2.4 CHAPTER Probability

Introduction

The probability has its origin in the problems dealing with games of chance such as gambling, coin tossing, die throwing and playing cards. In all these cases the outcome of a trial is uncertain. These days probability is widely used in business and economics in the field of prediction for future.

Lots of questions based on coins, die, playing cards and alphabetical arrangements are asked in various public sector examination.

Theory

Probability means the chance of occurrence of an event. In layman terms, we can say that it is likelyhood that something that is defined as an events will or will not occu mathematically,

$$\text{Probability (P)} = \frac{\text{Number of favourable events}}{\text{Total Number of events}}$$

for example probability of getting an even number after throwing a die can be calculated as.

$$(P) = \frac{\{2, 4, 6\}}{\{1, 2, 3, 4, 5, 6\}} = \frac{3}{6} = \frac{1}{2}$$

Were {2, 4, 6} are even numbers & hence favorable out comes and {1,2,3,4,5,6} etc. all 6 possible outcomes. Similarly, we can calculate probability of getting a number divisible by 3 out of 9 single digit number in following manner Probability

$$= \frac{\text{Number divisible by 3}}{\text{All nine single digit number}}$$

$$= \frac{\{3, 6, 9\}}{\{1, 2, 3, 4, 5, 6, 7, 8, 9\}} = \frac{3}{9} \Rightarrow \frac{1}{3}$$

The followings remark may be important for learning this chapter on probability

1. **Die:** A die is small cube used in games of chance. On its six faces dots are marked numbering (1, 2, 3, 4, 5, 6)
2. **Playing card:** A pack (or deck) of playing cards has 52 cards, divided into four suits:
   (1) Spades (2) Clubs
   (3) Hearts (4) Diamonds
   Each suit has 13 cards
   (a) Nine number cards 2, 3, 4, 5, 6, 7, 8, 9, 10
   (b) An Ace, a king, a Queen and a Jack are known as face cards
   Ace - A King - K
   Queen - Q Jack -J
   Spade and clubs are black faced cards while Hearts and Diamonds are red- faced cards.
   The King, Queens and Jacks are called court cards.
3. **Unbiased coin** - coin having two faces head **(H)** and Tail **(T)**

Some Basic Concepts

Random experiment

An experiment whose out come has to be among a set of events that are completely known but whose exact out come is unknown is random experiment.

Eg: (Throwing of a dice, tossing of a coin)

Sample Space

This is defined in the context of a random experiment and denotes the set representing all the possible outcomes of the random experiment.

Eg: 1. Sample Space when a coin is tossed is Head (H) or Tail (T)
2. Sample space when a dice is thrown is (1, 2, 3, 4, 5, 6)

Event

The set representing the desired out come of a random experiment is called event. Event is subset of a sample space for example: Probability of getting a number divisible by 5 in a single throw of a die if odd numbers are obtained.

Here sample space is (1, 2, 3, 4, 5, 6), event is (1, 3, 5) favourable outcome is (5)only.

$$\text{So probability} = \frac{\{5\}}{\{1, 3, 5\}} = \frac{1}{3}$$

Non Event

The outcome that is opposite of the desired event is the non-event.

Note: If the event occurs, the non-event does not ocur and vice versa.

Impossible Event

An event that can never occur is an impossible event. The probability of an impossible event is 0.

Eg. (Probability of occurance of 7 when a dice with 6 faces numbered 1-6 is thrown).

Mutually Exclusive Events

A set of event is mutually exclusive when the occurence of any one of them means that the other event cannot occur.

Eg: 1. If head appears on a coin tail will not appear and vice versa

2. If 5 occurs in a single throw of a die then (1, 2, 3, 4, 6) will never appear.

Equally likely Events

If two events have the same probability or chance of occurence they are called equally likely events.

Example: In a throw of a dice, the chance of 1 showing on the dice is equal to 2, is equal to 3, is equal to 4, is equal to 5, is equal to 6 appearing on the dice.

Exhaustive Set of Event

A set of events that includes all the possibilities of a sample space is said to be an exhaustive set of an events.

Example: In a throw of a dice the number is less than four or more than or equal to four.

Independent Events

An event is described as such if the occurence of an event has no effect on the probability of ocurence of another event.

Example: If the first child of a couple is girt there is no effect on the chances of the second child being a girl.

Conditional Probability

Probability of the occurence of an event A given that event B has already ocured.

This is denoted by P (A/B)

Example: The probability that in two throws of a dice we get a total of 7 or more given that in the first throw of the dices number 4 had occured.

Concept of AND and OR

Whenever we use AND as the natural conjunction joining two separate parts of event definitions, we replace the AND by the multiplication sign.

Example: If probability of passing in an exam is 1/2 for A and probability of passing the same is 1/3 for B then probability of passing of A and B is P(A)× P (B)

$$\text{i.e.: } \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Whenever we use OR as the natural conjunction joing two separate parts of the event definition, we replace the OR by their addition sign.

Example: If we have the probability of A winning a race as 2/3 and that of B as 1/6 then probability that either A or B wins a race is given by

$$P(A) + P(B) = \frac{2}{3} + \frac{1}{6} = \frac{5}{6}.$$

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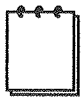

Solved Example

1. In throwing a fair dice, what is the probability of getting the number '3'?

   (a) $\frac{1}{3}$ (b) $\frac{1}{6}$

   (c) $\frac{1}{9}$ (d) $\frac{1}{12}$

2. Find the chance of throwing at least one ace in a simple throw with two dice.

   (a) $\frac{1}{12}$ (b) $\frac{1}{3}$

   (c) $\frac{1}{4}$ (d) $\frac{11}{36}$

3. Find the chance of drawing 2 blue balls in succession from a bag containing 5 red and 7 blue balls, if the balls are not being replaced.

   (a) $\frac{3}{13}$ (b) $\frac{21}{64}$

   (c) $\frac{7}{22}$ (d) $\frac{21}{61}$

4. If a card is picked up at random from a pack of 52 cards. Find the probability that it is

(i) a spade.

(a) $\frac{1}{9}$ (b) $\frac{1}{6}$

(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

(ii) a king or queen.

(a) $\frac{3}{13}$ (b) $\frac{2}{13}$

(c) $\frac{7}{52}$ (d) $\frac{1}{169}$

(iii) 'a spade' or 'a king' or 'a queen'

(a) $\frac{21}{52}$ (b) $\frac{5}{13}$

(c) $\frac{19}{52}$ (d) None of these

5. Three coins are tossed. What is the probability of getting 2 Tails and 1 Head

(a) $\frac{1}{2}$ (b) $\frac{3}{8}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

6. For the above question, the probability that there is at least one tail is :

(a) $\frac{2}{3}$ (b) $\frac{7}{8}$

(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

7. A bag contains 3 green and 7 white balls. Two balls are drawn from the bag in succession without replacement. What is the probability that both are white?

(a) $\frac{1}{7}$ (b) $\frac{5}{11}$

(c) $\frac{7}{11}$ (d) $\frac{7}{15}$

8. What is the probability of throwing a number greater than 2 with a fair dice?

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$

(c) 1 (d) $\frac{3}{5}$

9. Two fair coins are tossed. Find the probability of obtaining

(i) 2 Heads

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

(ii) 1 Head and 1 Tail

(a) $\frac{1}{4}$ (b) 1

(c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(iii) 2 Tails

(a) 1 (b) $\frac{1}{4}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

10. In rolling two dices, find the probability that

(i) there is at least one '6'

(a) $\frac{11}{36}$ (b) $\frac{22}{36}$

(c) $\frac{15}{36}$ (d) $\frac{29}{36}$

(ii) the sum is 5

(a) $\frac{1}{4}$ (b) $\frac{1}{9}$

(c) $\frac{1}{2}$ (d) $\frac{1}{6}$

11. From a bag containing 4 white and 5 black balls a man draws 3 at random. What are the odds against these being all black?

(a) $\frac{5}{37}$ (b) $\frac{37}{5}$

(c) $\frac{11}{13}$ (d) $\frac{13}{37}$

12. Two balls are to be drawn from a bag containing 8 grey and 3 blue balls. Find the chance that they will both be blue.

(a) $\frac{1}{5}$ (b) $\frac{3}{55}$

(c) $\frac{11}{15}$ (d) $\frac{14}{45}$

13. A bag contains four black and five red balls. If three balls from the bag are chosen at radom, what is the chance that they are all black?

(a) $\frac{1}{21}$ (b) $\frac{1}{20}$

(c) $\frac{2}{23}$ (d) $\frac{1}{9}$

14. If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits, what is the probability that the number formed is 35?

(a) $\frac{1}{10}$ (b) $\frac{1}{20}$

(c) $\frac{2}{11}$ (d) $\frac{1}{11}$

15. From a pack of 52 playing cards, three cards are drawn at random. Find the probability of drawing a king, a queen and jack.

(a) $\frac{16}{5525}$ (b) $\frac{1}{13^3}$

(c) $\frac{1}{14^3}$ (d) $\frac{1}{15^3}$

16. A bag contains 3 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both are black?

(a) $\frac{1}{8}$ (b) $\frac{7}{40}$

(c) $\frac{12}{40}$ (d) $\frac{13}{40}$

17. A bag contains 4 white and 2 black balls. Another contains 3 white and 5 black balls. If one ball is drawn from each bag, find the probability that

(i) both are white.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$

(ii) both are black.

(a) $\frac{3}{24}$ (b) $\frac{1}{24}$

(c) $\frac{3}{12}$ (d) $\frac{5}{24}$

(iii) one is white and one is black.

(a) $\frac{13}{24}$ (b) $\frac{15}{24}$

(c) $\frac{11}{21}$ (d) $\frac{1}{2}$

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Solutions

1. Ans. (b)
2. Ans. (d)

Ace is nothing but 6. we can get it in 11 different ways (1,6), (6,1), (2, 6), (6, 2), (3, 6), (6, 3), (4, 6), (6, 4), (5, 6), (6, 5), (6, 6)

$\Rightarrow \frac{11}{36}$ (Probability)

3. Ans. (c)

Probability of getting blue ball in single draw is $\frac{7}{12}$

and probability of getting blue ball in second draw is $\frac{6}{11}$ so $\frac{7}{12} \times \frac{6}{11} = \frac{7}{22}$

4. (i) Ans. (c)
   (ii) Ans. (b)
   (iii) Ans. (c)

(i) $\frac{13}{52} = \frac{1}{4}$

(ii) 4 king or 4 Queen = 8

$\Rightarrow \frac{8}{52} = \frac{2}{13}$

(iii) Spade = 13, King = 4 Queen = 4 but there will 1 King of spade & 1 Queen of spade so we have 19 to choose from $\Rightarrow \frac{19}{52}$

5. Ans. (b)

When three coins are tossed we get

HHH	or	(1)
HHT	or	(2)
HTT	or	(3)
HTH	or	(4)
THH	or	(5)
THT	or	(6)
TTT	or	(7)
TTH	or	(8)

We get 2T and one H in 3 cases so probability = $\frac{3}{8}$

6. Ans. (b)

For at least one tail, we have to exclude the case of all Heads {H, H, H} and rest 7 are favourable.

So $P = \frac{7}{8}$

7. Ans. (d)

To get both white

$$\frac{^7C_2}{^{10}C_2} = \frac{21}{45} = \frac{7}{15}$$

8. Ans. (a)

Number greater than 2 = { 3, 4, 5, 6}

So $P = \frac{4}{6} = \frac{2}{3}$

9. (i) Ans. (d)
(ii) Ans. (c)
(iii) Ans. (b)

When two coins are tossed then four out-comes are

HH, HT, TH, or TT

(i) $P = \frac{1}{4}$

(ii) $P = \frac{2}{4} = \frac{1}{2}$

(iii) $P = \frac{1}{4}$

10. (i) Ans. (a)
(ii) Ans. (b)

Rolling two dice gives following sums

1. Two (1, 1)
2. Three (1, 2), (2, 1)
3. Four (1, 3), (3, 1), (2, 2)
4. Five (1, 4), (2, 3), (3, 2), (4, 1)
5. Six (1, 5), (5, 1), (2, 4), (4, 2), (3, 3)
6. Seven (1, 6), (6, 1), (5, 2), (2, 5), (4, 3), (3, 4),
7. Eight (6, 2), (2, 6), (5 3), (3, 5), (4, 4)
8. Nine (6, 3), (3, 6), (5, 4), (4, 5)
9. Ten (4, 6), (4, 6), (5, 5)
10. Eleven (6, 5), (5, 6)
11. Twelve (6, 6)

(i) $P = \frac{11}{36}$

(ii) $P = \frac{4}{36} = \frac{1}{9}$

11. Ans. (b)

Probability that all 3 balls are black

$\frac{^5C_3}{^9C_3} = \frac{10}{84} = \frac{5}{42}$ out of 42 cases only 5 are favourable and 37 are not favourable so odd against these being all black is $\frac{37}{5}$

12. Ans. (b)

8 grey and 3 blue balls

probability that all are black $= \frac{^3C_2}{^{11}C_2} = \frac{3}{55}$

13. Ans. (a)

4 black and 5 red balls to get all black

$P = \frac{^4C_3}{^9C_3} = \frac{4}{84} = \frac{1}{21}$

14. Ans. (b)

Total two digit numbers formed = $^5P_2 = 20$, So probability this number being 35 is $\frac{1}{20}$

15. Ans. (a)

$$P = \frac{^4C_1 \times {}^4C_1 \times {}^4C_1}{^{52}C_3} = \frac{64}{22100} = \frac{16}{5525}$$

16. Ans. (b)

Here 3 red, 6 white & 7 black balls

$$P = \frac{^7C_2}{^{16}C_2} = \frac{21}{120} = \frac{7}{40}$$

17. (i) Ans. (c)
(ii) Ans. (d)
(iii) Ans. (a)

Bag 1 $\rightarrow$ 4 white & 2 black and

Bag 2 $\rightarrow$ 3 white & 5 black ball

(i) Probability of getting both white $\frac{4}{6} \times \frac{3}{8} = \frac{1}{4}$

(ii) Probability of geting both black

$= \frac{2}{6} \times \frac{5}{8} = \frac{5}{24}$

(iii) $\frac{4}{6} \times \frac{5}{8} + \frac{2}{6} \times \frac{3}{8} = \frac{13}{24}$

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I Practice Exercise: I

1. Three coins are tossed. Find the probability of no heads.

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$

(c) $\frac{1}{2}$ (d) None of these

2. A coin is tossed three times. Find the chance that head and tail show alternately.

(a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) None of these

3. In a single throw of two dice, find the probability of getting a total of 3 or 5.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

4. In a single throw of two dice, what is the probability of a doublet?

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$

(c) $\frac{1}{9}$ (d) $\frac{1}{18}$

5. What is the chance that a leap year, selected at random will contain 53 Sunday?

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$

(c) $\frac{3}{7}$ (d) $\frac{4}{7}$

6. Find the probability that in a random arrangement of letter of the words "UNIVERSITY" two 'I's do not come together.

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

7. An interger is chosen at random from first two hundred natural numbers. What is the probability that the integer chosen is divisible by 6 or 8?

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$

(c) $\frac{1}{2}$ (d) None of these.

8. In a simultaneous throw of two dice, find P (A or B) if A denotes the event 'a total of 11 and B denotes the event' 'an odd number on each die'.

(a) $\frac{11}{36}$ (b) $\frac{1}{4}$

(c) $\frac{5}{18}$ (d) $\frac{1}{6}$

9. A and B are mutually exclusive events of an experiment. If P ('not A') = 0.65, $P(A \cup B) = 0.65$ and P(B) = P, find the value of p.

(a) 0.70 (b) 0.30

(c) 0.63 (d) 0.35

10. The probabilities that a student will recive an A, B, C or D grade are 0.30, 0.38, 0.22 and 0.01, respectively. What is the probability that the student will receive at least B grade?

(a) 0.38 (b) 0.42

(c) 0.68 (d) None of these

11. A card is drawn from an ordinary pack and a gambler bets that it is a spade or an ace. What are the odds against his winning the bet?

(a) 9 : 4 (b) 4 : 9

(c) 5 : 9 (d) 9 : 5

12. A problem in Statistics is given to four students A, B, C and D. Their chances of solving it are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, and $\frac{1}{6}$, respectively. What is the probability that the problem will be solved?

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{5}$ (d) None of these

Direction for Question No. 13-14: An urn contains 25 balls numbered 1 to 25. Suppose getting an odd numbered ball is considered a 'success'. If two balls are drawn from the urn with replacement.

13. Find the probability of getting two successes.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

14. Find the probability of getting no success.

(a) $\frac{169}{625}$ (b) $\frac{312}{625}$

(c) $\frac{481}{625}$ (d) $\frac{144}{625}$

Direction for Question No. 15-18: A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's is 1/5.

15. What is the probability that only one of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{25}$ (d) $\frac{11}{35}$

16. What is the probability that both of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

17. What is the probability that none of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

18. What is the probability that at least one of them will be selected?

(a) $\frac{2}{7}$ (b) $\frac{1}{35}$

(c) $\frac{24}{35}$ (d) $\frac{11}{35}$

19. A man speaks truth in 80% of the cases and another in 90% of the cases. While stating the same fact, what is the probability that they contradict?

(a) $\frac{37}{50}$ (b) $\frac{13}{50}$

(c) $\frac{16}{50}$ (d) None of these

20. A can solve 90% of the problems given in a book and B solve 70%. What is the probability that atleast one of them will solve a problem selected at random from the book?

(a) $\frac{3}{100}$ (b) $\frac{97}{100}$

(c) $\frac{83}{100}$ (d) $\frac{17}{100}$

21. Find the probability that in a random arrangement of the letters of the word DAUGHTER, the letter D occupies the first place.

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$

(c) $\frac{3}{8}$ (d) $\frac{1}{2}$

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Solutions

1. **Ans. (b)**

Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}

Number of exhaustive cases = 8

P (no heads) = P (all tails)= $\frac{1}{8}$

(∵ there is only favourable case TTT).

2. **Ans. (b)**

Sample space S = {HHH, HHT, HTH, HTT, THT, TTH, THH, TTT}

Number of exhaustive cases = 8

Favourable case are HTH, THT

Number of favourable = 2.

∴ Required probability = $\frac{2}{8} = \frac{1}{4}$.

3. **Ans. (c)**

A total of 3 or 5 may be obtained in 6 ways, viz., (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1).

No. of exhaustive cases = 6 × 6 = 36.

∴ Probability of getting a total of 3

or 5 = $\frac{6}{36} = \frac{1}{6}$.

4. Ans. (a)

A 'doublet' means that both the dice show the same number on the upper most faces. Therefore, the outcomes, favourable to this event are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Thus, the number of favourable cases = 6.

Hence, $P(\text{doublet}) = \frac{6}{36} = \frac{1}{6}$

5. Ans. (b)

We know that a leap year has 366 days and thus a leap year has 52 weeks and 2 days over.

The two left over (successive days have the following likely cases:

(i) Sunday and Monday

(ii) Monday and Tuesday

(iii) Tuesday and Wednesday

(iv) Wednesday and Thursday

(v) Thursday and Friday

(vi) Friday and Saturday

(vii) Saturday and Sunday

$\therefore$ Number of exhaustive cases 'n' = 7.

Out of these, the favourable cases are ...(i) and (vii)

$\therefore$ Number of favourable cdases 'm' = 2

$\therefore$ Probability of having 53 Sunday = $\frac{2}{7}$.

3. Ans. (a)

Out of the letters in the word 'UNIVERSITY' two litters 'I' are alike.

$\therefore$ Number of permutations = $\frac{10!}{2}$ ...(i)

Number of words in which two 'I' are never together = Total number of words – Number of words in which two 'I' are together

$$\frac{10!}{2} - 9! = \frac{10! - 2.9!}{2} = \frac{9![10-2]}{2} = \frac{9!.8}{2} = 9!.4$$

$\therefore$ Required probability = $\frac{9.4}{10!/2} = \frac{9!.8}{10!}$

$= \frac{8}{10} = \frac{4}{5}$.

7. Ans. (a)

A : Integer chosen is divisible by 6B : Integer chosen is divisible by 8

$n(A) = 33,\ n(B) = 25, n(A \cap B) = 8, n(S) = 200$

$P(A) = \frac{33}{200},\ P(B) = \frac{25}{200}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{33}{200} + \frac{25}{200} - \frac{8}{200} = \frac{50}{200} = \frac{1}{4}$.

8. Ans. (a)

A:Getting total of 11 B: Getting odd number one each die

A = [(6, 5), (5, 6)]

B = [(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5 ,3), (5, 5)]

$P(A) = \frac{2}{36}, P(B) = \frac{9}{36}, P(A \cap B) = 0$

$\therefore$ Required probability

$= P(A) + P(B) - P(A \cap B)$

$= \frac{2}{36} + \frac{9}{36} - 0 = \frac{11}{36}$.

9. Ans. (b)

We know $P(A) = 1 - P(\bar{A})$

$= 1 - 0.65 = 0.35$ and

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 0.65 = 0.35 + P(B) - 0$

[$\because$ A and B are mutually exclusive events]

$\Rightarrow P(B) = 0.65 - 0.35 = 0.30.$

10. Ans. (c)

P(at least B grade) = P(B grade) + P(A grade) = 0.38 + 0.30 = 0.68.

11. Ans. (a)

Let A : a sapde is drawn and

B: an ace is drawn

Probability of winning the bet = P(A or B)

= P (A) + P(B) – P(A and B)

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{52}$

Probability of losing the bet $= 1 - \frac{4}{13} = \frac{9}{13}$

Odds against winning the bet = 9 : 4.

12. Ans. (b)

Probability that A fails to solve the problem is

$1 - \frac{1}{3} = \frac{2}{3}$

Probability that B fails to solve the problem is

$$1-\frac{1}{4}=\frac{3}{4}$$

Probability that C fails to solve the problem is

$$1-\frac{1}{5}=\frac{4}{5}$$

Probability that D fails to solve the problem is

$$1-\frac{1}{6}=\frac{5}{6}$$

Since the events are independent, the probability that all the four students fail to solve the problem is

$$\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}=\frac{1}{3}.$$

$\therefore$ The probability that the problem will be solved

$$=1-\frac{1}{3}=\frac{2}{3}.$$

13. Ans. (a)

Success: Getting odd number $P=\frac{13}{25}$

$$\Rightarrow q=1-P=1-\frac{13}{25}=\frac{12}{25}$$

P(two successes) = p × p

$$=\frac{13}{25}\times\frac{13}{25}=\frac{169}{625}.$$

14. Ans. (d)

$$P(\text{no success})=q\times q=\frac{12}{25}\left(\frac{12}{25}\right)=\frac{144}{625}.$$

15. Ans. (a)

A : Husband selected

B : Wife selected

$$P(A)=\frac{1}{7}\Rightarrow P(\bar{A})=1-P(A)=1-\frac{1}{7}=\frac{6}{7}$$

$$P(B)=\frac{1}{5}\Rightarrow P(\bar{B})=1-P(B)=1-\frac{1}{5}=\frac{4}{5}$$

P(only one of them will be selected)

$$=P(A)\times P(\bar{B})+P(B)P(\bar{A})$$

$$=\frac{1}{7}\left(\frac{4}{5}\right)+\frac{1}{5}\left(\frac{6}{7}\right)=\frac{4+6}{35}=\frac{10}{35}=\frac{2}{7}.$$

16. Ans. (b)

P(both of them will be selected)

$$=P(A)\times P(B)=\frac{1}{7}\times\frac{1}{5}=\frac{1}{35}.$$

17. Ans. (c)

P(none of them will be selected)

$$=P(\bar{A})P(\bar{B})=\frac{6}{7}\times\frac{4}{5}=\frac{24}{35}.$$

18. Ans. (d)

P(at least one of them will be selected)

$$=1-P(\bar{A})\times P(\bar{B})$$

$$=1-\frac{6}{7}\times\frac{4}{5}=1-\frac{24}{35}=\frac{11}{35}.$$

19. Ans. (b)

Let the two men be A and B.

A: A speaks truth

B: B speaks truth

$$P(A)=\frac{80}{100}\Rightarrow P(\bar{A})=1-P(A)=1-\frac{80}{100}=\frac{20}{100}$$

$$P(B)=\frac{90}{100}\Rightarrow P(\bar{B})=1-P(B)=1-\frac{90}{100}=\frac{10}{100}$$

$\therefore$ Required probability

$$=P(A)P(\bar{B})+P(B)P(\bar{A})$$

$$=\frac{80}{100}\times\frac{10}{100}+\frac{90}{100}\times\frac{20}{100}$$

$$=\frac{8+18}{100}=\frac{26}{100}=\frac{13}{50}.$$

20. Ans. (b)

A: a solves the problem;

B: B solves the problem.

$$P(A)=\frac{90}{100}\Rightarrow P(\bar{A})=1-P(A)=1-\frac{90}{100}=\frac{10}{100}$$

$$P(B)=\frac{70}{100}\Rightarrow P(\bar{B})=1-P(B)=1-\frac{70}{100}=\frac{30}{100}$$

Required probability $=1-P(\bar{A})P(\bar{B})$

$$=1-\frac{10}{100}\times\frac{30}{100}=1-\frac{3}{100}=\frac{97}{100}$$

21. Ans. (a)

If D occupies the first place

n(A) = 7!, Total words = n(S) = 8!

$\therefore$ Required probability

$$=P(A)=\frac{n(A)}{n(S)}=\frac{7!}{8!}=\frac{1}{8}.$$

□□□□

II Practice Exercise: II

1. The Probability of raining on day 1 is 0.2 and on day 2 is 0.3. What is the probability of raining on both the days?
   (a) 0.2 (b) 0.1
   (c) 0.06 (d) 0.25

2. A bag contains 5 red balls and 8 blue balls. It also contains 4 green and 7 black balls. If a ball is drawn at random, then find the probability that it is not green.
   (a) 5/6 (b) 1/4
   (c) 1/6 (d) 7/4

3. If the probability that A will live 15 years is $\frac{7}{8}$ and that B will live 15 years is $\frac{9}{10}$, then what is the probability that both will live after 15 years?
   (a) $\frac{1}{20}$ (b) $\frac{63}{80}$
   (c) $\frac{1}{5}$ (d) None of these

4. The probability that a student is not a swimmer is 1/5. Then the probability that out of the five students, exactly four are swimmers, is
   (a) ${}^5C_4\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)$ (b) $\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$
   (c) ${}^5C_1\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$ (d) None of these

5. If events A and B are independent and $P(A) = 0.15$, $P(A \cup B) = 0.45$ then $P(B) =$
   (a) $\frac{6}{13}$ (b) $\frac{6}{17}$
   (c) $\frac{6}{19}$ (d) $\frac{6}{23}$

6. Four different objects 1, 2, 3, 4 are distributed at random in four places marked 1, 2, 3, 4. What is the pobability that none of the objects occupy the place corresponding to its number?
   (a) 17/24 (b) 3/8
   (c) 1/2 (d) 5/8

7. If the probability of rain on any given day in Pune city is 50%, then what is the probability that it rains on exactly 3 days in a 5-day period?
   (a) 8/125 (b) 5/16
   (c) 8/25 (d) 2/25

8. The probability that an event A happens in one trial of an experiment is 0.4. Three independent trial of the experiment are formed. The probability that the event A happens at least once is
   (a) 0.934 (b) 0.784
   (c) 0.548 (d) 0.343

9. A number is chosen at random among the first 120 natural numbers. The probability of the number chosen being a multiple of 5 or 15 is
   (a) 1/5 (b) 1/6
   (c) 1/7 (d) 1/9

10. From a pack of 52 playing cards, two cards are drawn together at random. Calculate the probability of both the cards being Kings.
    (a) 1/15 (b) 25/57
    (c) 35/256 (d) None of these

11. From a box containing 60 standard and 40 substandard article, two articles are chosen one by one. What is the probability that one of them is standard and the other substandard?
    (a) $\frac{60}{100} \times \frac{40}{100}$ (b) $\frac{60}{100} \times \frac{39}{100}$
    (c) $\frac{16}{33}$ (d) 24%

Solutions

1. Ans. (c)
   $0.2 \times 0.3 = 0.06$

2. Ans. (a)
   The probability that the ball drawn is of green colour
   $= \frac{4}{24} = \frac{1}{6}$
   Probability that the ball drawn is not of green colour
   $= 1 - \frac{1}{6} = \frac{5}{6}$.

3. Ans. (b)

$P(A) \times P(B) = \frac{7}{8} \times \frac{9}{10} = \frac{63}{80}$.

4. Ans. (c)

5. Ans. (b)

$P(A \cap B) = P(A) \times P(B) = 0.15 \times P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.45 = 0.15 + P(B) - (0.15) \times P(B)$

$= 0.15 + P(B)(1 - 0.15)$

$= 0.15 + 0.85P(B)$

$\therefore \quad 0.85P(B) = 0.45 - 0.15 = 0.30$

$\therefore \quad P(B) = \frac{0.30}{0.85} = \frac{30}{85} = \frac{6}{17}$.

6. Ans. (c)

Let the four places be $\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$

Now object *i* cannot occupy the place i..... (A)

Suppose object 2 occpuies the place $\boxed{1}$. Then other placements can be done in 6 ways as follows:

(1)	2	1	3	4
(2)	2	1	4	3
(3)	2	3	1	4
(4)	2	3	4	1
(5)	2	4	1	3
(6)	2	4	3	1

Here out of the six ways, only three are permissible, because (1), (3) and (6) are not permissible because of the non-fulfilment of condition (A). Hence, required probability is $\frac{3}{6} = \frac{1}{2}$. Similarly you can allow objects 3 and 4 to occupy place 1 and in each case you can find that the probability is 1/2.

7. Ans. (b)

Probability that it rains on any day = $\frac{1}{2}$ and it doesn't rain = $1 - \frac{1}{2} = \frac{1}{2}$

Prob. that it it rains on exactly 3 days in a 5-day period.

$= {}^5C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$

${}^5C_3$ for choosing which three days among given five days. $\left(\frac{1}{2}\right)^3$ for that it rains on three days and $\left(\frac{1}{2}\right)^2$ for it doesn't rains on remaining 2 days.

8. Ans. (b)

Required probability

$= {}^3C_1(0.4)(0.6)^2 + {}^3C_2(0.4)^2(0.6) + {}^3C_3(0.4)^3$

$= 3(0.144) + 3(0.096) + 1(0.064) = 0.784$

Alternative

Probability that A happens = 0.4

Probability that A doesn't happen

= 1 – 0.4 = 0.6

So required probability

1 – Probability (that event does not happens i.e. not even one in three times)

= 1 – (0.6) × (0.6) (0.6)

= 1 – 0.216

= 0.784

9. Ans. (a)

10. Ans. (d)

Two cards can be drawnfrom a pack of 52 playing cards in ${}^{52}C_2$ ways,

i.e. $\frac{52 \times 51}{2} = 1326$ ways.

The event that two kings appear in a single draw can appear in ${}^4C_2$ ways, i.e. 6 ways.

$\therefore$ The probability that the two cards drawn from a pack of 52 cards are kings $= \frac{6}{1326} = \frac{1}{221}$.

11. Ans. (c)

Required probability

$= \frac{60}{100} \times \frac{40}{99} + \frac{40}{100} \times \frac{60}{99}$

$= \frac{4800}{100 \times 99} = \frac{16}{33}$.