7. PLASTIC ANALYSIS

LOAD FACTOR

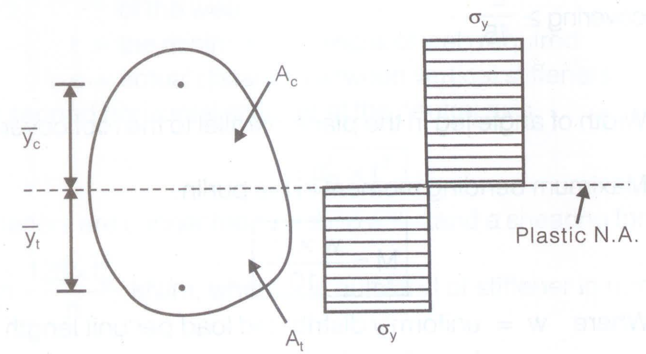
- The load factor (λ)

$$\lambda = \frac{\text{Collapse load}}{\text{Service load}} = \frac{P_c}{P}$$

SAVING IN MATERIAL

$$\% \text{ saving in material} = \left[1 - \frac{\text{Area required by plastic theory}}{\text{Area required by elastic theory}}\right] \times 100$$

PLASTIC SECTIONS MODULUS



- Plastic Section Modulus

$$Z_p = A_c\bar{y}_c + A_t\bar{y}_t$$

Remember

Total compressive force = Total tensile force
Note that in plastic analysis the Neutral axis divides the cross-section into two equal halves whereas in elastic analysis NA. Passes through C.G.
i.e., $A_c = A_t = A/2$, where A is total area

SHAPE FACTOR (α)

$$\alpha = \frac{M_p}{M_y} = \frac{Z_p}{Z_y}$$ Where Z_y is elastic section modulus

$$\text{Load factors } (\lambda) = \frac{P_c}{P}$$

$$\text{Load Factor} = \text{Factor of safety} \times \text{Shape factor} \qquad \lambda = FS \times \alpha$$

SHAPE FACTORS FOR DIFFERENT SHAPES

	Section	Shape Factor (α)
1.	Rectangular section	1.5
2.	(a) Triangular section (vertex upward)	2.34
	(b) Triangular section (vertex horizontal)	2.00
3.	Solid circular section	1.7
4.	Hollow circular section	$1.7 \times \frac{(1-K^3)}{(1-K^4)}$
5.	Thin circular ring solid	1.27
6.	(a) Diamond section (Rhombus)	2.00
	(b) Thin hollow rhombus	1.50
7.	I-section	
	(a) About strong axis	≈ 1.12
	(b) About weak axis	≈ 1.55
8.	T-section	≈ 1.90 to 1.95

Where K = Ratio of inner diameter to outer diameter.

RESERVE STRENGTH (Ψ)

- It is the ratio of the ultimate load W_u to the load at first yield W_y of the structure.

$$\psi = \frac{W_u}{W_y}$$

LENGTH OF PLASTIC HINGE (L_p)

- It is the length of the beam over which the moment is greater than the yield moment (M_y).

(a) For simply supported beam carrying concentrated load, length of plastic hinge is given by

$\boxed{L_P = \frac{L}{3}}$ (for rectangular section) $\boxed{L_P = \frac{L}{8}}$ (for I section)

$$\boxed{L_P = L\left[1 - \frac{1}{\alpha}\right]}$$

(b) For simply support beam carrying UDL length of plastic hinge is given by

$$\boxed{L_P = L\sqrt{1 - \frac{1}{\alpha}}}$$

Where α is shape factor and L is total length of the beam.

- The length of plastic hinge depends on loading and geometry.

COLLAPSE LOADS

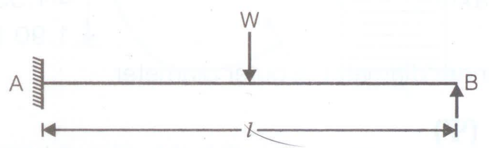
1. Simply supported beam with concentrated load at the center

$$\boxed{W_c = \frac{4M_p}{l}}$$

2. Simply supported beam with uniformly distributed load

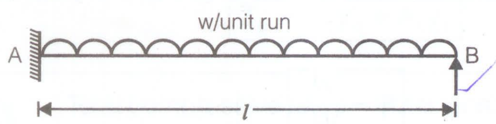
$$\cancel{\boxed{W_c = \frac{16M_p}{l}}} \quad \frac{8M_p}{l^2}$$

3. Propped cantilever with concentrated load at the center



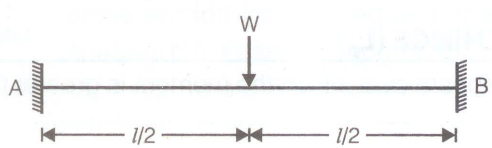
$$\boxed{W_c = \frac{6M_p}{l}}$$

4. Propped cantilever with uniformly distributed load



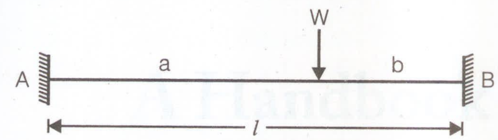
$$\boxed{W_c = \frac{11.656M_p}{l^2}}$$

5. Fixed beam with concentrated load at the center



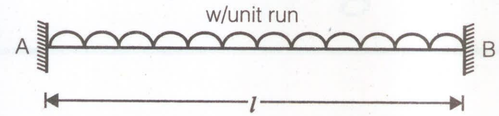
$$\boxed{W_c = \frac{8M_p}{l}}$$

6. Fixed beam with eccentric loading



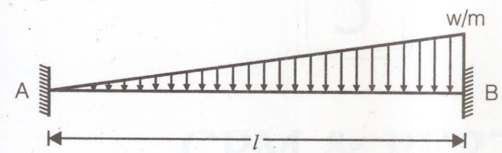
$$\boxed{W_c = \frac{2l}{ab}M_p}$$

7. Fixed beam with uniformly distributed load



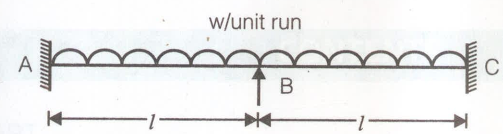
$$\boxed{W_c = \frac{16M_p}{l^2}}$$

8. Fixed beam with hydrostatic loading



$$\boxed{W_c = \frac{18\sqrt{3}\,M_p}{l^2}}$$

9. Continuous beam with uniformly distributed load



$$\boxed{W_c = \frac{11.656M_p}{l^2}}$$

The positions of the plastic hinges are one at the support B and one on each side of the central support at a distance of 0.414 l from A & C.

Remember

- **Upper bound theorem**

  It satisfies equilibrium and mechanism condition. $\boxed{P \geq P_u}$

- **Lower bound theorem**

  It satisfies equilibrium and yield condition. $\boxed{P \leq P_u}$

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