PHYSICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 23

Total Marks : 36

Max. Time : 39 min.

Topics : String Wave, Circular Motion, Projectile Motion, Geometrical Optics, Electrostatics, Center of Mass

Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.1 to Q.5	(3 marks, 3 min.)	[15, 15]
Subjective Questions ('–1' negative marking) Q.6	(4 marks, 5 min.)	[4, 5]
Comprehension ('–1' negative marking) Q.7 to Q.9	(3 marks, 3 min.)	[9, 9]
Match the Following (no negative marking) (2 × 4) Q.10	(8 marks, 10 min.)	[8, 10]

1. A string of length 1.5 m with its two ends clamped is vibrating in fundamental mode. Amplitude at the centre of the string is 4 mm. Minimum distance between the two points having amplitude 2 mm is:

(A) 1 m (B) 75 cm (C) 60 cm (D) 50 cm

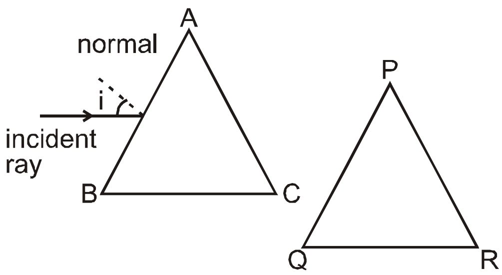
2. A particle is projected horizontaly with speed 10m/s from a certain point above ground. Find the tangential acceleration of particle at t = 2 sec. (Take g = 10 m/s^2).

(A) $\frac{10}{\sqrt{5}}$ (B) $\frac{25}{\sqrt{5}}$ (C) $4\sqrt{5}$ (D) $10\sqrt{5}$

3. A ball is thrown eastward across level ground. A wind blows horizontally to the east, and assume that the effect of wind is to provide a constant force to the east, equal in magnitude to the weight of the ball. The angle θ (with respect to horizontal) at which the ball should be projected so that it travels maximum horizontal distance is

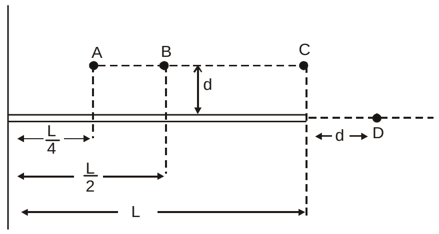
(A) 45° (B) 37° (C) 53° (D) 67.5°

4. Two equilateral glass prisms of refractive index $\sqrt{2}$ are placed as shown in figure. A ray is incident on side AB of left prism as shown in figure. This ray further suffers refraction at sides AC, PQ and PR in succession. The prisms are adjusted such that for each refraction the deviation is clockwise. Then the angle between sides AC and PQ of two prisms for net minimum deviation of incident ray is



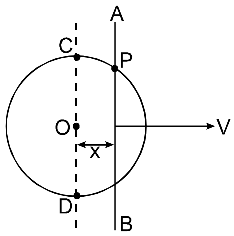
(A) 30° (B) 60°
(C) 90° (D) 120°

5. Figure given below shows uniformly positively charged, thin rod of length L and four points A, B, C and D at the same distance d from the rod, with position as marked. If V_A, V_B, V_C and V_D are their respective potentials then:



(A) $V_B > V_A > V_C > V_D$ (B) $V_B > V_A > V_C = V_D$
(C) $V_A = V_B > V_C = V_D$ (D) $V_D > V_B > V_A > V_C$

6. A rod AB is moving on a fixed circle of radius R with constant velocity 'v' as shown in figure. P is the point of intersection of the rod and the circle. At an instant the rod is at a distance $x = \frac{3R}{5}$ from centre of the circle. The velocity of the rod is perpendicular to the rod and the rod is always parallel to the diameter CD.

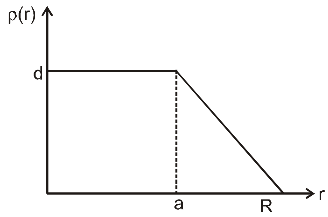


(a) Find the speed of point of intersection P.

(b) Find the angular speed of point of intersection P with respect to centre of the circle.

COMPREHENSION

The nuclear charge (Ze) is non–uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume] is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction. Figure **[JEE-2008 ; 12/163]**



7. The electric field at r = R is :

(A) independent of a
(B) directly proportional to a
(C) directly proportional to a^2
(D) inversely proportional to a

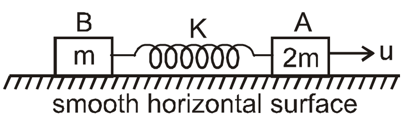
8. For a = 0, the value d (maximum value of ρ as shown in the figure) is :

(A) $\frac{3Ze^2}{4\pi R^3}$
(B) $\frac{3Ze}{\pi R^3}$
(C) $\frac{4Ze}{3\pi R^3}$
(D) $\frac{Ze}{3\pi R^3}$

9. The electric field within the nucleus is generally observed to be linearly dependent on r. This implies :

(A) a = 0
(B) $a = \frac{R}{2}$
(C) a = R
(D) $a = \frac{2R}{3}$

10. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant. In each situation of column I, certain statements are given and corresponding results are given in column II. Match the statements in column I corresponding results in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.



Column I	Column II
(A) The velocity of block A	(p) can never be zero
(B) The velocity of block B	(q) may be zero at certain instants of time
(C) The kinetic energy of system of two blocks	(r) is minimum at maximum compression of spring
(D) The potential energy of spring	(s) is maximum at maximum extension of spring

Answers Key

1. (A) **2.** (C) **3.** (D) **4.** (C)

5. (A) **6.** **(a)** $V_P = \frac{5}{4}$ V **(b)** $\omega = \frac{V_P}{R} = \frac{5V}{4R}$

7. (A) **8.** (B) **9.** (C)

10. (A) p (B) q (C) p,r (D) q,s

Hints & Solutions

1. $\lambda = 2\ell = 3$m

Equation of standing wave

$y = 2A \sin kx \cos \omega t$

$y = A$ as amplitude is 2A.

$A = 2A \sin kx$

$$\frac{2\pi}{\lambda} x = \frac{\pi}{6}$$

$$\Rightarrow \quad x_1 = \frac{1}{4} \text{m}$$

and $\frac{2\pi}{\lambda} . x = \frac{5\pi}{6}$

$\Rightarrow \quad x_2 = 1.25$ m $\Rightarrow x_2 - x_1 = 1$m

2. $\tan\theta = \frac{1}{2}$, $g\cos\theta = a_t$

$$10 \times \frac{20}{\sqrt{10^2 + 20^2}} = \frac{200}{\sqrt{100 + 400}} = \frac{20}{\sqrt{5}} \text{ m/s}^2.$$

3. Since time of flight depends only on vertical component of velocity and acceleration . Hence time of flight is

$T = \frac{2u_y}{g}$ where $u_x = \cos\theta$ and $u_y = u \sin\theta$

$\therefore$ In horizontal (x) direction

$d = u_x t + ½ gt^2$

$$= u \cos\theta \left(\frac{2u\sin\theta}{g}\right) + \frac{1}{2} g \left(\frac{2u\sin\theta}{g}\right)^2$$

$$= \frac{2u^2}{g} (\sin\theta \cos\theta + \sin^2\theta)$$

We want to maximise $f(\theta) = \cos\theta \sin\theta + \sin^2\theta$

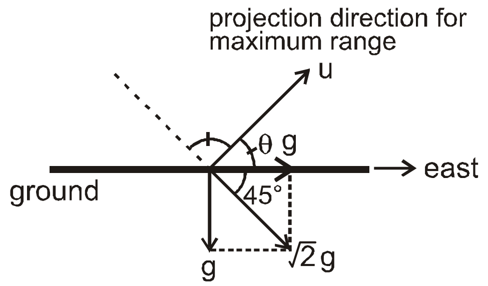
$\Rightarrow f'(\theta) = -\sin^2\theta + \cos^2\theta + 2 \sin\theta \cos\theta = 0$

$\Rightarrow \cos 2\theta + \sin 2\theta = 0 \qquad \Rightarrow \tan 2\theta = -1$

$$\text{or } 2\theta = \frac{3\pi}{4} \text{ or } \theta = \frac{3\pi}{8} = 67.5°$$

Alternate :

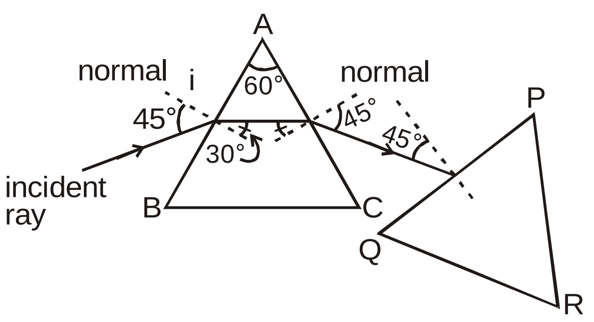
As shown in figure, the net acceleration of projectile makes on angle 45° with horizontal. For maximum range on horizontal plane, the angle of projection should be along angle bisector of horizontal and opposite direction of net acceleration of projectile.



$$\therefore \theta = \frac{135°}{2} = 67.5°$$

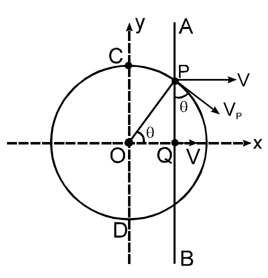
4. Let angle of incidence i for which deviation due to first prism is minimum, then $\sin i = n \sin 30°$ or i = 45°.

The net deviation shall be minimum if deviation due to each prism is minimum. From the ray diagram in figure, it is clear that angle between AC and PQ for net deviation to be minimum is 90°.



5. One can create a mental view of distribution of charge i.e. how much charge is nearer and how much is comparatively farther away.

6.



As a rod AB moves, the point 'P' will always lie on the circle.

$\therefore$ its velocity will be along the circle as shown by 'V_P' in the figure. If the point P has to lie on the rod 'AB' also then it should have component in 'x' direction as 'V'.

$\therefore \quad V_P \sin\theta = V \quad \Rightarrow \quad V_P = V \operatorname{cosec}\theta$

here $\cos\theta = \frac{x}{R} = \frac{1}{R} \cdot \frac{3R}{5} = \frac{3}{5}$

$\therefore \quad \sin\theta = \frac{4}{5} \quad \therefore \operatorname{cosec}\theta = \frac{5}{4}$

$\therefore \quad V_P = \frac{5}{4} V$ **...Ans.**

Sol. (b) $\omega = \frac{V_P}{R} = \frac{5V}{4R}$

ALTERNATIVE SOLUTION :

Sol. (a) Let 'P' have coordinate (x, y)

$x = R\cos\theta,\ y = R\sin\theta.$

$V_X = \frac{dx}{dt} = -R\sin\theta\,\frac{d\theta}{dt} = V$

$\Rightarrow \frac{d\theta}{dt} = \frac{-V}{R\sin\theta}$ and

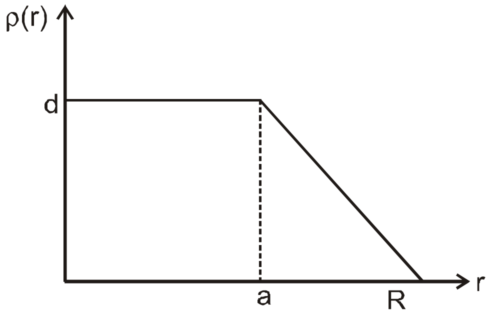
$V_Y = R\cos\theta\,\frac{d\theta}{dt} = R\cos\theta\left(-\frac{V}{R\sin\theta}\right) = -V\cot\theta$

$\therefore \quad V_P = \sqrt{V_x^2 + V_y^2} = \sqrt{V^2 + V^2\cot^2\theta}$

$= V\operatorname{cosec}\theta$ **...Ans.**

Sol. (b) $\omega = \frac{V_P}{R} = \frac{5V}{4R}$

7. (A) Electric field at r = R



$E = \frac{KQ}{R^2}$

where Q = Total charge within the nucleus = Ze

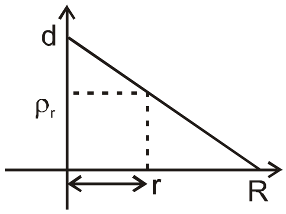
So $E = \frac{KZe}{R^2}$

So electric field is independent of a

8. $Q = \int \rho_r 4\pi r^2 dr$

for a = 0, $\frac{d}{R} = \frac{\rho_r}{R-r}$

$\therefore \quad \rho_r = \frac{d}{R}(R-r)$



or, $Q = \int_0^R \frac{d}{R}(R-r)\, 4\pi r^2\, dr$

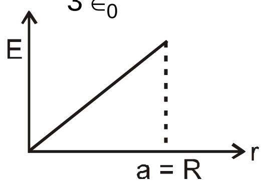
$= \frac{4\pi d}{R}\left[R\int_0^R r^2\, dr - \int_0^R r^3\, dr\right] = \frac{4\pi d}{R}\left[\frac{R^4}{3} - \frac{R^4}{4}\right]$

$= \frac{\pi d R^3}{3}$

$\therefore \quad Q = Ze = \frac{\pi d R^3}{3} \quad$ or $\quad d = \frac{3Ze}{\pi R^3}$

9. From the formula of uniformly (volume) charged solid sphere

$E = \frac{\rho r}{3\epsilon_0}$



For $E \propto r$, ρ should be constant throughout the volume of nucleus

This will be possible only when a = R.

10. (A) p (B) q (C) p,r (D) q,s

(A) If velocity of block A is zero, from conservation of momentum, speed of block B is 2u. Then K.E. of block B = $\frac{1}{2}m(2u)^2 = 2mu^2$ is greater than net mechanical energy of system. Since this is not possible, velocity of A can never be zero.

(B) Since initial velocity of B is zero, it shall be zero for many other instants of time.

(C) Since momentum of system is non-zero, K.E. of system cannot be zero. Also KE of system is minimum at maximum extension of spring.

(D) The potential energy of spring shall be zero whenever it comes to natural length. Also P.E. of spring is maximum at maximum extension of spring.