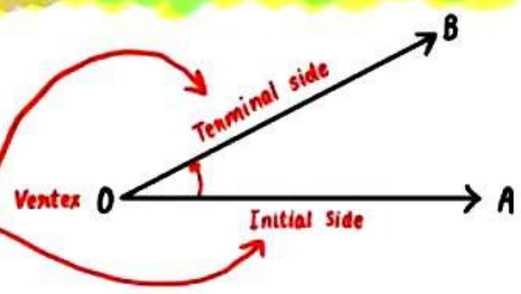
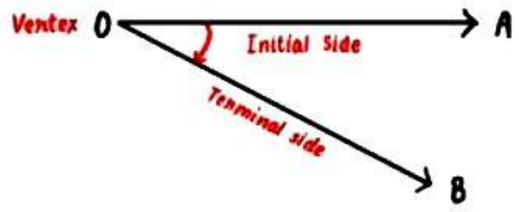
TRIGONOMETRY FUNCTIONS

- Angle is a measure of rotation of a given ray about its initial point.
- The original ray is called the initial side.
- The final position of the ray after rotation is called terminal side.



(Positive angle)

Rotation is anticlockwise



(Negative angle)

Rotation is clockwise

- $1^\circ = 60'$
- $1' = 60''$

1° (one degree) / $1'$ (one minute) / $1''$ (one second)

- $\theta = \frac{l}{r}$

θ = angle
l = an arc of length
r = radius of a circle

- 2π radian = 360°
- π radian = 180°
- Value of $\pi = \frac{22}{7} = 3.14$
- 1 radian = $\frac{180^\circ}{\pi}$
- $1^\circ = \frac{\pi}{180}$ radian

Radian measure = $\frac{\pi}{180}$ x Degree measure

Degree measure = $\frac{180}{\pi}$ x Radian measure

$\sin(-x) = -\sin x$ $\quad$ $\cos(-x) = \cos x$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$
$\sin(x-y) = \sin x \cos y - \cos x \sin y$
$\cos(x+y) = \cos x \cos y - \sin x \sin y$
$\cos(x-y) = \cos x \cos y + \sin x \sin y$

$\cos\left(\frac{\pi}{2} - x\right) = \sin x$ $\quad$ $\cos\left(\frac{\pi}{2} + x\right) = -\sin x$

$\sin\left(\frac{\pi}{2} - x\right) = \cos x$ $\quad$ $\sin\left(\frac{\pi}{2} + x\right) = \cos x$

$\cos(\pi - x) = -\cos x$ $\quad$ $\sin(\pi - x) = \sin x$

$\cos(\pi + x) = -\cos x$ $\quad$ $\sin(\pi + x) = -\sin x$

$\cos(2\pi - x) = \cos x$ $\quad$ $\sin(2\pi - x) = -\sin x$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$ $\quad$ $\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

$\cot(x+y) = \frac{\cot x \cot y - 1}{\cot y + \cot x}$ $\quad$ $\cot(x-y) = \frac{\cot x \cot y + 1}{\cot y - \cot x}$

- $\sin x = 0$ gives $x = n\pi$, where $n \in Z$
- $\cos x = 0$ gives $x = (2n+1)\frac{\pi}{2}$, where $n \in Z$
- $\sin x = \sin y$ implies $x = n\pi + (-1)^n y$, where $n \in Z$
- $\cos x = \cos y$ implies $x = 2n\pi \pm y$, where $n \in Z$
- $\tan x = \tan y$ implies $x = n\pi + y$, where $n \in Z$

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\sin 2x = 2 \sin x \cos x = \frac{2\tan x}{1 + \tan^2 x}$

$\cos 2x = \cos^2 x - \sin^2 x$ OR $\frac{1 - \tan^2 x}{1 + \tan^2 x}$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$\sin 3x = 3\sin x - 4\sin^3 x$
$\cos 3x = 4\cos^3 x - 3\cos x$
$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$

$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$

$2\cos x \cos y = \cos(x+y) + \cos(x-y)$
$-2\sin x \sin y = \cos(x+y) - \cos(x-y)$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$
$2\cos x \sin y = \sin(x+y) - \sin(x-y)$

☑ **Quadrantal angles** : All angles which are integral multiples of $\frac{\pi}{2}$ are called quadrantal angles.

$\sin x = 0$ implies $x = n\pi$, where n is any integer.

$\cos x = 0$ implies $x = (2n+1)\frac{\pi}{2}$, where n is any integer.

$\operatorname{cosec} x = \frac{1}{\sin x}$, $x \neq n\pi$, where n is any integer.

$\sec x = \frac{1}{\cos x}$, $x \neq (2n+1)\frac{\pi}{2}$, where n is any integer.

$\tan x = \frac{\sin x}{\cos x}$, $x \neq (2n+1)\frac{\pi}{2}$, where n is any integer.

$\cot x = \frac{\cos x}{\sin x}$, $x \neq n\pi$, where n is any integer.

☑ **Trigonometric Equations** : Equations involving trigonometric functions of a variable are called trigonometric Equations.

☑ **Principal solutions** : The solutions of a trigometric equation for which $0 \leq x \leq 2\pi$ are called principal solutions.

☑ **General solution** : The expression involving integer 'n' which give all solutions of a trigonometric equation is called the general function.

☑ **Trigonometry Table** :

θ	0°	30°	45°	60°	90°	180°	270°	360°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined	0	Not defined	0
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not defined	-1	Not defined
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined	-1	Not defined	1
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not defined	0	Not defined

☑ **Trigonometric functions in different quadrants** :