Sample Paper 13

Class- X Exam - 2022-23

Mathematics - Basic

Time Allowed: 3 Hours **Maximum Marks : 80**

General Instructions :

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

SECTION - A **20 marks**

(Section - A consists of 20 questions of 1 mark each.)

1. The value of $\frac{\cot A + \tan B}{\cot B + \tan A}$ is:

(a) cot A tan B (b) cot B tan A

(c) cot A (d) tan B 1

2. For any two positive integers '*a*' and '*b*', what is the value of HCF (*a*, *b*) × LCM (*a*, *b*)?

(a) ab^2 (b) a^2b

(c) $a \times b$ (d) $a + b$ 1

3. The mean of first 10 odd natural numbers is:

(a) 20 (b) 40

(c) 30 (d) 10 1

4. Using prime factorisation method, the HCF and LCM of 210 and 175 is:

(a) 35, 1000 (b) 34, 1050

(c) 35, 1050 (d) 35, 1010 1

5. If $2x$, $x + 10$, $3x + 2$ are in A.P., the value of x is:

(a) 6 (b) 7

(c) 9 (d) 10 1

6. By taking A = 90° and B = 30°, sin A cos B – cos A sin B is:

(a) 0 (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$ 1

7. The degree of the polynomial $(x + 1)(x^2 - x - x^4 + 1)$ is:

(a) 4 (b) 2

(c) 5 (d) 3 1

8. The ratio of the volume of a right circular cone to that of the volume of right circular cylinder, of equal diameter and height is:

(a) 4 : 3 (b) 1 : 4

(c) 2 : 3 (d) 1 : 3 1

9. The value of (cos 0° + sin 45° + sin 30°) (sin 90° – cos 45° + cos 60°) is:

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{5}{4}$ (d) $\frac{7}{4}$ 1

10. A quadratic polynomial sum of whose zeros is 3 and product is – 6 is:

(a) $x^2 - 3x - 6$ (b) $2x^2 + 3x + 6$

(c) $x^2 + 3x + 6$ (d) $x^2 - 6x - 3$ 1

11. The value of '*k*' for which the pair of linear equations $kx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is:

(a) 4 (b) 3

(c) 2 (d) None of these 1

12. The probability of getting a number which is neither prime nor composite in a single throw of a fair dice is:

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{1}{6}$ (d) $\frac{1}{4}$ 1

13. The mean of the following data is:

1, 7, 9, 3, 4, 5, 6

(a) 4 (b) 2

(c) 3 (d) 5 1

14. One card is drawn at random from a well shuffled deck of 52 cards. What is the probability to get a face card ?

(a) $\frac{1}{13}$ (b) $\frac{3}{13}$

(c) $\frac{2}{13}$ (d) $\frac{4}{13}$ 1

15. A solid cube is cut into two cuboids of equal volumes. The ratio of surface areas of the given cube and one of the resulting cuboid is:

(a) 2 : 3 (b) 1 : 3

(c) 3 : 2 (d) 3 : 1 1

16. An unbiased dice is rolled once. What is the probability of getting a prime number ?

(a) $\frac{3}{2}$ (b) $\frac{5}{2}$

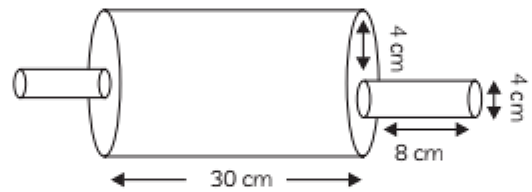
(c) $\frac{1}{3}$ (d) $\frac{1}{2}$ 1

17. If the ratio of the length of a rod to its shadow is $1 : \sqrt{3}$, then, angle of elevation of the sun is:

(a) 30° (b) 60°

(c) 45° (d) 90° 1

18. A rolling pin is made by joining three cylindrical pieces of wood, as shown in the figure:



Assuming that there is no wastage of wood, the volume of wood used in making the rolling pin is:

(a) 64π cm^3 (b) 280π cm^3

(c) 480π cm^3 (d) 544π cm^3 1

Direction for questions 19 and 20: In question number 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct option:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. Assertion (A) : The base radii of two right circular cylinders of the same height are in the ratio 3 : 5. The ratio of their curved surface area is 3 : 5.

Reason (R) : CSA of right circular cylinder is $2\pi r^2 h$. 1

20. Assertion (A): If the probability of the occurence of an event is $\frac{3}{7}$, then the probability of its non-occurence is $\frac{5}{7}$.

Reason (R) : $P(A) + P(\overline{A}) = 1$, when $P(\overline{A})$ is a complement of P(A). 1

SECTION - B **10 marks**

(Section - B consists of 5 questions of 2 marks each.)

21. Determine the A.P. whose 3[rd] term is 5 and the 7[th] term is 9. 2

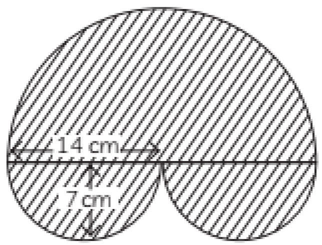
22. Find the zeros of $3x^2 - x - 4$. 2

23. Find the coordinates of the point which divides the line joining (1, –2) and (4, 7) internally in the ratio 1 : 2.

OR

Find the third vertex of a triangle, if two of its vertices are at (–3, 1) and (0, –2) and the centroid is at the origin. 2

24. Find the area of the shaded region:



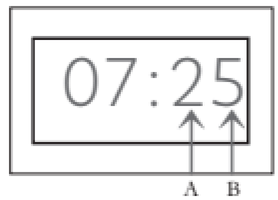
OR

Two dice are thrown simultaneously and the outcomes are noted. Find the probability that :

(A) doublets are obtained

(B) sum of numbers on the two dice is 5. 2

25. Amrish wakes up in the morning and notices that his digital clock reads 07: 25 am. After noon, he looks at the clock again.



What is the probability that:

(A) the number in column A is 4?

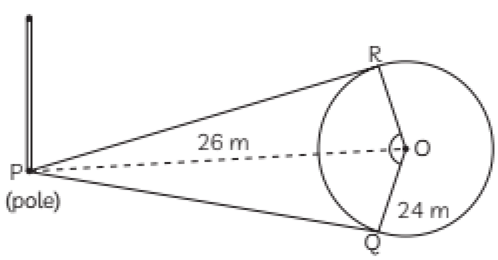
(B) the number in column B is 8? 2

SECTION - C **18 marks**

(Section - C consists of 6 questions of 3 marks each.)

26. On a morning walk, three girls step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps? 3

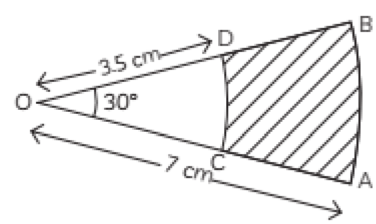
27. There is a circular park of radius 24 m and there is a pole at a distance of 26 m from the centre of the park as shown in the figure. It is planned to enclose the park by planting trees along line segments PQ and PR tangential to the park.



(A) Find the length of PQ and PR;

(B) If six trees are to be planted along each tangential line segments at equal distances, find the distance between any two consecutive trees. 3

28. In the figure, sectors of two concentric circles of radii 7 cm and 3.5 cm are shown.



Find the area of the shaded region. 3

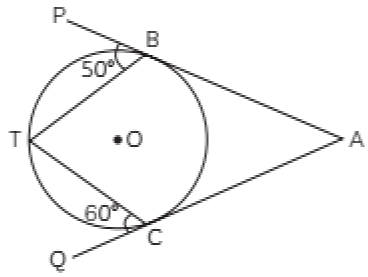
29. The sum of reciprocals of a child's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

OR

Solve for x and y:

$7x - 4y = 49; \quad 5x - 6y = 57$ 3

30. ABP and ACQ are two tangents to the circle with O as its centre in the given figure. If ∠TCQ = 60° and ∠TBP = 50°, then find the measure of ∠BTC.



3

31. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

OR

Find two numbers whose sum is 27 and product is 182. 3

SECTION - D

20 marks

(Section - D consists of 4 questions of 5 marks each.)

32. A number consists of two digits. When it is divided by the sum of the digits, the quotient is 6 with no remainder. When the number is diminished by 9, the digits are reversed. Find the number. 5

33. Amit and Prem were very good cricketers and also represented their school team at district and even state level. One day, after their match, they measured the height of the wickets and found it to be 28 inches. They marked a point P on the ground as shown in the figure below:

(A) If cot P = $\frac{3}{4}$, then find the length of P

(B) Find the value of cosec P.

(C) Find the value of $\frac{1 + \sin P}{1 + \cos P}$.

(D) Find the value of sec R.

(E) Find The value of coses^2 R – cot^2 R.

OR

Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point between them on the road , the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distance of the point from the two poles. 5

34. State and prove Basic Proportionality Theorem. 5

35. The mean of the following frequency distribution is 62.8 and the sum of all the frequencies is 50. Compute the missing frequencies f_1 and f_2.

Classes	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	5	f_1	10	f_2	7	8

OR

A test tube has a hemisphere-shaped lower portion and a cylindrical upper portion with the same radius. The test tube fills up to the point of being exactly full when $\frac{4554}{7}$ cu cm of water is poured, but when only 396 cu cm is added, 9 cm of the tube is left empty. Calcualte the test tube's radius and the cylindrical part's height. 5

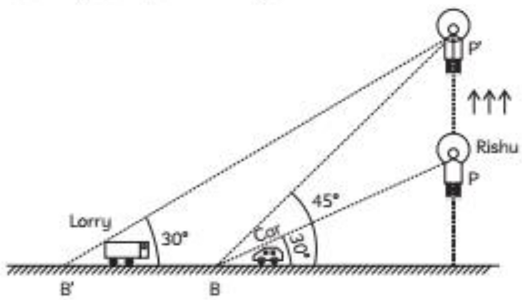
SECTION - E

12 marks

(Case Study Based Questions)

(Section - E consists of 3 questions. All are compulsory.)

36. *Rishu is riding in a hot air balloon. After reaching a point P, he spots a car parked at B on the ground at an angle of depression of 30º. The balloon rises further by 50 metres and now he spots the same car at an angle of depression of 45º and a lorry parked at B' at an angle of depression of 30º. (Use $\sqrt{3}$ = 1.73)*



The measurement of Rishu facing vertically is the height. Distance is defined as the measurement of car/lorry from a point in a horizontal direction. If an imaginary line is drawn from the observation point to the top edge of the car/lorry, a triangle is formed by the vertical, horizontal and imaginary line.

On the basis of the above information, answer the following questions:

(A) **If the height of the balloon at point P is '*h*' m and distance AB is '*x*' m, then find the relation between '*x*' and '*h*':** 1

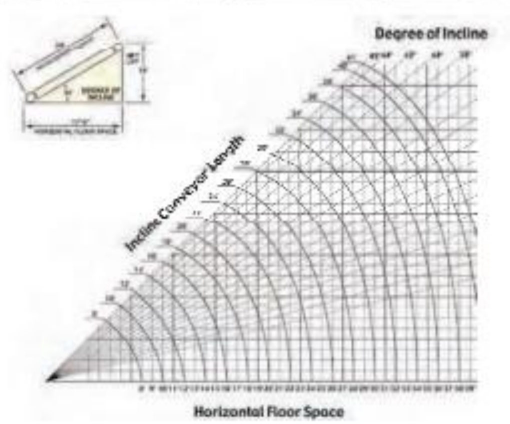
(B) **What is the relation between the height of the balloon at point P and distance AB.** 1

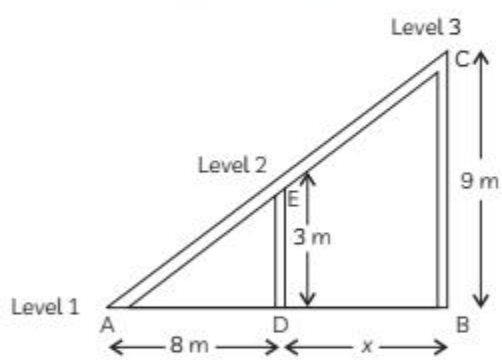
(C) **Find the height of the balloon at point P and the distance AB on the ground.**

OR

Find the distance B'B on the ground. 2

37. *A factory is using an inclined conveyor belt to transport its product from level 1 to level 2 which is 3 m above level 1 as shown in the figure below. The inclined conveyor is supported from one end to level 1 and from the other end to a post located 8 m away from level 1 supporting point.*





The factory wants to extend the conveyor belt to reach at a new level 3 which is 9 m above level 1 while maintaining the inclination angle.

On the basis of the above information, answer the following questions:

(A) Find the distance at which a new post is to be installed to support the conveyor belt at level 3. 1

(B) How much distance is extended from D to B? 1

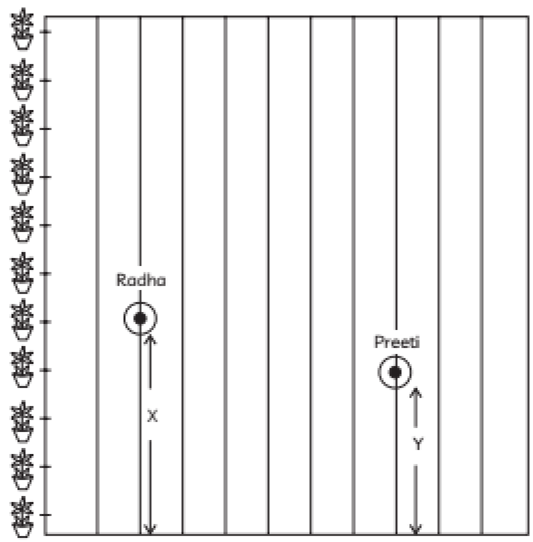
(C) Find the length of the conveyor belt up to level 3.

OR

Find the length of the conveyor belt up to level 2. 2

38. *To conduct sport day activities in the rectangular school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each along DB.*

100 flower pots have been placed at a distance of 1 m from each other along DA as shown in the figure below.



Radha runs $\frac{1}{4}$th of the distance DA on 2nd line and post a green flag at X . Preeti runs $\frac{1}{5}$th of the distance DA on other line post a red flag at Y.

On the basis of the above information, answer the following questions:

(A) Treating DB as x-axis and DA as y–axis, find the position of green flag. 1

(B) Treating DB as x-axis and DA as y–axis, find the position of red flag. 1

(C) Find the distance (in complete metres) between the two flags.

OR

Find the perimeter (in complete metres) of the triangular region OXY. 2

SOLUTION

SECTION - A

1. *(a) cot A tan B*

Explanation:

$$\frac{\cot A + \tan B}{\cot B + \tan A} = \frac{\frac{1}{\tan A} + \tan B}{\frac{1}{\tan B} + \tan A}$$

$$= \frac{1 + \tan A \tan B}{\tan A} \times \frac{\tan B}{1 + \tan A \tan B}$$

$$= \frac{\tan B}{\tan A} = \cot A \tan B$$

2. *(c) $a \times b$*

Explanation: We know the relation,

HCF $(a, b) \times$ LCM (a, b) = Product of a and b.

3. *(d) 10*

Explanation: First ten odd numbers are: 1, 3, 5, ..., 19.

$$\text{Their sum} = \frac{10}{2}(2 \times 1 + 9 \times 2) = 100$$

$$\text{Mean} = \frac{100}{10} = 10$$

4. *(c) 35, 1050*

Explanation: The prime factorisations of 210 and 175 are:

$$210 = 2 \times 3 \times 5 \times 7$$
$$175 = 5 \times 5 \times 7$$

So, HCF (210, 175) = 5 × 7 = 35; and

LCM (210, 175) = 2 × 3 × 5 × 7 × 5 = 1050

⚠ Caution

➥ *While calculating prime factors. Start with the lowest prime number.*

5. *(a) 6*

Explanation: Since $2x$, $x + 10$, $3x + 2$ are in A.P.,

$\therefore \quad 2(x + 10) = 2x + (3x + 2)$

$\Rightarrow \quad 2x + 20 = 5x + 2$

$\Rightarrow \quad 3x = 18$

$\Rightarrow \quad x = 6$

6. *(b) $\frac{\sqrt{3}}{2}$*

Explanation: sin A cos B – cos A sin B

$$= \sin 90° \cos 30° - \cos 90° \sin 30°$$

$$= 1 \times \frac{\sqrt{3}}{2} - 0 \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

7. *(c) 5*

Explanation: $(x + 1)(x^2 - x - x^4 + 1) = (x^3 - x^2 - x^5 + x) + (x^2 - x - x^4 + 1)$

$$= -x^5 - x^4 + x^3 + 1$$

So, the degree of the polynomial is 5.

⚠ Caution

➥ *While finding the degree of a polynomial, arrange the terms in descending powers.*

8. *(d) 1 : 3*

Explanation: Volume of a right circular cone

$= \frac{1}{3}\pi r^2 h$

Volume of a right circular cylinder = $\pi r^2 h$

So, required ratio is 1 : 3.

9. *(d) $\frac{7}{4}$*

Explanation: (cos 0° + sin 45° + sin 30°) (sin 90° – cos 45° + cos 60°)

$$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$= \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{9}{4} - \frac{1}{2} = \frac{7}{4}$$

10. *(a) $x^2 - 3x - 6$*

Explanation: Here, sum of zeros = 3

Product of zeros = – 6

$\therefore$ Quadratic polynomial is

x^2 – (sum of roots) x + product of roots

$\Rightarrow$ $x^2 - 3x - 6$, is the required quadratic polynomial.

11. *(d) None of these*

Explanation: The given pair of equations will have infinitely many solutions when

$$\frac{k}{6} = \frac{-1}{-2} = \frac{2}{3}$$

No value of k satisfy the above relation.

So, no value of k exist.

12. *(c) $\frac{1}{6}$*

Explanation: Total number of outcomes on rolling a fair dice = 6

$\because$ 1 is only the number which is neither prime nor composite.

So, required probability is $\frac{1}{6}$

13. *(d) 5*

Explanation: We know,

$$\text{Mean} = \frac{\text{Sum of observation}}{\text{Total no. of observation}}$$

$$= \frac{1 + 7 + 9 + 3 + 4 + 5 + 6}{7}$$

$$= \frac{35}{7} = 5$$

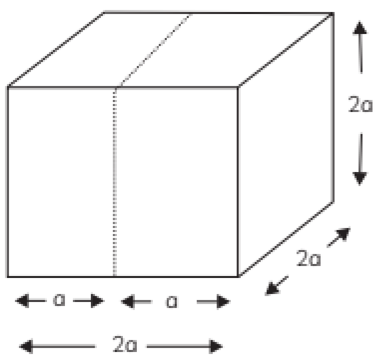
14. *(b)* $\frac{3}{13}$

Explanation: In a deck of 52 cards, number of face cards is 12.

So, the required probability = $\frac{12}{52}$, *i.e.* $\frac{3}{13}$.

15. *(c) 3 : 2*

Explanation: Since the cube is cut into two cuboids of equal volumes, so the two cuboids are equal.



Let the edge of the cube be $2a$ units and the cube be cut along its length.

∴ Dimensions of each cuboid formed are $a \times 2a \times 2a$.

So, Surface area of cuboid

$$= 2(a \times 2a + 2a \times 2a + 2a \times a)$$
$$= 16a^2$$

Also, Surface area of cube

$$= 6(2a)^2 = 24a^2$$

∴ Required ratio = $24a^2 : 16a^2 = 3 : 2$.

16. *(d)* $\frac{1}{2}$

Explanation: When a dice is rolled, the total outcomes are 1, 2, 3, 4, 5 and 6.

∴ Total outcomes = 6

Favourable outcomes = 2, 3, 5 *i.e.* 3

∴ P (getting a prime number) = $\frac{3}{6} = \frac{1}{2}$

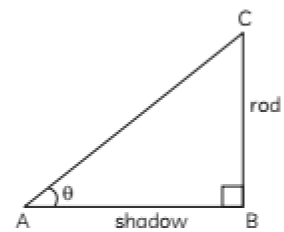
Hence, required probability is $\frac{1}{2}$.

⚠ Caution

➥ *Write all possible outcomes, before finding the probability of an event.*

17. *(a) 30°*

Explanation: Let θ be the angle of elevation of the sun.



Then, $\tan\theta = \frac{BC}{AB}$

$$= \frac{1}{\sqrt{3}}$$

$$\theta = 30°$$

18. *(d) 544π cm³*

Explanation: For the bigger cylinder:

$r_1 = 4$ cm

$h_1 = 30$ cm

For the two smaller cylinders:

$r_2 = \frac{4}{2} = 2$ cm,

$h_2 = 8$ cm

Now, Volume of wood used

= Volume of the pin

= Volume of bigger cylinder + 2x volume of smaller cylinder

$$= \pi r_1^2 h_1 + 2 \times \pi r_2^2 h_2$$

$$= \pi \times (4)^2 \times 30 + 2 \times \pi \times (2)^2 \times 8$$

$$= 480\pi + 64\pi$$

$$= 544\,\pi \text{ cm}^3$$

19. *(c) Assertion (A) is true but reason (R) is false.*

Explanation: Let r_1, h_1 and r_2, h_2 be the radii and the heights of the first and second cylinders respectively.

Then, $\frac{r_1}{r_2} = \frac{3}{5}$

and $h_1 = h_2 = h$ (say)

So, $\frac{\text{CSA of first cylinder}}{\text{CSA of second cylinder}} = \frac{2\neq r_1 h_1}{2\neq r_2 h_2}$

$$= \frac{r_1 h}{r_2 h}$$

$$= \frac{r_1}{r_2} = \frac{3}{5}$$

20. *(d) Assertion (A) is false but reason (R) is true.*

Explanation: For an event A, the P(A) + $P(\bar{A}) = 1$

Here, $P(A) = \frac{3}{7}$

Then, $P(\bar{A}) = 1 - \frac{3}{7}$

$$= \frac{4}{7}$$

SECTION - B

21. Here, $a_3 = a + 2d = 5$

and $a_7 = a + 6d = 9$

Solving the two equations, we get:

$$a = 3, \quad d = 1$$

So, required A.P. is 3, 4, 5, 6, ...

22. Let $p(x) = 3x^2 - x - 4$

$$= 3x^2 - 4x + 3x - 4$$

$$= x(3x - 4) + 1\,(3x - 4)$$

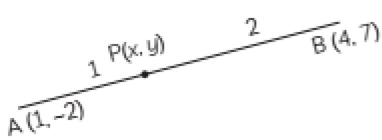
$$= (3x - 4)\,(x + 1)$$

So, the two zeros of $3x^2 - x - 4$ are $\frac{4}{3}$ and -1.

23. Let P(*x*, *y*) divide AB in the ratio 1 : 2. Then,

$$P(x,\ y) = P\left(\frac{2 \times 1 + 1 \times 4}{1 + 2}, \frac{2 \times (-2) + 1 \times 7}{1 + 2}\right)$$

$$= P\left(\frac{2 + 4}{3}, \frac{-4 + 7}{3}\right)$$



$$= P(2, 1)$$

Caution

➥ *In such problems, be clear about the ratio in which a particular point divides the given line, otherwise the points we get would be wrong.*

OR

Let the third vertex be (*x*, *y*). Then,

$$\left(\frac{x - 3 + 0}{3}, \frac{y + 1 - 2}{3}\right) = (0, 0)$$

$$\Rightarrow \quad \frac{x - 3}{3} = 0 \quad \text{and} \quad \frac{y - 1}{3} = 0$$

$$\Rightarrow \quad x = 3 \quad \text{and} \quad y = 1$$

Thus, the third vertex is (3, 1).

24. Area of the shaded region = Area of semi-circle of radius 14 cm + 2 × Area of semi-circle of radius 7 cm.

$$= \left(\frac{\pi}{2}(14)^2 + 2 \times \frac{\pi}{2}(7)^2\right) \text{ sq. cm}$$

$$= \left(\frac{1}{2} \times \frac{22}{7} \times 14 \times 14 + \frac{22}{7} \times 7 \times 7\right) \text{ sq. cm}$$

$$= (308 + 154) \text{ sq. cm}$$

$$= 462 \text{ sq. cm.}$$

OR

On throwing two dice together, Total number of outcomes = 36

(A) Favourable outcomes = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}.

⇒ Number of favourable outcomes = 6

$\therefore$ P (a doublet) = $\frac{6}{36}$ *i.e.,* $\frac{1}{6}$

(B) Favourable outcomes = {(1, 4), (2, 3), (3, 2), (4, 1)}.

⇒ Number of favourable outcomes = 4

$\therefore$ P (sum of 5 on two numbers)

$$= \frac{4}{36} \text{ i.e., } \frac{1}{9}$$

⚠ Caution

➥ *While finding the probability of any event, always write all favourable outcomes.*

25. (A) The number in column A can be 0, 1, 2, 3, 4 and 5.

So, $P(4) = \frac{1}{6}$

(B) The number in column B can be 0, 1, 2, 3, ..., 9.

So, $P(8) = \frac{1}{10}$

SECTION - C

26. Required minimum distance

= LCM (40, 42, 45)

$\because$ $40 = 2 \times 2 \times 2 \times 5$ i.e., $2^3 \times 5$

$42 = 2 \times 3 \times 7$

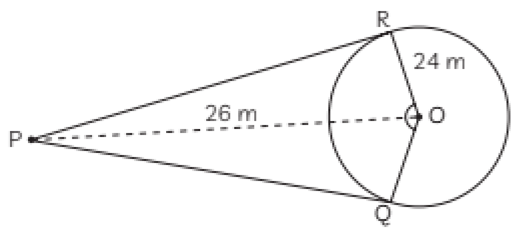
$45 = 3 \times 3 \times 5$ *i.e.,* $3^2 \times 5$

$\therefore$ LCM (40, 42, 45) $= 2^3 \times 3^2 \times 5 \times 7$

$= 2520$

27. (A) In right triangle PRO,

We have $PR = \sqrt{PO^2 - RO^2}$



$= \sqrt{26^2 - 24^2}$

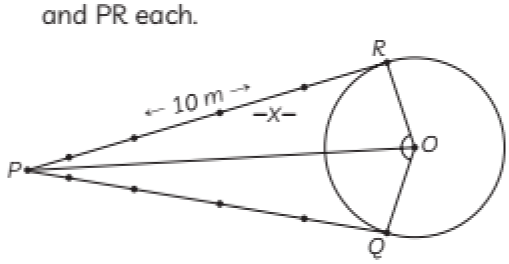
$= \sqrt{676 - 576}$

$= \sqrt{100}$

$= 10m$

$\Rightarrow$ PR = PQ = 10m

(B) As six trees are to be planted along PQ and PR each.



Let's assume the distance between consecutive trees is x.

Total trees are at 5 equal distances.

Hence, $5x = 10$

$x = 2$ m

28. Area of the shaded region

= Area of the sector of radius 7 cm.

– Area of the sector of radius 3.5 cm

$= \left(\frac{30°}{360°}\pi(7)^2 - \frac{30°}{360°}\pi(3.5)^2\right)$ sq. cm

$= \frac{30}{360} \times \frac{22}{7} \times (49 - 12.25)$ sq. cm

= 9.625 sq. cm

29. Let the present age of the child (in years) be x. Then,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{(x+5)+(x-3)}{(x-3)(x+5)} = \frac{1}{3}$$

$\Rightarrow 3(2x+2) = (x-3)(x+5)$

$\Rightarrow 6x + 6 = x^2 + 2x - 15$

$\Rightarrow x^2 - 4x - 21 = 0$

$\Rightarrow x^2 - 7x + 3x - 21 = 0$

$\Rightarrow x(x-7) + 3(x-7) = 0$

$\Rightarrow (x-7)(x+3) = 0$

$\Rightarrow x - 7 = 0$ or $x + 3 = 0$

$\Rightarrow x = 7$ or $x = -3$

($x \neq -3$, as age cannot be negative)

Thus, present age of child is 7 years.

OR

Given equations are:

$7x - 4y = 49$ ...(i)

$5x - 6y = 57$ ...(ii)

Multiplying eq. (i) by 5 and eq. (ii) by 7, we get:

$35x - 20y = 245$ ...(iii)

$35x - 42y = 399$ ...(iv)

Subtracting equation (iv) from equation (iii), we get

$\Rightarrow 22y = -154$

$\Rightarrow y = -7.$

Substituting $y = -7$ in eq. (i), we get

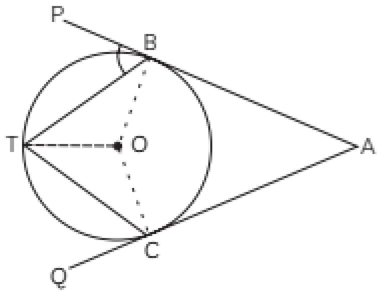
$7x + 28 = 49$

$\Rightarrow 7x = 21$

$\Rightarrow x = 3$

Thus, $x = 3$, $y = -7$ is the required solution.

30.



Join OB, OT and OC

We know, tangent is perpendicular to radius at the point of contact.

$\therefore$ OB $\perp$ AP and OC $\perp$ AQ

$\therefore$ $\angle OBP = 90°$

$\Rightarrow$ $\angle OBT + \angle TBP = 90°$

$\Rightarrow$ $\angle OBT + 50° = 90°$

$\Rightarrow$ $\angle OBT = 90° - 50° = 40°$

Similarly, $\angle OCQ = 90°$ and $\angle TCQ = 60°$

$\therefore$ $\angle OCT = 30°$

Now, in ΔOBT

OB = OT (Radii)

$\angle OTB = \angle OBT$

[Equal angles opposite to equal sides]

$\Rightarrow$ $\angle OTB = 40°$

Similarly in ΔOTC

OT = OC

31. Let the natural number be x. Then,

$$x + 12 = 160 \times \frac{1}{x}$$

$\Rightarrow$ $x^2 + 12x - 160 = 0$

$\Rightarrow$ $x^2 + 20x - 8x - 160 = 0$

$\Rightarrow$ $x(x + 20) - 8(x + 20) = 0$

$\Rightarrow$ $(x + 20)(x - 8) = 0$

$\Rightarrow$ $x + 20 = 0$

$\Rightarrow$ $x = -20$

(Rejected, as x is a natural number)

or $x - 8 = 0$

$\Rightarrow$ $x = 8$

Thus, the required natural number is 8.

OR

Let first number be x and let second number be $(27 - x)$

According to given condition, the product of two numbers is 182.

Therefore,

$x(27 - x) = 180$

$\Rightarrow$ $27x - x^2 = 182$

$\Rightarrow$ $x^2 - 27x + 182 = 0$

$\Rightarrow$ $x^2 - 14x - 13x + 182 = 0$

$\Rightarrow$ $x(x - 14) - 13(x - 14) = 0$

$\Rightarrow$ $(x - 14)(x - 13) = 0$

$\Rightarrow$ $x = 14, 13$

Therefore, the first number is equal to 14 or

SECTION - D

32. Let the number be ab, *i.e.*, $10a + b$

As per the question,

$\frac{10a + b}{a + b} = 6$ or $10a + b = 6a + 6b$

$\Rightarrow$ $4a = 5b$ ...(1)

$(10a + b) - 9 = 10b + a$

or $9a = 9b + 9$

or $a = b + 1$ ...(2)

Solving Eq. (1) and Eq. (2), we get

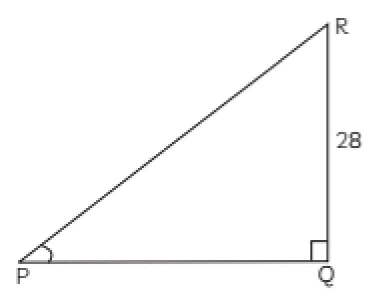
$b = 4$ and $a = 5$.

So, the required number is 54.

33. (A) It is given that QR = 28 inches and $\cot P = \frac{3}{4}$

We know that $\cot P = \frac{\text{Base}}{\text{Perpendicular}}$

$= \frac{PQ}{QR} = \frac{PQ}{28}$



Therefore, $\frac{PQ}{28}=\frac{3}{4}$

$\Rightarrow PQ = \frac{28 \times 3}{4} = 21$ inches.

(B) To evaluate cosec P, we will first find PR by applying Pythagoras theorem ΔPQR.

$$PR^2 = PQ^2 + QR^2$$
$$= 21^2 + 28^2$$
$$= 441 + 784$$
$$= 1225$$
$$\Rightarrow \quad PR = 35 \text{ inches}$$

$$\therefore \text{cosec } P = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{PR}{QR} = \frac{35}{28} = \frac{5}{4}$$

(C) $$\sin P = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$= \frac{PQ}{PR} = \frac{28}{35} = \frac{4}{5}$$

$$\cos P = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{21}{35} = \frac{3}{5}$$

$$\text{Therefore, } \frac{1+\sin P}{1+\cos P} = \frac{1+\frac{4}{5}}{1+\frac{3}{5}} = \frac{\frac{9}{5}}{\frac{8}{5}} = \frac{9}{8}$$

(D) $$\sec R = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{PR}{QR}$$

$$= \frac{35}{28} = \frac{5}{4}$$

(E) $$\text{cosec } R = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$$

$$= \frac{PR}{PQ} = \frac{35}{21} = \frac{5}{3}$$

$$\cot R = \frac{\text{Base}}{\text{Perpendicular}}$$

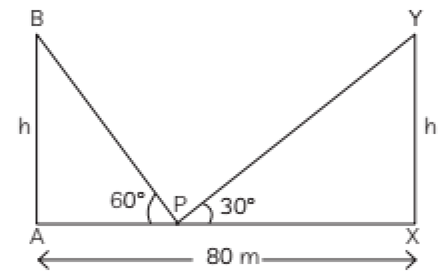
$$= \frac{QR}{PQ} = \frac{28}{21} = \frac{4}{3}$$

$$\therefore \text{Therefore, } \text{cosec}^2 R - \cot^2 R = \left(\frac{5}{3}\right)^2 = \left(\frac{4}{3}\right)^2$$

$$= \frac{25}{9} = \frac{16}{9} = \frac{9}{9} = 1$$

OR

In the figure, AB and XY are two poles of equal height, say 'h' metres.



Here, AX represents the width of the road and P is a point on the road.

In ΔBAP,

$$\frac{AB}{AP} = \tan 60°$$

$$\Rightarrow \quad \frac{h}{AP} = \sqrt{3} \text{ i.e. } AP = \frac{h}{\sqrt{3}} \quad ...(i)$$

In ΔYXP,

$$\frac{XY}{XP} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{XP} = \frac{1}{\sqrt{3}} \text{ i.e. } XP = h\sqrt{3} \quad ...(ii)$$

Adding the two, we get,

$$AP + XP = \frac{h}{\sqrt{3}} + h\sqrt{3}$$

$$\Rightarrow \quad AX = h\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \quad h\left(\frac{4}{\sqrt{3}}\right) = 80$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ metres}$$

Thus, the height of each pole is $20\sqrt{3}$ metres.

From eqn. (i) and (ii), we also have,

$$AP = \frac{20\sqrt{3}}{\sqrt{3}} \text{ i.e. } 20 \text{ metres}$$

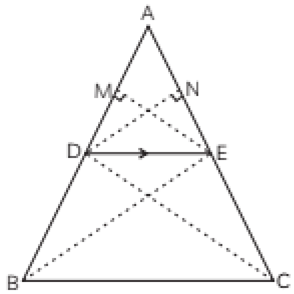
and $$XP = \sqrt{3} \times 20\sqrt{3}, \text{ i.e. } 60 \text{ metres.}$$

Thus, distance, of the point from the two poles are 20 metres and 60 metres..

34. **Statement :** If a line is drawn parallel to one-side of a triangle intersecting the other sides, the other two sides are divided in the same ratio.

Proof: Consider a ΔABC is which DE || BC.

To Prove: $\frac{AD}{BD} = \frac{AE}{EC}$



Construction : Draw EM ⊥ AB, DN ⊥ AC and join BE and CD.

Proof: We know,

$$\text{Area of triangle} = \frac{1}{2} \times AD \times EM$$

$$\text{Also, } \quad ar(\Delta ADE) = \frac{1}{2} \times AE \times DN$$

$$\text{Similarly, } \quad ar(\Delta BDE) = \frac{1}{2} \times BD \times EM$$

$$\text{And, } \quad ar(\Delta DEC) = \frac{1}{2} \times EC \times DN$$

Now, triangles BDE and DEC are on same base DE and lying between some paralleles DE and BC.

$$\therefore \quad ar(\Delta BDE) = ar(\Delta DEC)$$

$$\therefore \quad \frac{ar(\varnothing BDE)}{ar(\varnothing BDE)} = \frac{ar(\varnothing DEC)}{ar(\varnothing ADE)}$$

$$\Rightarrow \quad \frac{\frac{1}{2} \times BD \times EM}{\frac{1}{2} \times AD \times EM} = \frac{\frac{1}{2} \times EC \times DN}{\frac{1}{2} \times AE \times DN}$$

$$\Rightarrow \quad \frac{BD}{AD} = \frac{EC}{AE}$$

$$\text{or} \quad \frac{AD}{BD} = \frac{AE}{EC}$$

35. The frequency distribution for calculating the mean, for the given data is:

Classes	Frequency (f_i)	Class mark (x_i)	$d_i = x_i - A$ where A = 50	$f_i\ d_i$
0–20	5	10	– 40	– 200
20–40	f_1	30	– 20	$-20f_1$
40–60	10	50 = A	0	0
60–80	f_2	70	20	$20f_2$
80–100	7	90	40	280
100–120	8	110	60	480
$\Sigma f_i = 30 + f_1 + f_2$			$\Sigma f_i d_i = 560 + 20f_2 - 20f_1$	

We know that,

$$30 + f_1 + f_2 = 50$$

$$\Rightarrow \quad f_1 + f_2 = 20$$

$$\Rightarrow \quad f_2 = 20 - f_1 \quad \text{...(i)}$$

$$\text{Now, mean} = A + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$\Rightarrow \quad 62.8 = 50 + \frac{560 + 20f_2 - 20f_1}{50}$$

$$\Rightarrow \quad 12.8 = 50 + \frac{560 + 20(20-f_1) - 20f_1}{50}$$

[Using (i)]

$$\Rightarrow \quad 640 = 960 - 40\ f_1$$

$$\Rightarrow \quad 40\ f_1 = 320$$

$$f_1 = 8$$

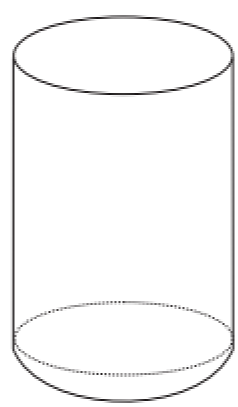
$$\therefore \quad f_2 = 20 - 8 = 12$$

$$\therefore \quad f_1 = 8, f_2 = 12.$$

OR

Volume of water that can fill the test tube

$$= \frac{4554}{7}\ cm^3$$



Let 'r' be the radius of cylinder and hemispherical part.

And let 'h' be height of cylinderical part

$$\therefore \quad \pi r^2 h + \frac{2}{3}\pi r^3 = \frac{4554}{7} \quad ...(i)$$

A.T.Q.,

$$\pi r^2 (h-9) + \frac{2}{3}\pi r^3 = 396$$

$$\Rightarrow \pi r^2 h - \pi r^2 \times 9 + \frac{2}{3}\pi r^3 = 396$$

$$\Rightarrow \pi r^2 h + \frac{2}{3}\pi r^3 - \pi r^2 d = 396$$

$$\Rightarrow \frac{4554}{7} - 9\pi r^2 = 396$$

$$\Rightarrow 9\pi r^2 = \frac{4554}{7} - 396$$

$$\Rightarrow \pi r^2 = \frac{506}{7} - 44$$

$$\Rightarrow r^2 = \frac{506-308}{7} \times \frac{7}{22}$$

$$= \frac{198}{22} = 9$$

$$\Rightarrow r = 3 \text{ cm}$$

From (i)

$$\pi \times 3^2 \times h + \frac{2}{3}\pi(3)^3 = \frac{4554}{7}$$

$$\Rightarrow 9\pi h + 18\pi = \frac{4554}{7}$$

$$\Rightarrow 9\pi h = \frac{4554}{7} - 18 \times \frac{22}{7}$$

$$\Rightarrow 9\pi h = \frac{4158}{7} = 594$$

$$\Rightarrow h = \frac{594}{9\times 22} \times 7$$

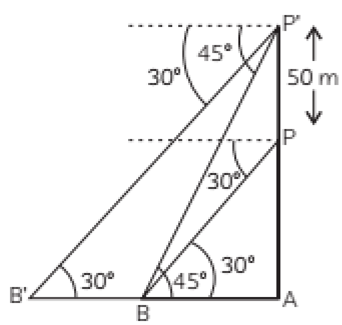
$$= \frac{66\times 7}{22}$$

$$= 21 \text{ cm}$$

Hence, radius of the test tube is 3 cm and height of cylinderical part is 21 cm.

SECTION - E

36.



(A) In ΔAPB,

$$\tan 30° = \frac{AP}{AB}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow x = \sqrt{3}h$$

(B) In ΔAP'B

$$\tan 45° = \frac{AP'}{AB}$$

$$\Rightarrow AB = AP'$$

$$\Rightarrow x = h + 50$$

(C) On solving equation obtained in (i) and (ii), we get

$$\sqrt{3}h = h + 50$$

$$\Rightarrow h(\sqrt{3}-1) = 50$$

$$\Rightarrow h = \frac{50}{0.732} = 68.25$$

In ΔAPB,

$$\tan 30° = \frac{AP}{AB}$$

$$\Rightarrow AB = \frac{AP}{\tan 30°} = \frac{68.25}{1/\sqrt{3}}$$

$$= 68.25 \times 1.732$$

$$= 118 \text{ m}$$

OR

In ΔAP'B'

$$\tan 30° = \frac{AP'}{AB'}$$

$$\frac{1}{\sqrt{3}} = \frac{68.25+50}{AB'}$$

$$\Rightarrow AB' = 118.25 \times 1.732$$

$$= 204.809$$

$$BB' = AB' - AB$$

$= 204.809 - 118$

$= 86.80 = 87$ m

37. (A) In ΔADE and ΔABC,

Since, both triangles are similar, then, their corresponding sides will be proportional.

Then $\frac{AD}{AB} = \frac{DE}{BC}$

$\Rightarrow \quad \frac{8}{AB} = \frac{3}{9}$

$\Rightarrow \quad AB = 24$ cm

(B) Distance extended = AB – AD

$= 24 - 8 = 16$ m

(C) Since, ΔABC is a right-angled at B.

$\therefore \; AC^2 = AB^2 + BC^2$

(by Pythagoras theorem)

$AC^2 = (24)^2 + 9^2$

$AC = \sqrt{676 + 81} = 25.63$

$= 26$ m

Then, distance need to be travelled to reach new level is 26 m

OR

In ΔADE, by pythagoras theorem

$AD^2 + DE^2 = AE^2$

$\Rightarrow \quad 8^2 + 3^2 = AE^2$

$\Rightarrow \quad AE^2 = 64 + 9$

$= 73$

$\Rightarrow \quad AE = \sqrt{73} = 8.5$ m

38. (A) Radha's distance on x-axis is 2 and on y-axis she is at $\frac{1}{4} \times 100 = 25$

Green flag coordinates are (2, 25)

(B) X-coordinate = 8

Y-coordinate = $\frac{1}{5} \times 100 = 20$

$\therefore$ Coordinates of red flag (8, 20)

(C) Coordinates of green flag is (2, 25)

$\therefore$ Coordinates of red flag is (8, 20)

$\therefore$ By distance formula

Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 2)^2 + (20 - 25)^2}$

$= \sqrt{36 + 25}$

$= \sqrt{61} = 7.8$ m

$= 8$ m (approx)

⚠ Caution

➥ *The distance formula* $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$. *It gives the same answer.*

OR

$OX = \sqrt{(2 - 0)^2 + (25 - 0)^2}$

$= \sqrt{4 + 625} = \sqrt{629}$

$= 25.07$

$OY = \sqrt{(8 - 0)^2 + (20 - 0)^2}$

$= \sqrt{64 + 400} = \sqrt{464}$

$= 21.54$

$XY = \sqrt{(8 - 2)^2 + (20 - 25)^2}$

$= \sqrt{36 + 25} = \sqrt{61}$

$= 7.81$

Perimeter = OX + OY + YY

$= 25.07 + 21.54 + 7.81$

$= 54.42$

$= 55$ m