

4. Inverse Trigonometric Functions

Exercise 4A

1. Question

Find the principal value of :

(i) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\sin^{-1}\left(\frac{1}{2}\right)$

(iii) $\cos^{-1}\left(\frac{1}{2}\right)$

(iv) $\tan^{-1}(1)$

(v) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(vi) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(vii) $\operatorname{cosec}^{-1}(\sqrt{2})$

Answer

NOTE:

Trigonometric Table

	$0^\circ (0)$	$30^\circ \left(\frac{\pi}{6}\right)$	$45^\circ \left(\frac{\pi}{4}\right)$	$60^\circ \left(\frac{\pi}{3}\right)$	$90^\circ \left(\frac{\pi}{2}\right)$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
cosec	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
cot	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

(i) Let $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

$\Rightarrow \frac{\sqrt{3}}{2} = \sin x$ [We know which value of x when placed in sin gives us this answer]

$\therefore x = \frac{\pi}{3}$

(ii) Let $\sin^{-1}\left(\frac{1}{2}\right) = x$

$\Rightarrow \frac{1}{2} = \sin x$ [We know which value of x when put in this expression will give us this result]

$$\Rightarrow x = \frac{\pi}{6}$$

$$(iii) \text{ Let } \cos^{-1}\left(\frac{1}{2}\right) = x$$

$$\Rightarrow \frac{1}{2} = \cos x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\therefore x = \frac{\pi}{3}$$

$$(iv) \text{ Let } \tan^{-1}(1) = x$$

$$\Rightarrow 1 = \tan x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\therefore x = \frac{\pi}{4}$$

$$(v) \text{ Let } \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \tan x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\therefore x = \frac{\pi}{6}$$

$$(vi) \text{ Let } \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = x$$

$$\Rightarrow \frac{2}{\sqrt{3}} = \sec x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\therefore x = \frac{\pi}{6}$$

$$(vii) \text{ Let } \operatorname{cosec}^{-1}(\sqrt{2}) = x$$

$$\Rightarrow \sqrt{2} = \operatorname{cosec} x$$

[We know which value of x when put in this expression will give us this result]

$$\therefore x = \frac{\pi}{4}$$

2. Question

Find the principal value of :

$$(i) \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right)$$

$$(ii) \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

$$(iii) \tan^{-1}(-\sqrt{3})$$

$$(iv) \sec^{-1}(-2)$$

$$(v) \operatorname{cosec}^{-1}(-\sqrt{2})$$

$$(vi) \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$$

Answer

$$(i) \text{ Let } \sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) = x$$

$$\Rightarrow -\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = x \text{ [Formula: } \sin^{-1}(-x) = -\sin^{-1} x \text{]}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = -\sin x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\therefore x = -\frac{\pi}{4}$$

$$\text{(ii) } \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \text{ [Formula: } \cos^{-1}(-x) = \pi - \cos^{-1} x \text{]}$$

$$\text{Let } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$$

$$\Rightarrow \left(\frac{\sqrt{3}}{2}\right) = \cos x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\therefore x = \frac{\pi}{6}$$

Putting this value back in the equation

$$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$$\text{(iii) Let } \tan^{-1}(-\sqrt{3}) = x$$

$$\Rightarrow -\tan^{-1}(\sqrt{3}) = x \text{ [Formula: } \tan^{-1}(-x) = -\tan^{-1}(x) \text{]}$$

$$\Rightarrow \sqrt{3} = -\tan x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\therefore x = \frac{-\pi}{3}$$

$$\text{(iv) } \sec^{-1}(-2) = \pi - \sec^{-1}(2) \dots \text{(i) [Formula: } \sec^{-1}(-x) = \pi - \sec^{-1}(x) \text{]}$$

$$\text{Let } \sec^{-1}(2) = x$$

$$\Rightarrow 2 = \sec x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\therefore x = \frac{\pi}{3}$$

Putting the value in (i)

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\text{(v) Let } \operatorname{cosec}^{-1}(-\sqrt{2}) = x$$

$$\Rightarrow -\operatorname{cosec}^{-1}(\sqrt{2}) = x \text{ [Formula: } \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x) \text{]}$$

$$\Rightarrow \sqrt{2} = -\operatorname{cosec} x$$

$$\therefore x = -\frac{\pi}{4}$$

$$\text{(vi) } \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) \dots \text{(i)}$$

$$\text{Let } \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = x$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \cot^{-1} x \text{ [We know which value of } x \text{ when put in this expression will give us this result]}$$

$$\Rightarrow x = \frac{\pi}{3}$$

Putting in (i)

$$\pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

3. Question

Evaluate $\cos \left\{ \cos^{-1} \left(\frac{-\sqrt{3}}{2} \right) + \frac{\pi}{6} \right\}$.

Answer

$$\cos \left\{ \pi - \frac{\pi}{6} + \frac{\pi}{6} \right\} \text{ [Refer to question 2(ii)]}$$

$$= \cos \{ \pi \}$$

$$= \cos \left(\frac{\pi}{2} + \frac{\pi}{2} \right)$$

$$= -1$$

4. Question

Evaluate $\sin \left\{ \frac{\pi}{2} - \left(\frac{-\pi}{3} \right) \right\}$

Answer

$$\sin \left(\frac{\pi}{2} + \frac{\pi}{3} \right)$$

$$= \sin \left(\frac{5\pi}{6} \right)$$

$$= \sin \left(\pi - \frac{\pi}{6} \right)$$

$$= \sin \frac{\pi}{6}$$

$$= \frac{1}{2}$$

Exercise 4B

1. Question

Find the principal value of each of the following :

$$\sin^{-1} \left(\frac{-1}{2} \right)$$

Answer

$$\sin^{-1} \left(\frac{-1}{2} \right) = -\sin^{-1} \left(\frac{1}{2} \right) \text{ [Formula: } \sin^{-1}(-x) = -\sin^{-1}(x) \text{]}$$

$$= -\frac{\pi}{6}$$

2. Question

Find the principal value of each of the following :

$$\cos^{-1} \left(\frac{-1}{2} \right)$$

Answer

$$\cos^{-1} \left(\frac{-1}{2} \right) = \pi - \cos^{-1} \left(\frac{1}{2} \right) \text{ [Formula: } \cos^{-1}(-x) = \pi - \cos^{-1}(x) \text{]}$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

3. Question

Find the principal value of each of the following :

$$\tan^{-1}(-1)$$

Answer

$$\tan^{-1}(-1) = -\tan^{-1}(1) \text{ [Formula: } \tan^{-1}(-x) = -\tan^{-1}(x) \text{]}$$

[We know that $\tan \frac{\pi}{4} = 1$, thus $\tan^{-1} \frac{\pi}{4} = 1$]

$$= -\frac{\pi}{4}$$

4. Question

Find the principal value of each of the following :

$$\sec^{-1}(-2)$$

Answer

$$\sec^{-1}(-2) = \pi - \sec^{-1}(2) \text{ [Formula: } \sec^{-1}(-x) = \pi - \sec^{-1}(x) \text{]}$$

$$= \pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

5. Question

Find the principal value of each of the following :

$$\operatorname{cosec}^{-1}(-\sqrt{2})$$

Answer

$$\operatorname{cosec}^{-1}(-\sqrt{2}) = -\operatorname{cosec}^{-1}(\sqrt{2}) \text{ [Formula: } \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}(x) \text{]}$$

$$= -\frac{\pi}{4}$$

This can also be solved as

$$\operatorname{cosec}^{-1}(-\sqrt{2})$$

Since cosec is negative in the third quadrant, the angle we are looking for will be in the third quadrant.

$$= \pi + \frac{\pi}{4}$$

$$= \frac{5\pi}{4}$$

6. Question

Find the principal value of each of the following :

$$\cot^{-1}(-1)$$

Answer

$$\cot^{-1}(-1) = \pi - \cot^{-1}(1) \text{ [Formula: } \cot^{-1}(-x) = \pi - \cot^{-1}(x) \text{]}$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{3\pi}{4}$$

7. Question

Find the principal value of each of the following :

$$\tan^{-1}(-\sqrt{3})$$

Answer

$$\tan^{-1}(-\sqrt{3}) = -\tan^{-1}(\sqrt{3}) \text{ [Formula: } \tan^{-1}(-x) = -\tan^{-1}(x) \text{]}$$

$$= -\frac{\pi}{3}$$

8. Question

Find the principal value of each of the following :

$$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$$

Answer

$$\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right) = \pi - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) \text{ [Formula: } \sec^{-1}(-x) = \pi - \sec^{-1}(x) \text{]}$$

$$= \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

9. Question

Find the principal value of each of the following :

$$\operatorname{cosec}^{-1}(2)$$

Answer

$$\operatorname{cosec}^{-1}(2)$$

Putting the value directly

$$= \frac{\pi}{6}$$

10. Question

Find the principal value of each of the following :

$$\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$$

Answer

$$\sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$$

[Formula: $\sin(\pi - x) = \sin x$]

$$= \sin^{-1}\left(\sin \frac{\pi}{3}\right)$$

[Formula: $\sin^{-1}(\sin x) = x$]

$$= \frac{\pi}{3}$$

11. Question

Find the principal value of each of the following :

$$\tan^{-1}\left(\tan \frac{3\pi}{4}\right)$$

Answer

$$\tan^{-1}\left(\tan \frac{3\pi}{4}\right) = \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{4}\right)\right)$$

[Formula: $\tan(\pi - x) = -\tan(x)$, as \tan is negative in the second quadrant.]

$$= \tan^{-1}\left(-\tan \frac{\pi}{4}\right)$$

[Formula: $\tan^{-1}(\tan x) = x$]

$$= -\frac{\pi}{4}$$

12. Question

Find the principal value of each of the following :

$$\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$$

Answer

$$\cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{6}\right)\right)$$

[Formula: $\cos(2\pi - x) = \cos(x)$, as \cos has a positive value in the fourth quadrant.]

$$= \cos^{-1}\left(\cos \frac{5\pi}{6}\right) \text{ [Formula: } \cos^{-1}(\cos x) = x$$

$$= \frac{5\pi}{6}$$

13. Question

Find the principal value of each of the following :

$$\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$$

Answer

$$\cos^{-1}\left(\cos \frac{13\pi}{6}\right) = \cos^{-1}\left(\cos\left(2\pi + \frac{\pi}{6}\right)\right)$$

[Formula: $\cos(2\pi + x) = \cos x$, \cos is positive in the first quadrant.]

$$= \cos^{-1}\left(\cos \frac{\pi}{6}\right) \text{ [Formula: } \cos^{-1}(\cos x) = x]$$

$$= \frac{\pi}{6}$$

14. Question

Find the principal value of each of the following :

$$\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$$

Answer

$$\tan^{-1}\left(\tan \frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{6}\right)\right)$$

[Formula: $\tan(\pi + x) = \tan x$, as \tan is positive in the third quadrant.]

$$= \tan^{-1}\left(\tan \frac{\pi}{6}\right) \text{ [Formula: } \tan^{-1}(\tan x) = x \text{]}$$

$$= \frac{\pi}{6}$$

15. Question

Find the principal value of each of the following :

$$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$$

Answer

$$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$$

Putting the value of $\tan^{-1}\sqrt{3}$ and using the formula

$$\cot^{-1}(-x) = \pi - \cot^{-1}x$$

$$= \frac{\pi}{3} - (\pi - \cot^{-1}(\sqrt{3}))$$

Putting the value of $\cot^{-1}(\sqrt{3})$

$$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$$

$$= \frac{\pi}{3} - \frac{5\pi}{6}$$

$$= -\frac{3\pi}{6}$$

$$= -\frac{\pi}{2}$$

16. Question

Find the principal value of each of the following :

$$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$$

Answer

$$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\} \text{ [Formula: } \sin^{-1}(-x) = -\sin^{-1}x \text{]}$$

$$= \sin\left\{\frac{\pi}{3} - \left(-\sin^{-1}\frac{1}{2}\right)\right\}$$

$$= \sin\left\{\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right\}$$

Putting value of $\sin^{-1}\left(\frac{1}{2}\right)$

$$= \sin\left\{\frac{\pi}{3} + \frac{\pi}{6}\right\}$$

$$= \sin \frac{3\pi}{6}$$

$$= \sin \frac{\pi}{2}$$

$$= 1$$

17. Question

Find the principal value of each of the following :

$$\cot(\tan^{-1}x + \cot^{-1}x)$$

Answer

$$\cot(\tan^{-1}x + \cot^{-1}x) = \cot\left(\frac{\pi}{2}\right) \text{ [Formula: } \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2} \text{]}$$

$$\text{Putting value of } \cot\left(\frac{\pi}{2}\right)$$

$$= 0$$

18. Question

Find the principal value of each of the following :

$$\operatorname{cosec}(\sin^{-1}x + \cos^{-1}x)$$

Answer

$$\operatorname{cosec}(\sin^{-1}x + \cos^{-1}x) = \operatorname{cosec} \frac{\pi}{2} \text{ [Formula: } \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \text{]}$$

$$\text{Putting the value of } \operatorname{cosec} \frac{\pi}{2}$$

$$= 1$$

19. Question

Find the principal value of each of the following :

$$\sin(\sec^{-1}x + \operatorname{cosec}^{-1}x)$$

Answer

$$\sin(\sec^{-1}x + \operatorname{cosec}^{-1}x) = \sin\left(\frac{\pi}{2}\right) \text{ [Formula: } \sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2} \text{]}$$

$$\text{Putting the value of } \sin\left(\frac{\pi}{2}\right)$$

$$= 1$$

20. Question

Find the principal value of each of the following :

$$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$$

Answer

Putting the values of the inverse trigonometric terms

$$\frac{\pi}{3} + 2 \times \frac{\pi}{6}$$

$$= \frac{\pi}{3} + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

21. Question

Find the principal value of each of the following :

$$\tan^{-1} 1 + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

Answer

[Formula: $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ and $\sin^{-1}(-x) = -\sin^{-1}(x)$]

$$\tan^{-1} 1 + \left(\pi - \cos^{-1}\left(\frac{1}{2}\right)\right) + \left(-\sin^{-1}\left(\frac{1}{2}\right)\right)$$

Putting the values for each of the inverse trigonometric terms

$$= \frac{\pi}{4} + \left(\pi - \frac{\pi}{3}\right) - \frac{\pi}{6}$$

$$= \frac{\pi}{12} + \frac{2\pi}{3}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

22. Question

Find the principal value of each of the following :

$$\sin^{-1}\left\{\sin\frac{3\pi}{5}\right\}$$

Answer

$$\sin^{-1}\left\{\sin\left(\frac{3\pi}{5}\right)\right\}$$

$$= \sin^{-1}\left\{\sin\left(\pi - \frac{2\pi}{5}\right)\right\}$$

[Formula: $\sin(\pi - x) = \sin x$, as sin is positive in the second quadrant.]

$$= \sin^{-1}\left\{\sin\frac{2\pi}{5}\right\} \text{ [Formula: } \sin^{-1}(\sin x) = x \text{]}$$

$$= \frac{2\pi}{5}$$

Exercise 4C

1 A. Question

Prove that:

$$\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x, x < 1$$

Answer

To Prove: $\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x$

Formula Used: $\tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$

Proof:

LHS = $\tan^{-1}\left(\frac{1+x}{1-x}\right) \dots (1)$

Let $x = \tan A \dots (2)$

Substituting (2) in (1),

$$\begin{aligned} \text{LHS} &= \tan^{-1} \left(\frac{1 + \tan A}{1 - \tan A} \right) \\ &= \tan^{-1} \left(\tan \left(\frac{\pi}{4} + A \right) \right) \\ &= \frac{\pi}{4} + A \end{aligned}$$

From (2), $A = \tan^{-1} x$,

$$\begin{aligned} \frac{\pi}{4} + A &= \frac{\pi}{4} + \tan^{-1} x \\ &= \text{RHS} \end{aligned}$$

Therefore, LHS = RHS

Hence proved.

1 B. Question

Prove that:

$$\tan^{-1} x + \cot^{-1} (x + 1) = \tan^{-1} (x^2 + x + 1)$$

Answer

To Prove: $\tan^{-1} x + \cot^{-1} (x + 1) = \tan^{-1} (x^2 + x + 1)$

Formula Used:

$$1) \cot^{-1} x = \tan^{-1} \frac{1}{x}$$

$$2) \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

Proof:

$$\text{LHS} = \tan^{-1} x + \cot^{-1} (x + 1) \dots (1)$$

$$= \tan^{-1} x + \tan^{-1} \frac{1}{(x + 1)}$$

$$= \tan^{-1} \left(\frac{x + \frac{1}{(x + 1)}}{1 - \left(x \times \frac{1}{(x + 1)} \right)} \right)$$

$$= \tan^{-1} \frac{x(x + 1) + 1}{x + 1 - x}$$

$$= \tan^{-1} (x^2 + x + 1)$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

2. Question

Prove that:

$$\sin^{-1} \left(2x\sqrt{1-x^2} \right) = 2\sin^{-1} x, |x| \leq \frac{1}{\sqrt{2}}$$

Answer

To Prove: $\sin^{-1}(2x\sqrt{1-x^2}) = 2 \sin^{-1} x$

Formula Used: $\sin 2A = 2 \times \sin A \times \cos A$

Proof:

$$\text{LHS} = \sin^{-1}(2x\sqrt{1-x^2}) \dots (1)$$

$$\text{Let } x = \sin A \dots (2)$$

Substituting (2) in (1),

$$\text{LHS} = \sin^{-1}(2 \sin A \sqrt{1 - \sin^2 A})$$

$$= \sin^{-1}(2 \times \sin A \times \cos A)$$

$$= \sin^{-1}(\sin 2A)$$

$$= 2A$$

$$\text{From (2), } A = \sin^{-1} x,$$

$$2A = 2 \sin^{-1} x$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

3 A. Question

Prove that:

$$\sin^{-1}(3x - 4x^3) = 3 \sin^{-1} x, |x| \leq \frac{1}{2}$$

Answer

To Prove: $\sin^{-1}(3x - 4x^3) = 3 \sin^{-1} x$

Formula Used: $\sin 3A = 3 \sin A - 4 \sin^3 A$

Proof:

$$\text{LHS} = \sin^{-1}(3x - 4x^3) \dots (1)$$

$$\text{Let } x = \sin A \dots (2)$$

Substituting (2) in (1),

$$\text{LHS} = \sin^{-1}(3 \sin A - 4 \sin^3 A)$$

$$= \sin^{-1}(\sin 3A)$$

$$= 3A$$

$$\text{From (2), } A = \sin^{-1} x,$$

$$3A = 3 \sin^{-1} x$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

3 B. Question

Prove that:

$$\cos^{-1}(4x^3 - 3x) = 3\cos^{-1} x, \frac{1}{2} \leq x \leq 1$$

Answer

To Prove: $\cos^{-1}(4x^3 - 3x) = 3\cos^{-1} x$

Formula Used: $\cos 3A = 4\cos^3 A - 3\cos A$

Proof:

$$\text{LHS} = \cos^{-1}(4x^3 - 3x) \dots (1)$$

$$\text{Let } x = \cos A \dots (2)$$

Substituting (2) in (1),

$$\text{LHS} = \cos^{-1}(4\cos^3 A - 3\cos A)$$

$$= \cos^{-1}(\cos 3A)$$

$$= 3A$$

$$\text{From (2), } A = \cos^{-1} x,$$

$$3A = 3\cos^{-1} x$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

3 C. Question

Prove that:

$$\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = 3\tan^{-1} x, |x| < \frac{1}{\sqrt{3}}$$

Answer

To Prove: $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) = 3\tan^{-1} x$

Formula Used: $\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

Proof:

$$\text{LHS} = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right) \dots (1)$$

$$\text{Let } x = \tan A \dots (2)$$

Substituting (2) in (1),

$$\text{LHS} = \tan^{-1}\left(\frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}\right)$$

$$= \tan^{-1}(\tan 3A)$$

$$= 3A$$

$$\text{From (2), } A = \tan^{-1} x,$$

$$3A = 3\tan^{-1} x$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

3 D. Question

Prove that:

$$\tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$$

Answer

To Prove: $\tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$

Formula Used: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

Proof:

$$\text{LHS} = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right) \dots (1)$$

$$= \tan^{-1} \left(\frac{x + \left(\frac{2x}{1-x^2} \right)}{1 - \left(x \times \left(\frac{2x}{1-x^2} \right) \right)} \right)$$

$$= \tan^{-1} \left(\frac{x(1-x^2) + 2x}{1-x^2-2x^2} \right)$$

$$= \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$$

= RHS

Therefore, LHS = RHS

Hence proved.

4 A. Question

Prove that:

$$\cos^{-1}(1-2x^2) = 2\sin^{-1} x$$

Answer

To Prove: $\cos^{-1}(1-2x^2) = 2\sin^{-1} x$

Formula Used: $\cos 2A = 1 - 2\sin^2 A$

Proof:

$$\text{LHS} = \cos^{-1}(1-2x^2) \dots (1)$$

$$\text{Let } x = \sin A \dots (2)$$

Substituting (2) in (1),

$$\text{LHS} = \cos^{-1}(1-2\sin^2 A)$$

$$= \cos^{-1}(\cos 2A)$$

$$= 2A$$

From (2), $A = \sin^{-1} x$,

$$2A = 2 \sin^{-1} x$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

4 B. Question

Prove that:

$$\cos^{-1}(2x^2 - 1) = 2\cos^{-1} x$$

Answer

$$\text{To Prove: } \cos^{-1}(2x^2 - 1) = 2\cos^{-1} x$$

$$\text{Formula Used: } \cos 2A = 2\cos^2 A - 1$$

Proof:

$$\text{LHS} = \cos^{-1}(2x^2 - 1) \dots (1)$$

$$\text{Let } x = \cos A \dots (2)$$

Substituting (2) in (1),

$$\text{LHS} = \cos^{-1}(2\cos^2 A - 1)$$

$$= \cos^{-1}(\cos 2A)$$

$$= 2A$$

$$\text{From (2), } A = \cos^{-1} x,$$

$$2A = 2\cos^{-1} x$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

4 C. Question

Prove that:

$$\sec^{-1}\left(\frac{1}{2x^2 - 1}\right) = 2\cos^{-1} x$$

Answer

$$\text{To Prove: } \sec^{-1}\left(\frac{1}{2x^2 - 1}\right) = 2\cos^{-1} x$$

Formula Used:

$$1) \cos 2A = 2\cos^2 A - 1$$

$$2) \cos^{-1} A = \sec^{-1}\left(\frac{1}{A}\right)$$

Proof:

$$\text{LHS} = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$$

$$= \cos^{-1}(2x^2 - 1) \dots (1)$$

$$\text{Let } x = \cos A \dots (2)$$

Substituting (2) in (1),

$$\text{LHS} = \cos^{-1}(2 \cos^2 A - 1)$$

$$= \cos^{-1}(\cos 2A)$$

$$= 2A$$

From (2), $A = \cos^{-1} x$,

$$2A = 2 \cos^{-1} x$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

4 D. Question

Prove that:

$$\cot^{-1}(\sqrt{1+x^2} - x) = \frac{\pi}{2} - \frac{1}{2} \cot^{-1} x$$

Answer

$$\text{To Prove: } \cot^{-1}(\sqrt{1+x^2} - x) = \frac{\pi}{2} - \frac{1}{2} \cot^{-1} x$$

Formula Used:

$$1) \tan\left(\frac{\pi}{4} + A\right) = \frac{1 + \tan A}{1 - \tan A}$$

$$2) \operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$3) 1 - \cos A = 2 \sin^2\left(\frac{A}{2}\right)$$

$$4) \sin A = 2 \sin\left(\frac{A}{2}\right) \cos\left(\frac{A}{2}\right)$$

Proof:

$$\text{LHS} = \cot^{-1}(\sqrt{1+x^2} - x)$$

Let $x = \cot A$

$$\text{LHS} = \cot^{-1}(\sqrt{1 + \cot^2 A} - \cot A)$$

$$= \cot^{-1}(\operatorname{cosec} A - \cot A)$$

$$= \cot^{-1}\left(\frac{1 - \cos A}{\sin A}\right)$$

$$= \cot^{-1}\left(\frac{2 \sin^2\left(\frac{A}{2}\right)}{2 \sin\left(\frac{A}{2}\right) \cos\left(\frac{A}{2}\right)}\right)$$

$$= \cot^{-1}\left(\tan\left(\frac{A}{2}\right)\right)$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\tan\left(\frac{A}{2}\right)\right)$$

$$= \frac{\pi}{2} - \frac{A}{2}$$

From (2), $A = \cot^{-1} x$,

$$\frac{\pi}{2} - \frac{A}{2} = \frac{\pi}{2} - \frac{1}{2} \cot^{-1} x$$

= RHS

Therefore, LHS = RHS

Hence proved.

5 A. Question

Prove that:

$$\tan^{-1} \left(\frac{\sqrt{x} + \sqrt{y}}{1 - \sqrt{xy}} \right) = \tan^{-1} \sqrt{x} + \tan^{-1} \sqrt{y}$$

Answer

To Prove: $\tan^{-1} \left(\frac{\sqrt{x} + \sqrt{y}}{1 - \sqrt{xy}} \right) = \tan^{-1} \sqrt{x} + \tan^{-1} \sqrt{y}$

We know that, $\tan A + \tan B = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Also, $\tan^{-1} \left(\frac{A+B}{1-AB} \right) = \tan^{-1} A + \tan^{-1} B$

Taking A = \sqrt{x} and B = \sqrt{y}

We get,

$$\tan^{-1} \left(\frac{\sqrt{x} + \sqrt{y}}{1 - \sqrt{xy}} \right) = \tan^{-1} \sqrt{x} + \tan^{-1} \sqrt{y}$$

Hence, Proved.

5 B. Question

Prove that:

$$\tan^{-1} \left(\frac{x + \sqrt{x}}{1 - x^{3/2}} \right) = \tan^{-1} x + \tan^{-1} \sqrt{x}$$

Answer

We know that,

$$\tan^{-1} \left(\frac{A+B}{1-AB} \right) = \tan^{-1} A + \tan^{-1} B$$

Now, taking A = x and B = \sqrt{x}

We get,

$$\tan^{-1} x + \tan^{-1} \sqrt{x} = \tan^{-1} \left(\frac{x + \sqrt{x}}{1 - x^{3/2}} \right)$$

As, $x \cdot x^{1/2} = x^{3/2}$

Hence, Proved.

5 C. Question

Prove that:

$$\tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right) = \frac{x}{2}$$

Answer

$$\text{To Prove: } \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right) = \frac{x}{2}$$

Formula Used:

$$1) \sin A = 2 \times \sin \frac{A}{2} \times \cos \frac{A}{2}$$

$$2) 1 + \cos A = 2 \cos^2 \frac{A}{2}$$

Proof:

$$\text{LHS} = \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right)$$

$$= \tan^{-1} \left(\frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} \right)$$

$$= \tan^{-1} \left(\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} \right)$$

$$= \tan^{-1} \left(\tan \frac{x}{2} \right)$$

$$= \frac{x}{2}$$

$$= \text{RHS}$$

Therefore LHS = RHS

Hence proved.

6 A. Question

Prove that:

$$\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \frac{3}{4}$$

Answer

$$\text{To Prove: } \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \frac{3}{4}$$

$$\text{Formula Used: } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

Proof:

$$\text{LHS} = \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11}$$

$$= \tan^{-1} \left(\frac{\frac{1}{2} + \frac{2}{11}}{1 - \left(\frac{1}{2} \times \frac{2}{11} \right)} \right)$$

$$= \tan^{-1} \left(\frac{11 + 4}{22 - 2} \right)$$

$$= \tan^{-1} \frac{15}{20}$$

$$= \tan^{-1} \frac{3}{4}$$

$$= \text{RHS}$$

Therefore LHS = RHS

Hence proved.

6 B. Question

Prove that:

$$\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24} = \tan^{-1} \frac{1}{2}$$

Answer

To Prove: $\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24} = \tan^{-1} \frac{1}{2}$

Formula Used: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

Proof:

$$\text{LHS} = \tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24}$$

$$= \tan^{-1} \left(\frac{\frac{2}{11} + \frac{7}{24}}{1 - \left(\frac{2}{11} \times \frac{7}{24} \right)} \right)$$

$$= \tan^{-1} \left(\frac{48 + 77}{264 - 14} \right)$$

$$= \tan^{-1} \frac{125}{250}$$

$$= \tan^{-1} \frac{1}{2}$$

= RHS

Therefore LHS = RHS

Hence proved.

6 C. Question

Prove that:

$$\tan^{-1} 1 + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \frac{\pi}{2}$$

Answer

To Prove: $\tan^{-1} 1 + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \frac{\pi}{2}$

Formula Used: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

Proof:

$$\text{LHS} = \tan^{-1} 1 + \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3}$$

$$= \tan^{-1} 1 + \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2} \times \frac{1}{3} \right)} \right)$$

$$= \tan^{-1} 1 + \tan^{-1} \left(\frac{5}{6-1} \right)$$

$$= \tan^{-1} 1 + \tan^{-1} 1$$

$$= \frac{\pi}{4} + \frac{\pi}{4}$$

$$= \frac{\pi}{2}$$

= RHS

Therefore LHS = RHS

Hence proved.

6 D. Question

Prove that:

$$2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$$

Answer

To Prove: $2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7} = \frac{\pi}{4}$

Formula Used: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

Proof:

$$\text{LHS} = 2 \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \left(\frac{\frac{1}{3} + \frac{1}{3}}{1 - \left(\frac{1}{3} \times \frac{1}{3} \right)} \right) + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \left(\frac{6}{9-1} \right) + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{1}{7}$$

$$= \tan^{-1} \left(\frac{\frac{3}{4} + \frac{1}{7}}{1 - \left(\frac{3}{4} \times \frac{1}{7} \right)} \right)$$

$$= \tan^{-1} \left(\frac{21+4}{28-3} \right)$$

$$= \tan^{-1} \frac{25}{25}$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

= RHS

Therefore LHS = RHS

Hence proved.

6 E. Question

Prove that:

$$\tan^{-1} 2 - \tan^{-1} 1 = \tan^{-1} \frac{1}{3}$$

Answer

To Prove: $\tan^{-1} 2 - \tan^{-1} 1 = \tan^{-1} \frac{1}{3}$

Formula Used: $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right)$ where $xy > -1$

Proof:

$$\text{LHS} = \tan^{-1} 2 - \tan^{-1} 1$$

$$= \tan^{-1} \left(\frac{2-1}{1+2} \right)$$

$$= \tan^{-1} \left(\frac{1}{3} \right)$$

$$= \text{RHS}$$

Therefore LHS = RHS

Hence proved.

6 F. Question

Prove that:

$$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$$

Answer

To Prove: $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$

Formula Used: $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ where $xy > 1$

Proof:

$$\text{LHS} = \tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$$

$$= \frac{\pi}{4} + \pi + \tan^{-1} \left(\frac{2+3}{1-(2 \times 3)} \right) \{ \text{since } 2 \times 3 = 6 > 1 \}$$

$$= \frac{5\pi}{4} + \tan^{-1} \left(\frac{5}{-5} \right)$$

$$= \frac{5\pi}{4} + \tan^{-1}(-1)$$

$$= \frac{5\pi}{4} - \frac{\pi}{4}$$

$$= \pi$$

$$= \text{RHS}$$

Therefore LHS = RHS

Hence proved.

6 G. Question

Prove that:

$$\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$$

Answer

$$\text{To Prove: } \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

$$\text{Formula Used: } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ where } xy < 1$$

Proof:

$$\text{LHS} = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{\frac{1}{5} + \frac{1}{8}}{1 - \left(\frac{1}{5} \times \frac{1}{8}\right)}\right)$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{8+5}{40-1}\right)$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\left(\frac{13}{39}\right)$$

$$= \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}$$

$$= \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2} \times \frac{1}{3}\right)}\right)$$

$$= \tan^{-1}\left(\frac{3+2}{6-1}\right)$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

$$= \text{RHS}$$

Therefore LHS = RHS

Hence proved.

6 H. Question

Prove that:

$$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3}$$

Answer

$$\text{To Prove: } \tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3} \Rightarrow 2\left(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}\right) = \tan^{-1}\frac{4}{3}$$

$$\text{Formula Used: } \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right) \text{ where } xy < 1$$

Proof:

$$\text{LHS} = 2\left(\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9}\right)$$

$$= 2\left(\tan^{-1}\left(\frac{\frac{1}{4} + \frac{2}{9}}{1 - \left(\frac{1}{4} \times \frac{2}{9}\right)}\right)\right)$$

$$= 2 \tan^{-1}\left(\frac{9+8}{36-2}\right)$$

$$\begin{aligned}
&= 2 \tan^{-1} \frac{17}{34} \\
&= 2 \tan^{-1} \frac{1}{2} \\
&= \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{2} \\
&= \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{2}}{1 - \left(\frac{1}{2} \times \frac{1}{2}\right)} \right) \\
&= \tan^{-1} \left(\frac{1}{\frac{4-1}{4}} \right) \\
&= \tan^{-1} \frac{4}{3} \\
&= \text{RHS}
\end{aligned}$$

Therefore LHS = RHS

Hence proved.

7 A. Question

Prove that:

$$\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$$

Answer

To Prove: $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$

Formula Used: $\cos^{-1} x + \cos^{-1} y = \cos^{-1}(xy - \sqrt{1-x^2} \times \sqrt{1-y^2})$

Proof:

$$\begin{aligned}
\text{LHS} &= \cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} \\
&= \cos^{-1} \left(\frac{4}{5} \times \frac{12}{13} - \sqrt{1 - \left(\frac{4}{5}\right)^2} \times \sqrt{1 - \left(\frac{12}{13}\right)^2} \right) \\
&= \cos^{-1} \left(\frac{48}{65} - \sqrt{1 - \frac{16}{25}} \times \sqrt{1 - \frac{144}{169}} \right) \\
&= \cos^{-1} \left(\frac{48}{65} - \left(\sqrt{\frac{25-16}{25}} \times \sqrt{\frac{169-144}{169}} \right) \right) \\
&= \cos^{-1} \left(\frac{48}{65} - \left(\sqrt{\frac{9}{25}} \times \sqrt{\frac{25}{169}} \right) \right) \\
&= \cos^{-1} \left(\frac{48}{65} - \frac{3}{13} \right)
\end{aligned}$$

$$= \cos^{-1}\left(\frac{48-15}{65}\right)$$

$$= \cos^{-1}\frac{33}{65}$$

= RHS

Therefore, LHS = RHS

Hence proved.

7 B. Question

Prove that:

$$\sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}} = \frac{\pi}{2}$$

Answer

To Prove: $\sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}} = \frac{\pi}{2}$

Formula Used: $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x \times \sqrt{1-y^2} + y \times \sqrt{1-x^2})$

Proof:

$$\text{LHS} = \sin^{-1}\frac{1}{\sqrt{5}} + \sin^{-1}\frac{2}{\sqrt{5}}$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{5}} \times \sqrt{1 - \left(\frac{2}{\sqrt{5}}\right)^2} + \frac{2}{\sqrt{5}} \times \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{5}} \times \sqrt{1 - \frac{4}{5}} + \frac{2}{\sqrt{5}} \times \sqrt{1 - \frac{1}{5}}\right)$$

$$= \sin^{-1}\left(\frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}} \times \frac{2}{\sqrt{5}}\right)$$

$$= \sin^{-1}\left(\frac{1}{5} + \frac{4}{5}\right)$$

$$= \sin^{-1}\frac{5}{5}$$

$$= \sin^{-1} 1$$

$$= \frac{\pi}{2}$$

= RHS

Therefore, LHS = RHS

Hence proved.

7 C. Question

Prove that:

$$\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \sin^{-1}\frac{56}{65}$$

Answer

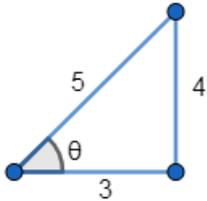
To Prove: $\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \sin^{-1}\frac{56}{65}$

Formula Used: $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x \times \sqrt{1-y^2} + y \times \sqrt{1-x^2})$

Proof:

LHS = $\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} \dots (1)$

Let $\cos\theta = \frac{3}{5}$



Therefore $\theta = \cos^{-1}\frac{3}{5} \dots (2)$

From the figure, $\sin\theta = \frac{4}{5}$

$\Rightarrow \theta = \sin^{-1}\frac{4}{5} \dots (3)$

From (2) and (3),

$\cos^{-1}\frac{3}{5} = \sin^{-1}\frac{4}{5}$

Substituting in (1), we get

$$\begin{aligned} \text{LHS} &= \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{12}{13} \\ &= \sin^{-1}\left(\frac{4}{5} \times \sqrt{1 - \left(\frac{12}{13}\right)^2} + \frac{12}{13} \times \sqrt{1 - \left(\frac{4}{5}\right)^2}\right) \\ &= \sin^{-1}\left(\frac{4}{5} \times \sqrt{1 - \frac{144}{169}} + \frac{12}{13} \times \sqrt{1 - \frac{16}{25}}\right) \\ &= \sin^{-1}\left(\frac{4}{5} \times \sqrt{\frac{25}{169}} + \frac{12}{13} \times \sqrt{\frac{9}{25}}\right) \\ &= \sin^{-1}\left(\frac{4}{5} \times \frac{5}{13} + \frac{12}{13} \times \frac{3}{5}\right) \\ &= \sin^{-1}\left(\frac{20}{65} + \frac{36}{65}\right) \\ &= \sin^{-1}\frac{56}{65} \end{aligned}$$

= RHS

Therefore, LHS = RHS

Hence proved.

7 D. Question

Prove that:

$$\cos^{-1} \frac{4}{5} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{27}{11}$$

Answer

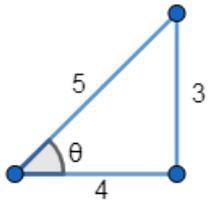
To Prove: $\cos^{-1} \frac{4}{5} + \sin^{-1} \frac{3}{5} = \sin^{-1} \frac{27}{11}$

Formula Used: $\sin^{-1} x + \sin^{-1} y = \sin^{-1}(x \times \sqrt{1-y^2} + y \times \sqrt{1-x^2})$

Proof:

$$\text{LHS} = \cos^{-1} \frac{4}{5} + \sin^{-1} \frac{3}{5} \dots (1)$$

$$\text{Let } \cos \theta = \frac{4}{5}$$



$$\text{Therefore } \theta = \cos^{-1} \frac{4}{5} \dots (2)$$

$$\text{From the figure, } \sin \theta = \frac{3}{5}$$

$$\Rightarrow \theta = \sin^{-1} \frac{3}{5} \dots (3)$$

From (2) and (3),

$$\cos^{-1} \frac{4}{5} = \sin^{-1} \frac{3}{5}$$

Substituting in (1), we get

$$\text{LHS} = \sin^{-1} \frac{3}{5} + \sin^{-1} \frac{3}{5}$$

$$= \sin^{-1} \left(2 \times \frac{3}{5} \times \sqrt{1 - \left(\frac{3}{5}\right)^2} \right)$$

$$= \sin^{-1} \left(2 \times \frac{3}{5} \times \sqrt{1 - \frac{9}{25}} \right)$$

$$= \sin^{-1} \left(2 \times \frac{3}{5} \times \sqrt{\frac{16}{25}} \right)$$

$$= \sin^{-1} \left(2 \times \frac{3}{5} \times \frac{4}{5} \right)$$

$$= \sin^{-1} \frac{24}{25}$$

7 E. Question

Prove that:

$$\tan^{-1} \frac{1}{3} + \sec^{-1} \frac{\sqrt{5}}{2} = \frac{\pi}{4}$$

Answer

To Prove: $\tan^{-1}\frac{1}{3} + \sec^{-1}\frac{\sqrt{5}}{2} = \frac{\pi}{4}$

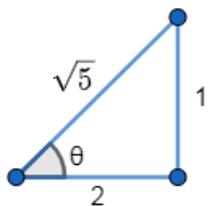
Formula Used: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ where $xy < 1$

Proof:

$$\text{LHS} = \tan^{-1}\frac{1}{3} + \sec^{-1}\frac{\sqrt{5}}{2} \dots (1)$$

$$\text{Let } \sec\theta = \frac{\sqrt{5}}{2}$$

$$\text{Therefore } \theta = \sec^{-1}\frac{\sqrt{5}}{2} \dots (2)$$



$$\text{From the figure, } \tan\theta = \frac{1}{2}$$

$$\Rightarrow \theta = \tan^{-1}\frac{1}{2} \dots (3)$$

From (2) and (3),

$$\sec^{-1}\frac{\sqrt{5}}{2} = \tan^{-1}\frac{1}{2}$$

Substituting in (1), we get

$$\text{LHS} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$$

$$= \tan^{-1}\left(\frac{\frac{1}{3} + \frac{1}{2}}{1 - \left(\frac{1}{3} \times \frac{1}{2}\right)}\right)$$

$$= \tan^{-1}\left(\frac{2+3}{6-1}\right)$$

$$= \tan^{-1}\frac{5}{5}$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

7 F. Question

Prove that:

$$\sin^{-1}\frac{1}{\sqrt{17}} + \cos^{-1}\frac{9}{\sqrt{85}} = \tan^{-1}\frac{1}{2}$$

Answer

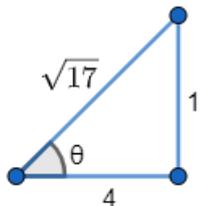
$$\text{To Prove: } \sin^{-1} \frac{1}{\sqrt{17}} + \cos^{-1} \frac{9}{\sqrt{85}} = \tan^{-1} \frac{1}{2}$$

Formula Used: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ where $xy < 1$

Proof:

$$\text{LHS} = \sin^{-1} \frac{1}{\sqrt{17}} + \cos^{-1} \frac{9}{\sqrt{85}} \dots (1)$$

$$\text{Let } \sin \theta = \frac{1}{\sqrt{17}}$$



$$\text{Therefore } \theta = \sin^{-1} \frac{1}{\sqrt{17}} \dots (2)$$

$$\text{From the figure, } \tan \theta = \frac{1}{4}$$

$$\Rightarrow \theta = \tan^{-1} \frac{1}{4} \dots (3)$$

From (2) and (3),

$$\sin^{-1} \frac{1}{\sqrt{17}} = \tan^{-1} \frac{1}{4} \dots (3)$$

$$\text{Now, let } \cos \theta = \frac{9}{\sqrt{85}}$$

$$\text{Therefore } \theta = \cos^{-1} \frac{9}{\sqrt{85}} \dots (4)$$

$$\text{From the figure, } \tan \theta = \frac{2}{9}$$

$$\Rightarrow \theta = \tan^{-1} \frac{2}{9} \dots (5)$$

From (4) and (5),

$$\cos^{-1} \frac{9}{\sqrt{85}} = \tan^{-1} \frac{2}{9} \dots (6)$$

Substituting (3) and (6) in (1), we get

$$\text{LHS} = \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9}$$

$$= \tan^{-1} \left(\frac{\frac{1}{4} + \frac{2}{9}}{1 - \left(\frac{1}{4} \times \frac{2}{9} \right)} \right)$$

$$= \tan^{-1} \left(\frac{9+8}{36-2} \right)$$

$$= \tan^{-1} \frac{17}{34}$$

$$= \tan^{-1} \frac{1}{2}$$

= RHS

Therefore, LHS = RHS

Hence proved.

7 G. Question

Prove that:

$$2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$$

Answer

To Prove: $2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$

Formula Used:

1) $2 \sin^{-1} x = \sin^{-1}(2x \times \sqrt{1-x^2})$

2) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ where $xy < 1$

Proof:

LHS = $2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} \dots (1)$

$$2 \sin^{-1} \frac{3}{5} = \sin^{-1} \left(2 \times \frac{3}{5} \times \sqrt{1 - \left(\frac{3}{5} \right)^2} \right)$$

$$= \sin^{-1} \left(\frac{6}{5} \times \frac{4}{5} \right)$$

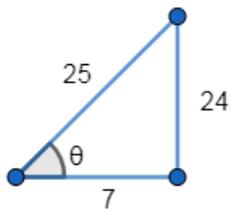
$$= \sin^{-1} \frac{24}{25} \dots (2)$$

Substituting (2) in (1), we get

LHS = $\sin^{-1} \frac{24}{25} - \tan^{-1} \frac{17}{31} \dots (3)$

Let $\sin \theta = \frac{24}{25}$

Therefore $\theta = \sin^{-1} \frac{24}{25} \dots (4)$



From the figure, $\tan \theta = \frac{24}{7}$

$$\Rightarrow \theta = \tan^{-1} \frac{24}{7} \dots (5)$$

From (4) and (5),

$$\sin^{-1} \frac{24}{25} = \tan^{-1} \frac{24}{7} \dots (6)$$

Substituting (6) in (3), we get

LHS = $\tan^{-1} \frac{24}{7} - \tan^{-1} \frac{17}{31}$

$$= \tan^{-1} \left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \left(\frac{24}{7} \times \frac{17}{31} \right)} \right)$$

$$= \tan^{-1} \left(\frac{744 - 119}{217 + 408} \right)$$

$$= \tan^{-1} \frac{625}{625}$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

$$= \text{RHS}$$

Therefore, LHS = RHS

Hence proved.

8 A. Question

Solve for x:

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$$

Answer

To find: value of x

Formula Used: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ where $xy < 1$

$$\text{Given: } \tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$$

$$\text{LHS} = \tan^{-1} \left(\frac{x+1+x-1}{1 - \{(x+1)(x-1)\}} \right)$$

$$= \tan^{-1} \frac{2x}{1 - (x^2 - x + x - 1)}$$

$$= \tan^{-1} \frac{2x}{2 - x^2}$$

$$\text{Therefore, } \tan^{-1} \frac{2x}{2-x^2} = \tan^{-1} \frac{8}{31}$$

Taking tangent on both sides, we get

$$\frac{2x}{2-x^2} = \frac{8}{31}$$

$$\Rightarrow 62x = 16 - 8x^2$$

$$\Rightarrow 8x^2 + 62x - 16 = 0$$

$$\Rightarrow 4x^2 + 31x - 8 = 0$$

$$\Rightarrow 4x^2 + 32x - x - 8 = 0$$

$$\Rightarrow 4x \times (x+8) - 1 \times (x+8) = 0$$

$$\Rightarrow (4x-1) \times (x+8) = 0$$

$$\Rightarrow x = \frac{1}{4} \text{ or } x = -8$$

Therefore, $x = \frac{1}{4}$ or $x = -8$ are the required values of x.

8 B. Question

Solve for x:

$$\tan^{-1}(2+x) + \tan^{-1}(2-x) = \tan^{-1} \frac{2}{3}$$

Answer

To find: value of x

Formula Used: $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$ where $xy < 1$

Given: $\tan^{-1}(2+x) + \tan^{-1}(2-x) = \tan^{-1} \frac{2}{3}$

$$\text{LHS} = \tan^{-1} \left(\frac{2+x+2-x}{1-((2+x) \times (2-x))} \right)$$

$$= \tan^{-1} \frac{4}{1 - (4 - 2x + 2x - x^2)}$$

$$= \tan^{-1} \frac{4}{x^2 - 3}$$

Therefore, $\tan^{-1} \frac{4}{x^2 - 3} = \tan^{-1} \frac{2}{3}$

Taking tangent on both sides, we get

$$\frac{4}{x^2 - 3} = \frac{2}{3}$$

$$\Rightarrow 12 = 2x^2 - 6$$

$$\Rightarrow 2x^2 = 18$$

$$\Rightarrow x^2 = 9$$

$$\Rightarrow x = 3 \text{ or } x = -3$$

Therefore, $x = \pm 3$ are the required values of x.

8 C. Question

Solve for x:

$$\cos(\sin^{-1} x) = \frac{1}{9}$$

Answer

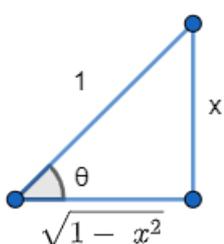
To find: value of x

Given: $\cos(\sin^{-1} x) = \frac{1}{9}$

LHS = $\cos(\sin^{-1} x) \dots (1)$

Let $\sin \theta = x$

Therefore $\theta = \sin^{-1} x \dots (2)$



From the figure, $\cos \theta = \sqrt{1-x^2}$

$$\Rightarrow \theta = \cos^{-1} \sqrt{1-x^2} \dots (3)$$

From (2) and (3),

$$\sin^{-1} x = \cos^{-1} \sqrt{1-x^2} \dots (4)$$

Substituting (4) in (1), we get

$$\text{LHS} = \cos(\cos^{-1} \sqrt{1-x^2})$$

$$= \sqrt{1-x^2}$$

$$\text{Therefore, } \sqrt{1-x^2} = \frac{1}{9}$$

Squaring and simplifying,

$$\Rightarrow 81 - 81x^2 = 1$$

$$\Rightarrow 81x^2 = 80$$

$$\Rightarrow x^2 = \frac{80}{81}$$

$$\Rightarrow x = \pm \frac{4\sqrt{5}}{9}$$

Therefore, $x = \pm \frac{4\sqrt{5}}{9}$ are the required values of x .

8 D. Question

Solve for x :

$$\cos(2\sin^{-1} x) = \frac{1}{9}$$

Answer

To find: value of x

$$\text{Formula Used: } 2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$$

$$\text{Given: } \cos(2\sin^{-1} x) = \frac{1}{9}$$

$$\text{LHS} = \cos(2\sin^{-1} x)$$

$$\text{Let } \theta = \sin^{-1} x$$

$$\text{So, } x = \sin \theta \dots (1)$$

$$\text{LHS} = \cos(2\theta)$$

$$= 1 - 2\sin^2 \theta$$

Substituting in the given equation,

$$1 - 2\sin^2 \theta = \frac{1}{9}$$

$$2\sin^2 \theta = \frac{8}{9}$$

$$\sin^2 \theta = \frac{4}{9}$$

Substituting in (1),

$$x^2 = \frac{4}{9}$$

$$x = \pm \frac{2}{3}$$

Therefore, $x = \pm \frac{2}{3}$ are the required values of x .

8 E. Question

Solve for x :

$$\sin^{-1} \frac{8}{x} + \sin^{-1} \frac{15}{x} = \frac{\pi}{2}$$

Answer

To find: value of x

$$\text{Given: } \sin^{-1} \frac{8}{x} + \sin^{-1} \frac{15}{x} = \frac{\pi}{2}$$

$$\text{We know } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\text{Let } \sin^{-1} \frac{8}{x} = P$$

$$\Rightarrow \sin P = \frac{8}{x}$$

$$\text{Therefore, } \cos P = \frac{\sqrt{x^2 - 64}}{x}$$

$$P = \cos^{-1} \frac{\sqrt{x^2 - 64}}{x}$$

$$\cos^{-1} \frac{\sqrt{x^2 - 64}}{x} + \sin^{-1} \frac{15}{x} = \frac{\pi}{2}$$

$$\text{Therefore, } \frac{\sqrt{x^2 - 64}}{x} = \frac{15}{x}$$

$$\Rightarrow \sqrt{x^2 - 64} = 15$$

Squaring both sides,

$$\Rightarrow x^2 - 64 = 225$$

$$\Rightarrow x^2 = 289$$

$$\Rightarrow x = \pm 17$$

Therefore, $x = \pm 17$ are the required values of x .

9 A. Question

Solve for x :

$$\cos(\sin^{-1} x) = \frac{1}{2}$$

Answer

To find: value of x

$$\text{Given: } \cos(\sin^{-1} x) = \frac{1}{2}$$

$$\text{LHS} = \cos(\sin^{-1} x)$$

$$= \cos(\cos^{-1}(\sqrt{1-x^2}))$$

$$= \sqrt{1-x^2}$$

$$\text{Therefore, } \sqrt{1-x^2} = \frac{1}{2}$$

Squaring both sides,

$$1-x^2 = \frac{1}{4}$$

$$x^2 = 1 - \frac{1}{4}$$

$$x^2 = \frac{3}{4}$$

$$x = \pm \frac{\sqrt{3}}{2}$$

Therefore, $x = \pm \frac{\sqrt{3}}{2}$ are the required values of x .

9 B. Question

Solve for x :

$$\tan^{-1} x = \sin^{-1} \frac{1}{\sqrt{2}}$$

Answer

To find: value of x

$$\text{Given: } \tan^{-1} x = \sin^{-1} \frac{1}{\sqrt{2}}$$

$$\text{We know that } \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\text{Therefore, } \frac{\pi}{4} = \sin^{-1} \frac{1}{\sqrt{2}}$$

Substituting in the given equation,

$$\tan^{-1} x = \frac{\pi}{4}$$

$$x = \tan \frac{\pi}{4}$$

$$\Rightarrow x = 1$$

Therefore, $x = 1$ is the required value of x .

9 C. Question

Solve for x :

$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$

Answer

$$\text{Given: } \sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$

$$\text{We know that } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\text{So, } \sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$$

Substituting in the given equation,

$$\frac{\pi}{2} - \cos^{-1}x - \cos^{-1}x = \frac{\pi}{6}$$

Rearranging,

$$2 \cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{6}$$

$$2 \cos^{-1}x = \frac{\pi}{3}$$

$$\cos^{-1}x = \frac{\pi}{6}$$

$$x = \frac{\sqrt{3}}{2}$$

Therefore, $x = \frac{\sqrt{3}}{2}$ is the required value of x .

Exercise 4D

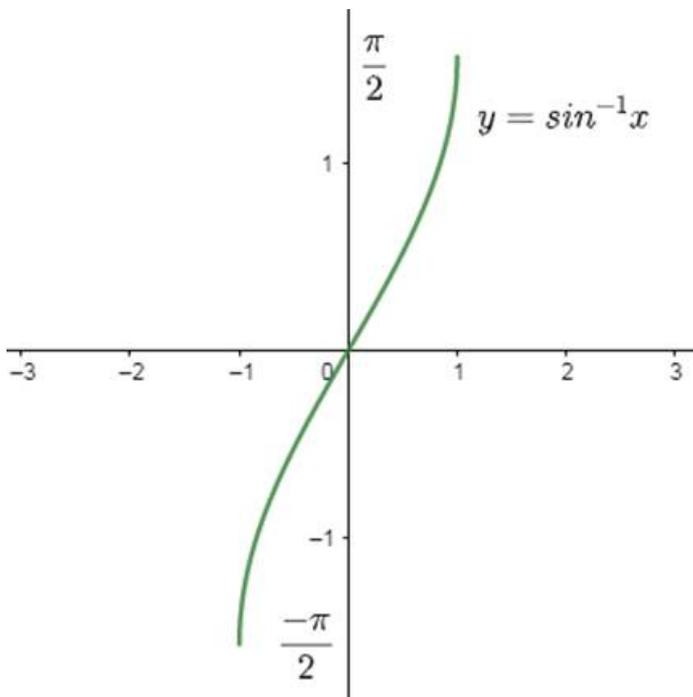
1. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$$\sin^{-1}x$$

Answer

Principal value branch of $\sin^{-1}x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$



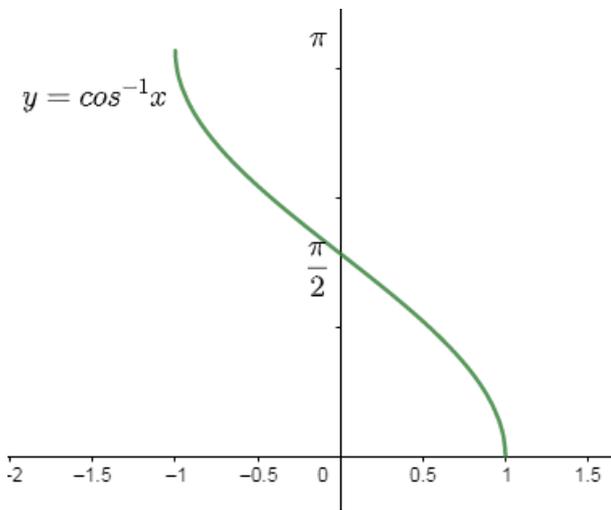
2. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$$\cos^{-1}x$$

Answer

Principal value branch of $\cos^{-1}x$ is $[0, \pi]$



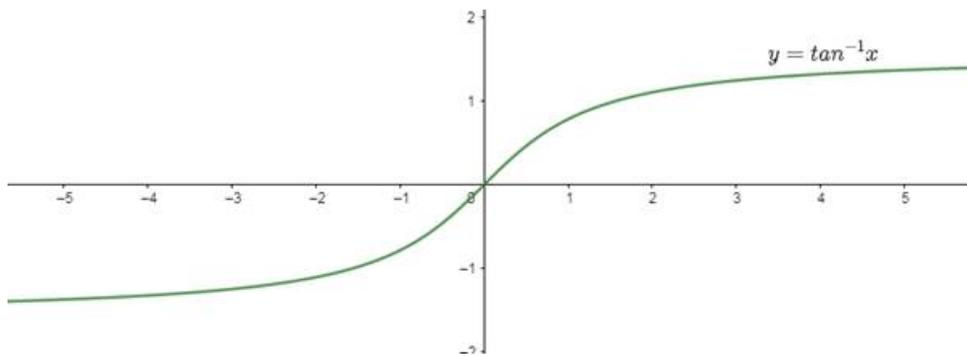
3. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\tan^{-1} x$

Answer

Principal value branch of $\tan^{-1} x$ is $[-\frac{\pi}{2}, \frac{\pi}{2}]$



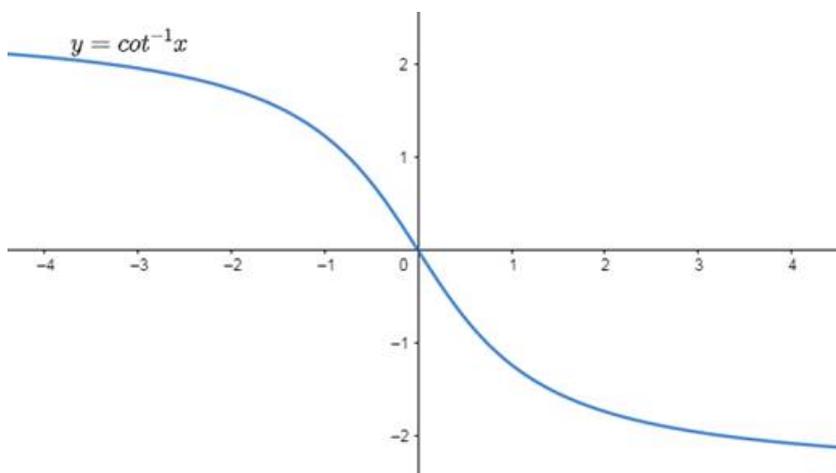
4. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\cot^{-1} x$

Answer

Principal value branch of $\cot^{-1} x$ is $(0, \pi)$



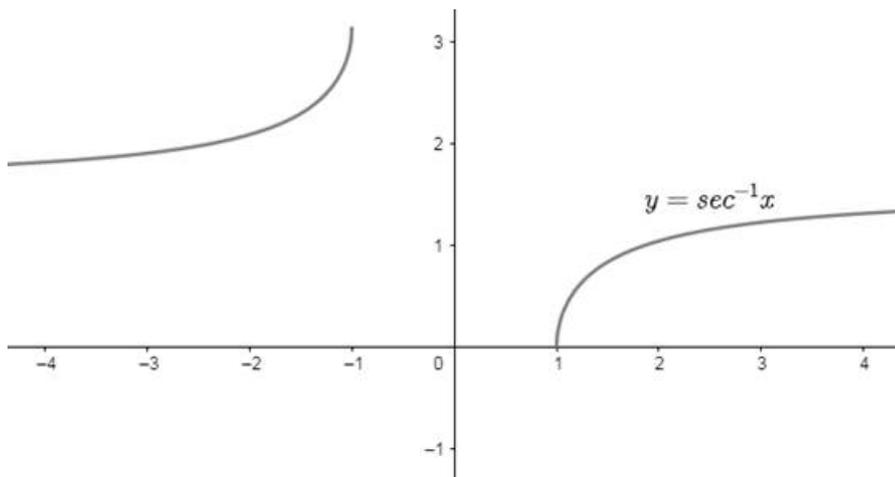
5. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\sec^{-1} x$

Answer

Principal value branch of $\sec^{-1} x$ is $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$



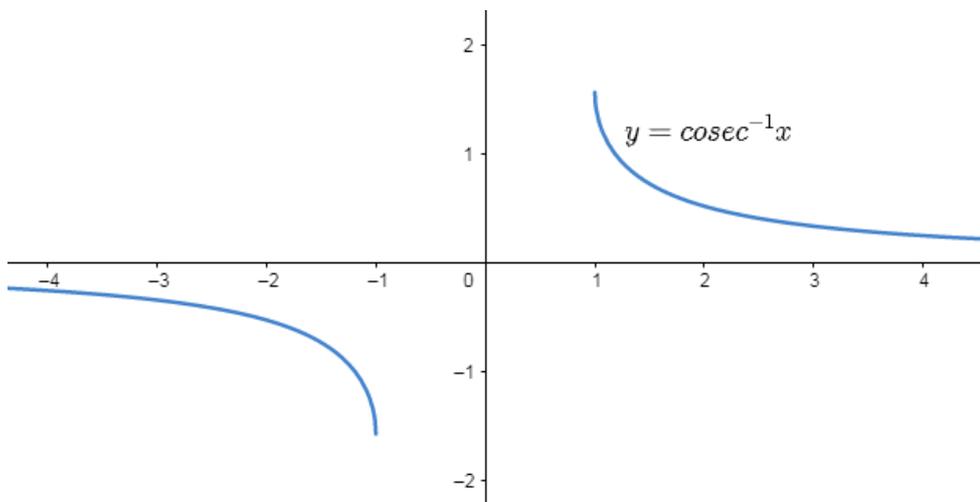
6. Question

Write down the interval for the principal-value branch of each of the following functions and draw its graph:

$\operatorname{cosec}^{-1} x$

Answer

Principal value branch of $\operatorname{cosec}^{-1} x$ is $\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$



Objective Questions

1. Question

Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is

A. $\frac{\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{7\pi}{6}$

D. none of these

Answer

To Find: The Principle value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Let the principle value be given by x

Now, let $x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\Rightarrow \cos x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos x = \cos\left(\frac{\pi}{6}\right) \left(\because \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow x = \frac{\pi}{6}$$

2. Question

Mark the tick against the correct answer in the following:

The principal value of $\operatorname{cosec}^{-1}(2)$ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. $\frac{5\pi}{6}$

Answer

To Find: The Principle value of $\operatorname{cosec}^{-1}(2)$

Let the principle value be given by x

Now, let $x = \operatorname{cosec}^{-1}(2)$

$$\Rightarrow \operatorname{cosec} x = 2$$

$$\Rightarrow \operatorname{cosec} x = \operatorname{cosec}\left(\frac{\pi}{6}\right) \left(\because \operatorname{cosec}\left(\frac{\pi}{6}\right) = 2\right)$$

$$\Rightarrow x = \frac{\pi}{6}$$

3. Question

Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$ is

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. $\frac{5\pi}{4}$

Answer

To Find: The Principle value of $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

Let the principle value be given by x

Now, let $x = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$$\Rightarrow \cos x = \frac{-1}{\sqrt{2}}$$

$$\Rightarrow \cos x = -\cos\left(\frac{\pi}{4}\right) (\because \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}})$$

$$\Rightarrow \cos x = \cos\left(\pi - \frac{\pi}{4}\right) (\because -\cos(\theta) = \cos(\pi - \theta))$$

$$\Rightarrow x = \frac{3\pi}{4}$$

4. Question

Mark the tick against the correct answer in the following:

The principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is

A. $\frac{-\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{7\pi}{6}$

D. none of these

Answer

To Find: The Principle value of $\sin^{-1}\left(\frac{-1}{2}\right)$

Let the principle value be given by x

Now, let $x = \sin^{-1}\left(\frac{-1}{2}\right)$

$$\Rightarrow \sin x = \frac{-1}{2}$$

$$\Rightarrow \sin x = -\sin\left(\frac{\pi}{6}\right) (\because \sin\left(\frac{\pi}{6}\right) = \frac{1}{2})$$

$$\Rightarrow \sin x = \sin\left(-\frac{\pi}{6}\right) (\because -\sin(\theta) = \sin(-\theta))$$

$$\Rightarrow x = -\frac{\pi}{6}$$

5. Question

Mark the tick against the correct answer in the following:

The principal value of $\cos^{-1}\left(\frac{-1}{2}\right)$ is

A. $\frac{-\pi}{3}$

B. $\frac{2\pi}{3}$

C. $\frac{4\pi}{3}$

D. $\frac{\pi}{3}$

Answer

To Find: The Principle value of $\cos^{-1}\left(\frac{-1}{2}\right)$

Let the principle value be given by x

Now, let $x = \cos^{-1}\left(\frac{-1}{2}\right)$

$$\Rightarrow \cos x = \frac{-1}{2}$$

$$\Rightarrow \cos x = -\cos\left(\frac{\pi}{3}\right) \quad (\because \cos\left(\frac{\pi}{3}\right) = \frac{1}{2})$$

$$\Rightarrow \cos x = \cos\left(\pi - \frac{\pi}{3}\right) \quad (\because -\cos(\theta) = \cos(\pi - \theta))$$

$$\Rightarrow x = \frac{2\pi}{3}$$

6. Question

Mark the tick against the correct answer in the following:

The principal value of $\tan^{-1}(-\sqrt{3})$ is

A. $\frac{2\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{-\pi}{3}$

D. none of these

Answer

To Find: The Principle value of $\tan^{-1}(-\sqrt{3})$

Let the principle value be given by x

Now, let $x = \tan^{-1}(-\sqrt{3})$

$$\Rightarrow \tan x = -\sqrt{3}$$

$$\Rightarrow \tan x = -\tan\left(\frac{\pi}{3}\right) (\because \tan\left(\frac{\pi}{3}\right) = \sqrt{3})$$

$$\Rightarrow \tan x = \tan\left(-\frac{\pi}{3}\right) (\because -\tan(\theta) = \tan(-\theta))$$

$$\Rightarrow x = -\frac{\pi}{3}$$

7. Question

Mark the tick against the correct answer in the following:

The principal value of $\cot^{-1}(-1)$ is

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{5\pi}{4}$

D. $\frac{3\pi}{4}$

Answer

To Find: The Principle value of $\cot^{-1}(-1)$

Let the principle value be given by x

Now, let $x = \cot^{-1}(-1)$

$$\Rightarrow \cot x = -1$$

$$\Rightarrow \cot x = -\cot\left(\frac{\pi}{4}\right) (\because \cot\left(\frac{\pi}{4}\right) = 1)$$

$$\Rightarrow \cot x = \cot\left(\pi - \frac{\pi}{4}\right) (\because -\cot(\theta) = \cot(\pi - \theta))$$

$$\Rightarrow x = \frac{3\pi}{4}$$

8. Question

Mark the tick against the correct answer in the following:

The principal value of $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is

A. $\frac{\pi}{6}$

B. $\frac{-\pi}{6}$

C. $\frac{5\pi}{6}$

D. $\frac{7\pi}{6}$

Answer

To Find: The Principle value of $\sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

Let the principle value be given by x

Now, let $x = \sec^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

$\Rightarrow \sec x = \frac{-2}{\sqrt{3}}$

$\Rightarrow \sec x = -\sec\left(\frac{\pi}{6}\right) (\because \sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}})$

$\Rightarrow \sec x = \sec\left(\pi - \frac{\pi}{6}\right) (\because -\sec(\theta) = \sec(\pi - \theta))$

$\Rightarrow x = \frac{5\pi}{6}$

9. Question

Mark the tick against the correct answer in the following:

The principal value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ is

A. $\frac{-\pi}{4}$

B. $\frac{3\pi}{4}$

C. $\frac{5\pi}{4}$

D. none of these

Answer

To Find: The Principle value of $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

Let the principle value be given by x

Now, let $x = \operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$

$\Rightarrow \operatorname{cosec} x = -\sqrt{2}$

$\Rightarrow \operatorname{cosec} x = -\operatorname{cosec}\left(\frac{\pi}{4}\right) (\because \operatorname{cosec}\left(\frac{\pi}{4}\right) = \sqrt{2})$

$\Rightarrow \operatorname{cosec} x = \operatorname{cosec}\left(-\frac{\pi}{4}\right) (\because -\operatorname{cosec}(\theta) = \operatorname{cosec}(-\theta))$

$\Rightarrow x = -\frac{\pi}{4}$

10. Question

Mark the tick against the correct answer in the following:

The principal value of $\cot^{-1}\left(-\sqrt{3}\right)$ is

A. $\frac{2\pi}{6}$

B. $\frac{\pi}{6}$

C. $\frac{7\pi}{6}$

D. $\frac{5\pi}{6}$

Answer

To Find: The Principle value of $\cot^{-1}(-\sqrt{3})$

Let the principle value be given by x

$$\text{Now, let } x = \cot^{-1}(-\sqrt{3})$$

$$\Rightarrow \cot x = -\sqrt{3}$$

$$\Rightarrow \cot x = -\cot\left(\frac{\pi}{6}\right) \quad (\because \cot\left(\frac{\pi}{6}\right) = \sqrt{3})$$

$$\Rightarrow \cot x = \cot\left(\pi - \frac{\pi}{6}\right) \quad (\because -\cot(\theta) = \cot(\pi - \theta))$$

$$\Rightarrow x = \frac{5\pi}{6}$$

11. Question

Mark the tick against the correct answer in the following:

The value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

A. $\frac{2\pi}{3}$

B. $\frac{5\pi}{3}$

C. $\frac{\pi}{3}$

D. none of these

Answer

To Find: The value of $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

$$\text{Now, let } x = \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$$

$$\Rightarrow \sin x = \sin\left(\frac{2\pi}{3}\right)$$

Here range of principle value of sine is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow x = \frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Hence for all values of x in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the value of

$$\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) \text{ is}$$

$$\Rightarrow \sin x = \sin\left(\pi - \frac{\pi}{3}\right) \left(\because \sin\left(\frac{2\pi}{3}\right) = \sin\left(\pi - \frac{\pi}{3}\right)\right)$$

$$\Rightarrow \sin x = \sin\left(\frac{\pi}{3}\right) \left(\because \sin(\pi - \theta) = \sin \theta \text{ as here } \theta \text{ lies in II quadrant and sine is positive}\right)$$

$$\Rightarrow x = \frac{\pi}{3}$$

12. Question

Mark the tick against the correct answer in the following:

The value of $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$ is

A. $\frac{13\pi}{6}$

B.

C. $\frac{5\pi}{6}$

D. $\frac{\pi}{6}$ $\frac{7\pi}{6}$

Answer

To Find: The value of $\cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right)$

Now, let $x = \cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right)$

$$\Rightarrow \cos x = \cos\left(\frac{13\pi}{6}\right)$$

Here, range of principle value of \cos is $[0, \pi]$

$$\Rightarrow x = \frac{13\pi}{6} \notin [0, \pi]$$

Hence for all values of x in range $[0, \pi]$, the value of

$$\cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right) \text{ is}$$

$$\Rightarrow \cos x = \cos\left(2\pi - \frac{\pi}{6}\right) \left(\because \cos\left(\frac{13\pi}{6}\right) = \cos\left(2\pi - \frac{\pi}{6}\right)\right)$$

$$\Rightarrow \cos x = \cos\left(\frac{\pi}{6}\right) \left(\because \cos(2\pi - \theta) = \cos \theta\right)$$

$$\Rightarrow x = \frac{\pi}{6}$$

13. Question

Mark the tick against the correct answer in the following:

The value of $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ is

A. $\frac{7\pi}{6}$

B. $\frac{5\pi}{6}$

C. $\frac{\pi}{6}$

D. none of these

AnswerTo Find: The value of $\tan^{-1}(\tan(\frac{7\pi}{6}))$ Now, let $x = \tan^{-1}(\tan(\frac{7\pi}{6}))$

$$\Rightarrow \tan x = \tan(\frac{7\pi}{6})$$

Here range of principle value of tan is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\Rightarrow x = \frac{7\pi}{6} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$$

Hence for all values of x in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the value of $\tan^{-1}(\tan(\frac{13\pi}{6}))$ is

$$\Rightarrow \tan x = \tan(\pi + \frac{\pi}{6}) (\because \tan(\frac{7\pi}{6}) = \tan(\pi + \frac{\pi}{6}))$$

$$\Rightarrow \tan x = \tan(\frac{\pi}{6}) (\because \tan(\pi + \theta) = \tan \theta)$$

$$\Rightarrow x = \frac{\pi}{6}$$

14. Question

Mark the tick against the correct answer in the following:

The value of $\cot^{-1}(\cot \frac{5\pi}{4})$ is

A. $\frac{\pi}{4}$

B. $\frac{-\pi}{4}$

C. $\frac{3\pi}{4}$

D. none of these

AnswerTo Find: The value of $\cot^{-1}(\cot(\frac{5\pi}{4}))$ Now, let $x = \cot^{-1}(\cot(\frac{5\pi}{4}))$

$$\Rightarrow \cot x = \cot\left(\frac{5\pi}{4}\right)$$

Here range of principle value of cot is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\Rightarrow x = \frac{5\pi}{4} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$$

Hence for all values of x in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the value of

$\cot^{-1}(\cot(\frac{5\pi}{4}))$ is

$$\Rightarrow \cot x = \cot\left(\pi + \frac{\pi}{4}\right) (\because \cot\left(\frac{5\pi}{4}\right) = \cot\left(\pi + \frac{\pi}{4}\right))$$

$$\Rightarrow \cot x = \cot\left(\frac{\pi}{4}\right) (\because \cot(\pi + \theta) = \cot \theta)$$

$$\Rightarrow x = \frac{\pi}{4}$$

15. Question

Mark the tick against the correct answer in the following:

The value of $\sec^{-1}\left(\sec\frac{8\pi}{5}\right)$ is

A. $\frac{2\pi}{5}$

B. $\frac{3\pi}{5}$

C. $\frac{8\pi}{5}$

D. none of these

Answer

To Find: The value of $\sec^{-1}\left(\sec\left(\frac{8\pi}{5}\right)\right)$

Now, let $x = \sec^{-1}\left(\sec\left(\frac{8\pi}{5}\right)\right)$

$$\Rightarrow \sec x = \sec\left(\frac{8\pi}{5}\right)$$

Here range of principle value of sec is $[0, \pi]$

$$\Rightarrow x = \frac{8\pi}{5} \notin [0, \pi]$$

Hence for all values of x in range $[0, \pi]$, the value of

$\sec^{-1}\left(\sec\left(\frac{8\pi}{5}\right)\right)$ is

$$\Rightarrow \sec x = \sec\left(2\pi - \frac{2\pi}{5}\right) (\because \sec\left(\frac{8\pi}{5}\right) = \sec\left(2\pi - \frac{2\pi}{5}\right))$$

$$\Rightarrow \sec x = \sec\left(\frac{2\pi}{5}\right) (\because \sec(2\pi - \theta) = \sec \theta)$$

$$\Rightarrow x = \frac{2\pi}{5}$$

16. Question

Mark the tick against the correct answer in the following:

The value of $\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{4\pi}{3}\right)$ is

A. $\frac{\pi}{3}$

B. $-\frac{\pi}{3}$

C. $\frac{2\pi}{3}$

D. none of these

Answer

To Find: The value of $\operatorname{cosec}^{-1}\left(\operatorname{cosec}\left(\frac{4\pi}{3}\right)\right)$

Now, let $x = \operatorname{cosec}^{-1}\left(\operatorname{cosec}\left(\frac{4\pi}{3}\right)\right)$

$$\Rightarrow \operatorname{cosec} x = \operatorname{cosec}\left(\frac{4\pi}{3}\right)$$

Here range of principle value of cosec is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\Rightarrow x = \frac{4\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Hence for all values of x in range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the value of

$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\left(\frac{4\pi}{3}\right)\right)$ is

$$\Rightarrow \operatorname{cosec} x = \operatorname{cosec}\left(\pi + \frac{\pi}{3}\right) \quad (\because \operatorname{cosec}\left(\frac{4\pi}{3}\right) = \operatorname{cosec}\left(\pi + \frac{\pi}{3}\right))$$

$$\Rightarrow \operatorname{cosec} x = \operatorname{cosec}\left(-\frac{\pi}{3}\right) \quad (\because \operatorname{cosec}(\pi + \theta) = \operatorname{cosec}(-\theta))$$

$$\Rightarrow x = -\frac{\pi}{3}$$

17. Question

Mark the tick against the correct answer in the following:

The value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is

A. $\frac{3\pi}{4}$

B. $\frac{\pi}{4}$

C. $-\frac{\pi}{4}$

D. none of these

Answer

To Find: The value of $\tan^{-1}(\tan(\frac{3\pi}{4}))$

Now, let $x = \tan^{-1}(\tan(\frac{3\pi}{4}))$

$$\Rightarrow \tan x = \tan(\frac{3\pi}{4})$$

Here range of principle value of tan is $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\Rightarrow x = \frac{3\pi}{4} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$$

Hence for all values of x in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$, the value of

$\tan^{-1}(\tan(\frac{3\pi}{4}))$ is

$$\Rightarrow \tan x = \tan(\pi - \frac{\pi}{4}) (\because \tan(\frac{3\pi}{4}) = \tan(\pi - \frac{\pi}{4}))$$

$$\Rightarrow \tan x = \tan(-\frac{\pi}{4}) (\because \tan(\pi - \theta) = \tan(-\theta))$$

$$\Rightarrow x = -\frac{\pi}{4}$$

18. Question

Mark the tick against the correct answer in the following:

$$\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right) = ?$$

A. 0

B. $\frac{2\pi}{3}$

C. $\frac{\pi}{2}$

D. π

Answer

To Find: The value of $\frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})$

Now, let $x = \frac{\pi}{3} - \sin^{-1}(\frac{-1}{2})$

$$\Rightarrow x = \frac{\pi}{3} - (-\sin^{-1}(\frac{1}{2})) (\because \sin(-\theta) = -\sin(\theta))$$

$$\Rightarrow x = \frac{\pi}{3} - (-\frac{\pi}{6}) (\because \sin \frac{\pi}{6} = \frac{1}{2})$$

$$\Rightarrow x = \frac{\pi}{3} + \frac{\pi}{6}$$

$$\Rightarrow x = \frac{3\pi}{6} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

19. Question

Mark the tick against the correct answer in the following:

The value of $\sin\left(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2}\right) = ?$

- A. 0
- B. 1
- C. -1
- D. none of these

Answer

To Find: The value of $\sin(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2})$

Now, let $x = \sin(\sin^{-1}\frac{1}{2} + \cos^{-1}\frac{1}{2})$

$$\Rightarrow x = \sin\left(\frac{\pi}{2}\right) (\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2})$$

$$\Rightarrow x = 1 (\because \sin\left(\frac{\pi}{2}\right) = 1)$$

20. Question

Mark the tick against the correct answer in the following:

If $x \neq 0$ then $\cos(\tan^{-1}x + \cot^{-1}x) = ?$

- A. -1
- B. 1
- C. 0
- D. none of these

Answer

Given: $x \neq 0$

To Find: The value of $\cos(\tan^{-1}x + \cot^{-1}x)$

Now, let $x = \cos(\tan^{-1}x + \cot^{-1}x)$

$$\Rightarrow x = \cos\left(\frac{\pi}{2}\right) (\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2})$$

$$\Rightarrow x = 0 (\because \cos\left(\frac{\pi}{2}\right) = 0)$$

21. Question

Mark the tick against the correct answer in the following:

The value of $\sin\left(\cos^{-1}\frac{3}{5}\right)$ is

- A. $\frac{2}{5}$
- B. $\frac{4}{5}$
- C. $\frac{-2}{5}$

D. none of these

Answer

To Find: The value of $\sin(\cos^{-1}\frac{3}{5})$

Now, let $x = \cos^{-1}\frac{3}{5}$

$$\Rightarrow \cos x = \frac{3}{5}$$

Now, $\sin x = \sqrt{1 - \cos^2 x}$

$$= \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$= \frac{4}{5}$$

$$\Rightarrow x = \sin^{-1}\frac{4}{5} = \cos^{-1}\frac{3}{5}$$

Therefore,

$$\sin(\cos^{-1}\frac{3}{5}) = \sin(\sin^{-1}\frac{4}{5})$$

Let, $Y = \sin(\sin^{-1}\frac{4}{5})$

$$\Rightarrow \sin^{-1} Y = \sin^{-1}\frac{4}{5}$$

$$\Rightarrow Y = \frac{4}{5}$$

22. Question

Mark the tick against the correct answer in the following:

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = ?$$

A. $\frac{4\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{5\pi}{3}$

D. π

Answer

To Find: The value of $\cos^{-1}(\cos(\frac{2\pi}{3})) + \sin^{-1}(\sin(\frac{2\pi}{3}))$

Here, consider $\cos^{-1}(\cos(\frac{2\pi}{3}))$ (\because the principle value of cos lies in the range $[0, \pi]$ and since $\frac{2\pi}{3} \in [0, \pi]$)

$$\Rightarrow \cos^{-1}(\cos(\frac{2\pi}{3})) = \frac{2\pi}{3}$$

Now, consider $\sin^{-1}(\sin(\frac{2\pi}{3}))$

Since here the principle value of sine lies in range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and since $\frac{2\pi}{3} \notin [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$\Rightarrow \sin^{-1}(\sin(\frac{2\pi}{3})) = \sin^{-1}(\sin(\pi - \frac{\pi}{3}))$$

$$= \sin^{-1}(\sin(\frac{\pi}{3}))$$

$$= \frac{\pi}{3}$$

Therefore,

$$\cos^{-1}(\cos(\frac{2\pi}{3})) + \sin^{-1}(\sin(\frac{2\pi}{3})) = \frac{2\pi}{3} + \frac{\pi}{3}$$

$$= \frac{3\pi}{3}$$

$$= \pi$$

23. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2) = ?$$

A. $\frac{\pi}{3}$

B. $\frac{-\pi}{3}$

C. $\frac{5\pi}{3}$

D. none of these

Answer

To Find: The value of $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

$$\text{Let } x = \tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$$

$$\Rightarrow x = \frac{\pi}{3} - [\pi - \sec^{-1}(2)] \quad (\because \tan(\frac{\pi}{3}) = \sqrt{3} \text{ and } \sec^{-1}(-\theta) = \pi - \sec^{-1}(\theta))$$

$$\Rightarrow x = \frac{\pi}{3} - [\pi - \frac{\pi}{3}]$$

$$\Rightarrow x = \frac{\pi}{3} - [\frac{2\pi}{3}]$$

$$\Rightarrow x = -\frac{\pi}{3}$$

24. Question

Mark the tick against the correct answer in the following:

$$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} = ?$$

A. $\frac{2\pi}{3}$

B. $\frac{3\pi}{2}$

C. 2π

D. none of these

Answer

To Find: The value of $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$

Now, let $x = \cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$

$\Rightarrow x = \frac{\pi}{3} + 2\left(\frac{\pi}{6}\right)$ ($\because \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$)

$\Rightarrow x = \frac{\pi}{3} + \frac{\pi}{3}$

$\Rightarrow x = \frac{2\pi}{3}$

25. Question

Mark the tick against the correct answer in the following:

$\tan^{-1} 1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right) = ?$

A. π

B. $\frac{2\pi}{3}$

C. $\frac{3\pi}{4}$

D. $\frac{\pi}{2}$

Answer

To Find: The value of $\tan^{-1} 1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

Now, let $x = \tan^{-1} 1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

$\Rightarrow x = \frac{\pi}{4} + [\pi - \cos^{-1}\left(\frac{1}{2}\right)] + [-\sin^{-1}\left(\frac{1}{2}\right)]$ ($\because \tan\left(\frac{\pi}{4}\right) = 1$ and $\cos^{-1}(-\theta) = [\pi - \cos^{-1}\theta]$ and $\sin^{-1}(-\theta) = -\sin^{-1}\theta$)

$\Rightarrow x = \frac{\pi}{4} + [\pi - \frac{\pi}{3}] + [-\frac{\pi}{6}]$

$\Rightarrow x = \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$\Rightarrow x = \frac{3\pi}{4}$

26. Question

Mark the tick against the correct answer in the following:

$\tan\left[2\tan^{-1}\frac{1}{5} - \frac{\pi}{4}\right] = ?$

A. $\frac{7}{17}$

B. $\frac{-7}{17}$

C. $\frac{7}{12}$

D. $\frac{-7}{12}$

Answer

To Find: The value of $\tan(2 \tan^{-1} \frac{1}{5} - \frac{\pi}{4})$

$$\text{Consider, } \tan(2 \tan^{-1} \frac{1}{5} - \frac{\pi}{4}) = \tan(\tan^{-1}(\frac{2(\frac{1}{5})}{1-(\frac{1}{5})^2}) - \frac{\pi}{4})$$

$$(\because 2 \tan^{-1} x = \tan^{-1}(\frac{2x}{1-x^2}))$$

$$= \tan(\tan^{-1}(\frac{\frac{2}{5}}{1-\frac{1}{25}}) - \frac{\pi}{4})$$

$$= \tan(\tan^{-1}(\frac{5}{12}) - \frac{\pi}{4})$$

$$= \tan(\tan^{-1}(\frac{5}{12}) - \tan^{-1}(1)) \quad (\because \tan(\frac{\pi}{4})=1)$$

$$= \tan(\tan^{-1}(\frac{\frac{5}{12}-1}{1+\frac{5}{12}}))$$

$$(\tan^{-1} x - \tan^{-1} y = \tan^{-1}(\frac{x-y}{1+xy}))$$

$$= \tan(\tan^{-1}(\frac{-7}{17}))$$

$$\tan(2 \tan^{-1} \frac{1}{5} - \frac{\pi}{4}) = \frac{-7}{17}$$

27. Question

Mark the tick against the correct answer in the following:

$$\tan \frac{1}{2} \left(\cos^{-1} \frac{\sqrt{5}}{3} \right) = ?$$

A. $\frac{(3-\sqrt{5})}{2}$

B. $\frac{(3+\sqrt{5})}{2}$

C. $\frac{(5-\sqrt{3})}{2}$

$$D. \frac{(5 + \sqrt{3})}{2}$$

Answer

To Find: The value of $\tan \frac{1}{2}(\cos^{-1} \frac{\sqrt{5}}{3})$

$$\text{Let , } x = \cos^{-1} \frac{\sqrt{5}}{3}$$

$$\Rightarrow \cos x = \frac{\sqrt{5}}{3}$$

Now, $\tan \frac{1}{2}(\cos^{-1} \frac{\sqrt{5}}{3})$ becomes

$$\tan \frac{1}{2}(\cos^{-1} \frac{\sqrt{5}}{3}) = \tan \frac{1}{2}(x) = \tan \frac{x}{2}$$

$$= \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$= \sqrt{\frac{1 - (\frac{\sqrt{5}}{3})}{1 + \frac{\sqrt{5}}{3}}}$$

$$= \sqrt{\frac{3 - \sqrt{5}}{3 + \sqrt{5}}}$$

$$= \sqrt{\frac{3 - \sqrt{5}}{3 + \sqrt{5}}} \times \sqrt{\frac{3 - \sqrt{5}}{3 - \sqrt{5}}}$$

$$\tan \frac{1}{2}(\cos^{-1} \frac{\sqrt{5}}{3}) = \frac{3 - \sqrt{5}}{2}$$

28. Question

Mark the tick against the correct answer in the following:

$$\sin \left(\cos^{-1} \frac{3}{5} \right) = ?$$

A. $\frac{3}{4}$

B. $\frac{4}{5}$

C. $\frac{3}{5}$

D. none of these

Answer

To Find: The value of $\sin(\cos^{-1} \frac{3}{5})$

$$\text{Let, } x = \cos^{-1} \frac{3}{5}$$

$$\Rightarrow \cos x = \frac{3}{5}$$

Now , $\sin(\cos^{-1}\frac{3}{5})$ becomes $\sin(x)$

Since we know that $\sin x = \sqrt{1 - \cos^2 x}$

$$= \sqrt{1 - (\frac{3}{5})^2}$$

$$\sin(\cos^{-1}\frac{3}{5}) = \sin x = \frac{4}{5}$$

29. Question

Mark the tick against the correct answer in the following:

$$\cos\left(\tan^{-1}\frac{3}{4}\right) = ?$$

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{4}{9}$

D. none of these

Answer

To Find: The value of $\cos(\tan^{-1}\frac{3}{4})$

$$\text{Let } x = \tan^{-1}\frac{3}{4}$$

$$\Rightarrow \tan x = \frac{3}{4}$$

$$\Rightarrow \tan x = \frac{3}{4} = \frac{\text{opposite side}}{\text{adjacent side}}$$

We know that by pythagorus theorem ,

$$(\text{Hypotenuse})^2 = (\text{opposite side})^2 + (\text{adjacent side})^2$$

Therefore, Hypotenuse = 5

$$\Rightarrow \cos x = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{4}{5}$$

Since here $x = \tan^{-1}\frac{3}{4}$ hence $\cos(\tan^{-1}\frac{3}{4})$ becomes $\cos x$

$$\text{Hence , } \cos(\tan^{-1}\frac{3}{4}) = \cos x = \frac{4}{5}$$

30. Question

Mark the tick against the correct answer in the following:

$$\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\} = ?$$

A. 1

B. 0

C. $\frac{-1}{2}$

D. none of these

Answer

To Find: The value of $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$

Let, $x = \sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$

$\Rightarrow x = \sin\left\{\frac{\pi}{3} - \left(-\sin^{-1}\frac{1}{2}\right)\right\}$ ($\because \sin^{-1}(-\theta) = -\sin\theta$)

$\Rightarrow x = \sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$

$\Rightarrow x = \sin\left(\frac{3\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

31. Question

Mark the tick against the correct answer in the following:

$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = ?$

A. $\frac{1}{\sqrt{5}}$

B. $\frac{2}{\sqrt{5}}$

C. $\frac{1}{\sqrt{10}}$

D. $\frac{2}{\sqrt{10}}$

Answer

To Find: The value of $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$

Let $x = \cos^{-1}\frac{4}{5}$

$\Rightarrow \cos x = \frac{4}{5}$

Therefore $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$ becomes $\sin\left(\frac{x}{2}\right)$, i.e. $\sin\left(\frac{x}{2}\right)$

We know that $\sin\left(\frac{x}{2}\right) = \sqrt{\frac{1-\cos x}{2}}$

$= \sqrt{\frac{1-\frac{4}{5}}{2}}$

$= \sqrt{\frac{\frac{1}{5}}{2}}$

$\sin\left(\frac{x}{2}\right) = \frac{1}{\sqrt{10}}$

32. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1}\left\{2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right\}=?$$

- A. $\frac{\pi}{3}$
- B. $\frac{\pi}{4}$
- C. $\frac{3\pi}{4}$
- D. $\frac{2\pi}{3}$

Answer

To Find: The value of $\tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$

$$\text{Let , } x = \tan^{-1}\{2\cos(2\sin^{-1}\frac{1}{2})\}$$

$$\Rightarrow x = \tan^{-1}\{2\cos(2(\frac{\pi}{6}))\} (\because \sin(\frac{\pi}{6}) = \frac{1}{2})$$

$$\Rightarrow x = \tan^{-1}(2\cos\frac{\pi}{3})$$

$$\Rightarrow x = \tan^{-1}(2(\frac{1}{2})) = \tan^{-1} 1 = \frac{\pi}{4} (\because \cos(\frac{\pi}{3}) = \frac{1}{2} \text{ and } \tan(\frac{\pi}{4}) = 1)$$

33. Question

Mark the tick against the correct answer in the following:

$$\text{If } \cot^{-1}\left(\frac{-1}{5}\right) = x \text{ then } \sin x = ?$$

- A. $\frac{1}{\sqrt{26}}$
- B. $\frac{5}{\sqrt{26}}$
- C. $\frac{1}{\sqrt{24}}$
- D. none of these

Answer

$$\text{Given: } \cot^{-1}\frac{-1}{5} = x$$

To Find: The value of $\sin x$

$$\text{Since , } x = \cot^{-1}\frac{-1}{5}$$

$$\Rightarrow \cot x = \frac{-1}{5} = \frac{\text{adjacent side}}{\text{opposite side}}$$

By pythagorus theroem ,

$$(\text{Hypotenuse})^2 = (\text{opposite side})^2 + (\text{adjacent side})^2$$

$$\text{Therefore, Hypotenuse} = \sqrt{26}$$

$$\Rightarrow \sin x = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{5}{\sqrt{26}}$$

34. Question

Mark the tick against the correct answer in the following:

$$\sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = ?$$

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. none of these

Answer

To Find: The value of $\sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$$\text{Let , } x = \sin^{-1}\left(\frac{-1}{2}\right) + 2\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$$

$$\Rightarrow x = -\sin^{-1}\left(\frac{1}{2}\right) + 2[\pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)] \quad (\because \sin^{-1}(-\theta) = -\sin^{-1}(\theta) \text{ and } \cos^{-1}(-\theta) = \pi - \cos^{-1}(\theta))$$

$$\Rightarrow x = -\left(\frac{\pi}{6}\right) + 2\left[\pi - \frac{\pi}{6}\right]$$

$$\Rightarrow x = -\left(\frac{\pi}{6}\right) + 2\left[\frac{5\pi}{6}\right]$$

$$\Rightarrow x = -\frac{\pi}{6} + \frac{5\pi}{3}$$

$$\Rightarrow x = \frac{3\pi}{2}$$

Tag:

35. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = ?$$

A. $\frac{\pi}{2}$

B. π

C. $\frac{3\pi}{2}$

D. $\frac{2\pi}{3}$

Answer

To Find: The value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

Let , $x = \tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$\Rightarrow x = -\tan^{-1}(1) + (\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right))$

$(\because \tan^{-1}(-\theta) = -\tan^{-1}(\theta) \text{ and } \cos^{-1}(-\theta) = \pi - \cos^{-1}(\theta))$

$\Rightarrow x = -\frac{\pi}{4} + (\pi - \frac{\pi}{4})$

$\Rightarrow x = -\frac{\pi}{4} + \frac{3\pi}{4}$

$\Rightarrow x = \frac{\pi}{2}$

36. Question

Mark the tick against the correct answer in the following:

$\cot(\tan^{-1}x + \cot^{-1}x) = ?$

- A. 1
- B. $\frac{1}{2}$
- C. 0
- D. none of these

Answer

To Find: The value of $\cot(\tan^{-1}x + \cot^{-1}x)$

Let , $x = \cot(\tan^{-1}x + \cot^{-1}x)$

$\Rightarrow x = \cot\left(\frac{\pi}{2}\right) (\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2})$

$\Rightarrow x = 0$

37. Question

Mark the tick against the correct answer in the following:

$\tan^{-1}1 + \tan^{-1}\frac{1}{3} = ?$

- A. $\tan^{-1}\frac{4}{3}$
- B. $\tan^{-1}\frac{2}{3}$
- C. $\tan^{-1}2$
- D. $\tan^{-1}3$

Answer

To Find: The value of $\tan^{-1} 1 + \tan^{-1} \frac{1}{3}$

Let , $x = \tan^{-1} 1 + \tan^{-1} \frac{1}{3}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

$$\Rightarrow \tan^{-1} 1 + \tan^{-1} \frac{1}{3} = \tan^{-1} \left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}} \right) = \tan^{-1} 2$$

38. Question

Mark the tick against the correct answer in the following:

$$\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = ?$$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. $\frac{2\pi}{3}$

Answer

To Find: The value of $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3}$

Let , $x = \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3}$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

$$\Rightarrow \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2} \times \frac{1}{3} \right)} \right) = \tan^{-1} 1 = \frac{\pi}{4}$$

39. Question

Mark the tick against the correct answer in the following:

$$2 \tan^{-1} \frac{1}{3} = ?$$

A. $\tan^{-1} \frac{3}{2}$

B. $\tan^{-1} \frac{3}{4}$

C. $\tan^{-1} \frac{4}{3}$

D. none of these

Answer

To Find: The value of $2 \tan^{-1} \frac{1}{3}$ i.e, $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3}$

$$\text{Let , } x = \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{3}$$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

$$\Rightarrow \tan^{-1} 1 + \tan^{-1} \frac{1}{3} = \tan^{-1} \left(\frac{\frac{1}{3} + \frac{1}{3}}{1 - \left(\frac{1}{3} \times \frac{1}{3} \right)} \right) = \tan^{-1} \frac{3}{4}$$

40. Question

Mark the tick against the correct answer in the following:

$$\cos \left(2 \tan^{-1} \frac{1}{2} \right) = ?$$

A. $\frac{3}{5}$

B. $\frac{4}{5}$

C. $\frac{7}{8}$

D. none of these

Answer

To Find: The value of $\cos \left(2 \tan^{-1} \frac{1}{2} \right)$

$$\text{Let , } x = \cos \left(2 \tan^{-1} \frac{1}{2} \right)$$

$$\Rightarrow x = \cos \left(\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{2} \right)$$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

$$\Rightarrow \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{2} = \tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{2}}{1 - \left(\frac{1}{2} \times \frac{1}{2} \right)} \right) = \tan^{-1} \frac{4}{3}$$

$$\Rightarrow x = \cos \left(\tan^{-1} \frac{4}{3} \right)$$

$$\text{Now , let } y = \tan^{-1} \frac{4}{3}$$

$$\Rightarrow \tan y = \frac{4}{3} = \frac{\text{opposite side}}{\text{adjacent side}}$$

By pythagorus theroem ,

$$(\text{Hypotenuse})^2 = (\text{opposite side})^2 + (\text{adjacent side})^2$$

Therefore, Hypotenuse = 5

$$\Rightarrow \cos \left(\tan^{-1} \frac{4}{3} \right) = \cos y = \frac{3}{5}$$

41. Question

Mark the tick against the correct answer in the following:

$$\sin \left[2 \tan^{-1} \frac{5}{8} \right]$$

A. $\frac{25}{64}$

B. $\frac{80}{89}$

C. $\frac{75}{128}$

D. none of these

Answer

To Find: The value of $\sin (2 \tan^{-1} \frac{5}{8})$

Let , $x = \sin(2 \tan^{-1} \frac{5}{8})$

We know that $2 \tan^{-1} x = \sin^{-1}(\frac{2x}{1+x^2})$

$$\Rightarrow x = \sin(\sin^{-1}(\frac{2(\frac{5}{8})}{1+(\frac{5}{8})^2})) = \sin(\sin^{-1}(\frac{80}{89})) = \frac{80}{89}$$

42. Question

Mark the tick against the correct answer in the following:

$$\sin \left[2 \sin^{-1} \frac{4}{5} \right]$$

A. $\frac{12}{25}$

B. $\frac{16}{25}$

C. $\frac{24}{25}$

D. None of these

Answer

To Find: The value of $\sin (2 \sin^{-1} \frac{4}{5})$

Let , $x = \sin^{-1} \frac{4}{5}$

$$\Rightarrow \sin x = \frac{4}{5}$$

We know that , $\cos x = \sqrt{1 - \sin^2 x}$

$$= \sqrt{1 - (\frac{4}{5})^2}$$

$$= \frac{3}{5}$$

Now since, $x = \sin^{-1} \frac{4}{5}$, hence $\sin (2 \sin^{-1} \frac{4}{5})$ becomes $\sin(2x)$

Here, $\sin(2x) = 2 \sin x \cos x$

$$= 2 \times \frac{4}{5} \times \frac{3}{5}$$

$$= \frac{24}{25}$$

43. Question

Mark the tick against the correct answer in the following:

If $\tan^{-1} x = \frac{\pi}{4} - \tan^{-1} \frac{1}{3}$ then $x = ?$

A. $\frac{1}{2}$

B. $\frac{1}{4}$

C. $\frac{1}{6}$

D. None of these

Answer

To Find: The value of $\tan^{-1} x = \frac{\pi}{4} - \tan^{-1} \frac{1}{3}$

Now, $\tan^{-1} x = \tan^{-1} 1 - \tan^{-1} \frac{1}{3}$ ($\because \tan \frac{\pi}{4} = 1$)

Since we know that $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right)$

$$\Rightarrow \tan^{-1} 1 + \tan^{-1} \frac{1}{3} = \tan^{-1} \left(\frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} \right) = \tan^{-1} \frac{1}{2}$$

$$\Rightarrow \tan^{-1} x = \tan^{-1} \frac{1}{2}$$

$$\Rightarrow x = \frac{1}{2}$$

44. Question

Mark the tick against the correct answer in the following:

If $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$ then $x = ?$

A. 1

B. -1

C. 0

D. $\frac{1}{2}$

Answer

To Find: The value of $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$

$$\text{Since we know that } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

$$\Rightarrow \tan^{-1}(1+x) + \tan^{-1}(1-x) = \tan^{-1} \left(\frac{(1+x)+(1-x)}{1-(1+x)(1-x)} \right)$$

$$= \tan^{-1} \left(\frac{2}{1-(1-x^2)} \right)$$

$$= \tan^{-1} \left(\frac{2}{x^2} \right)$$

$$\text{Here since } \tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} \left(\frac{2}{x^2} \right) = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} \left(\frac{2}{x^2} \right) = \tan^{-1}(\infty) \quad (\because \tan \frac{\pi}{2} = \infty)$$

$$\Rightarrow \frac{2}{x^2} = \infty$$

$$\Rightarrow x^2 = \frac{2}{\infty}$$

$$\Rightarrow x = 0$$

45. Question

Mark the tick against the correct answer in the following:

$$\text{If } \sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3} \text{ then } (\cos^{-1} x + \cos^{-1} y) = ?$$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. π

D. $\frac{2\pi}{3}$

Answer

$$\text{Given: } \sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$$

To Find: The value of $\cos^{-1} x + \cos^{-1} y$

$$\text{Since we know that } \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

$$\text{Similarly } \cos^{-1} y = \frac{\pi}{2} - \sin^{-1} y$$

$$\text{Now consider } \cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} - \sin^{-1} x + \frac{\pi}{2} - \sin^{-1} y$$

$$= \frac{2\pi}{2} - [\sin^{-1} x + \sin^{-1} y]$$

$$= \pi - \frac{2\pi}{3}$$

$$= \frac{\pi}{3}$$

46. Question

Mark the tick against the correct answer in the following:

$$(\tan^{-1} 2 + \tan^{-1} 3) = ?$$

A. $\frac{-\pi}{4}$

B. $\frac{\pi}{4}$

C. $\frac{3\pi}{4}$

D. π

Answer

To Find: The value of $\tan^{-1} 2 + \tan^{-1} 3$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

$$\Rightarrow \tan^{-1} 2 + \tan^{-1} 3 = \tan^{-1} \left(\frac{2+3}{1-(2 \times 3)} \right)$$

$$= \tan^{-1} \left(\frac{5}{-5} \right)$$

$$= \tan^{-1}(-1)$$

Since the principle value of \tan lies in the range $[0, \pi]$

$$\Rightarrow \tan^{-1}(-1) = \frac{3\pi}{4}$$

47. Question

Mark the tick against the correct answer in the following:

$$\text{If } \tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8 \text{ then } x = ?$$

A. $\frac{1}{3}$

B. $\frac{1}{5}$

C. 3

D. 5

Answer

$$\text{Given: } \tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8$$

To Find: The value of x

Here $\tan^{-1} x + \tan^{-1} 3 = \tan^{-1} 8$ can be written as

$$\tan^{-1} x = \tan^{-1} 8 - \tan^{-1} 3$$

Since we know that $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right)$

$$\begin{aligned}\tan^{-1} x &= \tan^{-1} 8 - \tan^{-1} 3 = \tan^{-1} \left(\frac{8-3}{1+(8 \times 3)} \right) \\ &= \tan^{-1} \left(\frac{5}{25} \right) \\ &= \tan^{-1} \left(\frac{1}{5} \right) \\ \Rightarrow x &= \frac{1}{5}\end{aligned}$$

48. Question

Mark the tick against the correct answer in the following:

$$\text{If } \tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4} \text{ then } x = ?$$

- A. $\frac{1}{2}$ or -2
- B. $\frac{1}{3}$ or -3
- C. $\frac{1}{4}$ or -2
- D. $\frac{1}{6}$ or -1

Answer

$$\text{Given: } \tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$$

To Find: The value of x

$$\text{Since we know that } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

$$\begin{aligned}\Rightarrow \tan^{-1} 3x + \tan^{-1} 2x &= \tan^{-1} \left(\frac{3x+2x}{1-(3x \times 2x)} \right) \\ &= \tan^{-1} \left(\frac{5x}{1-6x^2} \right)\end{aligned}$$

$$\text{Now since } \tan^{-1} 3x + \tan^{-1} 2x = \frac{\pi}{4}$$

$$\tan^{-1} 3x + \tan^{-1} 2x = \tan^{-1} 1 \quad (\because \tan \frac{\pi}{4} = 1)$$

$$\Rightarrow \tan^{-1} \left(\frac{5x}{1-6x^2} \right) = \tan^{-1} 1$$

$$\Rightarrow \frac{5x}{1-6x^2} = 1$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

$$\Rightarrow x = \frac{1}{6} \text{ or } x = -1$$

49. Question

Mark the tick against the correct answer in the following:

$$\tan \left\{ \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{2}{3} \right\} = ?$$

A. $\frac{13}{6}$

B. $\frac{17}{6}$

C. $\frac{19}{6}$

D. $\frac{23}{6}$

Answer

To Find: The value of $\tan \left\{ \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{2}{3} \right\}$

$$\text{Let } x = \cos^{-1} \frac{4}{5}$$

$$\Rightarrow \cos x = \frac{4}{5} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

By pythagorus theroem ,

$$(\text{Hypotenuse})^2 = (\text{opposite side})^2 + (\text{adjacent side})^2$$

Therefore , opposite side = 3

$$\Rightarrow \tan x = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{3}{4}$$

$$\Rightarrow x = \tan^{-1} \frac{3}{4}$$

$$\text{Now } \tan \left\{ \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{2}{3} \right\} = \tan \left\{ \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{2}{3} \right\}$$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

$$\tan \left\{ \tan^{-1} \frac{3}{4} + \tan^{-1} \frac{2}{3} \right\} = \tan \left(\tan^{-1} \left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \left(\frac{3}{4} \times \frac{2}{3} \right)} \right) \right)$$

$$= \tan \left(\tan^{-1} \left(\frac{17}{6} \right) \right)$$

$$= \frac{17}{6}$$

50. Question

Mark the tick against the correct answer in the following:

$$\cos^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4} = ?$$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$

D. $\frac{3\pi}{4}$

Answer

To Find: The value of $\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4}$

Now $\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4}$ can be written in terms of tan inverse as

$$\cot^{-1} 9 + \operatorname{cosec}^{-1} \frac{\sqrt{41}}{4} = \tan^{-1} \frac{1}{9} + \tan^{-1} \frac{4}{5}$$

Since we know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

$$\Rightarrow \tan^{-1} \frac{1}{9} + \tan^{-1} \frac{4}{5} = \tan^{-1} \left(\frac{\frac{1}{9} + \frac{4}{5}}{1 - \left(\frac{1}{9} \times \frac{4}{5} \right)} \right)$$

$$= \tan^{-1} \left(\frac{41}{41} \right)$$

$$= \tan^{-1}(1) = \frac{\pi}{4}$$

51. Question

Mark the tick against the correct answer in the following:

Range of $\sin^{-1} x$ is

A. $\left[0, \frac{\pi}{2} \right]$

B. $[0, \pi]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2} \right]$

D. None of these

Answer

To Find: The range of $\sin^{-1} x$

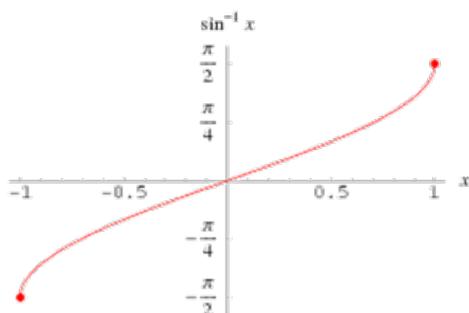
Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sin^{-1}(x)$ can be obtained from the graph of

$Y = \sin x$ by interchanging x and y axes. i.e, if (a, b) is a point on $Y = \sin x$ then (b, a) is

The point on the function $y = \sin^{-1}(x)$

Below is the Graph of range of $\sin^{-1}(x)$



From the graph, it is clear that the range of $\sin^{-1}(x)$ is restricted to the interval

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

52. Question

Mark the tick against the correct answer in the following:

Range of $\cos^{-1} x$ is

A. $[0, \pi]$

B. $\left[0, \frac{\pi}{2}\right]$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. None of these

Answer

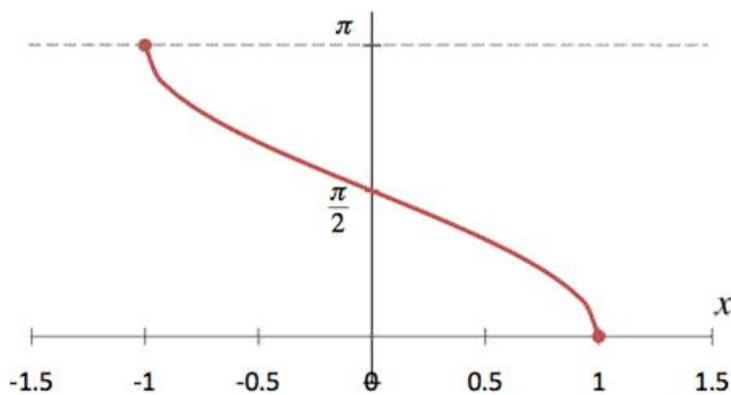
To Find: The range of $\cos^{-1} x$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \cos^{-1}(x)$ can be obtained from the graph of

$Y = \cos x$ by interchanging x and y axes. i.e, if (a, b) is a point on $Y = \cos x$ then (b, a) is the point on the function $y = \cos^{-1}(x)$

Below is the Graph of the range of $\cos^{-1}(x)$



From the graph, it is clear that the range of $\cos^{-1}(x)$ is restricted to the interval

$$[0, \pi]$$

53. Question

Mark the tick against the correct answer in the following:

Range of $\tan^{-1} x$ is

A. $\left(0, \frac{\pi}{2}\right)$

B. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

D. None of these

Answer

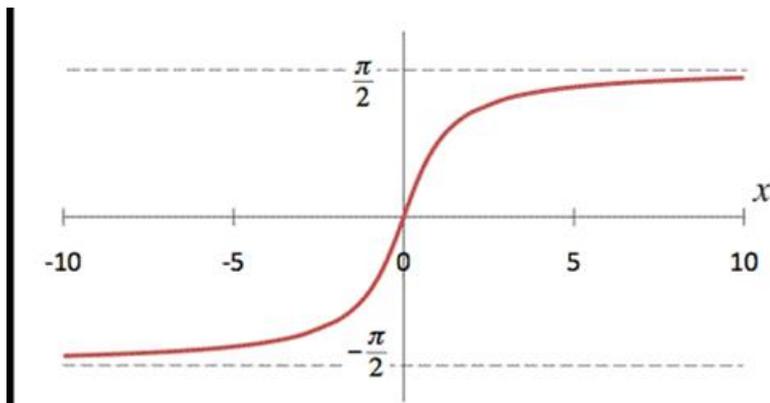
To Find: The range of $\tan^{-1} x$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \tan^{-1}(x)$ can be obtained from the graph of

$Y = \tan x$ by interchanging x and y axes. i.e, if (a,b) is a point on $Y = \tan x$ then (b,a) is the point on the function $y = \tan^{-1}(x)$

Below is the Graph of the range of $\tan^{-1}(x)$



From the graph, it is clear that the range of $\tan^{-1}(x)$ is restricted to any of the intervals like $[-\frac{3\pi}{2}, -\frac{\pi}{2}]$, $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $[\frac{\pi}{2}, \frac{3\pi}{2}]$ and so on. Hence the range is given by

$$(-\frac{\pi}{2}, \frac{\pi}{2}).$$

54. Question

Mark the tick against the correct answer in the following:

Range of $\sec^{-1} x$ is

A. $[0, \frac{\pi}{2}]$

B. $[0, \pi]$

C. $[0, \pi] - \left\{ \frac{\pi}{2} \right\}$

D. None of these

Answer

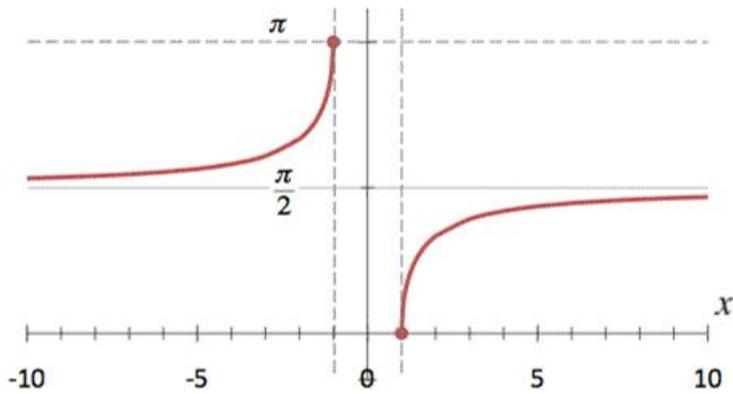
To Find: The range of $\sec^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sec^{-1}(x)$ can be obtained from the graph of

$Y = \sec x$ by interchanging x and y axes. i.e, if (a,b) is a point on $Y = \sec x$ then (b,a) is the point on the function $y = \sec^{-1}(x)$

Below is the Graph of the range of $\sec^{-1}(x)$



From the graph, it is clear that the range of $\sec^{-1}(x)$ is restricted to interval

$$[0, \pi] - \left\{ \frac{\pi}{2} \right\}$$

55. Question

Mark the tick against the correct answer in the following:

Range of $\operatorname{cosec}^{-1} x$ is

A. $\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$

B. $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$

C. $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$

D. None of these

Answer

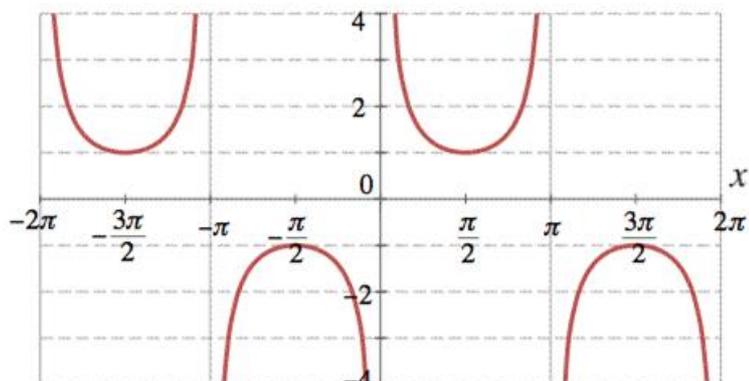
To Find: The range of $\operatorname{cosec}^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \operatorname{cosec}^{-1}(x)$ can be obtained from the graph of

$Y = \operatorname{cosec} x$ by interchanging x and y axes. i.e, if (a, b) is a point on $Y = \operatorname{cosec} x$ then (b, a) is the point on the function $y = \operatorname{cosec}^{-1}(x)$

Below is the Graph of the range of $\operatorname{cosec}^{-1}(x)$



From the graph it is clear that the range of $\operatorname{cosec}^{-1}(x)$ is restricted to interval

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

56. Question

Mark the tick against the correct answer in the following:

Domain of $\cos^{-1} x$ is

- A. $[0, 1]$
- B. $[-1, 1]$
- C. $[-1, 0]$
- D. None of these

Answer

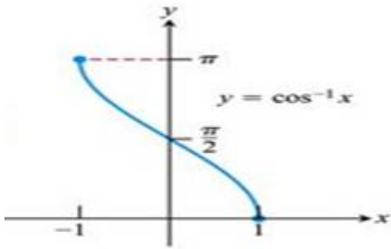
To Find: The Domain of $\cos^{-1}(x)$

Here, the inverse function of \cos is given by $y = f^{-1}(x)$

The graph of the function $y = \cos^{-1}(x)$ can be obtained from the graph of

$Y = \cos x$ by interchanging x and y axes. i.e, if (a, b) is a point on $Y = \cos x$ then (b, a) is the point on the function $y = \cos^{-1}(x)$

Below is the Graph of the domain of $\cos^{-1}(x)$



From the graph, it is clear that the domain of $\cos^{-1}(x)$ is $[-1, 1]$

57. Question

Mark the tick against the correct answer in the following:

Domain of $\sec^{-1} x$ is

- A. $[-1, 1]$
- B. $\mathbb{R} - \{0\}$
- C. $\mathbb{R} - [-1, 1]$
- D. $\mathbb{R} - \{-1, 1\}$

Answer

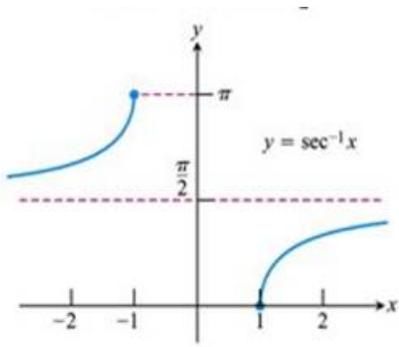
To Find: The Domain of $\sec^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \sec^{-1}(x)$ can be obtained from the graph of

$Y = \sec x$ by interchanging x and y axes. i.e, if (a, b) is a point on $Y = \sec x$ then (b, a) is the point on the function $y = \sec^{-1}(x)$

Below is the Graph of the domain of $\sec^{-1}(x)$



From the graph, it is clear that the domain of $\sec^{-1}(x)$ is a set of all real numbers excluding -1 and 1 i.e, $\mathbb{R} - [-1,1]$