Class XI Session 2023-24
Subject - Mathematics
Sample Question Paper - 1

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

Section A

1. cos 15° - sin 15° = ? **[1]**

a) $\frac{1}{2}$ b) $\frac{(\sqrt{2}-1)}{\sqrt{2}}$

c) $\frac{1}{\sqrt{2}}$ d) $\frac{(\sqrt{2}+1)}{\sqrt{2}}$

2. Let A = {a, b, c}, then the range of the relation R = {(a, b), (a, c), (b, c)} defined on A is **[1]**

a) {b, c} b) {c}

c) {a, b} d) {a, b, c}

3. The probability that a leap year will have 53 Fridays or 53 Saturdays is **[1]**

a) $\frac{2}{7}$ b) $\frac{3}{7}$

c) $\frac{1}{7}$ d) $\frac{4}{7}$

4. $\lim_{x \to \infty} \left(\sqrt{x^2+x+1}-x\right)$ is equal to **[1]**

a) $\frac{1}{2}$ b) 2

c) 0 d) -1

5. The equation of the straight line passing through the point (3, 2) and perpendicular to the line y = x is **[1]**

a) x + y = 1 b) x – y = 5

c) x + y = 5 d) x – y = 1

6. The number of subsets of a set containing n elements is **[1]**

a) 2^n-1 b) 2^n - 2

c) 2^n | d) n

7. If z is a complex number, then **[1]**

a) $|z^2| = |z|^2$ | b) $|z^2| < |z|^2$

c) $|z^2| \geq |z|^2$ | d) $|z^2| > |z|^2$

8. The domain of definition of f(x) = $\sqrt{4x - x^2}$ is **[1]**

a) R - [0, 4] | b) (0, 4)

c) $[0, 4]$ | d) R - (0, 4)

9. A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and third length is to be twice as long as the shortest. What are the possible lengths for the shortest board if the third piece is to be at least 5 cm longer than the second? **[1]**

a) $3 \leq x \leq 91$ | b) $3 \leq x \leq 5$

c) $5 \leq x \leq 91$ | d) $8 \leq x \leq 22$

10. Which is greater, sin 24° or cos 24° ? **[1]**

a) both are equal | b) cos 24°

c) sin 24° | d) cannot be compared

11. The number of proper subsets of the set {1, 2, 3} is : **[1]**

a) 6 | b) 7

c) 8 | d) 5

12. The sum of the infinite geometric series $\left(\frac{-5}{4} + \frac{5}{16} - \frac{5}{64} + \ldots\infty\right) =$? **[1]**

a) $\frac{-1}{4}$ | b) $\frac{5}{8}$

c) $\frac{1}{4}$ | d) -1

13. $\left\{\frac{c_1}{c_0} + 2\frac{C_2}{C_1} + 3\frac{C_3}{C_2} + \ldots + n \cdot \frac{C_n}{C_{n-1}}\right\}$ = ? **[1]**

a) $\frac{1}{2}n(n+1)$ | b) 2n

c) 2^{n-1} | d) 2^n

14. If $|x + 3| \geq 10$, then **[1]**

a) $x \in (-13, 7]$ | b) $x \in (-10, 7]$

c) $x \in (-\infty, -13] \cup [8, \infty)$ | d) $x \in (-\infty, -13] \cup [7, \infty)$

15. Let A and B be two non- empty subsets of a set X such that A is not a subset of B, then **[1]**

a) A and the complement of B are always non-disjoint | b) none of these

c) A and B are always disjoint | d) B is always a subset of A

16. In a circle of radius 14 cm an arc subtends an angle of 36° at the centre. The length of the arc is **[1]**

a) 7.7 cm | b) 8.8 cm

c) 9.1 cm | d) 6.6 cm

17. If x + iy = (1 + i) (1 + 2i) (1 + 3i), then $x^2 + y^2$ = **[1]**

a) None of these
b) 1
c) 100
d) 0

18. A five digit number divisible by 3 is to be formed using the numbers 0, 1, 2, 3, 4 and 5 without repetitions. The total number of ways this can be done is **[1]**
[Hint: 5 digit numbers can be formed using digits 0, 1, 2, 4, 5 or by using digits 1, 2, 3, 4, 5 since sum of digits in these cases is divisible by 3.]

a) 240
b) 3125
c) 216
d) 600

19. **Assertion (A):** The expansion of $(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$. **[1]**
Reason (R): If x = -1, then the above expansion is zero.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The proper measure of dispersion about the mean of a set of observations i.e. standard deviation is expressed as positive square root of the variance. **[1]**
Reason (R): The units of individual observations x_i and the unit of their mean are different that of variance. Since, variance involves sum of squares of $(x - \bar{x})$.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Draw the graph of the step function f(x) = [x]. **[2]**

OR

Find the domain of the real function: f(x) = $\frac{2x-3}{x^2+x-2}$

22. Differentiate: $(3x-5)(4x^2 - 3 + e^x)$. **[2]**

23. Find the equation of the circle, the coordinates of the end points of one of whose diameters are A (5, -3) and B (2, -4) **[2]**

OR

Find the equation of the parabola whose: focus is (0, 0) and the directrix is 2x - y - 1 = 0.

24. Write A = $\left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49}\right\}$ in set - builder form. **[2]**

25. Find the angle between the lines whose slopes are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$. **[2]**

Section C

26. For any sets A, B and C, prove that: $A \times (B \cap C) = (A \times B) \cap (A \times C)$ **[3]**

27. Solve the inequation $\frac{2x+4}{x-3} \leqslant 4$. **[3]**

28. Find the foot of perpendicular from the point (2, 3, -8) to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance from the given point to the line. **[3]**

OR

Find the distance between the point (-1, -5, -10) and the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane x - y + z = 5.

29. Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem. **[3]**

OR

If a and b are distinct integers, prove that a - b is a factor of $a^n - b^n$, whenever n is a positive integer.

30. Express the complex number $\left(-2-\frac{1}{3}i\right)^3$ in the form of a + ib. **[3]**

OR

Find the square root of $-2+2\sqrt{3}i$

31. For all sets A, B and C **[3]**

Is (A – B) ∩ (C – B) = (A ∩ C) – B?

Justify your answer.

Section D

32. A number is chosen from the numbers 1 to 100. Find the probability of its being divisible by 4 or 6. **[5]**

33. i. Find the derivative of $\frac{\sin x+\cos x}{\sin x-\cos x}$. **[5]**

ii. Let $f(x)=\begin{cases}x^2-1, & 0<x<2\\ 2x+3, & 2\leq x<3\end{cases}$, find quadratic equation whose roots are $\lim\limits_{x\to 2^-} f(x)$ and $\lim\limits_{x\to 2^+} f(x)$.

OR

Show that $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x+1}-x\right)\neq\lim\limits_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$.

34. Find the three numbers in GP, whose sum is 52 and sum of whose product in pairs is 624. **[5]**

35. Prove that $\cos 2x\cdot\cos\frac{x}{2}-\cos 3x\cdot\cos\frac{9x}{2}=\sin 5x\cdot\sin\frac{5x}{2}$ **[5]**

OR

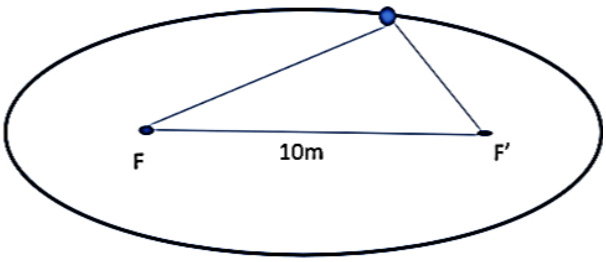
Prove that: cos 10° cos 30° cos 50° cos 70° = $\frac{3}{16}$.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A farmer wishes to install 2 handpumps in his field for watering.

The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.



(i) Name the curve traced by farmer and hence find the foci of curve.

(ii) Find the equation of curve traced by farmer.

(iii) Find the length of major axis, minor axis and eccentricity of curve along which farmer moves.

OR

Find the length of latus rectum.

37. **Read the text carefully and answer the questions:** **[4]**

For a group of 200 candidates, the mean and the standard deviation of scores were found to be 40 and 15 , respectively. Later on it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively.

Student	Eng	Hindi	S.St	Science	Maths
Ramu	39	59	84	80	41
Rajitha	79	92	68	38	75
Komala	41	60	38	71	82
Patil	77	77	87	75	42
Pursi	72	65	69	83	67
Gayathri	46	96	53	71	39

(i) Find the correct variance.

(ii) What is the formula of variance.

(iii) Find the correct mean.

OR

Find the sum of correct scores.

38. **Read the text carefully and answer the questions:** **[4]**

A state cricket authority has to choose a team of 11 members, to do it so the authority asks 2 coaches of a government academy to select the team members that have experience as well as the best performers in last 15 matches. They can make up a team of 11 cricketers amongst 15 possible candidates. In how many ways can the final eleven be selected from 15 cricket players if:

(i) Two of them being leg spinners, in how many ways can be the final eleven be selected from 15 cricket players if one and only one leg spinner must be included?

(ii) If there are 6 bowlers, 3 wicketkeepers, and 6 batsmen in all. In how many ways can be the final eleven be selected from 15 cricket players if 4 bowlers, 2 wicketkeepers and 5 batsmen are included.

Solution

Section A

1.

(c) $\frac{1}{\sqrt{2}}$

Explanation: cos 15° - sin 15° = cos(45° - 30°) - sin (45° - 30°)

= (cos 45° cos 30° + sin 45° sin 30°) - (sin 45° cos 30° - cos 45° sin 30°)

$= \left\{\left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)\right\} - \left\{\left(\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}} \times \frac{1}{2}\right)\right\}$

$= \frac{(\sqrt{3}+1)}{2\sqrt{2}} - \frac{(\sqrt{3}-1)}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

2. **(a)** {b, c}

Explanation: Since the range is represented by the y- coordinate of the ordered pair (x, y). Therefore, the range of the given relation is { b, c }.

3.

(b) $\frac{3}{7}$

Explanation: We know that a leap year has 366 days (i.e. 7 × 52 + 2) = 52 weeks and 2 extra days

The sample space for these 2 extra days is given below:

S = {(Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)}

There are 7 cases.

∴ n(S) = 7

Let E be the event that the leap year has 53 Fridays or 53 Saturdays.

E = {(Thursday, Friday), (Friday, Saturday), (Saturday, Sunday)}

i.e. n(E) = 3

∴ P(E) = $\frac{n(R)}{n(S)} = \frac{3}{7}$

Hence, the probability that a leap year has 53 Fridays or 53 Saturdays is $\frac{3}{7}$

4. **(a)** $\frac{1}{2}$

Explanation: Substitute x = $\frac{1}{t}$

$\Rightarrow \lim_{t \to 0} \frac{\sqrt{t^2+t+1}-1}{t}$

Using L' Hospital

$\lim_{t \to 0} \frac{\frac{2t+1}{2\sqrt{t^2+t+1}}}{1}$

= $\frac{1}{2}$

5.

(c) x + y = 5

Explanation: Here,it is the straight line passing through the point (3, 2)

and perpendicular to the line y = x

Suppose the equation of line 'L' is

$y - y_1 = m(x - x_1)$

Since, L is passing through the point (3, 2)

∴ y – 2 = m(x – 3) …(i)

Now, given eq. is y = x

Since, the above equation is in **y = mx + b** form

Therefore, the slope of this equation is 1

It is also given that line L and y = x are perpendicular to each other.

We know that, when two lines are perpendicular, then

$m_1 \times m_2 = -1$

$\therefore$ m $\times$ 1 = -1

$\Rightarrow$ m = -1

Substituting the value of m in eq. (i), we ob

y – 2 = (-1)(x – 3)

$\Rightarrow$ y – 2 = -x + 3

$\Rightarrow$ x + y = 3 + 2

$\Rightarrow$ x + y = 5

6.

(c) 2^n

Explanation: 2^n

The total number of subsets of a finite set consisting of n elements is 2^n.

7. **(a)** $|z^2| = |z|^2$

Explanation: If z is a complex number, then z = x + iyz = x + iy

|z| = |x + iy| and $|z|^2 = |x + iy|^2$

$\Rightarrow |z|^2 = x^2 + y^2$ ...(i)

and $z^2 = (x + iy)^2 = x^2 + i^2y^2 + i^2xy$

$\Rightarrow |z^2| = \sqrt{(x^2 - y^2)^2 + (2xy)^2}$

$\Rightarrow |z^2| = \sqrt{x^4 + y^4 - 2x^2y^2 + 4x^2y^2}$

$\Rightarrow |z^2| = \sqrt{x^4 + y^4 - 2x^2y^2} = \sqrt{(x^2 + y^2)^2}$

$\Rightarrow |z^2| = x^2 + y^2$ ..(ii)

From Eqs. (i) and (ii)

$|z|^2 = |z|^2$

8.

(c) $[0, 4]$

Explanation: Here, 4x - $x^2 \geq 0$

$x^2 - 4x \leq 0$

x(x - 4) $\leq$ 0

So, $x \in [0, 4]$

9.

(d) 8 $\leq$ x $\leq$ 22

Explanation: Let the length of the shortest piece be x cm. Then we have the length of the second and third pieces are x + 3 and 2x centimeters respectively.

According to the question,

x + (x + 3) + 2x $\leq$ 91

$\Rightarrow$ 4x + 3 $\leq$ 91

$\Rightarrow$ 4x $\leq$ 88

$\Rightarrow$ x $\leq$ 22

Also 2x $\geq$ (x + 3) + 5

$\Rightarrow$ 2x $\geq$ x + 8

$\Rightarrow$ x $\geq$ 8

$\Rightarrow$ 8 $\leq$ x $\leq$ 22

Hence the shortest piece may be atleast 8 cm long but it cannot be more than 22cm in length.

10.

(b) cos 24°

Explanation: cos 24° = cos (90° - 66°) = sin 66°.

In quadrant I, sin θ is increasing

$\therefore$ sin 66° > sin 24° $\Rightarrow$ cos 24° > sin 24°

11.

(b) 7

Explanation: The no. of proper subsets = 2^n - 1 = 2^3 - 1 = 7
Here n = no of elements of given set = 3.

12.
(d) -1
Explanation: Given, a = $\frac{-5}{4}$ and r = $\frac{5}{16} \times \frac{(-4)}{5} = \frac{-1}{4}$.
Clearly, |r| = $\frac{1}{4} < 1$.
$\therefore S_{-} = \frac{a}{(1-r)} = \frac{\left(\frac{-5}{4}\right)}{\left(1+\frac{1}{4}\right)} = \left(\frac{-5}{4} \times \frac{4}{5}\right) = -1$.

13. **(a)** $\frac{1}{2}n(n+1)$
Explanation: We know that $\frac{C_r}{C_{r1}} = \frac{n-r+1}{r}$,
Substituting r = 1,2,3,...,n, we obtain
$\frac{C_1}{C_0} + 2 \cdot \frac{C_2}{C_1} + 3 \cdot \frac{C_3}{C_2}$ + ... + $n \cdot \frac{C_n}{C_{n-1}}$ = n + (n -1) + (n - 2) + ... + 1= $\frac{1}{2}n(n+1)$.

14.
(d) x ∈ (–∞, – 13] ∪ [7, ∞)
Explanation: since |x + 3| $\geq$ 10 , ⇒ x + 3 $\leq$ – 10 or x + 3 $\geq$ 10
⇒ x $\leq$ – 13 or x $\geq$ 7
⇒ x ∈ (– ∞, – 13] ∪ [7, ∞)
solution set = (– ∞, – 13] ∪ [7, ∞)

15. **(a)** A and the complement of B are always non-disjoint
Explanation: Let x ∈ A, then x ∉ B as A is not a subset of B
∴ x ∈ A and x ∉ B
⇒ x ∈ A and x ∈ B'
⇒ x ∈ A ∩ B'
⇒ A and B' are non - disjoint.

16.
(b) 8.8 cm
Explanation: $\theta = \left(36 \times \frac{\pi}{180}\right)^c = \left(\frac{\pi}{5}\right)^c$ and r = 14 cm.
$\therefore l = r\theta = \left(14 \times \frac{\pi}{5}\right)$ cm $= \left(14 \times \frac{22}{7} \times \frac{1}{5}\right)$ cm $= \frac{44}{5}$cm = 8.8 cm

17.
(c) 100
Explanation: 100
∴ x + iy = (1 + i) (1 + 2i) (1 + 3i), then $x^2 + y^2$ =
Taking modulus on both the sides :
$|x+iy| = |(1+i)\ (1+2i)(1+3i)|$
$\Rightarrow |x+iy| = |1+i|\ \times |1+2i| \times |1+3i|$
$\Rightarrow \sqrt{x^2+y^2} = \sqrt{1^2+1^2}\ \sqrt{1^2+2^2}\sqrt{1^2+3^2}$
$\Rightarrow \sqrt{x^2+y^2} = \sqrt{2}\sqrt{5}\sqrt{10}$
$\Rightarrow \sqrt{x^2+y^2} = \sqrt{100}$
Squaring both the sides
⇒ $x^2 + y^2$ = 100

18.
(c) 216
Explanation: We know that a number is divisible by 3 when the sum of its digits is divisible by 3
If we take the digits 0, 1, 2, 4, 5, then the sum of the digits = 0 + 1 + 2 + 4 + 5 = 12 which is divisible by 3.
Therefore, the 5 digit numbers using the digits 0,1, 2, 4, and 5.

TTh	Th	H	T	O
4	4	3	2	1

= 4 × 4 × 3 × 2 × 1 = 96

and if we take the digits 1, 2, 3, 4, 5, then their sum = 1 + 2 + 3 + 4 + 5 = 15 divisible by 3

5 digit numbers can be formed using the digits 1, 2, 3, 4, 5 is 5! ways = $5 \times 4 \times 3 \times 2 \times 1$ = 120 ways

Therefore, Total number of ways = 96 + 120 = 216

19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Assertion:

$(1 + x)^n = n_{c_0} + n_{c_1}x + n_{c_2}x^2 \ldots + n_{c_n}x^n$

Reason:

$(1 + (-1))^n = n_{c_0}1^n + n_{c_1}(1)^{n-1}(-1)^1 + n_{c_2}(1)^{n-2}(-1)^2 + \ldots + {}^nc_n(1)^{n-n}(-1)^n$

$= n_{c_8} - n_{c_1} + n_{c_2} - n_{c_3} + \ldots (-1)^n n_{c_n}$

Each term will cancel each other

$\therefore (1 + (-1))^n = 0$

Reason is also the but not the correct explanation of Assertion.

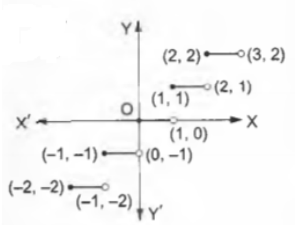
20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: In the calculation of variance, we find that the units of individual observations x_i and the unit of their mean $\bar{x}$ are different from that of variance, since variance involves the sum of squares of $(x_i - \bar{x})$.

For this reason, the proper measure of dispersion about the mean of a set of observations is expressed as positive square-root of the variance and is called standard deviation.

Section B

21. f(x) = [x].



As the definition of the function indicates,

for all x such that $-2 \leq x < -1$, we have f(x) = - 2 ;

for all x such that $-1 \leq x < 0$, we have f(x) = - 1;

for all x such that $0 \leq x < 1$, we have/(x) = 0;

for all x such that $1 \leq x < 2$, we have f(x) = 1,

and so on, f(x) = [x] = $\begin{cases} -2 \text{ when } x \in [-2,-1) \\ -1 \text{ when } x \in [-1,0) \\ 0 \text{ when } x \in [0,1) \\ 1 \text{ when } x \in [1,2) \\ \text{and so on.} \end{cases}$

Clearly, the function jumps at the points (-1 , -2), (0, -1), (1, 0), (2,1), etc.

In other words, the given function is discontinuous at each integral value of x.

OR

Here we are given that, f(x) = $\frac{2x-3}{x^2+x-2}$

We need to find where the function is defined.

To find the domain of the function f(x) we need to equate the denominator to 0.

Therefore,

$x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x + 2) - 1(x + 2) = 0$

$\Rightarrow (x + 2)(x - 1) = 0$

$\Rightarrow x = -2 \;\&\; x = 1$

It means that the denominator is zero when x = 1 and x = -2

So, the domain of the function is the set of all the real numbers except 1 and -2

The domain of the function, D{f(x)} = $(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$

22. Let u = (3x – 5) and v = $(4x^2 – 3 + e^x)$

$u' = \frac{du}{dx} = \frac{d(3x-5)}{dx} = 3$

$v' = \frac{dv}{dx} = \frac{d(4x^2-3+e^x)}{dx} = (8x + e^x)$

Put the above obtained values in the formula :-

(uv)′ = u′v + uv′ (Using product rule)

$[(3x – 5)(4x^2 – 3 + e^x)]' = 3\times(4x^2 – 3 + e^x) + (3x – 5)\times(8x + e^x)$

$= 12x^2 – 9 + 3e^x + 24x^2 + 3xe^x – 40x - 5e^x$

$= 36x^2 + x(3e^x – 40) – 9 - 2e^x$

23. The equation of a circle passing through the end points of diameters is:

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$ ...(1)

Substituting, values: $(x_1, y_1) = (5, - 3)$ & $(x_2, y_2) = (2, - 4)$ in equation (1)

We get: (x - 5)(x - 2) + (y + 3)(y + 4) = 0

$\Rightarrow x^2 - 2x - 5x + 10 + y^2 + 3y + 4y + 12 = 0$

$\Rightarrow x^2 + y^2 - 7x + 7y + 22 = 0$

OR

Let P (x, y) be any point on the parabola whose focus is S (0, 0) and the directrix is 2x - y - 1 = 0

Draw PM perpendicular to 2x - y - 1 = 0

Thus, we have:

SP = PM

$\Rightarrow SP^2 = PM^2$

$\Rightarrow (x - 0)^2 + (y - 0)^2 = \left|\frac{2x-y-1}{\sqrt{4+1}}\right|^2$

$\Rightarrow x^2 + y^2 = \left(\frac{2x-y-1}{\sqrt{5}}\right)^2$

$\Rightarrow 5x^2 + 5y^2 = 4x^2 + y^2 + 1 - 4xy + 2y - 4x$

$\Rightarrow x^2 + 4y^2 + 4xy - 2y + 4x - 1 = 0$

Which is the required equation of parabola.

24. Given,

$A = \left\{1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \frac{1}{36}, \frac{1}{49}\right\}$

$= \left\{\left(\frac{1}{1}\right)^2, \left(\frac{1}{2}\right)^2, \left(\frac{1}{3}\right)^2, \left(\frac{1}{4}\right)^2, \left(\frac{1}{5}\right)^2, \left(\frac{1}{6}\right)^2, \left(\frac{1}{7}\right)^2\right\}$

Thus in set builder form, it can be written as,

$A = \left\{x : x = \frac{1}{n^2}, n \in N\right\}$

25. To find out the angle between two lines, the angle is equal to the difference in θ.

The slope of a line = tanθ = $\left(\frac{y_2-y_1}{x_2-x_1}\right)$

Therefore ,slope of the first line = $\sqrt{3} = \tan\theta_1 \Rightarrow \tan\theta_1 = \sqrt{3}$

$\Rightarrow \theta_1 = \tan^{-1}(\sqrt{3})$

$\Rightarrow \theta_1 = 60°$

The slope of the second line = $\frac{1}{\sqrt{3}} = \tan\theta_2 \Rightarrow \theta_2 = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\Rightarrow \theta_1 = 30°$

Now the difference between the two lines is θ_1 - θ_2

= 60° - 30°

= 30°, which is the required angle

Section C

26. Here we are given that A, B and C three sets.

To prove: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let us consider, $(x, y) \in A \times (B \cap C)$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow (x \in A \text{ and } y \in B) \; (x \in A \text{ and } y \in C)$

$\Rightarrow (x, y) \in (A \times B) \text{ and } (x, y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

From above, we can say that,

$\Rightarrow A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ ....(i)

Let us consider again, $(a, b) \in (A \times B) \cap (A \times C)$

$\Rightarrow (a, b) \in (A \times B) \text{ and } (a, b) \in (A \times C)$

$\Rightarrow (a \in A \text{ and } b \in B) \text{ and } (a \in A \text{ and } b \in C)$

$\Rightarrow a \in A \text{ and } (b \in B \text{ and } b \in C)$

$\Rightarrow a \in A \text{ and } b \in (B \cap \mathrm{C})$

$\Rightarrow (a, b) \in A \times (B \cap C)$

From above, we can say that,

$\Rightarrow (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ ....(ii)

From (i) and (ii).

$A \times (B \cap C) = (A \times B) \cap (A \times C)$

Hence proved.

27. Here $\frac{2x+4}{x-3} \leqslant 4, x \neq 3$

$\Rightarrow \frac{2x+4}{x-3} - 4 \leqslant 0$

$\Rightarrow \frac{2x+4-4x+12}{x-3} \leqslant 0$

$\Rightarrow \frac{-2x+16}{x-3} \leqslant 0$

$\Rightarrow -2x + 16 \leqslant 0$

$\Rightarrow -2x \leqslant -16$

Dividing both sides by -2

$\Rightarrow x \geqslant 8$

the solution set of given in equation is $[8, \infty)$.

28. We have equation of line is $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$

$\Rightarrow \frac{x-4}{-2} = \frac{y}{6} = \frac{z-1}{-3} = \lambda$

$\Rightarrow x = -2\lambda + 4, y = 6\lambda \text{ and } z = -3\lambda + 1$

Let the coordinates of L be $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$, then, direction ratios of PL are proportional to $(4 - 2\lambda - 2, 6\lambda - 3, 1 - 3\lambda + 8)$ i.e., $(2 - 2\lambda, 6\lambda - 3, 9 - 3\lambda)$.

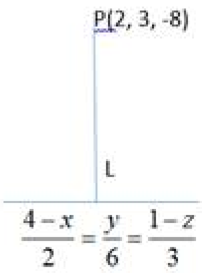
Also, direction ratios are proportional to -2, 6, -3. Since, PL is perpendicular to give line.

$\therefore -2(2 - 2\lambda) + 6(6\lambda - 3) - 3(9 - 3\lambda) = 0$

$\Rightarrow -4 + 4\lambda + 36\lambda - 18 - 27 + 9\lambda = 0$

$\Rightarrow 49\lambda = 49 \Rightarrow \lambda = 1$

So, the coordinates of L are $(4 - 2\lambda, 6\lambda, 1 - 3\lambda)$ i.e., (2, 6, -2).



Also, length of PL $= \sqrt{(2-2)^2 + (6-3)^2 + (-2+8)^2}$

$= \sqrt{0 + 9 + 36} = 3\sqrt{5} units$

OR

According to the question, $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ ....(i)

and $x - y + z = 5$ ....(ii)

$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = \lambda(\text{ say })$

$\Rightarrow x = 3\lambda + 2, \; y = 4\lambda - 1, \; z = 12\lambda + 2$

Point on the line is

$P(3\lambda+2, 4\lambda-1, 12\lambda+2)$....(iii)
P lies on the plane, so point P satisfy the plane.
Substitute (iii) in (ii) , we get
$\therefore (3\lambda+2)-(4\lambda-1)+(12\lambda+2)=5$
$\Rightarrow 3\lambda+2-4\lambda+1+12\lambda+2=5$
$\Rightarrow 11\lambda=0 \Rightarrow \lambda=0$
Put $\lambda=0$ in Eq.(iii), we get point of intersection p (2,-1,2).
Distance between points $(-1,-5,-10)\ and\ (2,-1,2)$
$=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$
$=\sqrt{9+16+144}=\sqrt{169}$
=13 units.

29. We have

$(3x^2 - 2ax + 3a^2)^3 = [(3x^2 - 2ax) + 3a^2)]^3$
$={}^3C_0(3x^2-2ax)^3+{}^3C_1(3x^2-2ax)^2(3a^2)+{}^3C_2(3x^2-2ax)(3a^2)^2+{}^3C_3(3a^2)^3$
$=(3x^2-2ax)^3+3\times 3a^2(3x^2-2ax)^2+3\times 9a^4(3x^2-2ax)+27a^6$
$=(27x^6-8a^3x^3-54ax^5+36a^2x^4)+9a^2(9x^4+4a^2x^2-12ax^3)+27a^4(3x^2-2ax)+27a^6$
$=27x^6-8a^3x^3-54ax^5+36a^2x^4+81a^2x^4+36a^4x^2-108a^3x^3+81a^4x^2-54a^5x+27a^6$
$=27x^6-54ax^5+117a^2x^4-116a^3x^3+117a^4x^2-54a^5x27a^6$

OR

We have $a^n = [(a - b) + b]^n$
$= {}^nC_0(a-b)^n+{}^nC_1(a-b)^{n-1}\cdot b+{}^nC_2(a-b)^{n-2}\cdot b^2+\ldots+{}^nC_{n-1}(a-b)b^{n-1}+{}^nC_nb^n$
$=(a-b)^n+{}^nC_1(a-b)^{n-1}\cdot b+{}^nC_2(a-b)^{n-2}\cdot b^2+\ldots+{}^nC_{n-1}(a-b)b^{n-1}+b^n$
$\Rightarrow\ a^n-b^n=(a-b)^n+{}^nC_1(a-b)^{n-1}\cdot b+{}^nC_2(a-b)^{n-2}\cdot b^2+\ldots+{}^nC_{n-1}(a-b)\,b^{n-1}$
$=(a-b)[(a-b)^{n-1}+{}^nC_1(a-b)^{n-2}\cdot b+{}^nC_2(a-b)^{n-3}\cdot b^2+\ldots+{}^nC_{n-1}b^{n-1}]$

Which shows that (a - b) is a factor of $a^n - b^n$.

30. $\left(-2-\frac{1}{3}i\right)^3=-\left(2+\frac{1}{3}i\right)^3$
$=-\left[(2)^3+\left(\frac{1}{3}i\right)^3+3\times(2)^2\times\frac{1}{3}i+3\times 2\times\left(\frac{1}{3}i\right)^2\right]$
$=-\left[8+\frac{1}{27}i^3+4i+\frac{2}{3}i^2\right]=-\left[8-\frac{1}{27}i+4i-\frac{2}{3}\right]\left[\begin{array}{c}\because i^3=-i\\ i^2=-1\end{array}\right]$
$=\left[\left(8-\frac{2}{3}\right)+\left(4-\frac{1}{27}\right)i\right.$
$=-\left[\frac{22}{3}+\frac{107}{27}i\right]=\frac{-22}{3}-\frac{107}{27}i$

OR

Let $x+yi=\sqrt{-2+2\sqrt{3}i}$
Squaring both sides, we get
$x^2 - y^2 + 2x\,yi = -2+2\sqrt{3}i$
Comparing the real and imaginary parts
$x^2 - y^2 = -2$ .... (i)
$2xy=2\sqrt{3}\Rightarrow xy=\sqrt{3}$
Now from the identity, we know
$(x^2 + y^2)^2 = (x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$
$=(-2)^2+4(\sqrt{3})^2$
= 4 + 12 = 16
$\therefore x^2+y^2=4$ .... (ii) [Neglecting (-) sign as $x^2 + y^2 > 0$]
Solving (i) and (ii), we get
$x^2 = 1$ and $y^2 = 3$
$\therefore x=\pm 1$ and $y=\pm\sqrt{3}$
Since the sign of xy is (+)
$\therefore$ if x = 1, $y=\sqrt{3}$

and if x = -1, $y=-\sqrt{3}$

$\therefore \sqrt{-2+2\sqrt{3}i}=\pm(1+\sqrt{3}i)$

31.

Suppose,x ∈ (A – B) ∩ (C – B)

⇒ x ∈ A – B and x ∈ C – B

⇒ (x ∈ A and x ∉ B) and (x ∈ C and x ∉ B)

⇒ (x ∈ A and x ∈ C) and x ∉ B

⇒ (x ∈ A ∩ C) and x ∉ B

⇒ x ∈ (A ∩ C) – B

Thus, (A – B) ∩ (C – B) ⊂ (A ∩ C) – B ... (1)

Now, conversely

Suppose, y ∈ (A ∩ C) – B

⇒ y ∈ (A ∩ C) and y ∉ B

⇒ (y ∈ A and y ∈ C) and (y ∉ B)

⇒ (y ∈ A and y ∉ B) and (y ∈ C and y ∉ B)

⇒ y ∈ (A – B) and y ∈ (C – B)

⇒ y ∈ (A – B) ∩ (C – B)

Therefore, (A ∩ C) – B ⊂ (A – B) ∩ (C – B) ... (2)

From (1) and (2),we get

(A – B) ∩ (C – B) = (A ∩ C) – B

Section D

32. We have to find the probability of its being divisible by 4 or 6.

Let A denote the event that the number is divisible by 4 and B denote the event that the number is divisible by 4.

To find : Probability that the number is both divisible by 4 or 6 = P(A or B)

The formula used : Probability = $\frac{\text{favourable number of outcomes}}{\text{total number of outcomes}}$

P(A or B) = P(A) + P(B) - P(A and B)

Numbers from 1 to 100 divisible by 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84, 88, 92, 96, 100.

There are 25 numbers from 1 to 100 divisible by 4

Favourable number of outcomes = 25

Total number of outcomes = 100 as there are 100 numbers from 1 to 100

P(A) = $\frac{25}{100}$

Numbers from 1 to 100 divisible by 6 are

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96

There are 16 numbers from 1 to 100 divisible by 6

Favourable number of outcomes = 16

Total number of outcomes = 100 as there are 100 numbers from 1 to 100

P(B) = $\frac{16}{100}$

Numbers from 1 to 100 divisible by both 4 and 6 are

12, 24, 36, 48, 60, 72, 84, 96

There are 8 numbers from 1 to 100 divisible by both 4 and 6

Favourable number of outcomes = 8

P(A and B) = $\frac{8}{100}$

P(A or B) = P(A) + P(B) - P(A and B)

P(A or B) = $\frac{25}{100}+\frac{16}{100}-\frac{8}{100}$

P(A or B) = $\frac{25+16-8}{100}=\frac{33}{100}$

P(A or B) = $\frac{33}{100}$

The probability that the number is both divisible by 4 or 6 = P(A or B) = $\frac{33}{100}$

33. i. Let $y=\frac{\sin x+\cos x}{\sin x-\cos x}$

On differentiating both sides of y w.r.t. x, we get

$$\frac{dy}{dx}=\frac{\left[(\sin x+\cos x)\frac{d}{dx}(\sin x+\cos x)-(\sin x+\cos x)\frac{d}{dx}(\sin x-\cos x)\right]}{(\sin x-\cos x)^2}$$

[by quotient rule of derivative]

$= \frac{[\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(cos x + sin]}{(\sin x - \cos x)^2}$

$= \frac{-(\cos x - \sin x)(\cos x - \sin x) - (\cos x + \sin x)^2}{(\sin x - \cos x)^2}$

$= \frac{-(\cos x - \sin x)^2 - (\cos x + \sin x)^2}{(\sin x - \cos x)^2}$

$= \frac{[-(\cos^2 x + \sin^2 x - 2\cos x \sin x) + (\cos^2 x + \sin^2 x + 2\cos x \sin x)]}{(\sin x - \cos x)^2}$

$= \frac{-[1+1]}{(\sin x - \cos x)^2} = \frac{-2}{(\sin x - \cos x)^2}$

ii. Given, $f(x) = \begin{cases} x^2 - 1, & 0 < x < 2 \\ 2x + 3, & 2 \leq x < 3 \end{cases}$

At x = 2,

RHL = $\lim\limits_{x \to 2^+} f(x)$

$= \lim\limits_{h \to 0} f(2 + h)$

$= \lim\limits_{h \to 0}$ 2(2 + h) + 3

= 2(2 + 0) + 3

= 4 + 3 = 7 = α [say]

[∵ f(x) = 2x + 3]

LHL = $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{h \to 0}$ f(2 - h)

$= \lim\limits_{h \to 0}$ $(2 - h)^2 - 1 = (2 - 0)^2 - 1$

= 4 - 1 = 3 = β [say] [∴ f(x) = x^2 - 1]

If a quadratic euation has root α and β, then the equation is

x^2 - (Sum of roots) x + Product of roots = 0

i.e., $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e., x^2 - (7 + 3)x + 7 × 3 = 0

⇒ x^2 - 10x + 21 = 0

OR

We have to show that

$\lim\limits_{x \to \infty}(\sqrt{x^2 + x + 1} - x) \neq \lim\limits_{x \to \infty}(\sqrt{x^2 + 1} - x)$

LHS:

$\lim\limits_{x \to \infty}((\sqrt{x^2 + x + 1} - x))$

Rationalising the numerator:

$\lim\limits_{x \to \infty}\left[\frac{(\sqrt{x^2+x+1} - x)(\sqrt{x^2+x+1} + x)}{(\sqrt{x^2+x+1} + x)}\right]$

= $\lim\limits_{x \to \infty}\left[\frac{(x^2+x+1) - x^2}{(\sqrt{x^2+x+1} + x)}\right]$

= $\lim\limits_{x \to \infty}\left[\frac{x+1}{(\sqrt{x^2+x+1} + x)}\right]$

Dividing the numerator and the denominator by x:

$\lim\limits_{x \to \infty}\left[\frac{1 + \frac{1}{x}}{\frac{\sqrt{x^2+x+1}}{x} + 1}\right]$

= $\lim\limits_{x \to \infty}\left[\frac{1 + \frac{1}{x}}{\sqrt{\frac{x^2+x+1}{x^2}} + 1}\right]$

= $\lim\limits_{x \to \infty}\left[\frac{1 + \frac{1}{x}}{\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} + 1}\right]$

When $x \to \infty$, then $\frac{1}{x} \to 0$

$\frac{1}{\sqrt{1} + 1}$

= $\frac{1}{2}$

RHS:

$\lim_{x\to\infty}(\sqrt{x^2+1}$ - x) [From $\infty - \infty$]

Rationalising the numerator:

$\lim_{x\to\infty}\left[\frac{(\sqrt{x^2+1}-x)(\sqrt{x^2+1}+x)}{(\sqrt{x^2+1}+x)}\right]$

= $\lim_{x\to\infty}\left[\frac{x^2+1-x^2}{(\sqrt{x^2+1}+x)}\right]$

= $\frac{1}{\infty}$

= 0

Hence, $\lim_{x\to\infty}[\sqrt{x^2+x+1}-x] \neq \lim_{x\to\infty}(\sqrt{x^2+1}$ - x)

34. Let the three numbers in GP be $\frac{a}{r}$, a, ar.

Sum of three numbers = 52 [given]

$\Rightarrow$ $\frac{a}{r}$ + a + ar = 52

$\Rightarrow$ a $\left(\frac{1}{r}+1+r\right)$ = 52 ...(i)

And sum of product in pair = 624

$\Rightarrow$ $\frac{a}{r}$ $\times$ a + a $\times$ ar + $\frac{a}{r}$ $\times$ ar = 624

$\Rightarrow$ a^2 $\left(\frac{1}{r}+r+1\right)$= 624 ...(ii)

On dividing Eqs. (ii) by (i), we get

a = $\frac{624}{52}$ $\Rightarrow$ a = 12

On putting a = 12 in Eq. (i), we get

12 ($\frac{1}{r}$ + r + 1) = 52

$\Rightarrow \frac{r^2+r+1}{r} = \frac{52}{12} \Rightarrow \frac{r^2+r+1}{r} = \frac{13}{3}$

$\Rightarrow$ 3r^2 +3r + 3 = 13r

$\Rightarrow$ 3r^2 - 10r + 3 = 0

$\Rightarrow$ (3r - 1) (r - 3) = 0

$\Rightarrow$ r = $\frac{1}{3}$ or r = 3

When r = $\frac{1}{3}$, then numers are $\frac{12}{\frac{1}{3}}$, 12, 12 $\times \frac{1}{3}$ i.e., 36, 12, 4.

When r = 3, then numbers are $\frac{12}{3}$, 12, 12 $\times \frac{1}{3}$ .i.e, 4, 12, 36.

35. LHS = $cos2x \times cos\ \frac{x}{2}\ - cos3x \times cos\ \frac{9x}{2}$

= $\frac{1}{2}$ [2 cos 2x $\times$ cos $\frac{x}{2}$ - 2 cos $\frac{9x}{2}$ $\times$ cos 3x] [multiplying numerator and denominator by 2]

= $\frac{1}{2}$ [cos $\left(2x+\frac{x}{2}\right)$ + cos $\left(2x-\frac{x}{2}\right)$ - cos $\left(\frac{9x}{2}+3x\right)$ - cos $\left(\frac{9x}{2}-3x\right)$] [$\because 2cos\ x \times cos\ y = cos(x+y) + cos(x-y)$]

= $\frac{1}{2}$ [cos $\frac{5x}{2}$ + cos $\frac{3x}{2}$ - cos $\frac{15x}{2}$ - cos $\frac{3x}{2}$]

= $\frac{1}{2}$ [cos $\frac{5x}{2}$ - cos $\frac{15x}{2}$]

= $\frac{1}{2}$ [- 2 sin $\left(\frac{\frac{5x}{2}+\frac{15x}{2}}{2}\right)$ sin $\left(\frac{\frac{5x}{2}-\frac{15x}{2}}{2}\right)$] [$\because cos\ x - cos\ y = -2sin\ \left(\frac{x+y}{2}\right)$ $\cdot$ sin$\left(\frac{x-y}{2}\right)$]

= - sin 5x sin $\left(\frac{-5x}{2}\right) = sin5x \times sin\ \frac{5x}{2}$ [$\because sin(-\theta) = -sin\,\theta$]

= RHS

$\therefore$ LHS = RHS

Hence proved.

OR

cos 10° cos 30° cos 50° cos 70° = $\frac{3}{16}$

LHS = cos 10° cos 30° cos 50° cos 70°

= cos 30° cos 10° cos 50° cos 70°

$= \frac{\sqrt{3}}{2}$ (cos 10° cos 50° cos 70°)

$= \frac{\sqrt{3}}{2}$ (cos 10° cos 50°) cos 70°

$= \frac{\sqrt{3}}{4}$ (2 cos 10° cos 50°) cos 70° [Multiplying and dividing by 2]

$= \frac{\sqrt{3}}{4}$ cos 70° {cos (50° + 10°) + cos (10° - 50°)} [Using 2 cos A cos B = cos (A + B) + cos (A - B)]

$= \frac{\sqrt{3}}{4}$ cos 70° {cos 60° + cos (-40°)}

$= \frac{\sqrt{3}}{4}$ cos 70° [$\frac{1}{2}$ + cos 40°] [∵ cos 60° = $\frac{1}{2}$ and cos (-x) = cos x]

$= \frac{\sqrt{3}}{8}$ cos 70° + $\frac{\sqrt{3}}{4}$ cos 70° cos 40°

$= \frac{\sqrt{3}}{8}$ cos 70° + $\frac{\sqrt{3}}{8}$ (2 cos 70° cos 40°)

$= \frac{\sqrt{3}}{8}$ [cos 70° + cos (70° + 40°) + cos (70° - 40°)]

$= \frac{\sqrt{3}}{8}$ [cos 70° + cos 110° + cos 30°]

$= \frac{\sqrt{3}}{8}$ [cos 70° + cos (180° - 70°) + $\frac{\sqrt{3}}{2}$] [∵ cos 30° = $\frac{\sqrt{3}}{2}$]

$= \frac{\sqrt{3}}{8}$ [cos 70° - cos 70° + $\frac{\sqrt{3}}{2}$] [∵ cos (180° - x) = - cos x]

$= \frac{\sqrt{3}}{8} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

= RHS

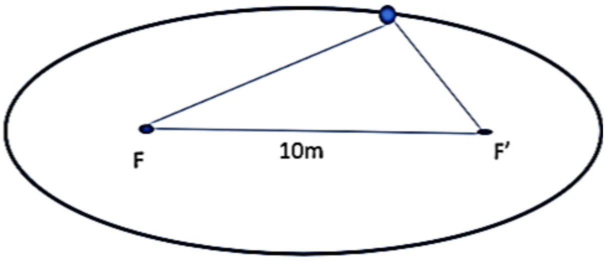
Hence proved.

Section E

36. **Read the text carefully and answer the questions:**

A farmer wishes to install 2 handpumps in his field for watering.

The farmer moves in the field while watering in such a way that the sum of distances between the farmer and each handpump is always 26m. Also, the distance between the hand pumps is 10 m.



(i) The curve traced by farmer is ellipse. Because An ellipse is the set of all points in a plane, the sum of whose distances from two fixed points in the plane is a constant.

Two positions of hand pumps are foci Distance between two foci = 2c = 10 Hence c = 5 Here foci lie on x axis & coordinates of foci = $(\pm c, 0)$

Hence coordinates of foci =$(\pm 5, 0)$

(ii) $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Sum of distances from the foci = 2a

Sum of distances between the farmer and each hand pump is = 26 = 2a

⇒ 2a = 26 ⇒ a = 13 m

Distance between the handpump = 10m = 2c

⇒ c = 5 m

$c^2 = a^2 - b^2$

⇒ $25 = 169 - b^2$

⇒ $b^2 = 144$

Equation is $\frac{x^2}{169} + \frac{y^2}{144} = 1$

(iii) Equation of ellipse is $\frac{x^2}{169} + \frac{y^2}{144} = 1$ comparing with standard equation of ellipse a=13, b= 12 and c= 5 (given)

Length of major axis = 2a = 2 × 13 = 26

Length of minor axis = 2b = 2 × 12 = 24

eccentricity $e = \frac{c}{a} = \frac{5}{13}$

OR

Equation of the ellipse is $\frac{x^2}{169}+\frac{y^2}{144}=1$ hence a = 13 and b = 12

length of latus rectum of ellipse is given by $\frac{2\,\mathrm{b}^2}{\mathrm{a}}=\frac{2\times 144}{13}$

37. **Read the text carefully and answer the questions:**

For a group of 200 candidates, the mean and the standard deviation of scores were found to be 40 and 15 , respectively. Later on it was discovered that the scores of 43 and 35 were misread as 34 and 53, respectively.

Student	Eng	Hindi	S.St	Science	Maths
Ramu	39	59	84	80	41
Rajitha	79	92	68	38	75
Komala	41	60	38	71	82
Patil	77	77	87	75	42
Pursi	72	65	69	83	67
Gayathri	46	96	53	71	39

(i) SD = σ = 15

$\Rightarrow$ Variance = 15^2 = 225

According to the formula,

Variance = $\left(\frac{1}{n}\sum x_{\mathrm{i}}{}^2\right)-\left(\frac{1}{n}\sum \mathrm{x}_{\mathrm{i}}\right)^2$

$\therefore \frac{1}{200}\sum x_{\mathrm{i}}^2-(40)^2$ = 225

$\Rightarrow \frac{1}{200}\sum (x_{\mathrm{i}})^2-1600$ = 225

$\Rightarrow \sum (x_{\mathrm{i}})^2$ = 200 $\times$ 1825 = 365000

This is an incorrect reading.

$\therefore$ Corrected $\sum (x_i)^2$ = 365000 - 34^2 - 53^2 + 43^2 + 35^2

= 365000 - 1156 - 2809 + 1849 + 1225

= 364109

Corrected variance = $\left(\frac{1}{n}\times \text{ Corrected } \sum x_i\right)$ - (Corrected mean)^2

= $\left(\frac{1}{200}\times 364109\right)-(39.955)^2$

= 1820.545 - 1596.402

= 224.14

(ii)

The formula of variance is $\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$.

(iii) Corrected mean = $\frac{\text{Corrected }\sum x_1}{200}$

= $\frac{7993}{200}$

= 39.955

OR

We have:

n = 200, $\bar{X}$ = 40, σ = 15

$\frac{1}{n}\sum x_{\mathrm{i}}=\bar{X}$

$\therefore \frac{1}{200}\sum x_i$ = 40

$\Rightarrow \sum x_i$ = 40 $\times$ 200 = 8000

Since the score was misread, this sum is incorrect.

$\Rightarrow$ Corrected $\sum x_i$ = 8000 - 34 - 53 + 43 + 35

= 8000 - 7

= 7993

38. **Read the text carefully and answer the questions:**

A state cricket authority has to choose a team of 11 members, to do it so the authority asks 2 coaches of a government academy to select the team members that have experience as well as the best performers in last 15 matches. They can make up a team of 11 cricketers amongst 15 possible candidates. In how many ways can the final eleven be selected from 15 cricket players if:

(i) Two of them being leg spinners, one and only one leg spinner must be included

Let's first find out possible ways to select players which are not leg spinner

There are two leg spinners out of 15 and one players must be leg spinner.

So, we have to select 10 players out of 13

Total possible ways to select 11 players out of 15 out of which one must be leg spinner out of 2 are ${}^{13}C_{10} \times {}^{2}C_{1}$

${}^{n}C_{r} = \frac{n!}{(n-r)!r!}$

$\Rightarrow {}^{13}C_{10} = \frac{13!}{(13-10)!10!}$

$\Rightarrow {}^{13}C_{10} = \frac{13!}{3!10!} = \frac{13\times12\times11\times10!}{3\times2\times1\times10!}$

$\Rightarrow {}^{13}C_{10} = \frac{13\times12\times11}{3\times2\times1} = 13 \times 6 \times 11$

$\Rightarrow {}^{13}C_{10} = 858$

${}^{2}C_{1} \times {}^{13}C_{10}$

$\Rightarrow 2 \times 858 = 1716$

Total possible ways to select 11 players out of 15 out of which one must be leg spinner out of 2 = 1716

(ii) number of ways of selecting 4 bowlers out of of 6 = ${}^{6}C_{4}$

$\Rightarrow {}^{6}C_{4} = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!} = \frac{6\times5\times4!}{2\times1\times4!} = 15$

number of ways of selecting 5 batsmen out of 6 = ${}^{6}C_{5} = 6$

number of ways of selecting 2 wicket keepers out of 3 = ${}^{3}C_{2} = {}^{3}C_{1} = 3$

$\Rightarrow {}^{6}C_{4} \times {}^{6}C_{5} \times {}^{3}C_{2}$

$\Rightarrow 15 \times 6 \times 3 = 270$

Total ways to select 4 bowlers, 2 wicketkeepers and 5 batsmen out of 6 bowlers, 3 wicketkeepers, and 6 batsmen in all are 270.