}$ $\left[\text{Because } \frac{1}{m^a} = m^{-a}\right]$

$= 4^{2-5}$ $\left[\text{Because } m^a \cdot m^b = m^{a+b}\right]$

$= 4^{-3}$ [Subtract]

$= \frac{1}{4^3}$ $\left[\text{Because } m^{-a} = \frac{1}{m^a}\right]$

$= \frac{1}{64}$

Therefore the answer is $\boxed{\frac{1}{64}}$

Answer 71e.

In the given expression, a quotient is raised to a power.

The property of the power of a quotient states that for any real numbers a and b, and integers m and n,

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0.$$

Apply the property by raising both the numerator and the denominator to the power.

$$\left(\frac{7^8}{7^9}\right)^{-2} = \frac{\left(7^8\right)^{-2}}{\left(7^9\right)^{-2}}$$

The property of the power of a power states that for any real number a, and integers m and n,

$(a^m)^n = a^{mn}$.

Apply the property and simplify.

$$\frac{\left(7^8\right)^{-2}}{\left(7^9\right)^{-2}} = \frac{7^{8(-2)}}{7^{9(-2)}}$$

$$= \frac{7^{-16}}{7^{-18}}$$

Now, rewrite the expression with positive exponents using the rule $a^{-m} = \frac{1}{a^m}, a \neq 0$.

$$\frac{7^{-16}}{7^{-18}} = \frac{7^{18}}{7^{16}}$$

The property of the quotient of a power states that for any real numbers a and b, and integers m and n,

$$\frac{a^m}{a^n} = a^{m-n}$$

Apply the property and simplify.

$$\frac{7^{18}}{7^{16}} = 7^{18-16}$$

$$= 7^2$$

$$= 49$$

The given expression evaluates to 49.

Answer 72e.

Evaluating the expression

$x^{1/2} \cdot x^{2/3}$

$= x^{1/2+2/3}$ [Because $m^a \cdot m^b = m^{a+b}$]

$= x^{7/6}$ [Add]

$= \sqrt[6]{x^7}$ [Because $m^{1/n} = \sqrt[n]{m}$]

$= x\sqrt[6]{x}$ [Find sixth roots]

Therefore the answer is $\boxed{x\sqrt[6]{x}}$

Answer 73e.

Multiply the exponents using the power of power property.

$$(m^9)^{-1/6} = m^{9(-1/6)}$$
$$= m^{-3/2}$$

Now, rewrite the expression with positive exponents using the rule $a^{-m} = \frac{1}{a^m}$, $a \neq 0$.

$$m^{-3/2} = \frac{1}{m^{3/2}}$$

The given expression evaluates to $\frac{1}{m^{3/2}}$.

Answer 74e.

Evaluating the expression

$\sqrt[3]{54x^6y^3}$

$= \sqrt[3]{2 \cdot 3 \cdot 3 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y}$ [Factor]

$= 3x^2y\sqrt[3]{2}$ [Find cubes roots]

Therefore the answer is $\boxed{3x^2y\sqrt[3]{2}}$

Answer 75e.

Raise each factor to the power using the power of a product property.

$$(n^{4/3} \cdot n^{2/5})^{1/6} = (n^{4/3})^{1/6} \cdot (n^{2/5})^{1/6}$$

Multiply the exponents using the power of a power property.

$$(n^{4/3})^{1/6} \cdot (n^{2/5})^{1/6} = n^{(4/3)(1/6)} \cdot n^{(2/5)(1/6)}$$
$$= n^{2/9} \cdot n^{1/15}$$

Add the exponents using the product of power property.

$$n^{2/9} \cdot n^{1/15} = n^{2/9+1/15}$$
$$= n^{13/45}$$

Therefore, the expression simplifies to $n^{13/45}$.

Answer 76e.

Evaluating the expression

$$\frac{x^{1/4}y^3}{x^{5/2}y^{1/2}}$$

$= x^{1/4} \cdot x^{-5/2} \cdot y^3 \cdot y^{-1/2}$ $\quad \left[\text{Because } \frac{1}{m^a} = m^{-a}\right]$

$= x^{1/4+(-5/2)}y^{3+(-1/2)}$ $\quad \left[\text{Because } m^a \cdot m^b = m^{a+b}\right]$

$= x^{-9/4}y^{5/2}$ $\quad$ [Simplify the exponents]

$= \frac{y^{5/2}}{x^{9/4}}$ $\quad \left[\text{Because } m^{-a} = \frac{1}{m^a}\right]$

$= \frac{y^2\sqrt{y}}{x^2\sqrt[4]{x}}$

Therefore the answer is $\boxed{\frac{y^2\sqrt{y}}{x^2\sqrt[4]{x}}}$

Answer 77e.

Apply the quotient property of radicals.

$$\sqrt[4]{\frac{x^{16}}{y^{12}}} = \frac{\sqrt[4]{x^{16}}}{\sqrt[4]{y^{12}}}$$

Simplify.

$$\frac{\sqrt[4]{x^{16}}}{\sqrt[4]{y^{12}}} = \frac{\sqrt[4]{\left(x^4\right)^4}}{\sqrt[4]{y^4 \cdot \left(y^4\right)^2}}$$
$$= \frac{x^4}{y \cdot y^3}$$

Add the exponents in the denominator.

$$\frac{x^4}{y \cdot y^3} = \frac{x^4}{y^4}$$

Therefore, the expression simplifies to $\frac{x^4}{y^4}$.

Answer 78e.

Evaluating the expression

$\left(\sqrt[5]{x^{10}} \cdot \sqrt[3]{x^{9}}\right)^2$

$= \left(x^{10/5} \cdot x^{9/3}\right)^2$ $\quad$ [Because $\sqrt[n]{m} = m^{1/n}$]

$= \left(x^{2} \cdot x^{3}\right)^2$ $\quad$ [Because $m^a \cdot m^b = m^{a+b}$]

$= \left(x^{2}\right)^2 \cdot \left(x^{3}\right)^2$ $\quad$ [Write $(a \cdot b)^2 = (a)^2 \cdot (b)^2$]

$= x^4 \cdot x^6$ $\quad$ [Power to power property]

$= x^{10}$ $\quad$ [Because $m^a \cdot m^b = m^{a+b}$]

Therefore the answer is $\boxed{x^{10}}$

Answer 79e.

Rewrite the given expression using the product property of radicals, $\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$.

$$\frac{5\sqrt{x} \cdot \sqrt{x^7}}{\sqrt[3]{250x^{16}}} = \frac{5\sqrt{x \cdot x^7}}{\sqrt[3]{250x^{16}}}$$

$$= \frac{5\sqrt{x^8}}{\sqrt[3]{250x^{16}}}$$

Simplify.

$$\frac{5\sqrt{x^8}}{\sqrt[3]{250x^{16}}} = \frac{5\sqrt{\left(x^4\right)^2}}{\sqrt[3]{250x^{16}}}$$

$$= \frac{5x^2}{\sqrt[3]{250x^{16}}}$$

Therefore, the given expression simplifies to $\dfrac{5x^2}{\sqrt[3]{250x^{16}}}$.