CBSE Test Paper 04

CH-10 Straight Lines

1. The locus of a point, whose abscissa and ordinate are always equal is
   a. $x - y = 0$
   b. $x + y + 1 = 0$
   c. $x + y = 1$
   d. none of these.
2. The area of the triangle whose sides are along the lines $x = 0$, $y = 0$ and $4x + 5y = 20$ is
   a. none of these
   b. 10
   c. 1/10
   d. 20
3. Two points (a , 0) and (0 , b) are joined by a straight line. Another point on this line is
   a. (- 3a , 2b)
   b. (a , - 2b)
   c. none of these
   d. (3a , - 2b)
4. The line $(p + 2q) x + (p - 3q) y = p - q$ for different values of p and q passes through the fixed point
   a. (2/5 , 3/5)
   b. (3/2 , 5/2)
   c. (2/5 , 2/5)
   d. (3/5 , 3/5)
5. The perpendicular distance of the origin from the line $3x + 4y + 1 = 0$ is
   a. none of these
   b. 1/5
   c. 1/2
   d. 1
6. Fill in the blanks:

   For specifying a straight line, the number of geometrical parameters should be

________.

7. Fill in the blanks:

   The equation of the line passing through (1, 2) and perpendicular to $x + y + 7 = 0$ is ________.

8. Find the new coordinates of point (3, -4), if the origin is shifted to (1, 2) by a translation.

9. Find the equation of a line which is parallel to Y-axis and passes through (-4, 3).

10. Find the point on X-axis which is equidistant from the points (3, 2) and (-5, - 2).

11. Reduce the following equation into slope intercept form and find their slopes and the y-intercepts.
    $x + 7y = 0$

12. Find the area of a ΔABC, whose vertices are A (6, 3), B (-3, 5) and C (4, - 2).

13. Find the values of β so that the point (0, β) lies on or inside the triangle having the sides $3x + y + 2 = 0$, $2x - 3y + 5 = 0$ and $x + 4y - 14 = 0$.

14. Find the equation of the line passing through (-3, 5) and perpendicular to the line through the points (2, 5) and (-3, 6).

15. Show that the perpendicular drawn from the point (4, 1) on the line segment joining (6, 5) and (2, -1) divides it internally in the ratio 8 : 5.

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Solution

1. (a) $x - y = 0$

   Explanation:

   The abscissa is equal to the ordinate implies x = y

   Hence the locus is x-y=0

2. (b) 10

   Explanation:

   The equation 4x + 5y = 20 can be written as $\frac{x}{5} + \frac{y}{4} = 1$

   This implies the intercepts cut by this line on the X and Y axes are 5 and 4 respectively.

   Hence the area of the triangle is 1/2 [5 x 4] = 10 square units

3. (d) (3a , - 2b)

   Explanation:

   The slope of the line joining the points (a,0) and (0,b) is [b-0]/[0-a] = -(b/a)

   Hence the equation of the line is y = (-b/a)x +b

   i.e; ay = -bx +ab

   Substituting the x coordinate 3a in the place of x in the above equation we get y = -2b

   Hence (3a,-2b) is another point on the line.

4. (a) (2/5 , 3/5)

   Explanation:

   Expanding the given equation

   px+2qx+py-3qy = p-q

   px+py+2qx-3qy = p-q

   p(x+y) -q(-2x+3y) = p-q

   Equating the coeffiecients of like terms

   x+y=1 and -2x+3y=1

   On solving both the equations we get,

   x = 2/5 and y = 3/5

   Hence the line passes through the fixed point (2/5.3/5)

5. (b) 1/5

   Explanation: The perpendicular distance from the origin to the line is given by
   $\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}$

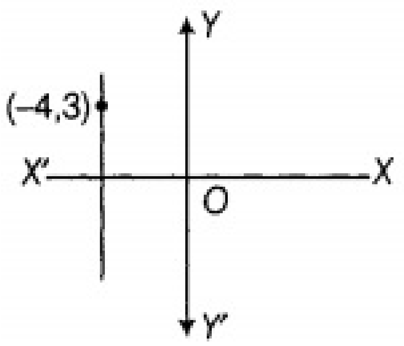
   For the given line c = 1, A = 3 and B = 4 and since it passes through the origin,

   Sustituting the values we get,
   $\frac{1}{\sqrt{9+16}}$ = 1/5

6. 2

7. y - x - 1 = 0

8. The coordinates of the new origin are h = 1, k = 2 and the original coordinates are given to be x = 3, y = - 4.
   The transformation relation between the old coordinates (x, y) and the new coordinates (X, Y) are given by
   x = X + h i.e., X = x - h ...(i)
   and y = Y + k i.e., Y= y - k ...(ii)
   On substituting the values x = 3, y =- 4 in Eqs. (i) and (ii),
   we get,
   X = 3 - 1 = 2 and Y = - 4 - 2 = -6
   Hence, the coordinates of point (3, -4) in the new system are (2, -6).

9. The equation of a line which is parallel to Y-axis and passes through (-4, 3) is
   x = -4
   

10. Let the point on X-axis be P(x, 0) and A = (3, 2) and B = (-5, -2).
    Since, P is equidistant from A and B. So,

PA = PB $\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow (3 - x)^2 + (2 - 0)^2 = (-5 - x)^2 + (-2 - 0)^2$ $\left[\because \text{ distance } = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$

$\Rightarrow 9 + x^2 - 6x + 4 = 25 + x^2 + 10x + 4$

$\Rightarrow 16x + 16 = 0 \Rightarrow x = -1$

$\therefore$ Thus, point on X-axis is (-1, 0)

11. Here x + 7 y = 0

    $\Rightarrow 7y = -x$

    $y = \frac{-1}{7}x \Rightarrow y = \frac{-1}{7}x + 0$ which is required slope intercept form.
    Comparing it with y = mx +c, we have
    $m = \frac{-1}{7}$ and c = 0

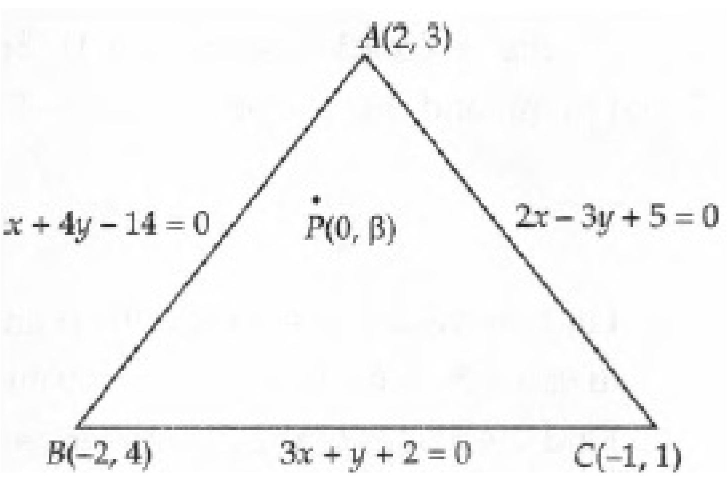
12. Area of ΔABC

    $= \frac{1}{2}[x_1(y_2 - y_3)+x_2(y_3 - y_1)+x_3(y_1 - y_2)]$

    Here, $(x_1, y_1) = (6, 3)$, $(x_2, y_2) = (-3, 5)$ and $(x_3, y_3) = (4, -2)$

    $\therefore$ Area of $\Delta ABC = \frac{1}{2}$ [6(5+2)+(-3)(-2-3)+4(3-5)]
    $= \frac{1}{2}|6(7) - 3(-5) + 4(-2)|$
    $= \frac{1}{2}|42 + 15 - 8| = \frac{1}{2}|49| = \frac{49}{2}$ sq units
    Hence, area of ΔABC is $\frac{49}{2}$ sq units.

13. Let ABC be the given triangle. The coordinates of the vertices of the triangle ABC are marked in Fig. The point P (0, β) will lie inside or on the triangle ABC if the following three conditions hold simultaneously:



Now,

A and P will lie on the same side of BC, if

(3 $\times$ 2 + 3 + 2) (3 $\times$ 0 + β + 2) > 0

$\Rightarrow$ 11 (β + 2) $\geq$ 0

$\Rightarrow$ β + 2 $\geq$ 0

$\Rightarrow$ β $\geq$ -2 ....(i)

B and P will lie on the same side of AC, if

(2 $\times$ - 2 - 3 $\times$ 4 + 5) (2 $\times$ 0 - 3p + 5) $\geq$ 0

$\Rightarrow$ -11 (- 3β + 5) $\geq$ 0

$\Rightarrow$ 3β -5 $\geq$ 0

$\Rightarrow$ $\beta \geq \frac{5}{3}$

C and P will lie on the same side of AB, if

(-1 + 1 $\times$ 4 - 14) (0 + 4β - 14) $\geq$ 0

$\Rightarrow$ -22 (2β - 7) $\geq$ 0

$\Rightarrow$ 2β - 7 $\leq$ 0

$\Rightarrow$ $\beta \leq \frac{7}{2}$

From (i), (ii) and (iii), we obtain that $\frac{5}{3} \leq \beta \leq \frac{7}{2}$ i.e., $\beta \in$ [5/3, 7/2]

i. A and P lie on the same side of BC
ii. B and P lie on the same side of AC
iii. C and P lie on the same side of AB.

14. Let A (2, 5) and B(-3, 6) be any two points.

$\therefore$ Slope of AB $= \frac{6-5}{-3-2} = -\frac{1}{5}$

Since the required line is perpendicular to AB.

$\therefore$ Slope of required line m = 5.

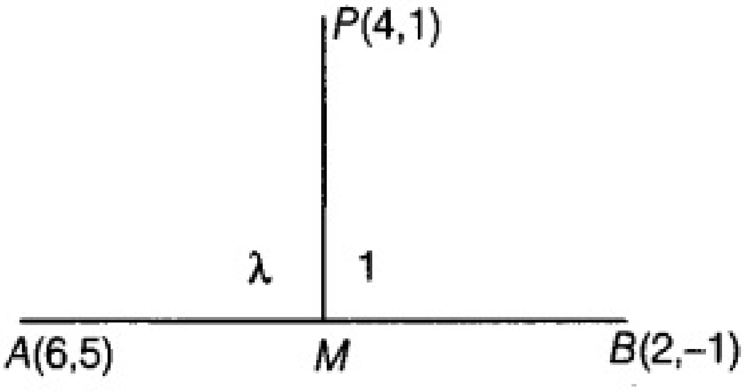
Now the required line passing through point (-3, 5) having slope 5

$\therefore$ y - 5 = 5(x + 3) $\Rightarrow$ y - 5 = 5x + 15

$\Rightarrow$ 5x - y + 20 = 0

15. Suppose perpendicular drawn from P(4, 1) on the line joining A(6, 5) and B(2, -1) meets AB at M. Let, m be the slope of PM. Then,

$PM \perp AB$



$\Rightarrow$ Slope of PM $\times$ Slope of AB = -1

$\Rightarrow \quad m \times \frac{-1-5}{2-6} = -1$

$\Rightarrow \quad m \times \frac{3}{2} = -1 \quad \Rightarrow \quad m = \frac{-2}{3}$

Clearly, PM passes through P(4, 1) and has slope $m = \frac{-2}{3}$.

So, its equation is

$y - 1 = \frac{-2}{3}(x - 4)$

$\Rightarrow$ 2x + 3y - 11 = 0 ...(i)

Suppose M divides line segment AB in the ratio $\lambda : 1$

Then, coordinates of M are

$\left(\frac{2\lambda+6}{\lambda+1}, \frac{-\lambda+5}{\lambda+1}\right)$

Since, M lies on line PM whose equation is 2x + 3y - 11 = 0. So, it will satisfies it.

$\therefore \quad 2\left(\frac{2\lambda+6}{\lambda+1}\right) + 3\left(\frac{-\lambda+5}{\lambda+1}\right) - 11 = 0$

$\Rightarrow \quad 4\lambda + 12 - 3\lambda + 15 - 11\lambda - 11 = 0$

$\Rightarrow \quad -10\lambda + 16 = 0 \quad \Rightarrow \quad \lambda = \frac{8}{5}$

Hence, M divides AB internally in the ratio 8 : 5.