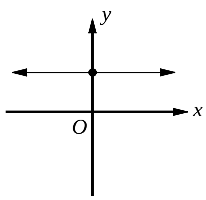
# CHAPTER 13
# Polynomial and Radical Functions

## 13-1. Polynomial Functions and Their Graphs

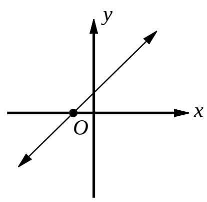
A **polynomial function** is a function of the form $\boldsymbol{f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0}$,
in which the coefficients $a_n, a_{n-1}, \cdots, a_2, a_1, a_0$ are real numbers and $n$ is a nonnegative integer.
The degree of a polynomial function is its greatest exponent of $x$. The graphs of several polynomial functions are shown below. The maximum number of zeros, which are the $x$-intercepts, is equal to the degree of the function.

Constant Function



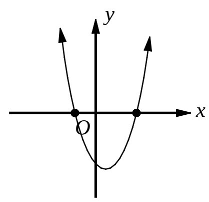

$f(x) = 2$
No real solution

Linear Function



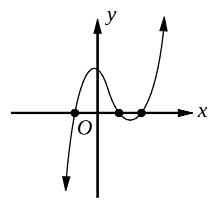

$f(x) = x + 1$
One real solution

Quadratic Function



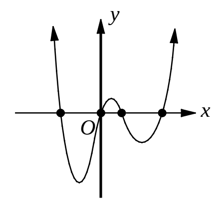

$f(x) = x^2 - x - 2$
Two real solutions

Cubic Function



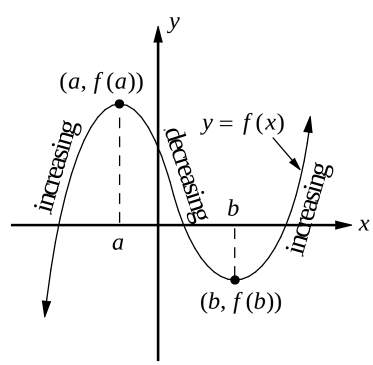

$f(x) = x^3 - 2x^2 - x + 2$
Three real solutions

Quartic Function




$f(x) = x^4 - x^3 - 4x^2 + 4x$
Four real solutions

A function $f$ is **increasing** on an interval if the value of $f$ increases as $x$ increases in the interval.
A function $f$ is **decreasing** on an interval if the value of $f$ decreases as $x$ increases in the interval.
In the graph shown at the right, function $f$ increases on the intervals $(-\infty, a)$ and $(b, \infty)$, and decreases on the interval $(a, b)$. At a point where the graph changes from increasing to decreasing, $f$ has a local **maximum value**, and at a point where the graph changes from decreasing to increasing, $f$ has a local **minimum value**.

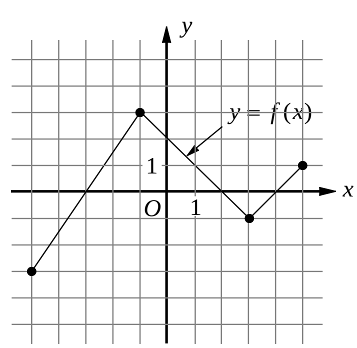



Example 1 □ The complete graph of function $f$ is shown at the right.

a. Find the $x$-intercepts of $f(x)$.

b. For what value of $x$ is the value of $f(x)$ at its maximum?

c. Find the interval where $f(x)$ is strictly decreasing.




Solution □ a. The $x$-intercepts are $-3$, $2$, and $4$.

b. The value of $f(x)$ is maximum at $x = -1$.

c. $f(x)$ is strictly decreasing between $-1$ and $3$.

## Exercises - Polynomial Functions and Their Graphs

**1**

The graph of $f(x) = ax^3 + x^2 - 18x - 9$ intersects the $x$-axis at $(3,0)$. What is the value of $a$?

A) $-1$

B) $0$

C) $1$

D) $2$

**2**

In the $xy$-plane, the graph of function $f$ has $x$-intercepts at $-7$, $-5$, and $5$. Which of the following could define $f$?

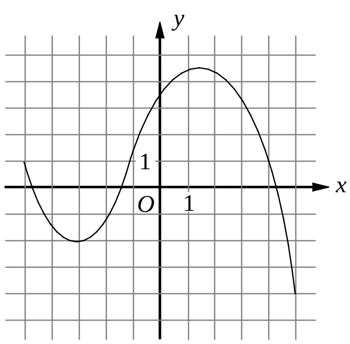
A) $f(x) = (x-7)(x^2-25)$

B) $f(x) = (x-7)(x^2+25)$

C) $f(x) = (x+7)(x^2-25)$

D) $f(x) = (x+7)(x^2+25)$

**3**




What is the minimum value of the function graphed on the $xy$-plane above, for $-5 \le x \le 5$?

A) $-4$

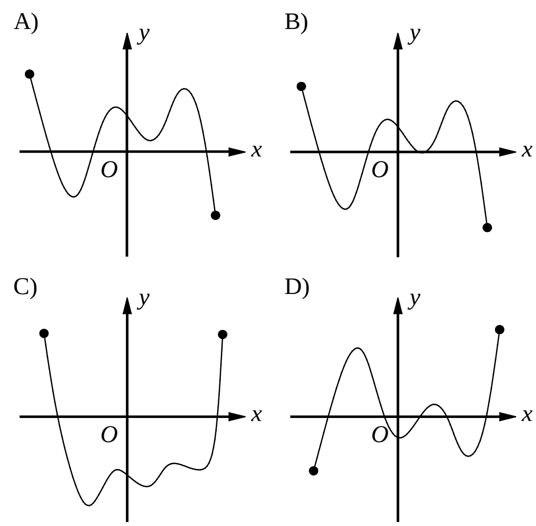
B) $-3$

C) $-2$

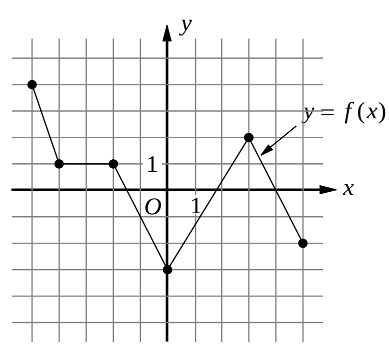
D) $-\infty$

**4**

If function $f$ has four distinct zeros, which of the following could represent the complete graph of $f$ in the $xy$-plane?




**5**




The complete graph of function $f$ is shown on the $xy$-plane above, for $-5 \le x \le 5$. Which of the following is/are true?

I. $f$ is strictly decreasing for $-5 < x < 0$.

II. $f(-3) = 1$

III. $f$ is minimum at $x = 5$.

A) I only

B) II only

C) III only

D) I and II only

## 13-2. Remainder Theorem and Factor Theorem

**Remainder Theorem**

If a polynomial $f(x)$ is divided by $x-c$, the remainder is $f(c)$.
Since Dividend = Divisor × Quotient + Remainder ,
$f(x)=(x-c)q(x)+f(c)$, inwhich $q(x)$ is the quotient.

**Factor Theorem**

The polynomial $f(x)$ has $x-c$ as a factor if and only if $f(c)=0$.

The following statements are equivalent for a polynomial $f(x)$ and a real number $c$:

- $c$ is a **solution** to the equation $f(x)=0$.
- $c$ is a **zero** of $f(x)$.
- $c$ is a **root** of $f(x)$.
- $x-c$ is a **factor** of $f(x)$.
- $f(x)$ is **divisible** by $x-c$.
- $c$ is an $\boldsymbol{x}$**-intercept** of the graph of $f(x)$.

Example 1 □ Find the remainder of $f(x)=x^3+x^2-6x-7$ divided by $x+2$.

Solution □ To find the remainder of $f(x)$ divided by $x+2=x-(-2)$,
evaluate $f(-2)$. $f(-2)=(-2)^3+(-2)^2-6(-2)-7=1$
By the remainder theorem the remainder is 1.

Example 2 □ Find the value of $a$ if $x-3$ is a factor of $f(x)=x^3-11x+a$.

Solution □ If $x-3$ is a factor of $f(x)$, then $f(3)=0$
$f(3)=(3)^3-11(3)+a=0 \Rightarrow -6+a=0 \Rightarrow a=6$

Example 3 □ Find the value of $k$ if $f(x)=3(x^2+3x-4)-8(x-k)$ is divisible by $x$.

Solution □ If $f(x)$ is divisible by $x$, since $x=x-0$, $f(0)=0$ by the factor theorem.
$f(0)=3(0^2+3(0)-4)-8(0-k)=-12+8k=0$
$8k=12 \Rightarrow k=\frac{12}{8}=\frac{3}{2}$

Example 4□ Find the $x$-intercepts of $f$ if $f(x)=2x^2+x-10$.

Solution □ $f(x)=2x^2+x-10=(2x+5)(x-2)$ Factor.
$(2x+5)(x-2)=0$ Let $f(x)=0$.
$2x+5=0$ or $x-2=0$ Zero Product Property
$x=-\frac{5}{2}$ or $x=2$ Solve.
The $x$-intercepts of $f$ are $-\frac{5}{2}$ and $2$.

## Exercises - Polynomial Functions and Their Graphs

**1**

If $-1$ and $1$ are two real roots of the polynomial function $f(x) = ax^3 + bx^2 + cx + d$ and $(0,3)$ is the $y$- intercept of graph of $f$, what is the value of $b$?

A) $-3$

B) $-1$

C) $2$

D) $4$

**2**

What is the remainder of polynomial $p(x) = 81x^5 - 121x^3 - 36$ divided by $x+1$?

A) $-76$

B) $-36$

C) $4$

D) $6$

**3**

If $x-2$ is a factor of polynomial $p(x) = a(x^3 - 2x) + b(x^2 - 5)$, which of the following must be true?

A) $a+b=0$

B) $2a-b=0$

C) $2a+b=0$

D) $4a-b=0$

**4**

| $x$ | $f(x)$ |
|---|---|
| $-4$ | $-10$ |
| $-3$ | $0$ |
| $-1$ | $-4$ |
| $2$ | $20$ |

The function $f$ is defined by a polynomial. Some values of $x$ and $f(x)$ are shown in the table above. Which of the following must be a factor of $f(x)$?

A) $x+4$

B) $x+3$

C) $x+1$

D) $x-2$

**5**

$$x^3 - 8x^2 + 3x - 24 = 0$$

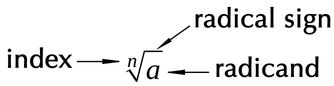
For what real value of $x$ is the equation above true?

**6**

If $x > 0$, what is the solution to the equation $x^4 - 8x^2 = 9$?

## 13-3. Radical Expressions

The symbol $\sqrt[n]{a}$ is called a **radical**. Each part of a radical is given a name as indicated below.




**Definition of $n$th root**

For any real numbers $x$ or $a$, and any positive integer $n$, if $x^n = a$, then $x$ is an $n$th root of $a$.
If $n$ is even, $x = \pm\sqrt[n]{a}$. If $n$ is odd, $x = \sqrt[n]{a}$.

**Definition of $a^{\frac{1}{2}}$ and $a^{\frac{1}{3}}$**

For any nonnegative number $a$, $a^{\frac{1}{2}} = \sqrt{a}$. For any real number $a$, $a^{\frac{1}{3}} = \sqrt[3]{a}$.

**Product and Quotient Property of Radicals**

For any nonnegative number $a$ or $b$, $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

For any real number $a$ or $b$, $\sqrt[3]{ab} = \sqrt[3]{a}\sqrt[3]{b}$ and $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$ if $b \neq 0$.

Example 1 □ Solve.

a. $(x-5)^4 = 16$  b. $x^3 + 1 = -26$

Solution □ a. $(x-5)^4 = 16$

$x - 5 = \pm\sqrt[4]{16}$ Definition of $n$th root, for when $n$ is even.

$x - 5 = \pm 2$ $\sqrt[4]{16} = \sqrt[4]{2^4} = 2$

$x = 5 \pm 2$ Add 5 to each side.

$x = 7$ or $x = 3$ Answer

b. $x^3 + 1 = -26$

$x^3 = -27$ Subtract 1 from each side.

$x = \sqrt[3]{-27} = (-27)^{\frac{1}{3}} = ((-3)^3)^{\frac{1}{3}}$ Definition of $n$th root, for when $n$ is odd.

$= -3$ Answer

Example 2 □ Simplify.

a. $\sqrt{50}\sqrt{6}$  b. $\sqrt{18a^2b^3}$

Solution □ a. $\sqrt{50}\sqrt{6} = \sqrt{25}\sqrt{2}\sqrt{2}\sqrt{3}$  $\sqrt{50} = \sqrt{25}\sqrt{2}$, $\sqrt{6} = \sqrt{2}\sqrt{3}$

$= 5 \cdot 2 \cdot \sqrt{3} = 10\sqrt{3}$

b. $\sqrt{18a^2b^3} = \sqrt{3^2 \cdot 2 \cdot a^2 \cdot b^2 \cdot b}$

$= \sqrt{3^2} \cdot \sqrt{2} \cdot \sqrt{a^2} \cdot \sqrt{b^2} \cdot \sqrt{b}$  $\sqrt{ab} = \sqrt{a}\sqrt{b}$

$= 3 \cdot \sqrt{2} \cdot a \cdot b \cdot \sqrt{b} = 3ab\sqrt{2b}$  $\sqrt{3^2} = 3$, $\sqrt{a^2} = a$, $\sqrt{b^2} = b$

A method used to eliminate radicals from a denominator is called **rationalizing the denominator**. Binomials of the form $\sqrt{a}+\sqrt{b}$ and $\sqrt{a}-\sqrt{b}$ are called **conjugates**. The product of conjugates is always an integer if $a$ and $b$ are integers. You can use conjugates to rationalize denominators.

**Adding and Subtracting Radical Expressions**

Radical expressions in which the radicands are alike can be added or subtracted in the same way that like monomials are added or subtracted.

**Multiplying Radical Expressions**

Multiplying two radical expressions with different radicands is similar to multiplying binomials.

Example 3 □ Simplify.

a. $\dfrac{1}{2-\sqrt{3}}$ b. $(\sqrt{6}-\sqrt{2})(\sqrt{3}+1)$

c. $\sqrt{50}-\sqrt{18}+\sqrt{8}$ d. $\sqrt{6}-\dfrac{\sqrt{2}}{\sqrt{3}}+\dfrac{\sqrt{3}}{\sqrt{2}}$

Solution □ a. $\dfrac{1}{2-\sqrt{3}}=\dfrac{1}{2-\sqrt{3}}\cdot\dfrac{2+\sqrt{3}}{2+\sqrt{3}}$ The conjugate of $2-\sqrt{3}$ is $2+\sqrt{3}$.

$=\dfrac{2+\sqrt{3}}{2^2-(\sqrt{3})^2}$ $(a-b)(a+b)=a^2-b^2$

$=\dfrac{2+\sqrt{3}}{4-3}=2+\sqrt{3}$

b. $(\sqrt{6}-\sqrt{2})(\sqrt{3}+1)$

$=\sqrt{6}\cdot\sqrt{3}+\sqrt{6}\cdot 1-\sqrt{2}\cdot\sqrt{3}-\sqrt{2}\cdot 1$ FOIL

$=\sqrt{2}\cdot\sqrt{3}\cdot\sqrt{3}+\sqrt{6}-\sqrt{6}-\sqrt{2}$ $\sqrt{6}=\sqrt{2}\cdot\sqrt{3}$

$=3\sqrt{2}-\sqrt{2}=2\sqrt{2}$

c. $\sqrt{50}-\sqrt{18}+\sqrt{8}$

$=\sqrt{25\cdot 2}-\sqrt{9\cdot 2}+\sqrt{4\cdot 2}$

$=\sqrt{25}\sqrt{2}-\sqrt{9}\sqrt{2}+\sqrt{4}\sqrt{2}$ $\sqrt{ab}=\sqrt{a}\sqrt{b}$

$=5\sqrt{2}-3\sqrt{2}+2\sqrt{2}$

$=4\sqrt{2}$ Combine like radicals.

d. $\sqrt{6}-\dfrac{\sqrt{2}}{\sqrt{3}}+\dfrac{\sqrt{3}}{\sqrt{2}}$

$=\sqrt{6}-\dfrac{\sqrt{2}}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}+\dfrac{\sqrt{3}}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}}$ Rationalize the denominator.

$=\sqrt{6}-\dfrac{\sqrt{6}}{3}+\dfrac{\sqrt{6}}{2}$ Simplify.

$=\sqrt{6}(1-\dfrac{1}{3}+\dfrac{1}{2})$ Factor.

$=\dfrac{7\sqrt{6}}{6}$

## Exercises - Radical Expressions

**1**

Which of the following is equal to $a^{-\frac{1}{2}}$ ?

A) $-\sqrt{a}$

B) $\dfrac{1}{\sqrt{a}}$

C) $-\dfrac{1}{\sqrt{a}}$

D) $\dfrac{1}{a^2}$

**2**

Which of the following is equal to $\dfrac{1}{3-2\sqrt{2}}$ ?

A) $3-\sqrt{2}$

B) $3+\sqrt{2}$

C) $3+2\sqrt{2}$

D) $3+4\sqrt{2}$

**3**

If $(x+1)^3=-64$, what is the value of $x$ ?

A) $-6$

B) $-5$

C) $-4$

D) $-3$

**4**

Which of the following is equal to $\sqrt{8}+\sqrt{18}-\sqrt{32}$ ?

A) $\sqrt{2}$

B) $2\sqrt{2}$

C) $3\sqrt{2}$

D) $\sqrt{3}$

**5**

Which of the following is equal to $(1+\sqrt{3})(2-\sqrt{3})$ ?

A) $1-\sqrt{3}$

B) $1+\sqrt{3}$

C) $-1-\sqrt{3}$

D) $-1+\sqrt{3}$

**6**

Which of the following is equal to $b^{\frac{5}{3}}$ ?

A) $b\cdot\sqrt{b}$

B) $b\cdot\sqrt{b^{\frac{1}{3}}}$

C) $b\cdot\sqrt[3]{b}$

D) $b\cdot\sqrt[3]{b^2}$

## 13-4. Solving Radical Equations

An equation which contains a radical with a variable in the radicand is called a **radical equation**.
To solve such an equation, first isolate the radical on one side of the equation. Then square each side of the equation to eliminate the radical.

Example 1 □ Solve each equation.

a. $\sqrt{5-2x}=3$

b. $4+\sqrt{\frac{1}{2}x}=7$

Solution □ a. $(\sqrt{5-2x})^2=(3)^2$ Square each side.

$5-2x=9$

$-2x=4$ Subtract 5 from each side.

$x=-2$ Divide each side by $-2$.

b. $4+\sqrt{\frac{1}{2}x}=7$ Original Equation

$\sqrt{\frac{1}{2}x}=3$ Subtract 3 from each side.

$(\sqrt{\frac{1}{2}x})^2=(3)^2$ Square each side.

$\frac{1}{2}x=9$ Simplify.

$x=18$ Multiply each side by 2.

When you square both sides of a radical equation, the resulting equation may have a solution that is not a solution of the original equation. Such a solution is called an **extraneous solution**. Therefore, you must check all the possible solutions in the original equation and disregard the extraneous solutions.

Example 2 □ Solve $\sqrt{x+2}=x$.

Solution □ a. $\sqrt{x+2}=x$ Original equation

$(\sqrt{x+2})^2=(x)^2$ Square each side.

$x+2=x^2$ Simplify.

$0=x^2-x-2$ Subtract $x$ and 2 from each side.

$0=(x-2)(x+1)$ Factor.

$x-2=0$ or $x+1=0$ Zero Product Property

$x=2$ or $x=-1$ Solve.

Check the results by substituting 2 and $-1$ for $x$ in the original equation.

| Check: | |
|---|---|
| $\sqrt{x+2}=x$ | $\sqrt{x+2}=x$ |
| $\sqrt{2+2}=2$ | $\sqrt{-1+2}=-1$ |
| $\sqrt{4}=2$ | $\sqrt{1}=-1$ |
| $2=2$ ✓ True | $\sqrt{1}\neq-1$ ✕ False |

Since $-1$ does not satisfy the original equation, 2 is the only solution.

## Exercises - Solving Radical Equations

**1**

$$11-\sqrt{2x+3}=8$$

What is the solution set of the equation above?

A) 0

B) 3

C) 6

D) 9

**2**

$$\sqrt{-3x+4}=7$$

What is the solution set of the equation above?

A) $-15$

B) $-12$

C) $-8$

D) $-6$

**3**

$$\sqrt{x+18}=x-2$$

What is the solution set of the equation above?

A) $\{-2\}$

B) $\{7\}$

C) $\{-2,7\}$

D) $\{2,-7\}$

**4**

$$\sqrt{5x-12}=3\sqrt{2}$$

What is the solution set of the equation above?

A) 2

B) 4

C) 6

D) 8

**5**

If $a=\sqrt{3}$ and $\sqrt{2-3x}=\frac{1}{3}a$, what is the value of $x$?

**6**

If $k=8-\sqrt{2}$ and $\sqrt[3]{x-k}=-2$, what is the value of $x^2$?

## 13-5. Complex Numbers

**Definition of $i$**

$i=\sqrt{-1}$ and $i^2=-1$

For real numbers $a$ and $b$, the expression $a+bi$ is called a **complex number**.
Number $a$ is called the **real part** and number $b$ is called the **imaginary part** of the complex number $a+bi$.

To add or subtract complex numbers, combine the real parts and combine the imaginary parts.

$(a+bi)+(c+di)=(a+c)+(b+d)i$

$(a+bi)-(c+di)=(a-c)+(b-d)i$

Example 1 □ Simplify.

a. $i^{35}$

b. $\sqrt{-5}\cdot\sqrt{-10}$

c. $(4-3i)+(5+4i)$

d. $2(-3+i)-5(1-i)$

Solution □

a. $i^{35}=i\cdot i^{34}$ — $a^m\cdot a^n=a^{m+n}$

$=i\cdot(i^2)^{17}$ — $(a^m)^n=a^{mn}$

$=i\cdot(-1)^{17}$ — $i^2=-1$

$=-i$ — $(-1)^{17}=-1$

b. $\sqrt{-5}\cdot\sqrt{-10}$

$=(i\cdot\sqrt{5})(i\sqrt{10})$ — $\sqrt{-5}=\sqrt{5}\cdot\sqrt{-1}=i\sqrt{5}$, $\sqrt{-10}=i\sqrt{10}$

$=i^2\sqrt{50}$ — Multiply.

$=(-1)(\sqrt{25}\cdot\sqrt{2})$ — $i^2=-1$, $\sqrt{50}=\sqrt{25}\cdot\sqrt{2}$

$=-5\sqrt{2}$ — Simplify.

c. $(4-3i)+(5+4i)$

$=(4+5)+(-3i+4i)$ — Combine the real parts and the imaginary parts.

$=9+i$ — Simplify.

d. $2(-3+i)-5(1-i)$

$=-6+2i-5+5i$ — Multiply.

$=-11+7i$ — Simplify.

Example 2 □ Solve $3x^2+75=0$.

Solution □ $3x^2+75=0$

$3x^2=-75$ — Subtract 75 from each side.

$x^2=-25$ — Divide each side by 3.

$x=\pm\sqrt{-25}$ — Take the square root of each side.

$x=\pm\sqrt{25}\sqrt{-1}$ — Product Property of Radicals

$x=\pm 5i$ — $\sqrt{-1}=i$

To multiply two complex numbers, use the FOIL method and use the fact that $i^2 = -1$.

$(a+bi)(c+di) = ac + adi + bci + bdi^2 = (ac-bd) + (ad+bc)i$

Imaginary numbers of the form $a+bi$ and $a-bi$ are called **complex conjugates**, and their product is the real number $a^2+b^2$. This fact can be used to simplify the quotient of two imaginary numbers.

**Definition of Equal Complex Numbers**
Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.
$a+bi = c+di$ if and only if $a=c$ and $b=d$.

Example 3 □ Simplify.

a. $(6-i)(2+3i)$

b. $(\sqrt{3}+\sqrt{-2})(\sqrt{3}-\sqrt{-2})$

c. $\dfrac{10}{(1+3i)}$

d. $\dfrac{2+3i}{4-3i}$

Solution □ a. $(6-i)(2+3i)$

| | |
|---|---|
| $=12+18i-2i-3i^2$ | FOIL |
| $=12+16i-3(-1)$ | $i^2=-1$ |
| $=15+16i$ | Simplify. |

b. $(\sqrt{3}-\sqrt{-2})(\sqrt{3}+\sqrt{-2})$

| | |
|---|---|
| $=(\sqrt{3}+i\sqrt{2})(\sqrt{3}-i\sqrt{2})$ | $\sqrt{-2}=i\sqrt{2}$ |
| $=(\sqrt{3})^2-(i\sqrt{2})^2$ | $(a+b)(a-b)=a^2-b^2$ |
| $=3-2i^2$ | |
| $=5$ | $i^2=-1$ |

c. $\dfrac{10}{(1+3i)}$

| | |
|---|---|
| $=\dfrac{10}{(1+3i)}\cdot\dfrac{(1-3i)}{(1-3i)}$ | Rationalize the denominator. |
| $=\dfrac{10(1-3i)}{1-9i^2}$ | $(a+b)(a-b)=a^2-b^2$ |
| $=\dfrac{\cancel{10}(1-3i)}{\cancel{10}}$ | $i^2=-1$ |
| $=1-3i$ | Simplify. |

d. $\dfrac{2+3i}{4-3i}$

| | |
|---|---|
| $=\dfrac{2+3i}{4-3i}\cdot\dfrac{4+3i}{4+3i}$ | Rationalize the denominator. |
| $=\dfrac{8+6i+12i+9i^2}{16-9i^2}$ | FOIL |
| $=\dfrac{-1+18i}{25}$ | $i^2=-1$ |

## Exercises - Complex Numbers

**1**

Which of the following is equal to $\sqrt{-1}-\sqrt{-4}+\sqrt{-9}$ ?

A) $i$

B) $2i$

C) $3i$

D) $4i$

**2**

Which of the following is equal to $\sqrt{-2}\cdot\sqrt{-8}$ ?

A) $-4i$

B) $4i$

C) $-4$

D) $4$

**3**

Which of the following complex numbers is equal to $\dfrac{3-i}{3+i}$ ?

A) $\dfrac{9}{10}-\dfrac{3i}{5}$

B) $\dfrac{9}{10}+\dfrac{3i}{5}$

C) $\dfrac{3}{5}-\dfrac{3i}{5}$

D) $\dfrac{4}{5}-\dfrac{3i}{5}$

**4**

Which of the following is equal to $\dfrac{1}{2}(5i-3)-\dfrac{1}{3}(4i+5)$ ?

A) $\dfrac{3}{2}i-\dfrac{5}{2}$

B) $\dfrac{7}{6}i-\dfrac{7}{3}$

C) $\dfrac{7}{6}i-\dfrac{19}{6}$

D) $\dfrac{5}{6}i-\dfrac{17}{6}$

**5**

If $(4+i)^2=a+bi$, what is the value of $a+b$ ?

**6**

If the expression $\dfrac{3-i}{1-2i}$ is rewritten in the form $a+bi$, in which $a$ and $b$ are real numbers, what is the value of $a+b$ ?

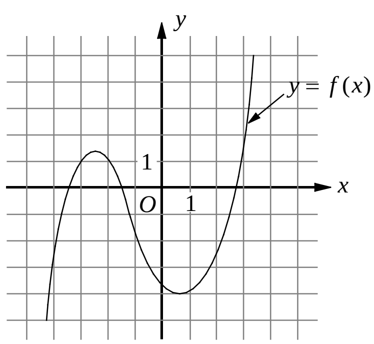
# Chapter 13 Practice Test

**1**

If the graph of $f(x) = 2x^3 + bx^2 + 4x - 4$ intersects the $x$-axis at $(\frac{1}{2}, 0)$, and $(-2, k)$ lies on the graph of $f$, what is the value of $k$?

A) $-4$
B) $-2$
C) $0$
D) $2$

**2**




The function $y = f(x)$ is graphed on the $xy$-plane above. If $k$ is a constant such that the equation $f(x) = k$ has one real solution, which of the following could be the value of $k$?

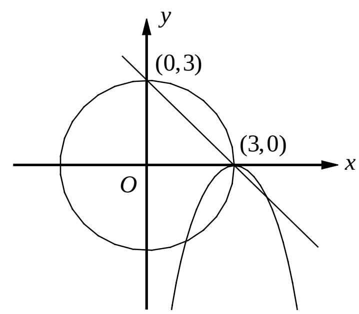
A) $-3$
B) $-1$
C) $1$
D) $3$

**3**

What is the value of $a$ if $x + 2$ is a factor of $f(x) = -(x^3 + 3x^2) - 4(x - a)$?

A) $-2$
B) $-1$
C) $0$
D) $1$

**4**




$$x^2 + y^2 = 9$$
$$y = -(x-3)^2$$
$$x + y = 3$$

A system of three equations and their graphs on the $xy$-plane are shown above. How many solutions does the system have?

A) 1
B) 2
C) 3
D) 4

**5**

Which of the following complex numbers is equivalent to $\dfrac{(1-i)^2}{1+i}$?

A) $-\dfrac{i}{2}-\dfrac{1}{2}$

B) $-\dfrac{i}{2}+\dfrac{1}{2}$

C) $-i-1$

D) $-i+1$

**6**

Which of the following is equal to $a\sqrt[3]{a}$?

A) $a^{\frac{2}{3}}$

B) $a^{\frac{4}{3}}$

C) $a^{\frac{5}{3}}$

D) $a^{\frac{7}{3}}$

**7**

$$p(x)=-2x^3+4x^2-10x$$
$$q(x)=x^2-2x+5$$

The polynomials $p(x)$ and $q(x)$ are defined above. Which of the following polynomials is divisible by $x-1$?

A) $f(x)=p(x)-\dfrac{1}{2}q(x)$

B) $g(x)=-\dfrac{1}{2}p(x)-q(x)$

C) $h(x)=-p(x)+\dfrac{1}{2}q(x)$

D) $k(x)=\dfrac{1}{2}p(x)+q(x)$

**8**

$$\sqrt{2x+6}=x+3$$

What is the solution set of the equation above?

A) $\{-3\}$

B) $\{-1\}$

C) $\{-3,2\}$

D) $\{-3,-1\}$

**9**

What is the remainder when polynomial $p(x)=24x^3-36x^2+14$ is divided by $x-\dfrac{1}{2}$?

A) 4

B) 6

C) 8

D) 10

**10**

The function $f$ is defined by a polynomial. If $x+2$, $x+1$, and $x-1$ are factors of $f$, which of the following table could define $f$?

A)

| $x$ | $f(x)$ |
|---|---|
| $-2$ | 4 |
| $-1$ | 0 |
| 1 | 0 |
| 2 | 0 |

B)

| $x$ | $f(x)$ |
|---|---|
| $-2$ | 0 |
| $-1$ | 4 |
| 1 | 0 |
| 2 | 0 |

C)

| $x$ | $f(x)$ |
|---|---|
| $-2$ | 0 |
| $-1$ | 0 |
| 1 | 4 |
| 2 | 0 |

D)

| $x$ | $f(x)$ |
|---|---|
| $-2$ | 0 |
| $-1$ | 0 |
| 1 | 0 |
| 2 | 4 |

**Answer Key**

Section 13-1

| | | | | |
|---|---|---|---|---|
| 1. D | 2. C | 3. A | 4. B | 5. B |

Section 13-2

| | | | | |
|---|---|---|---|---|
| 1. A | 2. C | 3. D | 4. B | 5. 8 |
| 6. 3 | | | | |

Section 13-3

| | | | | |
|---|---|---|---|---|
| 1. B | 2. C | 3. B | 4. A | 5. D |
| 6. D | | | | |

Section 13-4

| | | | | |
|---|---|---|---|---|
| 1. B | 2. A | 3. B | 4. C | 5. $\frac{5}{9}$ |
| 6. 2 | | | | |

Section 13-5

| | | | | |
|---|---|---|---|---|
| 1. B | 2. C | 3. D | 4. C | 5. 23 |
| 6. 2 | | | | |

Chapter 13 Practice Test

| | | | | |
|---|---|---|---|---|
| 1. C | 2. D | 3. B | 4. A | 5. C |
| 6. B | 7. B | 8. D | 9. C | 10. D |

**Answers and Explanations**

**Section 13-1**

1. D

$f(x) = ax^3 + x^2 - 18x - 9$

If point $(3,0)$ lies on the graph of $f$, substitute 0 for $f$ and 3 for $x$.

$0 = a(3)^3 + (3)^2 - 18(3) - 9$.

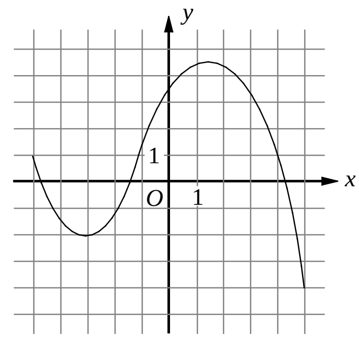
$0 = 27a - 54$

$2 = a$

2. C

If the graph of a polynomial function $f$ has an $x$-intercept at $a$, then $(x-a)$ is a factor of $f(x)$. Since the graph of function $f$ has $x$-intercepts at $-7$, $-5$, and $5$, $(x+7)$, $(x+5)$, and $(x-5)$ must each be a factor of $f(x)$. Therefore, $f(x) = (x+7)(x+5)(x-5) = (x+7)(x^2-5)$.

3. A

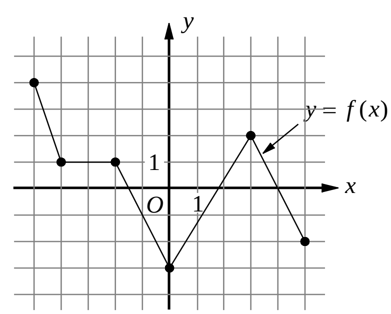



The minimum value of a graphed function is the minimum $y$-value of all the points on the graph. For the graph shown, when $x = -3$, $y = -2$ and when $x = 5$, $y = -4$, so the minimum is at $(5,-4)$ and the minimum value is $-4$.

4. B

A zero of a function corresponds to an $x$-intercept of the graph of the function on the $xy$-plane. Only the graph in choice B has four $x$-intercepts. Therefore, it has the four distinct zeros of function $f$.

5. B




I. $f$ is not strictly decreasing for $-5 < x < 0$, because on the interval $-4 < x < -2$, $f$ is not decreasing.

Roman numeral I is not true.

II. The coordinates $(-3,1)$ is on the graph of $f$, therefore, $f(-3) = 1$

Roman numeral II is true.

III. For the graph shown, when $x = 0$, $y = -3$ and when $x = 5$, $y = -2$, so $f$ is minimum at $x = 0$.

Roman numeral III is not true.

## Section 13-2

1. A

If $-1$ and $1$ are two real roots of the polynomial function, then $f(-1)=0$ and $f(1)=0$. Thus

$f(-1)=a(-1)^3+b(-1)^2+c(-1)+d=0$ and

$f(1)=a(1)^3+b(1)^2+c(1)+d=0$.

Simplify the two equations and add them to each other.

$$\begin{array}{rl} & -a+b-c+d=0 \\ + & \underline{\;a+b+c+d=0\;} \\ & 2b \quad +2d=0 \end{array}$$ or $b+d=0$.

Also $f(0)=3$, since the graph of the polynomial passes through $(0,3)$.

$f(0)=a(0)^3+b(0)^2+c(0)+d=3$ implies $d=3$.

Substituting $d=3$ in the equation $b+d=0$ gives $b+3=0$, or $b=-3$.

2. C

If polynomial $p(x)=81x^5-121x^3-36$ is divided by $x+1$, the remainder is $p(-1)$.

$p(-1)=81(-1)^5-121(-1)^3-36=4$

The remainder is 4.

3. D

If $x-2$ is a factor for polynomial $p(x)$, then $p(2)=0$.

$p(x)=a(x^3-2x)+b(x^2-5)$

$p(2)=a(2^3-2(2))+b(2^2-5)$

$=a(8-4)+b(4-5)$

$=4a-b=0$

4. B

If $(x-a)$ is a factor of $f(x)$, then $f(a)$ must be equal to 0. Based on the table, $f(-3)=0$.

Therefore, $x+3$ must be a factor of $f(x)$.

5. 8

| | |
|---|---|
| $x^3-8x^2+3x-24=0$ | |
| $(x^3-8x^2)+(3x-24)=0$ | Group terms. |
| $x^2(x-8)+3(x-8)=0$ | Factor out the GCF. |
| $(x^2+3)(x-8)=0$ | Distributive Property |
| $x^2+3=0$ or $x-8=0$ | Solutions |

Since $x^2+3=0$ does not have a real solution, $x-8=0$, or $x=8$, is the only solution that makes the equation true.

6. 3

| | |
|---|---|
| $x^4-8x^2=9$ | |
| $x^4-8x^2-9=0$ | Make one side 0. |
| $(x^2-9)(x^2+1)=0$ | Factor. |
| $(x+3)(x-3)(x^2+1)=0$ | Factor. |

Since $x^2+1=0$ does not have a real solution, the solutions for $x$ are $x=-3$ and $x=3$. Since it is given that $x>0$, $x=3$ is the only solution to the equation.

## Section 13-3

1. B

$$a^{-\frac{1}{2}}=\frac{1}{a^{\frac{1}{2}}}=\frac{1}{\sqrt{a}}$$

2. C

| | |
|---|---|
| $\dfrac{1}{3-2\sqrt{2}}$ | |
| $=\dfrac{1}{3-2\sqrt{2}}\cdot\dfrac{3+2\sqrt{2}}{3+2\sqrt{2}}$ | Multiply the conjugate of of the denominator. |
| $=\dfrac{3+2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$ | $(a-b)(a+b)=a^2-b^2$ |
| $=\dfrac{3+2\sqrt{2}}{9-8}$ | Simplify. |
| $=3+2\sqrt{2}$ | |

3. B

| | |
|---|---|
| $(x+1)^3=-64$ | |
| $x+1=\sqrt[3]{-64}$ | Definition of cube root. |
| $x+1=-4$ | $\sqrt[3]{-64}=(-64)^{\frac{1}{3}}=-4$ |
| $x=-5$ | Subtract 1 from each side. |

4. A

$\sqrt{8}+\sqrt{18}-\sqrt{32}$

$=\sqrt{4}\sqrt{2}+\sqrt{9}\sqrt{2}-\sqrt{16}\sqrt{2}$

$=2\sqrt{2}+3\sqrt{2}-4\sqrt{2}$

$=\sqrt{2}$

5. D

$(1+\sqrt{3})(2-\sqrt{3})$

$= 2-\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}$ FOIL

$= 2+\sqrt{3}-3$ Combine like radicals.

$= -1+\sqrt{3}$ Simplify.

6. D

$b^{\frac{5}{3}} = b^1 \cdot b^{\frac{2}{3}} = b \cdot (b^2)^{\frac{1}{3}} = b \cdot \sqrt[3]{b^2}$

**Section 13-4**

1. B

$11-\sqrt{2x+3} = 8$

$11-\sqrt{2x+3}-11 = 8-11$ Subtract 11 from each side.

$-\sqrt{2x+3} = -3$ Simplify.

$(-\sqrt{2x+3})^2 = (-3)^2$ Square each side.

$2x+3 = 9$ Simplify.

$2x = 6$ Subtract 3 from each side.

$x = 3$ Divide each side by 2.

2. A

$\sqrt{-3x+4} = 7$

$(\sqrt{-3x+4})^2 = (7)^2$ Square each side.

$-3x+4 = 49$ Simplify.

$-3x = 45$ Subtract 4 from each side.

$x = -15$ Divide each side by $-3$.

3. B

$\sqrt{x+18} = x-2$

$(\sqrt{x+18})^2 = (x-2)^2$ Square each side.

$x+18 = x^2-4x+4$ Simplify.

$0 = x^2-5x-14$ Make one side 0.

$0 = (x-7)(x+2)$ Factor.

$0 = x-7$ or $0 = x+2$ Zero Product Property

$7 = x$ or $-2 = x$

Check each $x$-value in the original equation.

$\sqrt{7+18} = 7-2$ $x = 7$

$\sqrt{25} = 5$ Simplify.

$5 = 5$ True

$\sqrt{-2+18} = -2-2$ $x = -2$

$\sqrt{16} = -4$ Simplify.

$4 = -4$ False

Thus, 7 is the only solution.

4. C

$\sqrt{5x-12} = 3\sqrt{2}$

$(\sqrt{5x-12})^2 = (3\sqrt{2})^2$ Square each side.

$5x-12 = 18$ Simplify.

$5x = 30$ Add 12 to each side.

$x = 6$ Divide by 5 on each side.

5. $\frac{5}{9}$

$\sqrt{2-3x} = \frac{1}{3}a$

$\sqrt{2-3x} = \frac{1}{3}\sqrt{3}$ $a = \sqrt{3}$

$(\sqrt{2-3x})^2 = (\frac{1}{3}\sqrt{3})^2$ Square each side.

$2-3x = \frac{1}{3}$ Simplify.

$-3x = -\frac{5}{3}$ Subtract 2 from each side.

$-\frac{1}{3}(-3x) = -\frac{1}{3}(-\frac{5}{3})$ Multiply each side by $-\frac{1}{3}$.

$x = \frac{5}{9}$ Simplify.

6. 2

$\sqrt[3]{x-k} = -2$

$(\sqrt[3]{x-k})^3 = (-2)^3$ Cube each side.

$x-k = -8$ Simplify.

$x-(8-\sqrt{2}) = -8$ $k = 8-\sqrt{2}$

$x-8+\sqrt{2} = -8$ Simplify.

$x+\sqrt{2} = 0$ Add 8 to each side.

$x = -\sqrt{2}$ Subtract $\sqrt{2}$.

$(x)^2 = (-\sqrt{2})^2$ Square each side.

$x^2 = 2$ Simplify.

**Section 13-5**

1. B

$\sqrt{-1}-\sqrt{-4}+\sqrt{-9}$

$= i - i\sqrt{4}+i\sqrt{9}$ $i = \sqrt{-1}$

$= i-2i+3i$

$= 2i$

2. C

$\sqrt{-2}\cdot\sqrt{-8}$

$= i\sqrt{2}\cdot i\sqrt{8}$ $\qquad$ $\sqrt{-2}=i\sqrt{2}$ , $\sqrt{-8}=i\sqrt{8}$

$= i^2\sqrt{16}$

$= -4$ $\qquad$ $i^2=-1$

3. D

$\dfrac{3-i}{3+i}$

$= \dfrac{3-i}{3+i}\cdot\dfrac{3-i}{3-i}$ $\qquad$ Rationalize the denominator.

$= \dfrac{9-6i+i^2}{9-i^2}$ $\qquad$ FOIL

$= \dfrac{9-6i-1}{9+1}$ $\qquad$ $i^2=-1$

$= \dfrac{8-6i}{10}$ $\qquad$ Simplify.

$= \dfrac{4-3i}{5}$ or $\dfrac{4}{5}-\dfrac{3i}{5}$

4 . C

$\dfrac{1}{2}(5i-3)-\dfrac{1}{3}(4i+5)$

$= \dfrac{5}{2}i-\dfrac{3}{2}-\dfrac{4i}{3}-\dfrac{5}{3}$ $\qquad$ Distributive Property

$= \dfrac{15}{6}i-\dfrac{9}{6}-\dfrac{8i}{6}-\dfrac{10}{6}$ $\qquad$ 6 is the GCD.

$= \dfrac{7}{6}i-\dfrac{19}{6}$ $\qquad$ Simplify.

5. 23

$(4+i)^2 = a+bi$

$16+8i+i^2 = a+bi$ $\qquad$ FOIL

$16+8i-1 = a+bi$ $\qquad$ $i^2=-1$

$15+8i = a+bi$ $\qquad$ Simplify.

$15 = a$ and $8 = b$ $\qquad$ Definition of Equal Complex Numbers

Therefore, $a+b=15+8=23$ .

6. 2

$\dfrac{3-i}{1-2i}=\dfrac{3-i}{1-2i}\cdot\dfrac{1+2i}{1+2i}=\dfrac{3+6i-i-2i^2}{1-4i^2}$

$=\dfrac{3+6i-i+2}{1+4}=\dfrac{5+5i}{5}=1+i=a+bi$

Therefore, $a=1$ and $b=1$, and $a+b=1+1=2$ .

**Chapter 13 Practice Test**

1. C

$f(x)=2x^3+bx^2+4x-4$

$f(\frac{1}{2})=0$ because the graph of $f$ intersects the $x$- axis at $(\frac{1}{2},0)$ .

$f(\frac{1}{2})=2(\frac{1}{2})^3+b(\frac{1}{2})^2+4(\frac{1}{2})-4=0$

Solving the equation for $b$ gives $b=7$ .

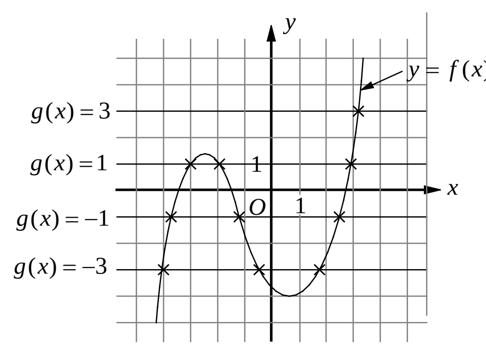
Thus $f(x)=2x^3+7x^2+4x-4$ .

Also $k=f(-2)$ , because $(-2,k)$ lies on the graph of $f$ .

$k=f(-2)=2(-2)^3+7(-2)^2+4(-2)-4$

Solving the equation for $k$ gives $k=0$ .

2. D




$g(x)=-3$ has 3 points of intersection with $y=f(x)$ , so there are 3 real solutions.

$g(x)=-1$ has 3 points of intersection with $y=f(x)$ , so there are 3 real solutions.

$g(x)=1$ has 3 points of intersection with $y=f(x)$ , so there are 3 real solutions.

$g(x)=3$ has 1 point of intersection with $y=f(x)$ , so there is 1 real solution.

Choice D is correct

3. B

If $x+2$ is a factor of $f(x)=-(x^3+3x^2)-4(x-a)$ , then $f(-2)=0$ .

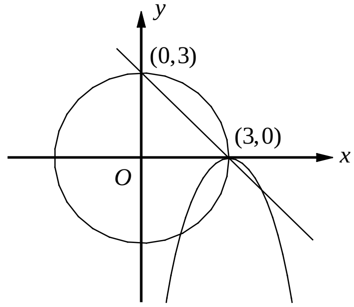
$f(-2)=-((-2)^3+3(-2)^2)-4(-2-a)=0$

$-(-8+12)+8+4a=0$

$4+4a=0$

$a=-1$

4. A




The solutions to the system of equations are the points where the circle, parabola, and line all intersect. That point is $(3,0)$ and is therefore the only solution to the system.

5. C

$$\frac{(1-i)^2}{1+i}$$

| | |
|---|---|
| $=\frac{1-2i+i^2}{1+i}$ | FOIL the numerator. |
| $=\frac{1-2i-1}{1+i}$ | $i^2=-1$ |
| $=\frac{-2i}{1+i}$ | Simplify. |
| $=\frac{-2i}{1+i}\cdot\frac{1-i}{1-i}$ | Rationalize the denominator. |
| $=\frac{-2i+2i^2}{1-i^2}$ | FOIL |
| $=\frac{-2i-2}{2}$ | $i^2=-1$ |
| $=-i-1$ | |

6. B

$$a\sqrt[3]{a}=a\cdot a^{\frac{1}{3}}=a^{1+\frac{1}{3}}=a^{\frac{4}{3}}$$

7. B

$p(x)=-2x^3+4x^2-10x$

$q(x)=x^2-2x+5$

In $p(x)$, factoring out the GCF, $-2x$, yields $p(x)=-2x(x^2-2x+5)=-2x\cdot q(x)$.

Let's check each answer choice.

A) $f(x)=p(x)-\frac{1}{2}q(x)$

$$=-2x\cdot q(x)-\frac{1}{2}q(x)=(-2x-\frac{1}{2})q(x)$$

$q(x)$ is not a factor of $x-1$ and $(-2x-\frac{1}{2})$ is not a factor of $x-1$. $f(x)$ is not divisible by $x-1$.

B) $g(x)=-\frac{1}{2}p(x)-q(x)$

$$=-\frac{1}{2}[-2x\cdot q(x)]-q(x)=(x-1)q(x)$$

Since $g(x)$ is $x-1$ times $q(x)$, $g(x)$ is divisible by $x-1$.

Choices C and D are incorrect because $x-1$ is not a factor of the polynomials $h(x)$ and $k(x)$.

8. D

| | |
|---|---|
| $\sqrt{2x+6}=x+3$ | |
| $(\sqrt{2x+6})^2=(x+3)^2$ | Square each side. |
| $2x+6=x^2+6x+9$ | Simplify. |
| $x^2+4x+3=0$ | Make one side 0. |
| $(x+1)(x+3)=0$ | Factor. |
| $x+1=0$ or $x+3=0$ | Zero Product Property |
| $x=-1$ or $x=-3$ | |

Check each $x$- value in the original equation.

| | |
|---|---|
| $\sqrt{2(-1)+6}=-1+3$ | $x=-1$ |
| $\sqrt{4}=2$ | Simplify. |
| $2=2$ | True |
| $\sqrt{2(-3)+6}=-3+3$ | $x=-3$ |
| $0=0$ | True |

Thus, $-1$ and $-3$ are both solutions to the equation.

9. C

Use the remainder theorem.

$$p(\frac{1}{2})=24(\frac{1}{2})^3-36(\frac{1}{2})^2+14=8$$

Therefore, the remainder of polynomial $p(x)=24x^3-36x^2+14$ divided by $x-\frac{1}{2}$ is 8.

10. D

If $(x-a)$ is a factor of $f(x)$, then $f(a)$ must equal to 0. Thus, if $x+2$, $x+1$ and $x-1$ are factors of $f$, we have $f(-2)=f(-1)=f(1)=0$.

Choice D is correct.