Exercise 6.2

Answer 1E.

(a)

Let x be a variable, a be a base with $a>0$ and $y=f(x)$ be an equation which consists of the exponential function $f(x)=a^x$.

Therefore, $\boxed{y=a^x}$ represents an equation which defines an exponential function.

(b)

To find the domain of the function $f(x)=a^x$:

For all real numbers, the exponential function a^x is well defined. Therefore, the domain of the function $f(x)=a^x$ would be $\boxed{\mathbb{R}}$ or $\boxed{(-\infty,\infty)}$.

(c)

Here, $a>0$ in the exponential function $f(x)=a^x$, then for any real values x, the value of $f(x)$ does not equal to negative.

So it is always positive.

Also, a^x approaches to infinity when the value x approaches to infinity.

Suppose if $a=1$, then $f(x)=1^x=1$ for all x, but at the same time

$f(0)=1$ for any value of a.

Therefore, the range of the function $f(x)=a^x$ is $\boxed{(0,\infty)}$ or all positive integers if $a\neq 1$.

(d)

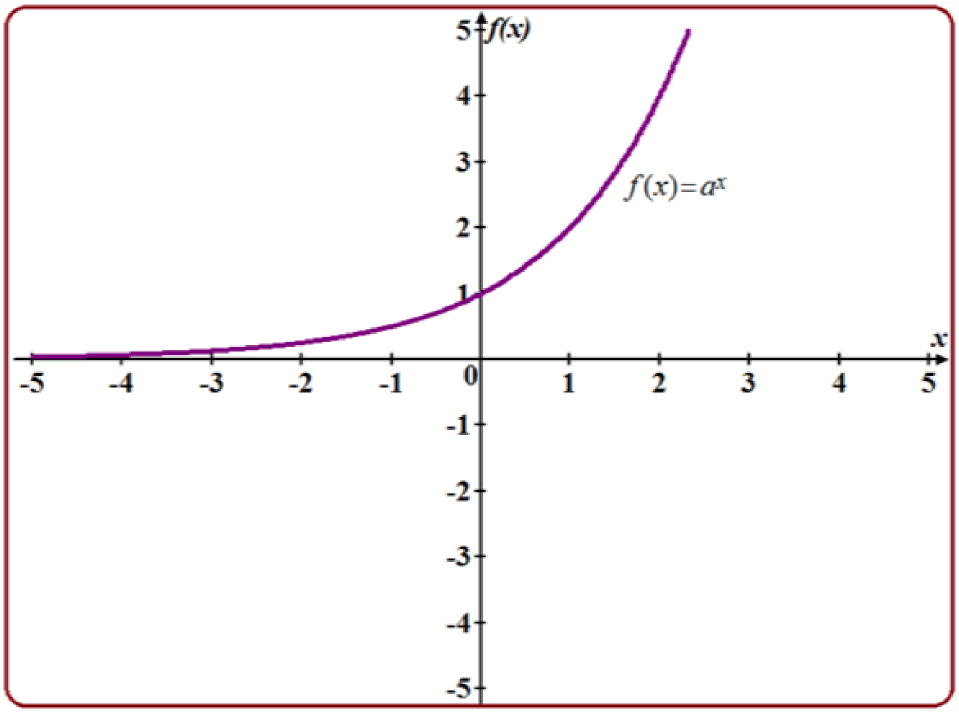
To sketch a general shape of the graphs for the exponential function in below case:

(i)

Here, $a>1$

To draw a graph, letting $a=2$ which satisfies $a>1$.

Graph of the function $f(x)=2^x$ is as shown below:



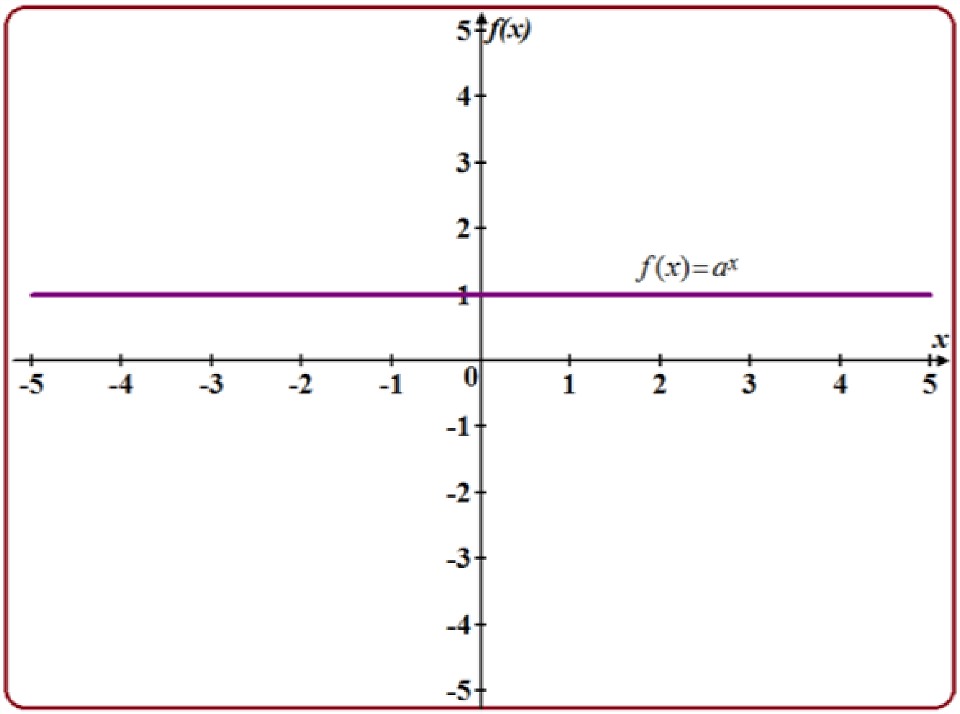
This represents some general form of the exponential function $f(x)=a^x$ if $a>1$.

To sketch a general shape of the graphs for the exponential function in below case:

(ii)

Here, $a=1$

Graph of the function $f(x)=1^x$ is as shown below:



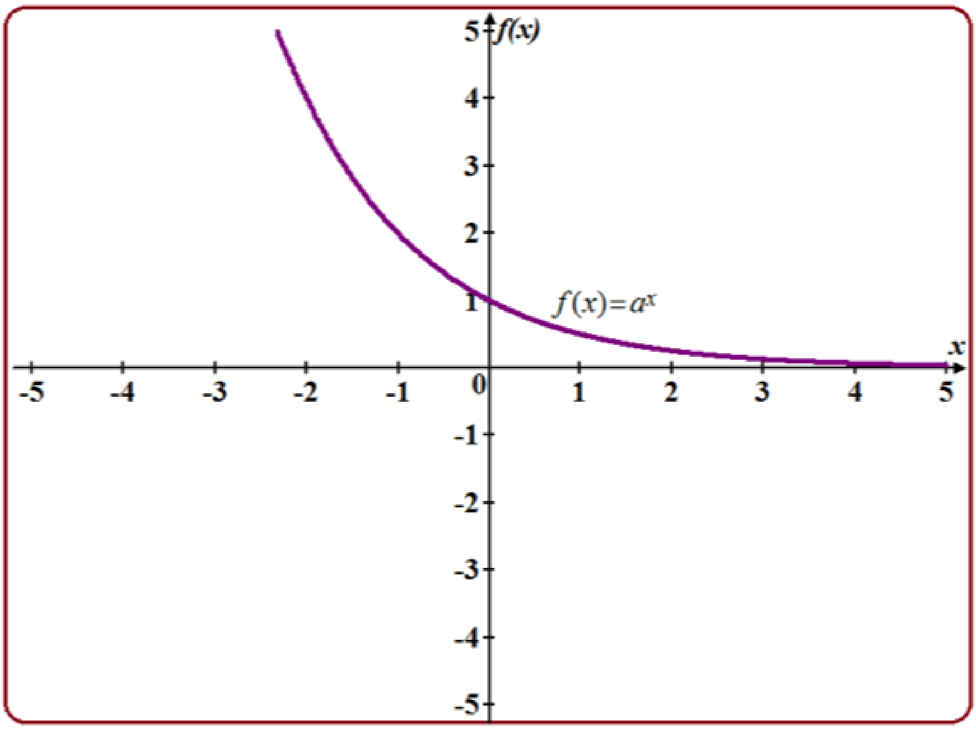
To sketch a general shape of the graphs for the exponential function in below case:

(iii)

Here, $0<a<1$

To draw a graph, letting $a=0.5$ which satisfies $0<a<1$.

Graph of the function $f(x)=(0.5)^x$ is as shown below:



This represents some general form of the exponential function $f(x)=a^x$ if $0<a<1$.

Answer 2E.

(a)

The objective to define the number e.

The number *e* is a constant number that is the base of natural logarithmic function: $f(x)=\ln x$.

In other words, the number *e* is a number such that $\ln e=1$.

(b)

The value of the number *e* is approximately equal to 2.7183.

(c)

The natural exponential function is an exponential function of the form: $f(x)=e^x$, where, *e* is the constant number which is approximately equal to 2.7183.

The inverse function of this exponential function is the natural logarithmic function or the logarithmic function to the base *e*: $g(x)=\ln x$.

Graph the function $f(x)=e^x$ by hand. If $x=0$, the function is

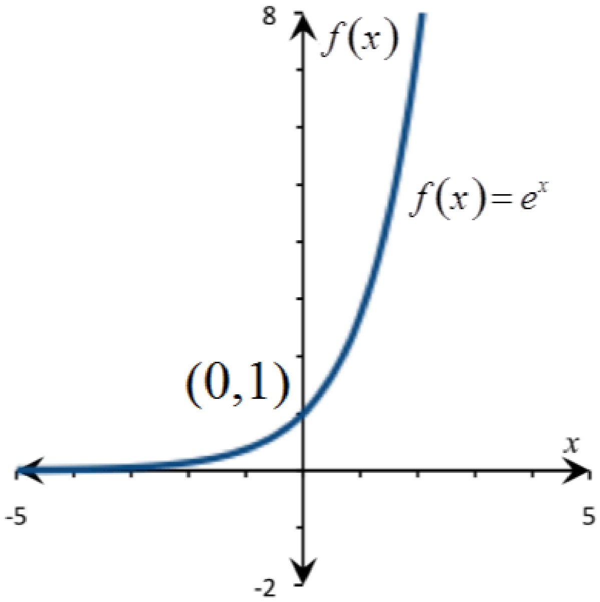
$$\begin{aligned} f(0) &= e^0 \\ &= 1 \end{aligned}$$

Thus, the graph crosses the *y*-axis at the point $(0,1)$.

Use the definition of *f* to find other points on the graph.

x	$f(x)$	(x, y)
2	$f(-2)=e^{-2}\approx 0.135$	$(-2,0.135)$
1	$f(-1)=e^{-1}\approx 0.368$	$(-1,0.368)$
0	$f(0)=e^{0}=1$	$(0,1)$
1	$f(1)=e^{1}\approx 2.718$	$(1,2.718)$
2	$f(2)=e^{2}\approx 7.389$	$(2,7.389)$

Sketch the graph of the function.



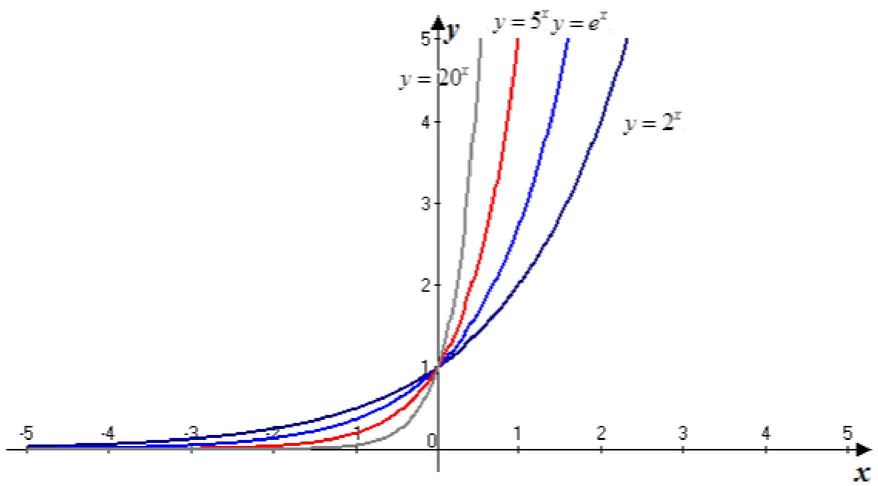
Answer 3E.

Consider the following functions:

$y=2^x, y=e^x, y=5^x, y=20^x$

The object is to graph the function on the same screen and how they related.

The graphs of the function $y=2^x, y=e^x, y=5^x, y=20^x$ are as shown below:



Observe that all approach o as $x \to -\infty$, all pass through $(0,1)$, and all are increasing.

The larger the base, the faster the rate of increase.

Answer 4E.

Consider the given functions;

$y = e^x$

$y = e^{-x}$

$y = 8^x$

$y = 8^{-x}$

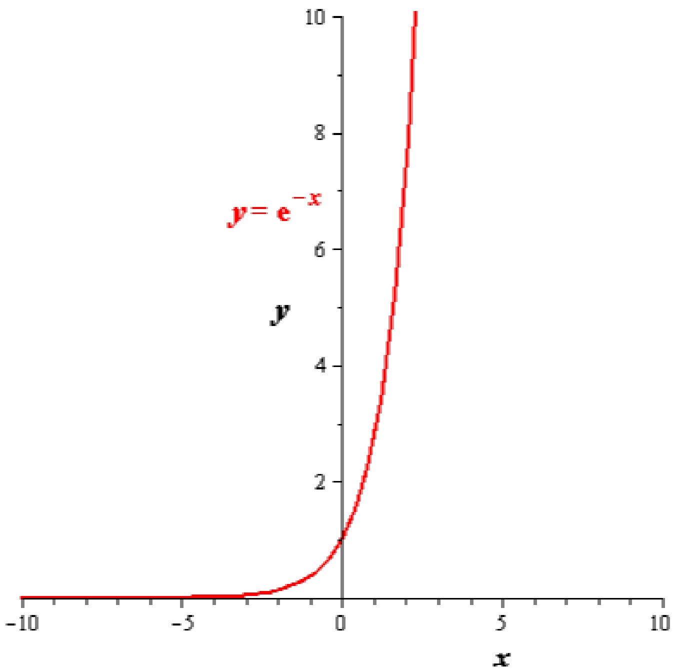
The objective is to make the graph of these four functions and to explain how these graphs are related to each other.

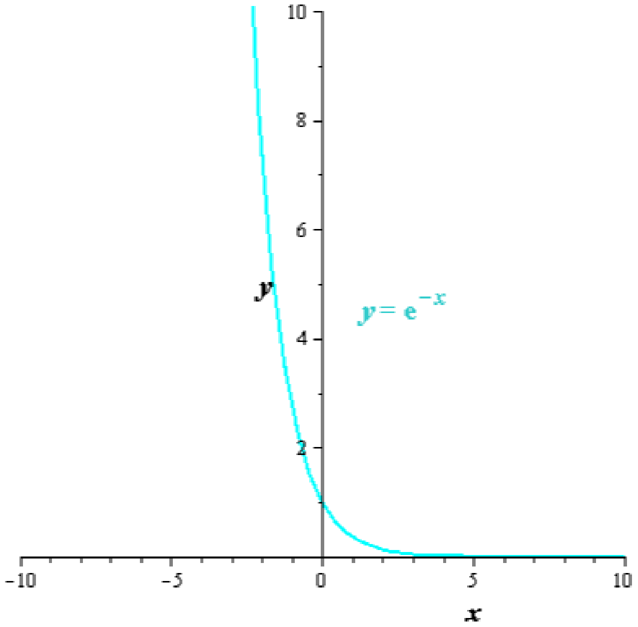
Given below are the graphs of the given function.

Graph of;

$y = e^x$

$y = e^{-x}$





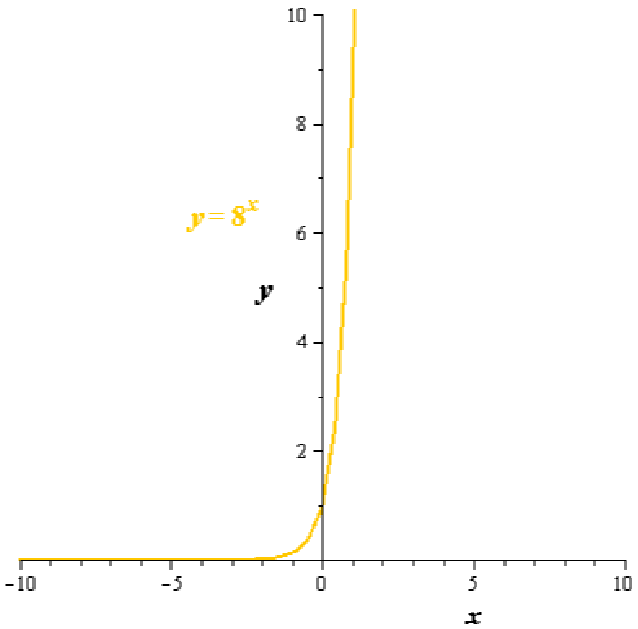
It can be seen easily from the graph that $y=e^{-x}$ is the reflection of $y=e^{x}$ about *y*-axis.

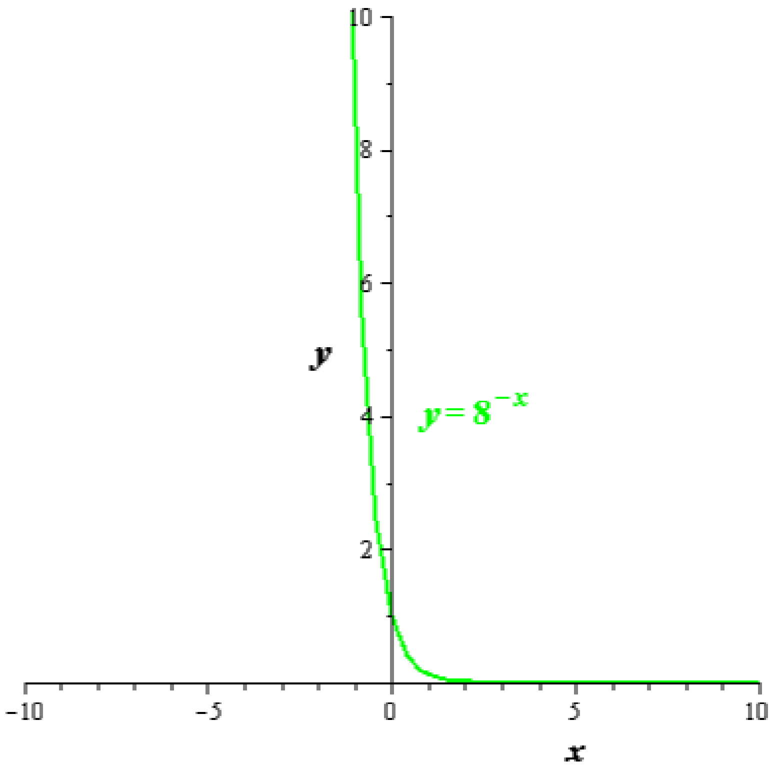
From the graph see that $y=e^{x}$ is an increasing function where as $y=e^{-x}$ is a decreasing function.

Graph of;

$y=8^{x}$

$y=8^{-x}$





Again, from the graph it can be seen easily that $y=8^x$ is an increasing function whereas $y=8^{-x}$ is a decreasing function.

Also, $y=8^{-x}$ is the reflection of $y=8^x$ about the *y*-axis

And one more relation between graphs is that $y=8^x$ is increasing more rapidly than $y=e^x$ because $e<8$.

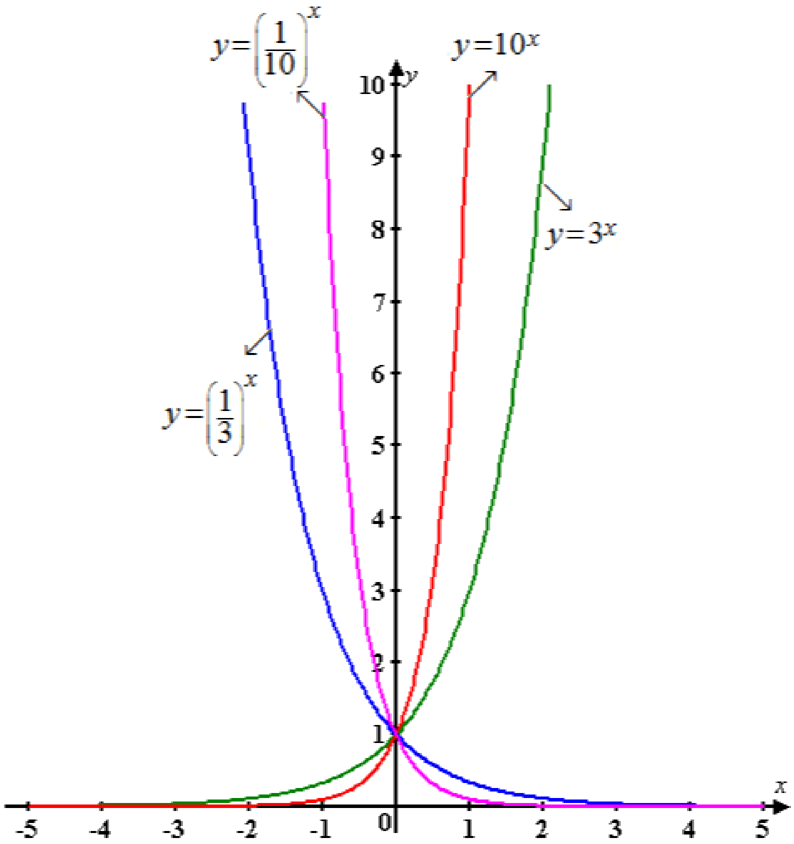
Answer 5E.

Consider the functions,

$$y=3^x, y=10^x, y=\left(\frac{1}{3}\right)^x, y=\left(\frac{1}{10}\right)^x.$$

The object is to graph the function on the same screen and how they related.

The graphs of the function $y=3^x, y=\left(\frac{1}{3}\right)^x, y=10^x, y=\left(\frac{1}{10}\right)^x$ are shown below:



Observe that the graph $y=\left(\frac{1}{3}\right)^x$ is the reflection of the graph of $y=3^x$ about the $y-$axis; also the graph $y=\left(\frac{1}{10}\right)^x$ is the reflection of the graph of $y=10^x$ about the $y-$axis.

Answer 9E.

Consider the function,

$y=-2^{-x}$.

The object is to draw a rough sketch of the function $y=-2^{-x}$.

Plug $x=-2,-1,0,1,2$ in $y=-2^{-x}$ then the values of y are calculated below:

$$\begin{aligned} y&=-2^{-(-2)} \\ &=-2^2 \\ &=-4 \end{aligned}$$

$$\begin{aligned} y&=-2^{-(-1)} \\ &=-2^1 \\ &=-2 \end{aligned}$$

$$\begin{aligned} y&=-2^{-0} \\ &=-1 \end{aligned}$$

$$\begin{aligned} y&=-2^{-1} \\ &=-\frac{1}{2} \end{aligned}$$

$$\begin{aligned} y&=-2^{-2} \\ &=-\frac{1}{2^2} \\ &=-\frac{1}{4} \end{aligned}$$

So the points on the graph are $(-2,-4),(-1,-2),(0,-1),\left(1,\frac{-1}{2}\right),\left(2,\frac{-1}{4}\right)$.

Graph the points $(-2,-4),(-1,-2),(0,-1),\left(1,\frac{-1}{2}\right),\left(2,\frac{-1}{4}\right)$ on the coordinate system and the graph of the function $y=-2^{-x}$ is shown below:

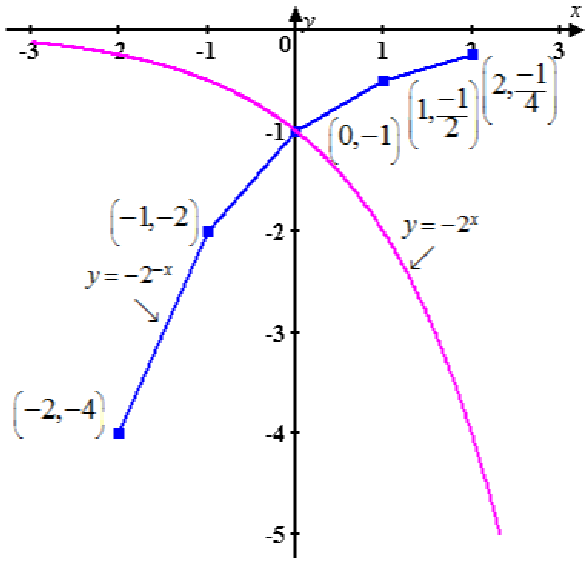


Figure1

Observe the figure 1; it indicates that the exponential function $y=-2^{-x}$ is the reflection of the graph of $y=-2^{x}$ about the $y-$axis.

The graph of $y=-2^{-x}$ is the reflection of the graph of $y=2^{-x}$ about the $x-$axis.

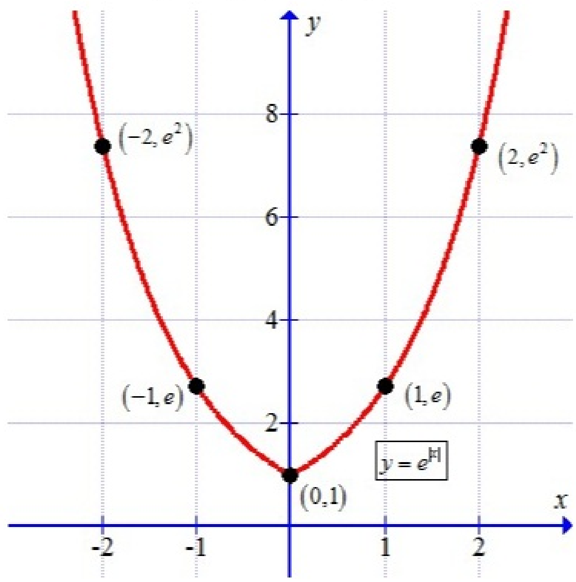
Answer 10E.

Consider the function: $y=e^{|x|}$

To draw the graph of the function $y=e^{|x|}$, consider the table having the values of y for different values of x as shown below.

x	$y=e^{\|x\|}$	(x,y)
0	$y=e^{0}$	$(0,1)$
1	$y=e^{1}$	$(1,e)$
-1	$y=e^{1}$	$(-1,e)$
2	$y=e^{2}$	$(2,e^{2})$
-2	$y=e^{-2}$	$(2,e^{2})$

Plot the above points on the graph sheet, and then join those points with smooth curve as shown below to get the graph of $y=e^{|x|}$.



Answer 11E.

Consider the function $y=1-\frac{1}{2}e^{-x}$.

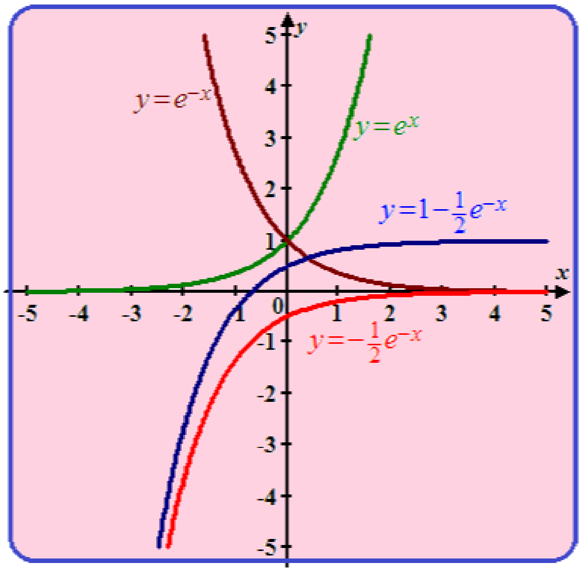
To sketch the graph of the function, begin by sketch the graph of $y=e^{x}$.

The graph of $y=e^{-x}$ is obtained by multiplying the exponent by -1 which reflects the curve horizontally about the *y*-axis.

The graph of $y=-\frac{1}{2}e^{-x}$, is obtained by multiplying the function $y=e^{-x}$ by $-\frac{1}{2}$ which compresses the graph vertically.

Finally the graph of the function $y=1-\frac{1}{2}e^{-x}$ is obtained by shifting $y=-\frac{1}{2}e^{-x}$ 1 unit upward.

The graph of the function and its transformations is as follows:



Answer 12E.

Consider the following function:

$y=2\left(1-e^{x}\right)$

The objective is to sketch the given function.

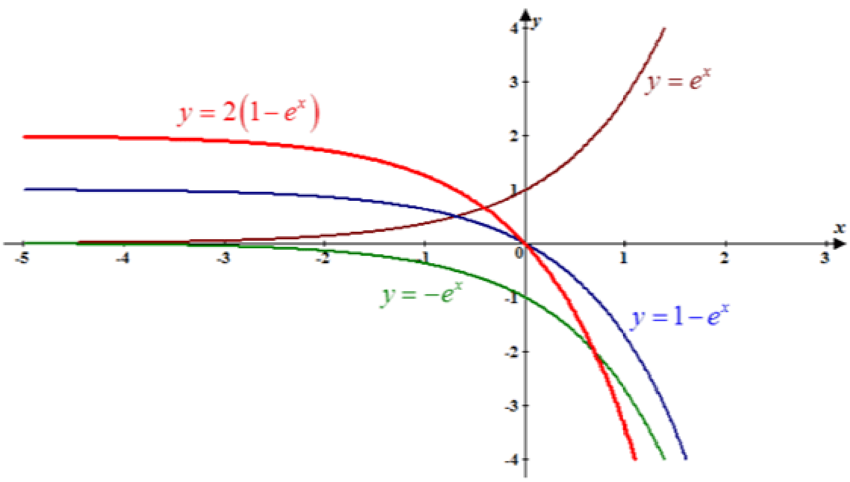
To sketch the graph of the function, begin by sketch the graph of $y=e^{x}$.

The graph of $y=-e^{x}$ is obtained by reflecting the function $y=e^{x}$ about the *x*-axis.

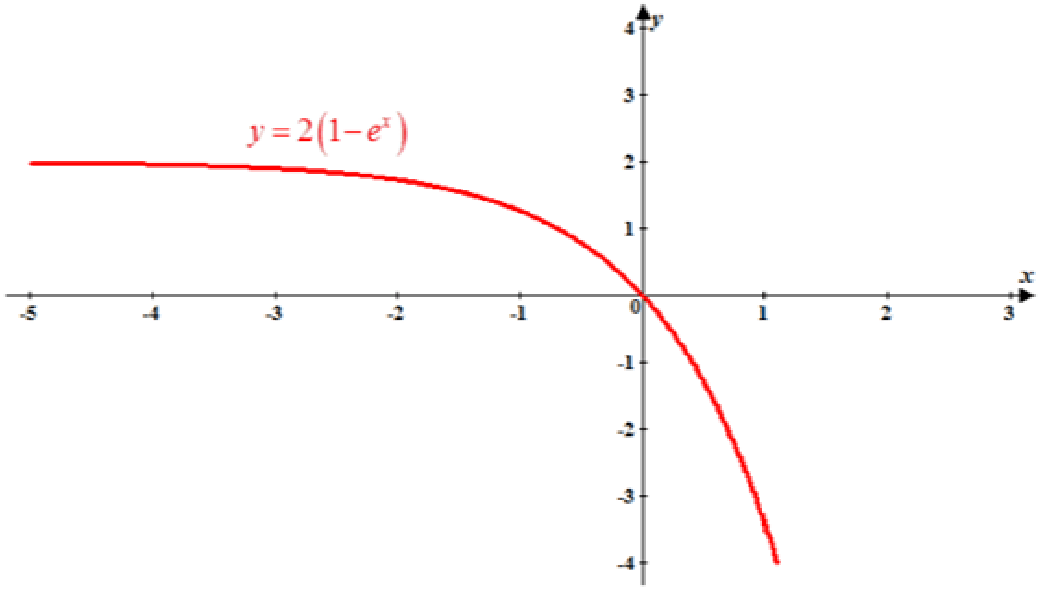
The graph of the function $y=1-e^{x}$ is obtained by shifting $y=-e^{x}$ to 1 unit upward.

Finally the graph of $y=2\left(1-e^{x}\right)$ is obtained by stretching $y=1-e^{x}$ vertically by a factor of 2.

The graph of the function and its transformations is as follows:



Hence, the graph of the function $y=2(1-e^x)$ is shown below:



Answer 13E.

(A) The equation of given graph is $y=e^x$

If the graph of $y=e^x$ is sifted 2 units downward then equation of the resulting graph will be in the form of $y=f(x)-c$

Here $c=2$ and $f(x)=e^x$

Therefore the equation of resulting graph is

$$\boxed{y=e^x-2}$$

(B) If the graph is shifted 2 units to the right then the equation of resulting graph is obtained by replacing x by $(x-2)$ in the original equation of the curve therefore the equation of resulting graph is $\boxed{y=e^{x-2}}$

(C) If the graph is reflected about x- axis then the equation of resulting graph will be in the form of $y=-f(x)$

Here $f(x)=e^x$

Therefore, the equation of resulting graph is $y=-e^x$

(D) If the graph is reflected about the y- axis then the equation of resulting graph is obtained by substituting $-x$ for x.

Therefore, the equation of resulting graph is $\boxed{y=e^{-x}}$

(E) If the graph is reflected about the x- axis then the equation of resulting graph is $y=-e^x$.

Now if the graph is further reflected about the y- axis then the equation of resulting graph is obtained by substituting $-x$ for x.

Therefore the equation of resulting graph is $\boxed{y=-e^{-x}}$

Answer 14E.

Consider the following function:

$y=e^x$

The objective is to find the equation of the graph that result from the reflecting about the given lines.

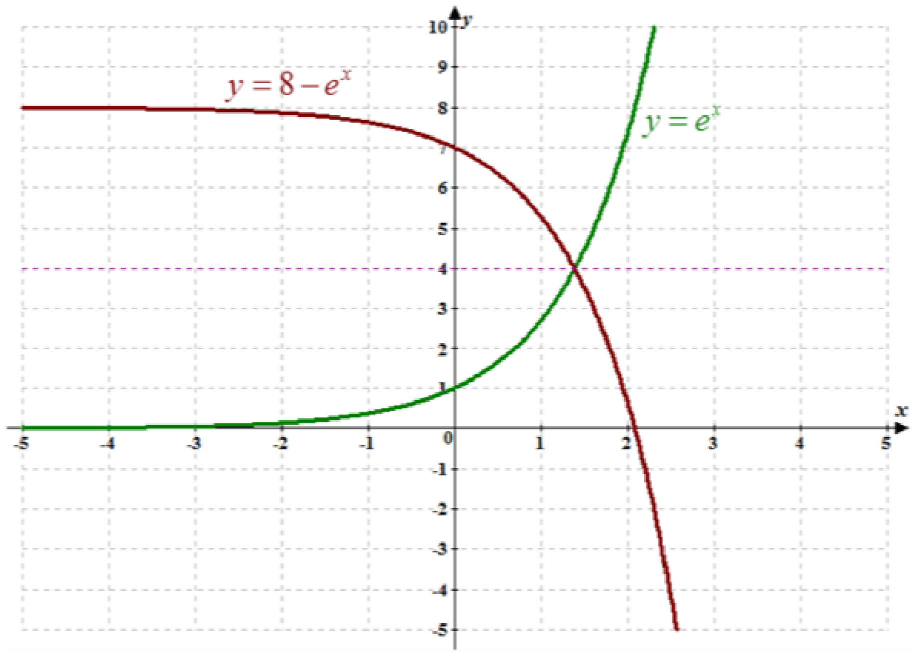
(a)

Reflecting about the line $y=4$:

This reflection consists of first reflecting the graph about the x-axis. That is the graph with $y=-e^x$ and then shifting this graph $2\cdot 4=8$ units upward.

So, the equation becomes after reflecting is $\boxed{y=8-e^x}$.

The sketch of the function $y=e^x$ and $y=8-e^x$ is shown below:



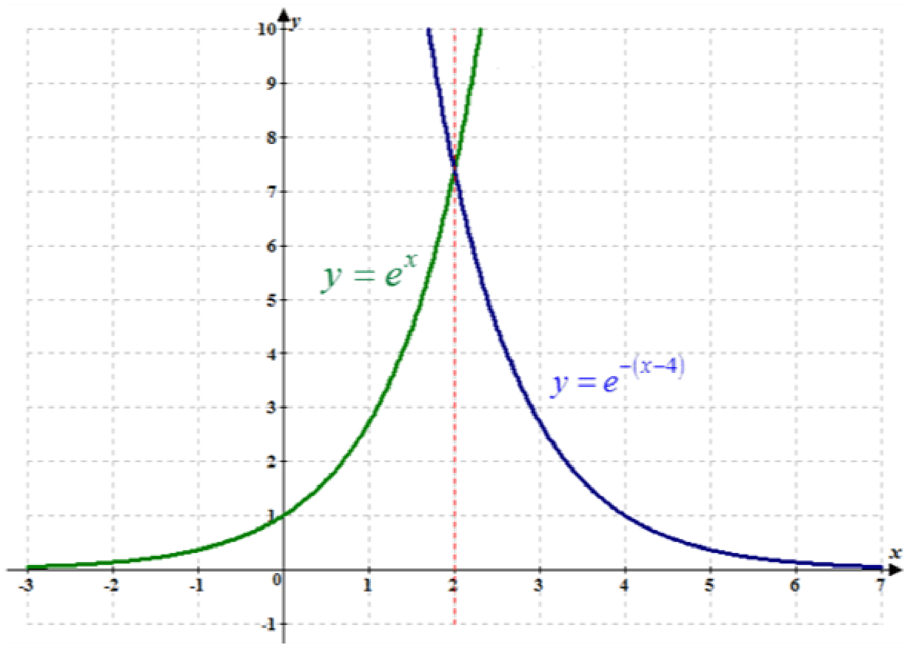
(b)

Reflecting about the line $x=2$:

This reflection consists of first reflecting the graph about the *y*-axis. That is the graph with equation $y=-e^x$ and then shifting this graph $2\cdot 2=4$ units to the right.

So, the equation becomes after reflecting is $\boxed{y=e^{-(x-4)}}$.

The sketch of the function $y=e^x$ and $y=e^{-(x-4)}$ is shown below:



Answer 15E:

(a) Given $f(x)=\dfrac{1-e^{x^2}}{1-e^{1-x^2}}$

So, the function is not defined when $1-e^{1-x^2}=0$

$\Rightarrow \quad 1-x^2=0$

$\Rightarrow \quad x=\pm 1$

Therefore $\boxed{\begin{aligned}&\text{Domain of } f=R-\{-1,1\}\\ &\qquad =(-\infty,-1)\cup(-1,1)\cup(1,\infty)\end{aligned}}$

(b) Given $f(x)=\dfrac{1+k}{e^{\cos x}}$

Since $e^{\cos x}$ is not equal to zero for any x,

$\boxed{\begin{aligned}&\text{Domain of } f=R\\ &\qquad =(-\infty,\infty)\end{aligned}}$

Answer 16E.

(A) We have to find the domain of $g(t)=\sin\left(e^{-t}\right)$.

The domain of $g(t)$ is the set of those values of t for which $g(t)$ is finite or exists.

Since e^{-t} is always defined. And the value of $\sin\left(e^{-t}\right)$ will oscillate between -1 to $+1$. so $\sin\left(e^{-t}\right)$ is defined for all t.

Therefore the domain of $g(t)$ is $\boxed{(-\infty,\infty)}$

(B) We have to find the domain of $g(t)=\sqrt{1-2^t}$.

The domain of $g(t)$ is the set of those values of t for which $g(t)$ is finite or exists.

Now $g(t)$ exists if $1-2^t\ge 0$

$\Rightarrow \quad 1\ge 2^t$

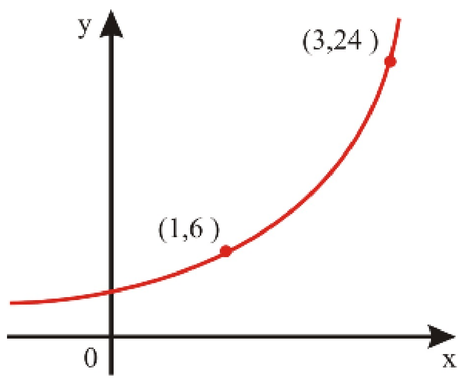
$\Rightarrow \quad 2^t\le 1$

$\Rightarrow \quad t\le \log_2 1$

$\Rightarrow \quad t\le 0 \qquad$ since $\log_2 1=0$

Therefore, the domain of $g(t)$ is $\boxed{(-\infty,0]}$

Answer 17E.



Given $f(x)=Ca^x$

From the graph it is clear that (3,24) lies on the graph. So it will satisfy the equation of graph $f(x)=Ca^x$.

Putting x = 3 and f(x) = 24, we get,

$24=Ca^3 \qquad$ ----- (1)

Also from the graph the point (1, 6) lies on the graph.

Therefore, putting x = 1 and f(x) = 6

We get $6=Ca^1 \qquad$ ------ (2)

Dividing equation (1) by equation (2), we have

$$\frac{24}{6}=\frac{Ca^3}{Ca}$$

$\Rightarrow \quad a^2=4$

$\Rightarrow \quad a=2,-2$

Since, ‘a’ can not be negative, as ‘a’ is positive in the exponential function.

Thus, a = 2.

Putting value of a in equation (2) we have

$$6 = C \times 2$$

$$\Rightarrow \quad C = \frac{6}{2} = 3$$

Therefore, the exponential function is

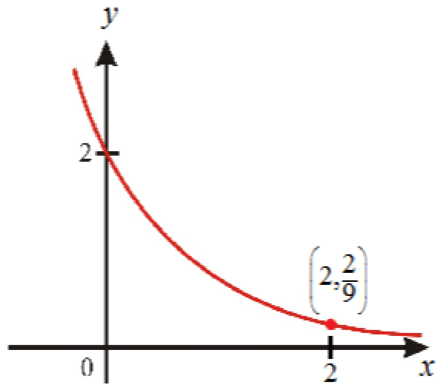
$$f(x) = Ca^x = 3 \cdot 2^x$$

Hence

$$\boxed{f(x) = 3 \cdot 2^x}$$

Answer 18E.

Consider the graph:



Need to find the exponential function in the form of $f(x) = Ca^x$ which satisfies the given graph shown above.

From the graph it is clear that the y – intercept of the curve is 2, therefore, the point (0, 2) will lie on the curve.

So this point will satisfy the equation of the curve $f(x) = Ca^x$.

Therefore, put $x = 0$ and $y = f(0) = 2$ in $f(x) = Ca^x$ we get,

$$2 = Ca^0$$

$$C = 2$$

Also the graph passes through the point (2, 2/9).

Therefore, put $x = 2$ and $f(x) = \frac{2}{9}$ in $f(x) = Ca^x$.

$$f(x) = Ca^x$$

$$\frac{2}{9} = Ca^2$$

$$\frac{2}{9} = 2a^2 \quad [\text{From equation (1), } C = 2]$$

$$a^2 = \frac{1}{9}$$

$$a = \frac{1}{3}, -\frac{1}{3}$$

Since, $a > 0$ therefore, we take $a = \frac{1}{3}$.

Substitute $a = \frac{1}{3}$ and $C = 2$ in the exponential function $f(x) = Ca^x$, get

Thus, $f(x) = Ca^x$

$$= 2 \cdot \left(\frac{1}{3}\right)^x$$

$$= 2\left(3^{-x}\right)$$

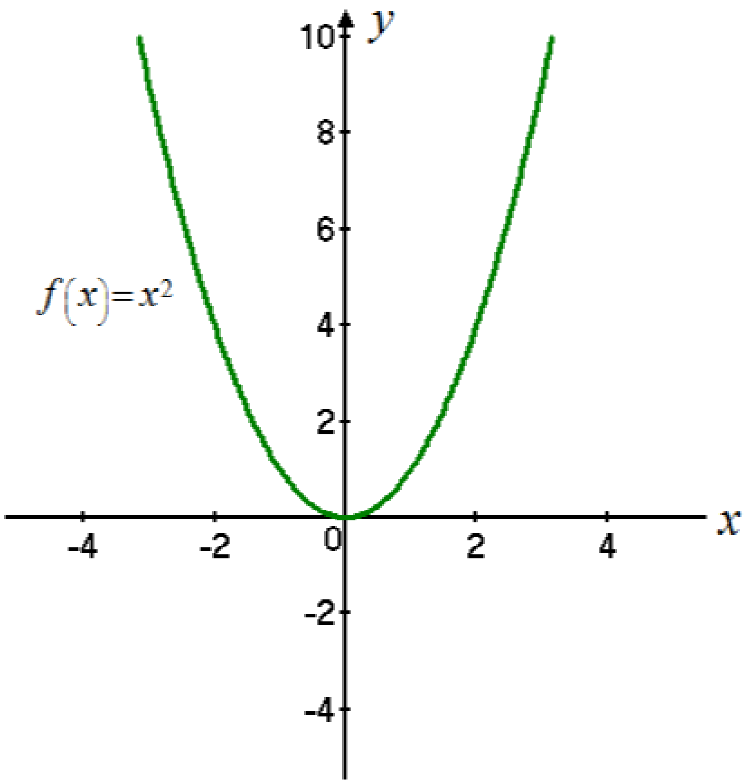
Hence the required exponential function is $\boxed{f(x) = 2\left(3^{-x}\right)}$.

Answer 19E.

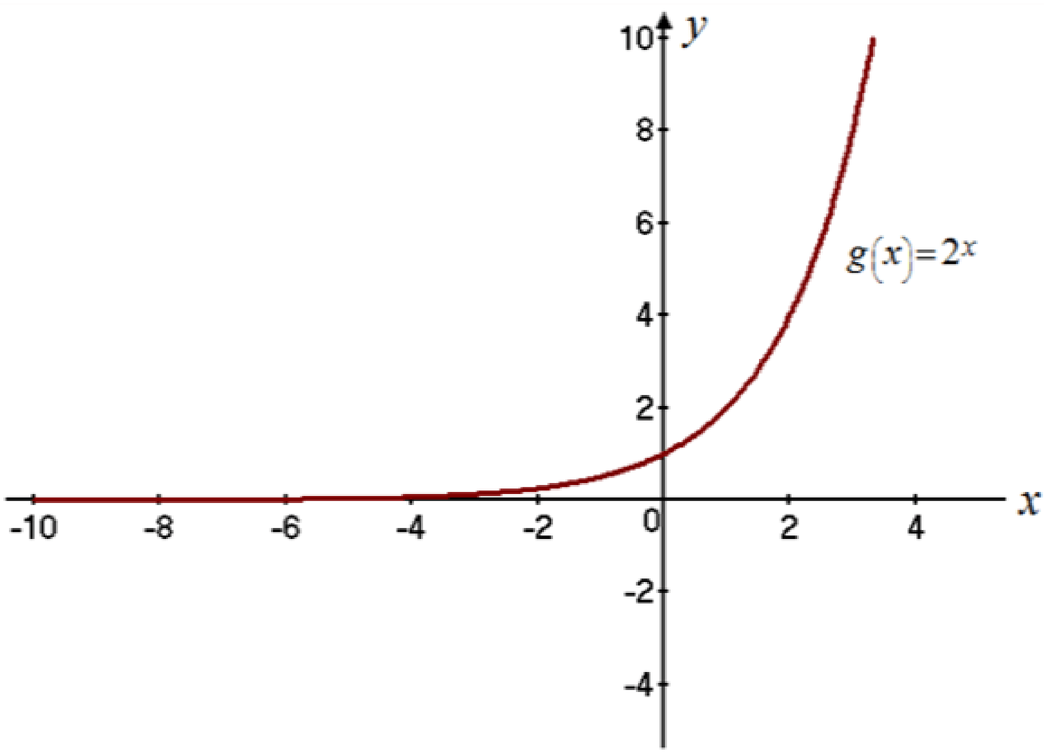
Suppose the graphs of $f(x)=x^2$ and $g(x)=2^x$ are drawn on a coordinate grid where the unit of measurement is 1 inch.

Need to prove, at a distance 2 ft. to the right of the origin, the height of the graph of f is 48 ft. but the height of the graph of g is about 265 mi.

Sketch the graph of $f(x)=x^2$:



Sketch the graph of $g(x)=2^x$:



Given distance is 2 ft to the right of the origin.

1 ft. = 12 inches

So, 2 ft. = 24 inches

Substitute $x = 24$ in $f(x) = x^2$.

$f(24) = (24)^2 \text{ inches}$

= 576 inches

$= \frac{576}{12}\text{ ft}$

$f(24) = 48\text{ ft}$

Therefore, the height of the graph of f is $\boxed{48\text{ ft}}$.

Now substitute $x = 24$ in $g(x) = 2^x$.

$g(x) = 2^x$

$g(24) = 2^{24}\text{ inches}$

$g(x) = \frac{2^{24}}{12}\text{ ft}$

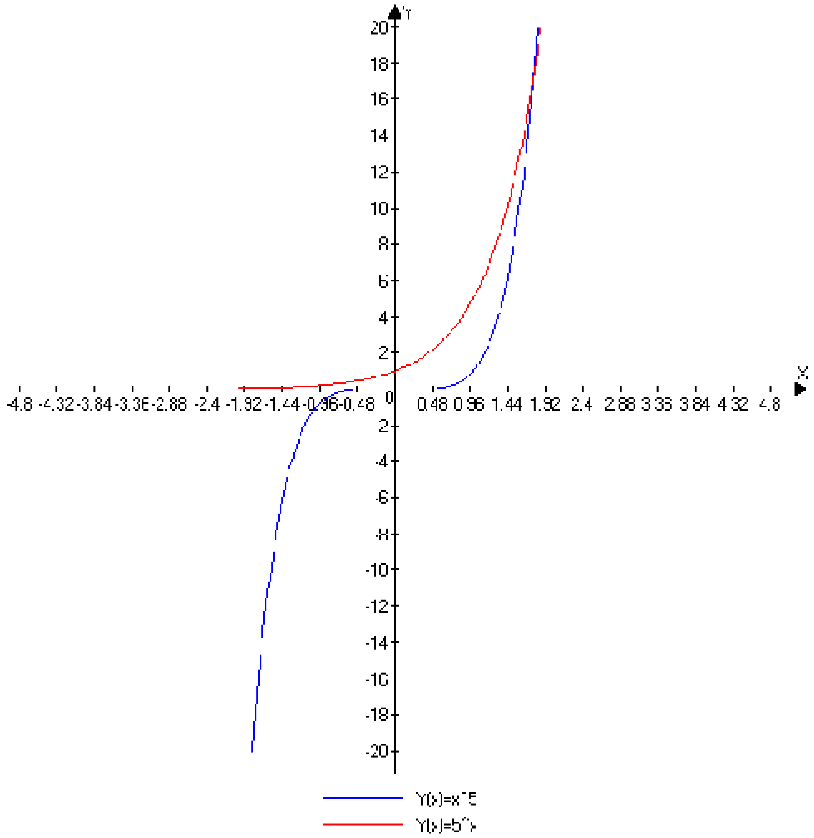
$= \frac{2^{24}}{(12 \times 5280)}\text{miles}$ [1 mile = 5280 ft.]

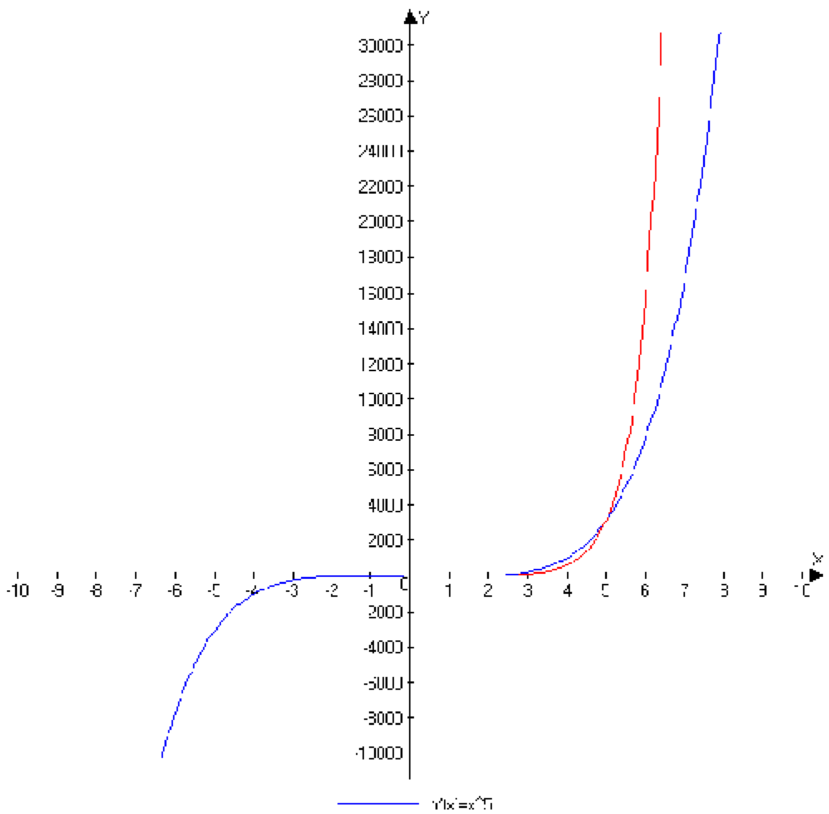
$g(x) \approx 265\,\text{miles}$

Therefore, the height of the graph of g is $\boxed{265\,\text{miles}}$.

Answer 20E.

The graphs of $f(x) = x^5$ and $g(x) = 5^x$ in various viewing rectangles are as follows:





From the above figures, we follow that the given functions have two points of intersection (1.772, 17.1) and (5,3265). Also from the graphs we follow that the rate of growth of 5^x is more than that of x^5

From the above figures, we follow that the given functions have two points of intersection (1.772, 17.1) and (5,3265). Also from the graphs we follow that the rate of growth of 5^x is more than that of x^5

Answer 23E.

Note that if a > 1 then,

$$\lim_{x\to\infty} a^x = \infty$$

In given problem:

$$a = 1.001$$

And

$$1.001 > 1$$

Therefore,

$$\lim_{x\to\infty}(1.001)^x = \infty$$

Hence the required limit is:

$$\boxed{\lim_{x\to\infty}(1.001)^x = \infty}$$

Answer 24E.

Consider the limit $\lim_{x\to-\infty}(1.001)^x$.

Recollect the definition,

If $a > 1$, then $\lim_{x\to-\infty} a^x = 0$ and $\lim_{x\to\infty} a^x = \infty$.

Since $1.001 > 1$, so by definition $\lim_{x\to-\infty}(1.001)^x = 0$.

Therefore,

$$\lim_{x\to-\infty}(1.001)^x = \boxed{0}.$$

Answer 25E.

We have to find the limit

$$\lim_{x\to\infty}\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}}$$

Dividing both numerator and denominator by e^{3x}

$$\lim_{x\to\infty}\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}} = \lim_{x\to\infty}\frac{\left(e^{3x}-e^{-3x}\right)/e^{3x}}{\left(e^{3x}+e^{-3x}\right)/e^{3x}}$$

$$= \lim_{x\to\infty}\frac{1-e^{-6x}}{1+e^{-6x}}$$

Now let $u = -6x$

Then $u \to -\infty$ as $x \to \infty$

So, $\lim_{x\to\infty} = \frac{1-e^{-6x}}{1+e^{-6x}} = \lim_{u\to-\infty}\frac{1-e^u}{1+e^u}$

Therefore, $\lim_{x\to\infty}\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}} = \lim_{u\to-\infty}\frac{1-e^u}{1+e^u}$

$$= \frac{\lim_{u\to-\infty}1-\lim_{u\to-\infty}e^u}{\lim_{u\to-\infty}1+\lim_{u\to-\infty}e^u} \qquad \text{[Using limit laws]}$$

$$= \frac{1-0}{1+0} = 1$$

[By the properties of natural exponential function we have $\lim_{t\to-\infty} e^t = 0$]

[And since $\lim_{t\to\infty} k = k$, k is a constant.]

Thus, $\boxed{\lim_{x\to\infty}\frac{e^{3x}-e^{-3x}}{e^{3x}+e^{-3x}} = 1}$

Answer 26E.

Consider the following limit:

$$\lim_{x \to \infty} e^{-x^2}$$

Find the limit.

Since there is a negative exponent value, rewrite the limit as follows:

$\lim_{x \to \infty} \frac{1}{e^{x^2}}$ Since $e^{-ax} = \frac{1}{e^{ax}}$

Let f be a function defined on some interval (a, ∞). Then $\lim_{x \to \infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

The exponential function $f(x) = e^x$ is an increasing continuous function with domain $\mathbb{R}$ and range $(0, \infty)$. Thus $e^x > 0$ for all x.

In this case, the denominator is getting larger as x approaches infinity.

As the denominator gets larger, the value of the fraction gets closer to 0.

$$\lim_{x \to \infty} \frac{1}{e^{x^2}} = 0$$

Thus, the limit $\lim_{x \to \infty} \frac{1}{e^{x^2}}$ is $\boxed{0}$.

Answer 27E.

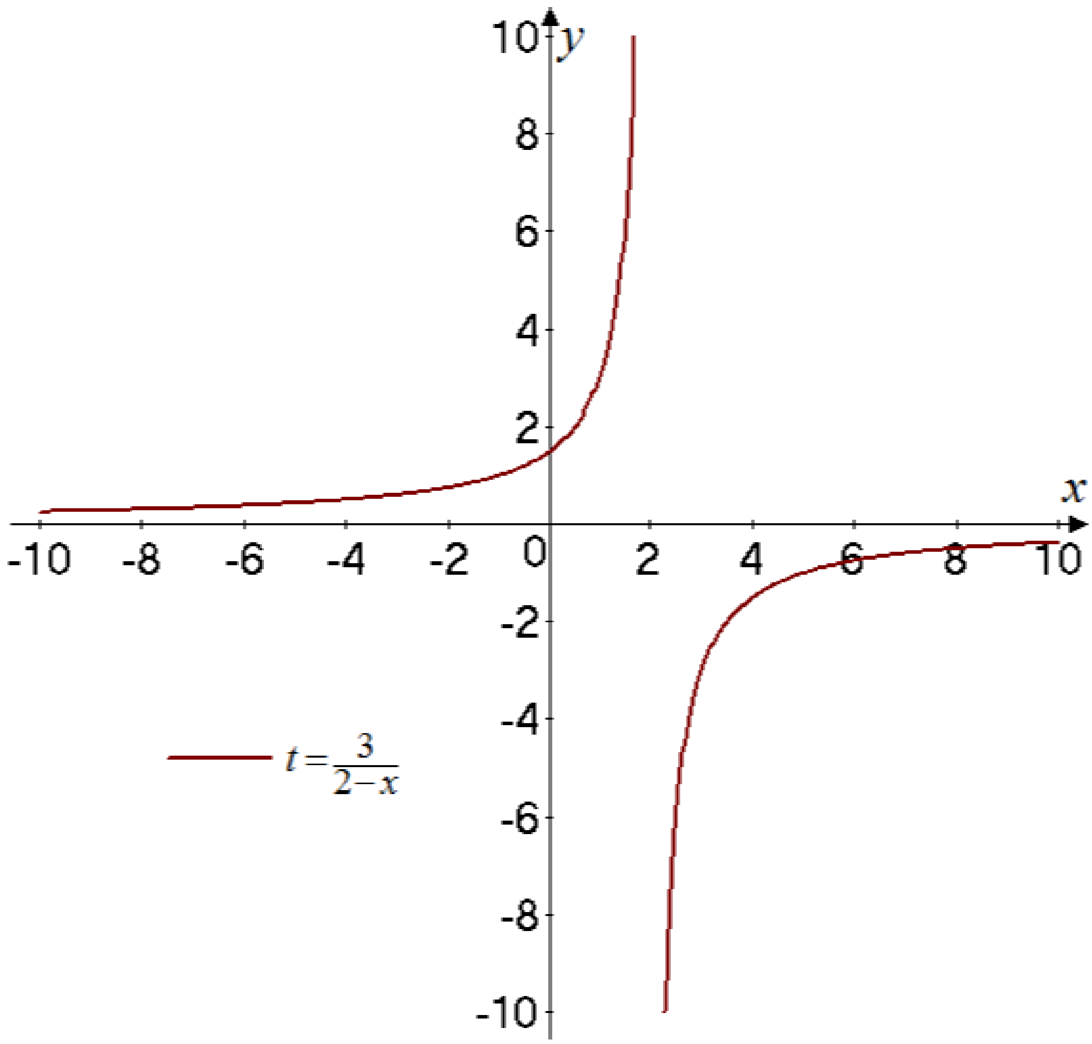
Consider the following limit:

$$\lim_{x \to 2^+} e^{\frac{3}{2-x}}.$$

As $x \to 2^+$, $\frac{3}{2-x} \to -\infty$.

As x tends to 2 from the right side, t tends to negative infinity.

The following graph confirms this result:



Thus, $\lim_{x\to 2^+} e^{\frac{3}{2-x}} = e^{-\infty}$

$=0$

Therefore, $\lim_{x\to 2^+} e^{\frac{3}{2-x}} = \boxed{0}$.

Answer 28E.

Consider the following limit:

$\lim_{x\to 2^-} e^{\frac{3}{2-x}}$

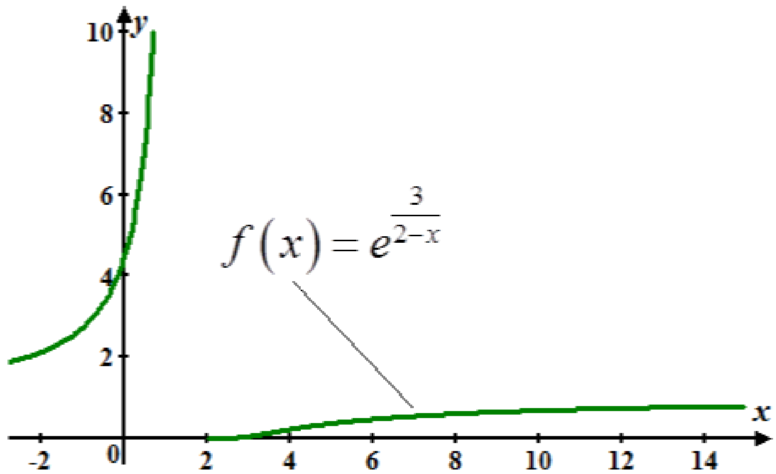
The objective is to find the given limit.

If x is close to 2 and smaller than 2, then the denominator $2-x$ is positive small number, so the quotient $\frac{3}{2-x}$ is large positive number.

Thus, $e^{\frac{3}{2-x}}$ approaches to ∞.

Hence, the limit is $\lim_{x\to 2^-} e^{\frac{3}{2-x}} = \boxed{\infty}$.

Observe the sketch of the function $f(x)=e^{\frac{3}{2-x}}$ is as follows:



Answer 29E.

Consider the following limit:

$\lim_{x\to\infty}\left(e^{-2x}\cos x\right)$.

Rewrite the limit as follows:

$\lim_{x\to\infty}\left(e^{-2x}\cos x\right)=\lim_{x\to\infty}\left(\frac{\cos x}{e^{2x}}\right)$ Since $e^{-ax}=\frac{1}{e^{ax}}$

The exponential function is an increasing continuous function with domain $\mathbb{R}$ and range $(0,\infty)$.

Let f be a function defined on some interval (a,∞). Then $\lim_{x\to\infty}f(x)=L$ means that the values of $f(x)$ can be made arbitrarily close to L by taking x sufficiently large.

If $\lim_{x\to a}f(x)=0$, and if $g(x)$ is bounded, then $\lim_{x\to a}f(x)g(x)=0$.

Here, $\lim_{x\to\infty}\frac{1}{e^{2x}}=\frac{1}{e^{2(\infty)}}$

$=0$ Since $e^{-\infty}=0$

And $-1\le\cos x\le 1$

So, the function $\cos x$ is bounded.

$\lim_{x\to\infty}\left(e^{-2x}\cos x\right)=0$

Therefore, the limit $\lim_{x\to\infty}\left(e^{-2x}\cos x\right)$ is $\boxed{0}$.

Answer 30E.

Consider the following limit:

$\lim_{x\to\left(\frac{\pi}{2}\right)^+}\left(e^{\tan x}\right)$

Evaluate, $\lim_{x\to(\pi/2)^+}e^{\tan x}$.

Let, $y=tanx$.

So $y\to-\infty,$ as $x\to(\pi/2)^+$.

Since $\lim_{x\to(\pi/2)^+}\tan x=-\infty$, so $\lim_{x\to(\pi/2)^+}e^{\tan x}=\lim_{y\to-\infty}e^y$.

$=0$

Therefore, $\boxed{\lim_{x\to(\pi/2)^+}e^{\tan x}=0}$.

Method 2:

Consider the following limit:

$$\lim_{x\to\left(\frac{\pi}{2}\right)^{+}}\left(e^{\tan x}\right)$$

As $x\to\left(\frac{\pi}{2}\right)^{+}$, $h\to 0, x=\frac{\pi}{2}+h.$

Substitute these values in the limit.

$$\lim_{x\to\left(\frac{\pi}{2}\right)^{+}}\left(e^{\tan x}\right)=\lim_{h\to 0}\left(e^{\tan\left(\frac{\pi}{2}+h\right)}\right)$$

$$=\lim_{h\to 0}\left(e^{-\coth}\right)$$

$$=e^{-\infty}$$

$$=\frac{1}{e^{\infty}}$$

$$=\frac{1}{\infty}$$

$$=0$$

Therefore, $\boxed{\lim_{x\to(\pi/2)^{+}} e^{\tan x}=0}$.

Answer 31E.

Given $f(x)=e^5$, a constant function

$$f'(x)=\frac{d}{dx}(f(x))$$

$$=\frac{d}{dx}(e^5)$$

$$=0$$

Therefore $\boxed{f'(x)=0}$

Answer 32E.

Given $k(r)=e^r+r^e$

$$\frac{d}{dr}(k(r))=k'(r)$$

$$=\frac{d}{dr}(e^r+r^e)$$

$$=\frac{d}{dr}(e^r)+\frac{d}{dr}(r^e)$$

$$=e^r+er^{e-1}$$

Therefore $\boxed{k'(r)=e^r+er^{e-1}}$

Answer 33E.

Consider the function

$$f(x)=(x^3+2x)e^x.$$

Use the product rule to differentiate the above function.

Product Rule: if g and h are both differentiable, then

$$\frac{d}{dx}[g(x)h(x)]=g(x)\frac{d}{dx}[h(x)]+h(x)\frac{d}{dx}[g(x)] \quad \text{...... (1)}$$

Now differentiate $f(x)=(x^3+2x)e^x$ with respect to x,

$$f'(x)=\frac{d}{dx}\left[(x^3+2x)e^x\right]$$

Let $g(x)=(x^3+2x)$ and $h(x)=e^x$

Now substitute $g(x)$ and $h(x)$ into the equation **(1)**, get

$$f'(x) = (x^3 + 2x)\frac{d}{dx}(e^x) + e^x \frac{d}{dx}(x^3 + 2x)$$

$$= (x^3 + 2x)\frac{d}{dx}(e^x) + e^x\left(\frac{d}{dx}(x^3) + \frac{d}{dx}(2x)\right)$$

$$= (x^3 + 2x)e^x + e^x(3x^2 + 2) \text{ Use } \begin{cases} \frac{d}{dx}(e^x) = e^x \text{ and} \\ \frac{d}{dx}(x^n) = n \cdot x^{n-1} \end{cases}$$

$$= e^x(x^3 + 2x + 3x^2 + 2)$$

$$= e^x(x^3 + 3x^2 + 2x + 2)$$

Therefore,

$$\boxed{f'(x) = e^x(x^3 + 3x^2 + 2x + 2)}.$$

Answer 34E.

Consider the function $y = \dfrac{e^x}{1 - e^x}$

Note that, if

$$y = \frac{u}{v}$$

then $\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

And $\dfrac{d}{dx}(e^x) = e^x$

Therefore,

$$y = \frac{e^x}{1 - e^x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{1 - e^x}\right)$$

$$= \frac{(1 - e^x)\frac{d}{dx}(e^x) - e^x \cdot \frac{d}{dx}(1 - e^x)}{(1 - e^x)^2}$$

$$= \frac{(1 - e^x)e^x - e^x(-e^x)}{(1 - e^x)^2}$$

$$= \frac{e^x - e^{2x} + e^{2x}}{(1 - e^x)^2}$$

$$= \frac{e^x}{(1 - e^x)^2}$$

Hence, the derivative of y is $y' = \boxed{\dfrac{e^x}{(1 - e^x)^2}}$.

Answer 35E.

$$y = e^{ax^3}$$

$$\frac{dy}{dx} = \frac{d}{dx}e^{ax^3}$$

$$= e^{ax^3}.\frac{d}{dx}\left(ax^3\right) \qquad \text{(Chain rule)}$$

$$= e^{ax^3}.3ax^2$$

$$\boxed{\frac{dy}{dx} = 3ax^2e^{ax^3}}$$

Answer 36E.

Given $y = e^{-2t}\cos 4t$

On differentiating y, $y' = \frac{d}{dt}\left(e^{-2t}\cos 4t\right)$

$$= e^{-2t}\frac{d}{dt}(\cos 4t) + \left[\frac{d}{dt}\left(e^{-2t}\right)\right]\cos 4t$$

$$= e^{-2t}(-4\sin 4t) + (-2)e^{-2t}\cos 4t$$

$$= (-4\sin 4t - 2\cos 4t)e^{-2t}$$

Therefore $\boxed{\begin{aligned} y' &= \frac{dy}{dt} \\ &= (-4\sin 4t - 2\cos 4t)e^{-2t}\end{aligned}}$

Answer 37E.

Given $y = xe^{-kx}$

On differentiating,

$$y' = \frac{d}{dx}\left(xe^{-kx}\right)$$

$$= x\cdot\frac{d}{dx}\left(e^{-kx}\right) + \left(\frac{d}{dx}(x)\right)e^{-kx}$$

$$= x\left(-ke^{-kx}\right) + e^{-kx}$$

$$= -kxe^{-kx} + e^{-kx}$$

$$= (1-kx)e^{-kx}$$

Therefore $\boxed{y' = (1-kx)e^{-kx}}$

Answer 38E.

Given $y = \frac{1}{s+ke^s}$

Differentiating y with respect to s,

$$y'(s) = \frac{d}{ds}\left(s+ke^s\right)^{-1}$$

$$= \frac{-1}{\left(s+ke^s\right)^2}\frac{d}{dr}\left(s+ke^s\right)$$

$$= \frac{-1}{\left(s+ke^s\right)^2}\left(1+ke^s\right)$$

Therefore $\boxed{y' = \frac{-\left(1+ke^s\right)}{\left(s+ke^s\right)^2}}$

Answer 39E.

$$f(u) = e^{1/u}$$

Differentiating with respect to u.

$$f'(u) = \frac{d}{du}\left(e^{1/u}\right)$$

$$= e^{1/u}.\frac{d}{du}\left(\frac{1}{u}\right)$$

$$= e^{1/u}.(-1)\frac{1}{u^2}$$

$$\boxed{f'(u) = -\frac{e^{1/u}}{u^2}}$$

Answer 40E.

Consider the function,

$f(t)=\sin(e^t)+e^{\sin t}$

Differentiate the function with respect to t, get

$f'(t)=\frac{d}{dt}\left(\sin(e^t)+e^{\sin t}\right)$

$=\frac{d}{dt}(\sin e^t)+\frac{d}{dt}(e^{\sin t})$ Use $\frac{d}{dx}(u+v)=\frac{d}{dx}(u)+\frac{d}{dx}(v)$

$=\cos e^t\cdot\frac{d}{dt}(e^t)+e^{\sin t}\cdot\frac{d}{dt}(\sin t)$ Use $\frac{d}{dx}(e^u)=e^u\frac{d}{dx}(u)$

$=\cos e^t\cdot e^t+e^{\sin t}\cdot\cos t$ Since $\frac{d}{dt}(e^t)=e^t$ and $\frac{d}{dt}(\sin t)=\cos t$

$=e^t\cos e^t+\cos t(e^{\sin t})$

Therefore, the simplified derivative is $f'(t)=\boxed{e^t\cos e^t+\cos t(e^{\sin t})}$

Answer 41E.

$F(t)=e^{t\sin 2t}$

Differentiating with respect to t.

$F'(t)=\frac{d}{dt}\left(e^{t\sin 2t}\right)$

$=e^{t\sin 2t}\frac{d}{dt}(t\sin 2t)$ [Chain rule]

$=e^{t\sin 2t}\left[t.\frac{d}{dt}\sin 2t+\sin 2t\frac{d}{dt}t\right]$ [Product rule]

$=e^{t\sin 2t}\left[t.\cos 2t.2+\sin 2t\right]$

$=e^{t\sin 2t}(2t.\cos 2t+\sin 2t)$

$\boxed{F'(t)=e^{t\sin 2t}(2t.\cos 2t+\sin 2t)}$

Answer 42E.

Given $y=x^2e^{-1/x}$

On differentiation,

$y'=\frac{d}{dx}\left(x^2e^{-1/x}\right)$

$=\frac{d}{dx}(x^2)e^{-1/x}+x^2\cdot\frac{d}{dx}\left(e^{-1/x}\right)$

$=2xe^{-1/x}+x^2\cdot e^{-1/x}\cdot\frac{d}{dx}\left(\frac{-1}{x}\right)$

$=2xe^{-1/x}+x^2e^{-1/x}\cdot\frac{1}{x^2}$

$=2xe^{-1/x}+e^{-1/x}$

$=(1+2x)e^{-1/x}$

Therefore $\boxed{y'=(1+2x)e^{-1/x}}$

Answer 43E.

$y=\sqrt{1+2e^{3x}}$

$\frac{dy}{dx}=\frac{d}{dx}\left(1+2e^{3x}\right)^{1/2}$

$=\frac{1}{2}\frac{1}{\sqrt{(1+2e^{3x})}}\frac{d}{dx}\left(1+2e^{3x}\right)$ [Chain rule]

$=\frac{1}{2\sqrt{(1+2e^{3x})}}\left[2e^{3x}.3\right]$ [Chain rule]

$\boxed{\frac{dy}{dx}=\frac{3e^{3x}}{\sqrt{(1+2e^{3x})}}}$

Answer 44E.

$$y = e^{k\tan\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{k\tan\sqrt{x}}\right]$$

$$= e^{k\tan\sqrt{x}} . \frac{d}{dx}\left[k\tan\sqrt{x}\right] \qquad \text{[Chain rule]}$$

$$= e^{k\tan\sqrt{x}} . k\sec^2\sqrt{x} . \frac{1}{2\sqrt{x}} \qquad \text{[Chain rule]}$$

$$y' = \boxed{\frac{ke^{k\tan\sqrt{x}}\sec^2\sqrt{x}}{2\sqrt{x}}}$$

Answer 45E.

Consider the following function:

$$y = e^{e^x}$$

The objective is to differentiate the function, use the chain rule formula.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Let $u = e^x$, then $y = e^u$

Differentiate u with respect to x.

$$\frac{du}{dx} = e^x$$

And differentiate y with respect to u,

$$\frac{dy}{du} = e^u$$

Find $\frac{dy}{dx}$:

Substitute the values of $\frac{du}{dx}$ and $\frac{dy}{du}$ in chain rule formula.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= e^u \cdot e^x$$

$$= e^{e^x} \cdot e^x \left(u = e^x\right)$$

$$= e^x e^{e^x}$$

Therefore, the differentiate function is, $\boxed{e^x e^{e^x}}$.

Answer 46E.

Consider the following function:

$$y = \frac{e^u - e^{-u}}{e^u + e^{-u}}$$

Write the function as follows:

$$y = \frac{e^u - \frac{1}{e^u}}{e^u + \frac{1}{e^u}}$$

The objective is to differentiate the function *y*.

Use the chain rule,

$$\frac{dy}{du} = \frac{dy}{dt} \cdot \frac{dt}{du}$$

Let $t = e^u$ then the function be,

$$y = \frac{t - \frac{1}{t}}{t + \frac{1}{t}}$$
$$= \frac{t^2 - 1}{t^2 + 1}$$

Differentiate *y* with respect to '*t*'.

$$\frac{d}{dt}(y) = \frac{d}{dt}\left(\frac{t^2 - 1}{t^2 + 1}\right)$$

Use the dividing rule formula,

$$\frac{d}{dt}\left(\frac{h(t)}{g(t)}\right) = \frac{g(t) \cdot h'(t) - h(t) \cdot g'(t)}{\left[g(t)\right]^2}$$

Use dividing formula with $h(t) = t^2 - 1$ and $g(t) = t^2 + 1$.

$$\frac{dy}{dt} = \frac{(t^2+1) \cdot (2t) - (t^2-1) \cdot (2t)}{(t^2+1)^2}$$
$$= \frac{2t^3 + 2t - 2t^3 + 2t}{(t^2+1)^2}$$
$$= \frac{4t}{(t^2+1)^2}$$

And let $t = e^u$ then the function be,

Differentiate with respect to *u*,

$$\frac{dt}{du} = e^u$$

By chain rule formula substitute the values $\frac{dy}{dt}$ and $\frac{dt}{du}$ in $\frac{dy}{du}$.

$$\frac{dy}{du} = \frac{dy}{dt} \cdot \frac{dt}{du}$$
$$= \frac{4t}{(t^2+1)^2} \cdot e^u \quad \left(\text{since } t = e^u\right)$$
$$= \frac{4e^u \cdot e^u}{(e^{2u}+1)^2}$$
$$= \frac{4e^{2u}}{(e^{2u}+1)^2}$$

Therefore, the differentiate of *y* is $\boxed{\frac{dy}{du} = \frac{4e^{2u}}{(e^{2u}+1)^2}}$

Answer 47E.

$$y=\frac{ae^x+b}{ce^x+d}$$

Differentiating with respect to x by quotient rule

$$\frac{dy}{dx}=\frac{d}{dx}\left[\frac{ae^x+b}{ce^x+d}\right]$$

$$=\frac{\left(ce^x+d\right)\frac{d}{dx}\left(ae^x+b\right)-\left(ae^x+b\right)\frac{d}{dx}\left(ce^x+d\right)}{\left(ce^x+d\right)^2}$$

$$=\frac{\left(ce^x+d\right)\left(ae^x\right)-\left(ae^x+b\right)\left(ce^x\right)}{\left(ce^x+d\right)^2}$$

$$=\frac{\cancel{ae^x ce^x}+dae^x-\cancel{ce^x ae^x}-bce^x}{\left(ce^x+d\right)^2}$$

Or $\quad \frac{dy}{dx}=\frac{dae^x-bce^x}{\left(ce^x+d\right)^2}$

Or $\quad \boxed{\frac{dy}{dx}=\frac{(ad-bc)e^x}{\left(ce^x+d\right)^2}}$

Answer 48E.

Consider the following function:

$y=\sqrt{1+xe^{-2x}}$.

Rewrite the function as follows:

$$y=\left(1+xe^{-2x}\right)^{\frac{1}{2}}$$

Find the derivative of the function.

Let us assume $u=1+xe^{-2x}$.

Then $y=u^{\frac{1}{2}}$

Now differentiate $y=u^{\frac{1}{2}}$ with respect to *x*.

$$y'=\frac{d}{dx}\left(u^{\frac{1}{2}}\right)$$

$$=\frac{1}{2}u^{\frac{1}{2}-1}\cdot\frac{du}{dx}$$

$$=\frac{1}{2}u^{\frac{-1}{2}}\cdot\frac{du}{dx}$$

Use $\frac{d}{dx}u^n=nu^{n-1}\cdot\frac{du}{dx}$

$=\frac{1}{2}\left(1+xe^{-2x}\right)^{-1/2}\cdot\frac{d}{dx}\left(1+xe^{-2x}\right)$ Back substitute $u=1+xe^{-2x}$

$=\frac{1}{2}\left(1+xe^{-2x}\right)^{-1/2}\left[\frac{d}{dx}(1)+\frac{d}{dx}\left(xe^{-2x}\right)\right]$ Use $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$

$=\frac{1}{2}\left(1+xe^{-2x}\right)^{-1/2}\left[0+\frac{d}{dx}\left(xe^{-2x}\right)\right]$ ...... (1)

Now find the derivative of $\frac{d}{dx}\left(xe^{-2x}\right)$.

Use product rule: $\frac{d}{dx}(u \cdot v) = u' \cdot v + u \cdot v'$

In this case $u = x$ and $v = e^{-2x}$.

So their respective derivatives are $u' = 1$ and $v' = -2e^{-2x}$.

Substitute these values into $\frac{d}{dx}(u \cdot v) = u' \cdot v + u \cdot v'$.

$$\frac{d}{dx}\left(xe^{-2x}\right) = 1 \cdot e^{-2x} + x\left(-2e^{-2x}\right)$$
$$\frac{d}{dx}\left(xe^{-2x}\right) = e^{-2x} - 2xe^{-2x}$$
$$= e^{-2x}(1-2x)$$

Now simplify equation (1) further as follows:

$$\frac{1}{2}\left(1+xe^{-2x}\right)^{-1/2}\left[0+\frac{d}{dx}\left(xe^{-2x}\right)\right] = \frac{1}{2}\left(1+xe^{-2x}\right)^{-1/2}\left[e^{-2x}(1-2x)\right]$$
$$= \frac{e^{-2x}(1-2x)}{2}\left(1+xe^{-2x}\right)^{-1/2}$$
$$= \frac{(1-2x)e^{-2x}}{2\sqrt{1+xe^{-2x}}}$$

Therefore, the simplified derivative is $\boxed{y' = \frac{(1-2x)e^{-2x}}{2\sqrt{1+xe^{-2x}}}}$.

Answer 49E.

Consider the function,

$$y = \cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)$$

The objective is to find the differentiate the given function.

Differentiate the function, with respect to *x* as follows:

$$y' = \frac{dy}{dx}$$
$$= \frac{d}{dx}\left[\cos\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\right]$$
$$= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right) \cdot \frac{d}{dx}\left(\frac{1-e^{2x}}{1+e^{2x}}\right) \quad \cdots\cdots (1)$$

Use $\frac{d}{dx}(\cos u) = -(\sin u)u'$

Simplify the equation (1) as follows:

$$y' = -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\left[\frac{\left(1+e^{2x}\right)\cdot\frac{d}{dx}\left(1-e^{2x}\right)-\left(1-e^{2x}\right)\cdot\frac{d}{dx}\left(1+e^{2x}\right)}{\left(1+e^{2x}\right)^2}\right]$$

Use $\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{uv'-vu'}{v^2}$

$$= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\left[\frac{\left(1+e^{2x}\right)\cdot\left[\frac{d}{dx}(1)-\frac{d}{dx}\left(e^{2x}\right)\right]-\left(1-e^{2x}\right)\cdot\left[\frac{d}{dx}(1)+\frac{d}{dx}\left(e^{2x}\right)\right]}{\left(1+e^{2x}\right)^2}\right]$$

$$= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\left[\frac{\left(1+e^{2x}\right)\left(-2e^{2x}\right)-\left(1-e^{2x}\right)\left(2e^{2x}\right)}{\left(1+e^{2x}\right)^2}\right]$$

Use $\frac{d}{dx}(\text{constant})=0, \frac{d}{dx}\left(e^{u}\right)=e^{u}u'$

$$= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\left[\frac{-2e^{2x}-2e^{2x}\cdot e^{2x}-2e^{2x}+2e^{2x}\cdot e^{2x}}{\left(1+e^{2x}\right)^2}\right]$$

$$= -\sin\left(\frac{1-e^{2x}}{1+e^{2x}}\right)\left[\frac{-4e^{2x}}{\left(1+e^{2x}\right)^2}\right]$$

$$= -\frac{4e^{2x}}{\left(1+e^{2x}\right)^2}\sin\left(\frac{e^{2x}-1}{1+e^{2x}}\right)$$

Therefore, the derivative of the given function is,

$$y' = \boxed{-\frac{4e^{2x}}{\left(1+e^{2x}\right)^2}\sin\left(\frac{e^{2x}-1}{1+e^{2x}}\right)}.$$

Answer 50E.

Consider the function,

$$f(t)=\sin^2\left(e^{\sin^2 t}\right).$$

Need to find the derivative of the function.

Use chain rule: $\frac{d}{dx}\left(f\left(g(x)\right)\right)=f'\left(g(x)\right)g'(x)$.

Let $e^{\sin^2 t}=u$

Differentiate on both sides, we have

$$\frac{du}{dt}=\frac{d}{dt}e^{\sin^2 t}$$

$$=e^{\sin^2 t}\frac{d}{dt}\left(\sin^2 t\right)$$

$$=e^{\sin^2 t}\left(2\sin t\cos t\right)$$

$$=\sin(2t)e^{\sin^2 t}$$

Use the substitution $e^{\sin^2 t}=u$ then, the function can be reduced as,

$$f(t)=\sin^2(u)$$

Differentiate the above function on both sides, we have

$$\begin{aligned} f'(t) &= \frac{d}{du}\sin^2(u) \\ &= 2\sin u \frac{d}{dx}\left[\sin(u)\right]\frac{du}{dt} \\ &= 2\sin u\cos u\frac{du}{dt} \\ &= \sin 2u\frac{du}{dt} \end{aligned}$$

Plug $\frac{du}{dt} = \sin(2t)e^{\sin^2 t}$ in the above result, we have

$$\begin{aligned} f'(t) &= \sin 2u\frac{du}{dt} \\ &= \sin 2u\sin(2t)e^{\sin^2 t} && \text{Use } \frac{du}{dt} = \sin(2t)e^{\sin^2 t} \\ &= \sin\left(2e^{\sin^2 t}\right)\sin(2t)e^{\sin^2 t} && \text{Use } u = e^{\sin^2 t} \end{aligned}$$

Hence, the derivative of the function $f(t) = \sin^2\left(e^{\sin^2 t}\right)$ is,

$$f'(t) = \boxed{\sin\left(2e^{\sin^2 t}\right)\sin(2t)e^{\sin^2 t}}.$$

Answer 51E.

The equation of given curve is

$$y = e^{2x}\cos\pi x$$

Differentiating with respect to x, we get,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}\left(e^{2x}\cos\pi x\right) \\ &= e^{2x}\frac{d}{dx}\cos\pi x + \cos\pi x\frac{d}{dx}e^{2x} && \text{[product rule]} \\ &= e^{2x}\left(-\sin\pi x\right)\cdot\pi + \cos\pi x\cdot\left(e^{2x}\right)\cdot 2 && \text{[chain rule]} \\ &= e^{2x}\left[-\pi\sin\pi x + 2\cos\pi x\right] \end{aligned}$$

Slope of tangent $= \frac{dy}{dx}$

Slope of tangent at point $(0,1)$

$$\begin{aligned} &= \left(\frac{dy}{dx}\right)_{(0,1)} \\ &= e^{2\times 0}\left[-\pi\sin(\pi\times 0) + 2\cos(\pi\times 0)\right] \\ &= e^0\left[-\pi\sin 0 + 2\cos 0\right] \\ &= 1\left[-\pi\times 0 + 2\times 1\right] \\ &= 2 \end{aligned}$$

The equation of tangent to the given curve at point (x_1, y_1) is

$$y - y_1 = \left(\frac{dy}{dx}\right)(x - x_1)$$

Therefore, the equation of tangent to the given curve at point $(0,1)$ is

$$y - 1 = 2(x - 0)$$

$$\Rightarrow \quad y - 1 = 2x$$

$$\Rightarrow \quad 2x - y + 1 = 0 \quad \text{or } y = 2x + 1$$

Hence,

$$\boxed{\begin{array}{l}\text{Equation of tangent is} \\ \quad y = 2x + 1\end{array}}$$

Answer 52E.

The equation of curve is

$$y = \frac{e^x}{x}$$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\frac{e^x}{x} \qquad \text{By "Quotient rule"}$$

$$= \frac{x\frac{d}{dx}e^x - e^x\frac{d}{dx}x}{x^2}$$

$$= \frac{xe^x - e^x \cdot 1}{x^2}$$

$$= \frac{e^x(x-1)}{x^2}$$

The slope of the tangent $= \frac{dy}{dx}$

Therefore, the slope of the tangent at point (1,e)

$$= \left(\frac{dy}{dx}\right)_{(1,e)} = \frac{e^1(1-1)}{1^2} = 0$$

The equation of tangent at point (x_1, y_1) to the curve $y = f(x)$ is

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

Therefore, the equation of tangent to the given curve at point (1,e) is

$$y - e = 0(x-1)$$

$$\Rightarrow \quad y - e = 0 \quad \text{or} \quad y = e$$

Hence,

The equation of the tangent line is y = e.

Answer 53E.

Consider the following function:

$$e^{x/y} = x - y$$

Differentiate the equation with respect to x.

$$\frac{d}{dx}\left(e^{x/y}\right) = \frac{d}{dx}(x-y)$$

$$e^{x/y}\frac{d}{dx}\left(\frac{x}{y}\right) = 1 - \frac{dy}{dx} \quad \text{Use Chain Rule for left hand side.}$$

$$e^{x/y}\left(\frac{y \cdot 1 - x \cdot \frac{dy}{dx}}{y^2}\right) = 1 - \frac{dy}{dx} \quad \text{Use Quotient Rule for differentiation.}$$

$$e^{x/y}\left(y - x\frac{dy}{dx}\right) = y^2\left(1 - \frac{dy}{dx}\right) \quad \text{Multiply each side by } y^2$$

Continue simplifying the equation.

$$e^{x/y}y - e^{x/y}x\frac{dy}{dx} = y^2 - y^2\frac{dy}{dx}$$

$$y^2\frac{dy}{dx} - e^{x/y}x\frac{dy}{dx} = y^2 - e^{x/y}y$$

$$\left(y^2 - xe^{x/y}\right)\frac{dy}{dx} = y^2 - e^{x/y}y$$

$$\frac{dy}{dx} = \frac{y^2 - e^{x/y}y}{y^2 - xe^{x/y}}$$

$$= \frac{y\left(y - e^{x/y}\right)}{y^2 - xe^{x/y}}$$

Hence, $y' = \boxed{\frac{y\left(y - e^{x/y}\right)}{y^2 - xe^{x/y}}}$.

Answer 54E.

The equation of the given curve is

$$xe^y + ye^x = 1$$

Differentiating with respect to x, we get,

$$\frac{d}{dx}xe^y + \frac{d}{dx}ye^x = \frac{d}{dx}1$$

$$\Rightarrow \quad x\frac{d}{dx}e^y + e^y\frac{d}{dx}x + y\frac{d}{dx}e^x + e^x\frac{d}{dx}y = 0 \qquad \text{[Product rule]}$$

$$\Rightarrow \quad x\cdot\left[e^y\frac{dy}{dx}\right]^{*} + e^y + y\cdot e^x + e^x\left[\frac{dy}{dx}\right]^{*} = 0 \qquad \text{[Chain rule]}$$

$$\Rightarrow \quad \frac{dy}{dx}\left[xe^y + e^x\right] = -\left[e^y + ye^x\right]$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{-\left(e^y + ye^x\right)}{\left(xe^y + e^x\right)}$$

The slope of tangent line $= \frac{dy}{dx}$

The slope of tangent line to the given curve at point (0, 1) is

$$= \left(\frac{dy}{dx}\right)_{(0,\,1)} = -\frac{\left(e^1 + 1\cdot e^0\right)}{\left(0\cdot e^1 + e^0\right)} = -\frac{(e+1)}{1}$$

We know that the equation of tangent line at point (x_1, y_1) is

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

Therefore, the equation of tangent to the given curve at point (0, 1) is

$$y - 1 = -(e+1)(x-0)$$

$$\Rightarrow \quad y - 1 = -(e+1)x$$

$$\Rightarrow \quad y = -(e+1)x + 1$$

Hence,

The equation of tangent is $y + (e+1)x = 1$

Answer 55E.

$$y = e^x + e^{-x/2}$$

$$y' = \frac{d}{dx}\left(e^x + e^{-x/2}\right)$$

$$y' = e^x + e^{-x/2}\cdot\left(-\frac{1}{2}\right) = \left(e^x - \frac{1}{2}e^{-x/2}\right)$$

$$y'' = \frac{d}{dx}\left[e^x - \frac{1}{2}e^{-x/2}\right] = \left[e^x - \frac{1}{2}e^{-x/2}\left(-\frac{1}{2}\right)\right]$$

$$= \left(e^x + \frac{1}{4}e^{-x/2}\right)$$

Left hand side

$$2y'' - y' - y = 2e^x + \frac{1}{2}e^{-x/2} - e^x + \frac{1}{2}e^{-x/2} - e^x - e^{-x/2}$$

$0 =$ Right hand side

Hence proved

Answer 56E.

$$y = Ae^{-x} + Bxe^{-x}$$

$$y' = \frac{d}{dx}\left[Ae^{-x} + Bxe^{-x}\right]$$

$$= A\frac{d}{dx}e^{-x} + B\frac{d}{dx}\left[xe^{-x}\right]$$

$$= Ae^{-x}(-1) + B\left(xe^{-x}(-1) + e^{-x}\right)$$

$$= -Ae^{-x} - Bxe^{-x} + Be^{-x}$$

$$y'' = \frac{d}{dx}\left[-Ae^{-x} - Bxe^{-x} + Be^{-x}\right]$$

$$= -A\frac{d}{dx}e^{-x} - B\frac{d}{dx}\left(xe^{-x}\right) + B\frac{d}{dx}e^{-x}$$

$$= -Ae^{-x}(-1) - B\left[xe^{-x}(-1) + e^{-x}\right] + Be^{-x}(-1)$$

$$= Ae^{-x} + Be^{-x}x - Be^{-x} - Be^{-x}$$

Left hand side

$$y'' + 2y' + y = \cancel{Ae^{-x}} + \cancel{Be^{-x}x} - \cancel{Be^{-x}} - \cancel{Be^{-x}} - \cancel{2Ae^{-x}} - \cancel{2Be^{-x}x} + \cancel{2Be^{-x}} + \cancel{Ae^{-x}} + \cancel{Bxe^{-x}}$$

$= 0$
$=$ Right hand side
Hence proved

Answer 57E.

Given $y = e^{rx}$
Differentiating with respect to x,

$$y' = \frac{d}{dx} e^{rx}$$

$$\Rightarrow \quad y' = e^{rx} \cdot r$$

Again differentiating with respect to x, we get,

$$y'' = \frac{d}{dx} \cdot re^{rx}$$
$$= r \cdot e^{rx} \cdot r$$
$$= r^2 e^{rx}$$

If y is a solution of the equation

$y'' + 6y' + 8y = 0$ then putting values of y, y' and y'' we get

$$r^2 e^{rx} + 6re^{rx} + 8e^{rx} = 0$$

$$\Rightarrow \quad e^{rx}\left[r^2 + 6r + 8\right] = 0$$

Since $e^{rx} \neq 0$ therefore, $r^2 + 6r + 8 = 0$

$$\Rightarrow \quad r^2 + 4r + 2r + 8 = 0$$

$$\Rightarrow \quad r(r+4) + 2(r+4) = 0$$

$$\Rightarrow \quad (r+4)(r+2) = 0$$

If $r + 4 = 0 \Rightarrow r = -4$
If $r + 2 = 0 \Rightarrow r = -2$
Hence,

$$\boxed{\text{Possible values of r are } -4 \text{ and } -2.}$$

Answer 58E.

If $y = e^{\lambda x}$
Then differentiating both sides with respect to x, we get,

$$y' = \frac{d}{dx} e^{\lambda x} = e^{\lambda x} \cdot \lambda$$

$$y' = \lambda e^{\lambda x}$$

Again differentiating with respect to x,

$$y'' = \frac{d}{dx} \lambda e^{\lambda x}$$
$$= \lambda \frac{d}{dx} e^{\lambda x}$$
$$= \lambda e^{\lambda x} \cdot \lambda$$
$$= \lambda^2 e^{\lambda x}$$

Now putting y, y' and y'' in the given equation $y'' = y + y'$ we get,

$$\lambda^2 e^{\lambda x} = e^{\lambda x} + \lambda e^{\lambda x}$$

$$\Rightarrow \quad \lambda^2 e^{\lambda x} - \lambda e^{\lambda x} - e^{\lambda x} = 0$$

$$\Rightarrow \quad e^{\lambda x}\left[\lambda^2 - \lambda - 1\right] = 0$$

Since $e^{\lambda x} \neq 0$ therefore,

$$\lambda^2 - \lambda - 1 = 0$$

$$\Rightarrow \quad \lambda = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 1(-1)}}{2 \times 1}$$

$$= \frac{1 \pm \sqrt{1+4}}{2}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

Hence

$$\boxed{\lambda = \frac{1 \pm \sqrt{5}}{2}}$$

Answer 59E.

Given $f(x)=e^{2x}$

Differentiating with respect to x, we get,

$$f'(x)=\frac{d}{dx}f(x)=\frac{d}{dx}e^{2x}$$
$$=e^{2x}\cdot 2$$
$$=2e^{2x}$$

Again differentiating with respect to x,

$$f''(x)=\frac{d}{dx}f'(x)=\frac{d}{dx}2e^{2x}$$
$$=2\frac{d}{dx}e^{2x}$$
$$=2\cdot e^{2x}$$
$$=2^2e^{2x}$$

Again differentiating with respect to x,

$$f'''(x)=\frac{d}{dx}f''(x)=\frac{d}{dx}2^2e^{2x}$$
$$=2^2\frac{d}{dx}e^{2x}$$
$$=2^2\cdot e^{2x}\cdot 2$$
$$=2^3e^{2x}$$

Proceeding in the same way, differentiating n times, we get by inspection

$$f^n(x)=2^ne^{2x}$$

Hence

$$\boxed{f^n(x)=2^ne^{2x}}$$

Answer 60E.

We have to find the thousandth derivative of $f(x)=xe^{-x}$

We have $f(x)=xe^{-x}$

Differentiating with respect to x,

$$f'(x)=\frac{d}{dx}f(x)=\frac{d}{dx}\left(xe^{-x}\right)$$
$$=x\frac{d}{dx}e^{-x}+e^{-x}\frac{d}{dx}x \quad \text{using product rule.}$$
$$=xe^{-x}\frac{d}{dx}(-x)+e^{-x}.1$$
$$=xe^{-x}(-1)+e^{-x}$$
$$=e^{-x}(1-x)$$

We have $f'(x)=e^{-x}(1-x)$

Differentiating with respect to x,

$$f''(x)=\frac{d}{dx}f'(x)=\frac{d}{dx}e^{-x}(1-x)$$
$$=e^{-x}\frac{d}{dx}(1-x)+(1-x)\frac{d}{dx}e^{-x} \quad \text{using product rule}$$
$$=e^{-x}(0-1)+(1-x)e^{-x}\frac{d}{dx}(-x)$$
$$=-e^{-x}+(1-x)e^{-x}(-1)$$
$$=e^{-x}[-1-1+x]$$
$$=-e^{-x}(2-x)$$
$$=(-1)^1e^{-x}(2-x)$$
$$=(-1)^{2-1}e^{-x}(2-x)$$

We have $f''=(-1)^1(e)^{-x}(2-x)$.

Differentiating with respect to x,

$$f'''(x)=\frac{d}{dx}f''(x)=\frac{d}{dx}(-1)^1e^{-x}(2-x)$$

$$=(-1)^1\frac{d}{dx}e^{-x}(2-x)$$

$$=(-1)^1\left[e^{-x}\frac{d}{dx}(2-x)+(2-x)\frac{d}{dx}e^{-x}\right) \qquad \text{using product rule.}$$

$$=(-1)^1\left[e^{-x}(0-1)+(2-x)e^{-x}\frac{d}{dx}(-x)\right)$$

$$=(-1)^1\left[-e^{-x}+(2-x)e^{-x}(-1)\right)$$

$$=(-1)^1e^{-x}[-1-2+x]$$

$$=(-1)^1e^{-x}(-3+x)$$

$$=(-1)^1e^{-x}(-1)(3-x)$$

$$=(-1)^2e^{-x}(3-x)$$

$$=(-1)^3e^{-x}(3-x)$$

Now, we have $f'''(x)=(-1)^2e^{-x}(3-x)$.

Differentiating with respect to x, we get,

$$f^{iv}(x)=\frac{d}{dx}f'''(x)=\frac{d}{dx}(-1)^2e^{-x}(3-x)$$

$$=(-1)^2\frac{d}{dx}e^{-x}(3-x)$$

$$=(-1)^2\left[e^{-x}\frac{d}{dx}(3-x)+(3-x)\frac{d}{dx}e^{-x}\right]$$

$$=(-1)^2\left[e^{-x}(0-1)+(3-x)e^{-x}\frac{d}{dx}(-x)\right]$$

$$=(-1)^2\left[-e^{-x}(3-x)e^{-x}(-1)\right]$$

$$=(-1)^2e^{-x}[-1-3+x]$$

$$=(-1)^2e^{-x}(-1)[4-x]$$

$$=(-1)^3e^{-x}(4-x)$$

$$=(-1)^{4-1}e^{-x}(4-x)$$

Proceeding in the same way we find 1000^{th} derivative of $f(x)$ by inspection as

$$f^{1000}(x)=(-1)^{1000-1}e^{-x}(1000-x)$$

$$=(-1)^{999}e^{-x}(1000-x)$$

$$=-e^{-x}(1000-x)$$

$$=\boxed{e^{-x}(x-1000)}$$

Answer 61E.

(a)

Consider the function,

$f(x)=e^x+x$.

Recall the Intermediate Value Theorem:

If f is continuous on the closed interval [*a*, *b*] and *N* is any number between $f(a)$ and $f(b)$, where $f(a)\neq f(b)$, then there exists a number *c* in (*a*, *b*) such that $f(c)=N$.

The function $f(x)=e^x+x$ is continuous because ex and *x* are both continuous on R.

For our convenience, consider the interval $[-1,1]$

So the functional value is,

$$\begin{aligned} f(-1)&=e^{-1}-1 \\ &=-0.63212 \end{aligned}$$

And,

$$\begin{aligned} f(1)&=e^{1}+1 \\ &=3.7183 \end{aligned}$$

Moreover, since $f(-1)<0<f(1)$, there is a number *c* in $(-1,1)$ such that $e^x+x=0$.

Hence by the Intermediate Value Theorem, the equation $e^x+x=0$ has root.

(b)

Now find the root of the equation $e^x+x=0$ correct to six decimal places using Newton's method.

Recall the *n*th approximation x_n of Newton's method,

$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$

Differentiate the function $f(x)=e^x+x$ with respect to x.

$$\begin{aligned} \frac{d}{dx}f(x)&=\frac{d}{dx}(e^x+x) \\ f'(x)&=e^x+1 \end{aligned}$$

Starting with $x_1=-1$, find x_2 using Newton's method for the function $f(x)=e^x+x$

$$\begin{aligned} x_2&=x_1-\frac{f(x_1)}{f'(x_1)} \\ &=-1-\frac{e^{-1}-1}{e^{-1}+1} \\ &=-1-(-0.46211) \\ &=-0.53789 \end{aligned}$$

Now find next iteration as,

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= -0.53789 - \frac{e^{-0.53789} - 0.53789}{e^{-0.53789} + 1}$$
$$= -0.53789 - (0.029097)$$
$$= -0.566987$$

Now find next iteration as,

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$
$$= -0.566987 - \frac{e^{-0.566987} - 0.566987}{e^{-0.566987} + 1}$$
$$= -0.566987 - (0.0001562)$$
$$= -0.567143$$

Find next iteration as,

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)}$$
$$= -0.567143 - \frac{e^{-0.567143} - 0.567143}{e^{-0.567137} + 1}$$
$$= -0.567143 - (0.00000629)$$
$$= -0.567143$$

Observe that, the last two iterations both 4th and 5th are same for six decimal places.

Hence, the root of the equation $f(x) = e^x + x$ is

$\boxed{x = -0.567143}$.

Answer 65E.

The given function is

$$f(x) = x - e^x$$

Differentiating with respect to x,

$$f'(x) = \frac{d}{dx}(x - e^x)$$
$$= \frac{d}{dx}x - \frac{d}{dx}e^x$$
$$= 1 - e^x$$

For critical points $f'(x) = 0$

$\Rightarrow \quad 1 - e^x = 0$

$\Rightarrow \quad e^x = 1$

$\Rightarrow \quad x = 0$

Here $f'(x)$ will be increasing if $f'(x) > 0$

$\Rightarrow \quad 1 - e^x > 0 \quad \Rightarrow \quad 1 > e^x$

$\Rightarrow \quad e^x < 1$

$\Rightarrow \quad x < \ln 1 \quad$ i.e. $x < 0$

And $f'(x)$ will be decreasing if $f'(x) < 0$

$\Rightarrow \quad 1 - e^x < 0$

$\Rightarrow \quad 1 < e^x$

$\Rightarrow \quad e^x > 1$

$\Rightarrow \quad x > \ln 1 \quad \Rightarrow \quad x > 0$

We see that f(x) is increasing for x < 0 and decreasing for x > 0. Therefore, by first derivative test for absolute extreme values, f(x) has an absolute maximum value at x = 0.

And absolute maximum value of f(x) is

$$f(x) = 0 - e^0$$
$$= 0 - 1$$
$$= -1$$

Hence,

$$\boxed{\text{Absolute maximum value of } f(0) = -1}$$

Answer 66E.

Consider the function,

$$g(x)=\frac{e^x}{x}, x>0.$$

Need to find the absolute minimum of the above function.

Differentiate the function $g(x)=\frac{e^x}{x}$ on both sides, we have

$$g'(x)=\frac{xe^x-e^x\cdot 1}{x^2} \qquad \text{Use } \frac{d}{dx}\left(\frac{u}{v}\right)=\frac{vu'-uv'}{v^2}$$

$$=\frac{e^x(x-1)}{x^2}$$

For critical points set $g'(x)=0$, it follows that

$$\frac{e^x(x-1)}{x^2}=0$$

$$x-1=0 \qquad \text{Since } e^x\neq 0$$

$$x=1$$

Again differentiate the function $g'(x)=\frac{e^x(x-1)}{x^2}$ on both sides, we have

$$g''(x)=\frac{d}{dx}\left[\frac{e^x(x-1)}{x^2}\right]$$

$$=\frac{x^2\frac{d}{dx}\left(e^x(x-1)\right)-e^x(x-1)\frac{d}{dx}\left(x^2\right)}{x^4}$$

$$=\frac{x^2\left[e^x\cdot 1+(x-1)e^x\right]-e^x(x-1)2x}{x^4}$$

$$=\frac{x\left[e^x\cdot 1+(x-1)e^x\right]-e^x(x-1)2}{x^3}$$

$$=\frac{xe^x+x^2e^x-xe^x-2xe^x+2e^x}{x^3}$$

$$=\frac{e^x\left(x^2-2x+2\right)}{x^3}$$

At $x=1$, then

$$g''(1)=\frac{e^1\left(1^2-2(1)+2\right)}{(1)^3}$$

$$=e$$

$$>0$$

Here, $g''(x)>0$ when $x=1$.

So the function has absolute minimum at $x=1$.

Plug $x=1$ in $g(x)=\frac{e^x}{x}$, we have

$$g(1)=\frac{e^1}{1}$$

$$=e$$

Hence, the absolute minimum of the function is $\boxed{e}$.

Answer 67E.

Consider the following function and the interval:

$f(x) = xe^{-x^2/8}, [-1,4]$.

To find the absolute maximum and absolute minimum values of the function f.

Find the derivative of the function and then equate it to zero.

First find $\frac{d}{dx}(f(x))$.

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(xe^{-x^2/8}\right)$$
$$= x\cdot\frac{d}{dx}\left(e^{-x^2/8}\right) + \left[\frac{d}{dx}(x)\right]e^{-x^2/8} \quad \text{Use } \frac{d}{dx}(uv) = uv' + vu'$$
$$= x\cdot e^{-x^2/8}\left(\frac{-2x}{8}\right) + e^{-x^2/8}$$
$$= e^{-x^2/8}\left(-\frac{x^2}{4}+1\right)$$
$$= e^{-x^2/8}\left(\frac{-x^2+4}{4}\right)$$

Now equate $\frac{d}{dx}(f(x))$ to zero.

That is,

$$e^{-x^2/8}\left(\frac{-x^2+4}{4}\right) = 0$$
$$-x^2+4 = 0$$
$$x^2 = 4$$
$$x = \pm 2$$

But only $+2 \in [-1,4]$.

Hence, the function $f(x)$ has either maximum or minimum at $x = 2$.

Now again differentiate the function with respect to *x*,

$$f''(x) = \frac{d}{dx}(f'(x))$$
$$= \frac{d}{dx}\left[e^{-x^2/8}\left(-\frac{x^2}{4}+1\right)\right]$$
$$= e^{-x^2/8}\left(\frac{-2x}{4}\right) + e^{-x^2/8}\left(\frac{-2x}{8}\right)\cdot\left(-\frac{x^2}{4}+1\right) \quad \text{Use } \frac{d}{dx}(uv) = uv' + vu'$$
$$= \frac{-x}{2}e^{-x^2/8} - \frac{x}{4}\left(1-\frac{x^2}{4}\right)e^{-x^2/8}$$

Substitute $x = 2$ in the function $f''(x) = \frac{-x}{2}e^{-x^2/8} - \frac{x}{4}\left(1-\frac{x^2}{4}\right)e^{-x^2/8}$.

$$f''(2) = \frac{-2}{2}e^{-(2)^2/8} - \frac{2}{4}\left(1-\frac{2^2}{4}\right)e^{-(2)^2/8}$$
$$= -e^{-1/2} - \left(\frac{1}{2}(1-1)\right)e^{-1/2}$$
$$= -e^{-1/2} - \frac{1}{2}(0)e^{-1/2}$$
$$= -e^{-1/2} < 0$$

The function $f(x)$ has absolute maximum at $x = 2$ in $[-1,4]$.

The absolute maximum value of $f(x)$ in the interval $[-1,4]$ is as follows:

$$f(x) = xe^{-x^2/8}$$
$$f(2) = 2e^{-4/8}$$
$$f(2) = 2e^{-1/2}$$

The absolute minimum value of $f(x)$ in the interval $[-1,4]$ is as follows:

$$f(x) = xe^{-x^2/8}$$
$$\begin{aligned} f(-1) &= (-1)e^{-(-1)^2/8} \\ &= -e^{-\frac{1}{8}} \\ &= \frac{-1}{\sqrt[8]{e}} \end{aligned}$$

Therefore, the absolute maximum and minimum values of $f(x)$ in the interval $[-1,4]$ is

$\boxed{f(2) = 2e^{-1/2}}$, $\boxed{f(-1) = \frac{-1}{\sqrt[8]{e}}}$.

Answer 68E.

Consider the following function:

$$f(x) = x^2 e^{-\frac{x}{2}}$$

The objective is to find the absolute maximum and absolute minimum of the function $f(x)$ on the interval $[-1,6]$.

Differentiate the function $f(x)$ with respect to *x*.

$$\begin{aligned} f'(x) &= x^2 e^{-\frac{x}{2}}\left(-\frac{1}{2}\right) + e^{-\frac{x}{2}}(2x) \\ &= e^{-\frac{x}{2}}\left(2x - \left(\frac{1}{2}\right)x^2\right) \end{aligned}$$

To find the critical number, $f'(x) = 0$

$$e^{-\frac{x}{2}}\left(2x - \left(\frac{1}{2}\right)x^2\right) = 0$$

Here, $e^{-\frac{x}{2}}$ is always positive, and $\left(2x - \left(\frac{1}{2}\right)x^2\right) = 0$

Take x is common on both sides,

$$x\left(2 - \left(\frac{1}{2}\right)x\right) = 0$$
$$x\left(\frac{4-x}{2}\right) = 0$$

$x = 0$ or $x = 4$

Evaluate $f(x)$ at each critical number in the interval and at the endpoints. $f(0)=(0)^2 e^0$
$=0$

$$f(4)=4^2\cdot e^{-\left(\frac{4}{2}\right)}$$
$$=16\cdot e^{-2}$$
$$=16\cdot(0.135)$$
$$=2.165$$

$$f(-1)=(-1)^2\cdot e^{-\left(\frac{-1}{2}\right)}$$
$$=1\cdot e^{\frac{1}{2}}$$
$$=1.648$$

$$f(6)=6^2\cdot e^{-\left(\frac{6}{2}\right)}$$
$$=36\cdot e^{-3}$$
$$=36(0.0498)$$
$$=1.7928$$

The largest value obtained in the interval $[-1,6]$ is absolute maximum of the function.

Absolute maximum value of the function $f(x)$ is $\boxed{2.165}$ on $[-1,6]$.

The smallest value obtained in the interval $[-1,6]$ is absolute minimum of the function.

Absolute minimum value of the function $f(x)$ is $\boxed{0}$ on $[-1,6]$.

Answer 69E.

Consider the following function:

$$f(x)=(1-x)e^{-x}$$

(a)

Find the first derivative of the function.

Differentiate the function $f(x)$ with respect to *x*.

$$f'(x)=\frac{d}{dx}(1-x)e^{-x}$$

Use product rule to differentiate the function.

$$\frac{d}{dx}(uv)=u\frac{d}{dx}(v)+v\frac{d}{dx}(u)$$

$f'(x)=(1-x)\frac{d}{dx}\left(e^{-x}\right)+e^{-x}\frac{d}{dx}(1-x)$ Use product rule

$=(1-x)\left(-e^{-x}\right)+e^{-x}(-1)$ Since $\frac{d}{dx}\left(e^{-ax}\right)=-e^{-ax}\frac{d}{dx}(ax)$

$$=-(1-x)\left(e^{-x}\right)-e^{-x}$$

$=-\left(e^{-x}\right)(1-x+1)$ Simplify

$$f'(x)=-\left(e^{-x}\right)(2-x)$$

$$f'(x)=\left(e^{-x}\right)(x-2)$$

Find the intervals of increase.

A function is said to be increasing, if $f'(x) > 0$ on an interval.

$$f'(x) > 0$$
$$(e^{-x})(x-2) > 0$$
$$(x-2) > 0$$
$$x > 2$$

By the increasing Test, the curve is increasing, when $x > 2$.

Therefore, the function is increasing on the intervals $\boxed{(2,\infty)}$.

Find the intervals of decrease.

A function is said to be decreasing, if $f'(x) < 0$ on an interval.

$$f'(x) < 0$$
$$(e^{-x})(x-2) < 0$$
$$(x-2) < 0$$
$$x < 2$$

By the decreasing Test, the curve is decreasing, when $x < 2$.

Therefore, the function is decreasing in the interval $\boxed{(-\infty,2)}$.

(b)

Find the intervals of concavity.

The concavity test

The graph of the function is concave upward, if $f''(x) > 0$ on some interval *I*.

The graph of the function is concave downward, if $f''(x) < 0$ on some interval *I*.

Use the Concavity Test.

Use the product Rule to find the second derivative of f.

$$f'(x) = (e^{-x})(x-2)$$
$$f''(x) = \frac{d}{dx}\left[(e^{-x})(x-2)\right]$$

Use product rule to differentiate the function.

$$= (e^{-x})\cdot\frac{d}{dx}(x-2) + (x-2)\cdot\frac{d}{dx}(e^{-x}) \text{ Since } \frac{d}{dx}(uv) = u\frac{d}{dx}(v) + v\frac{d}{dx}(u)$$
$$= (e^{-x})\cdot(1) + (x-2)\cdot(-e^{-x})$$
$$= e^{-x} - e^{-x}(x-2)$$
$$= e^{-x}(1-x+2)$$
$$= e^{-x}(3-x)$$

Therefore, $f''(x) = e^{-x}(3-x)$.

The graph of the function is concave upward, if $f''(x) > 0$ on some interval *I*.

$$f''(x) > 0$$
$$e^{-x}(3-x) > 0$$
$$(3-x) > 0$$
$$3 > x$$

Therefore, the curve is concave upward in the interval $\boxed{(-\infty,3)}$.

The graph of the function is concave downward, if $f''(x)<0$ on some interval *I*.

$f''(x)<0$

$e^{-x}(3-x)<0$

$(3-x)<0$

$3<x$

Therefore, the curve is concave downward in the interval $\boxed{(3,\infty)}$.

(c)

Find the point of inflection.

Equate $f''(x)$ to zero.

$$\begin{aligned} f''(x)&=0 \\ e^{-x}(3-x)&=0 \\ 3-x&=0 \\ x&=3 \end{aligned}$$

Hence, the curve changes its position at $x=3$.

$f(x)=(1-x)e^{-x}$

$f(3)=(1-3)e^{-3}$

$=-2e^{-3}$

Therefore, the inflection point is $\boxed{(3,-2e^{-3})}$.

Answer 70E.

Consider the function

$$f(x)=\frac{e^x}{x^2}$$

(a)

Find the first derivative of the function. To do so, differentiate the function $f(x)$ with respect to *x*.

Use quotient rule to differentiate the function. State the product rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)\frac{d}{dx}[f(x)]-f(x)\frac{d}{dx}[g(x)]}{(g(x))^2}$$

$$f'(x)=\frac{x^2\frac{d}{dx}(e^x)+e^x\frac{d}{dx}(x^2)}{x^4} \quad \text{Use quotient rule}$$

$$=\frac{x^2e^x+e^x(2x)}{x^4}$$

$$=\frac{x(x-2)e^x}{x^4} \quad \text{Simplify}$$

$$=\frac{(x-2)e^x}{x^3}$$

Increasing/Decreasing Test:

A function is said to be increasing if $f'(x)>0$ on an interval and is decreasing if $f'(x)<0$ on an interval.

To apply I/D test first check for the sign of the terms of the expression.

When $x<0$, the numerator and denominator are both negative; when $0<x<2$, the numerator is negative and the denominator is positive; and when $x>2$, the numerator and denominator are both positive. Thus, the first derivative is positive when $x<0$ or $x>2$ and negative when $0<x<2$.

By the Increasing/Decreasing Test, the curve is increasing when $x<0$ or $x>2$ and decreasing when $0<x<2$.

Therefore, the function is increasing on the intervals $\boxed{(-\infty,0),(2,\infty)}$ and decreasing in the interval $\boxed{(0,2)}$.

(b)

The concavity test:

The graph of the function is concave upward if $f''(x)>0$ on some interval *I*.

The graph of the function is concave down ward if $f''(x)<0$ on some interval *I*.

Use the Concavity Test. Use the Quotient Rule to find the second derivative of f.

$$f''(x)=\frac{d}{dx}\left[\frac{(x-2)e^x}{x^3}\right]$$

$$=\frac{x^3\cdot\frac{d}{dx}\left[(x-2)e^x\right]-(x-2)e^x\cdot\frac{d}{dx}(x^3)}{(x^3)^2}$$

$$=\frac{x^3\left[(x-2)e^x+e^x\right]-3x^2(x-2)e^x}{x^6}$$

$$=\frac{x^3(x-1)e^x-3x^2(x-2)e^x}{x^6}$$

Continue evaluating the second derivative.

$$f''(x)=\frac{(x^4-x^3)e^x+(-3x^3+6x^2)e^x}{x^6}$$

$$=\frac{(x^4-4x^3+6x^2)e^x}{x^6}$$

$$=\frac{x^2(x^2-4x+6)e^x}{x^6}$$

$$=\frac{(x^2-4x+6)e^x}{x^4}$$

The function $y=x^2-4x+6$ is always positive.

Therefore, the curve is concave upward for all real numbers.

(c)

The curve has no points of inflection since $f''(x)\neq 0$ for any values of *x*.

Answer 71E.

Consider

$y=e^{-1/(x+1)}$ ...... (1)

A.

Domain:

Domain is set of all values that x take so that the function $f(x)$ is defined.

The function is an exponential function defined for all real numbers.

Domain of the function is the set of all real numbers.

B.

Intercepts:

The x-intercepts are values obtained by substituting $y=0$.

Substitute $y=0$ in (1)

$$e^{-1/(x+1)}=0$$

$$\frac{-1}{x+1}=0$$

So there are no x- intercepts.

The y-intercepts are values obtained by substituting $x=0$.

Substitute $x=0$ in (1)

$$\begin{aligned}y&=e^{-1/(0+1)}\\&=e^{-1}\\&=\frac{1}{e}\end{aligned}$$

Therefore, the y intercept is $\left(0,\frac{1}{e}\right)$.

C.

The function is neither an even function nor an odd function.

And also it is not periodic.

The symmetry is none

D.

Vertical Asymptote is the x-value at which the function is undefined.

The given curve has a vertical asymptote at $x=-1$ because the function not defined for this value.

Horizontal Asymptote is the value the function cannot take.

As x –8 or 8 $-1/(x+1)$ approaches 0, so $e^{-1/(x+1)}$ approaches 1.

Therefore, the curve has a horizontal asymptote at $y=1$.

E.

The intervals of increase and decrease are determined by the critical points of the function. The critical points of the function occur when the derivative of the function equals zero.

Find the derivative of the function (1)

$$f(x)=e^{-1/(x+1)} \quad y=f(x)$$

$$f'(x)=e^{\frac{-1}{x+1}}\frac{d}{dx}\left(\frac{-1}{x+1x}\right) \text{ Use chain rule ...... (2)}$$

$$=e^{\frac{-1}{x+1}}\left(\frac{1}{(x+1)^2}\right)$$

The numerator and denominator of the derivative are positive when the function is defined (i.e. when $x\neq-1$), so the curve is increasing when $x\neq-1$.

Use increasing/decreasing test of first derivative:

$$f'(-1)=\frac{2}{e^2}-\frac{1}{e}<0$$

$$f'(0)=2e^0-e^0=2-1>0$$

Therefore, the function is increasing on the interval $\boxed{(-\infty,-1)\cup(-1,\infty)}$.

F.

The value of f' does not change its sign. So, the function has no local minimum or minimum.

G.

The intervals of concavity and the inflection points are determined by the second derivative of the function. The inflection points occur where the second derivative of the function equals zero.

Find the second derivative of the function (1)

Differentiate (2) with respect to *x* on both sides.

$$f''(x)=\frac{d}{dx}\left[\frac{e^{-1/(x+1)}}{(x+1)^2}\right]$$

$$=\frac{(x+1)^2\cdot\frac{d}{dx}\left[e^{-1/(x+1)}\right]-e^{-1/(x+1)}\cdot\frac{d}{dx}\left[(x+1)^2\right]}{(x+1)^4}$$

$$=\frac{e^{-1/(x+1)}-2(x+1)e^{-1/(x+1)}}{(x+1)^4}$$

$$=\frac{(-2x-1)e^{-1/(x+1)}}{(x+1)^4}$$

Equate $y''(x)$ to zero:

$$f''(x)=0$$

$$\frac{(-2x-1)e^{-1/(x+1)}}{(x+1)^4}=0$$

$$(-2x-1)=0$$

$$2x+1=0$$

$$2x=-1$$

$$x=\frac{-1}{2}$$

The second derivative test,

The curve changes is concave upward if $f''(x)>0$ and concave downward if $f''(x)<0$

And the inflection points are the points at which the curve changes its direction of concavity.

Concavity: To check concavity, apply the second derivative test.

For $x<-\frac{1}{2}$

$$f''(-2)=\frac{(-2(-2)-1)e^{-1/((-2)+1)}}{((-2)+1)^4}>0$$

For $-1<x<-\frac{1}{2}$

$$f''(-0.75)=\frac{(-2(-0.75)-1)e^{-1/((-0.75)+1)}}{((-0.75)+1)^4}>0$$

For $x>-\frac{1}{2}$

$$f''(0)=\frac{(-2(0)-1)e^{-1/((0)+1)}}{((0)+1)^4}<0$$

The curve is concave downward for $x > \frac{-1}{2}$ and concave upward for $x < \frac{-1}{2}$ and $-1 < x < -\frac{1}{2}$.

The curve changes its position at $x = \frac{-1}{2}$.

$$f\left(-\frac{1}{2}\right) = e^{-\frac{1}{\left(-\frac{1}{2}+1\right)}}$$

$$= e^{-\frac{1}{\frac{1}{2}}}$$

$$= e^{-2}$$

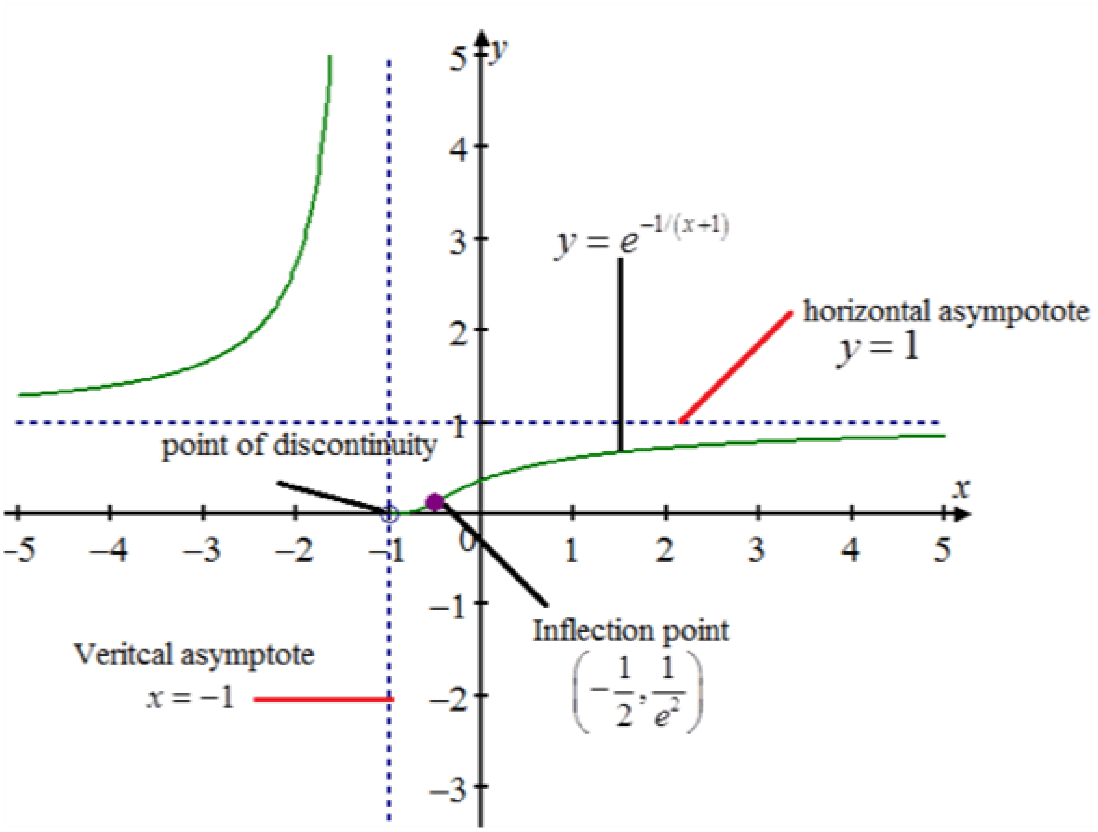
$$= \frac{1}{e^2}$$

Therefore, the inflection point is $\boxed{\left(-\frac{1}{2}, \frac{1}{e^2}\right)}$.

H.

Graphing:

Use the information from A to G to sketch curve of the function.



Answer 72E.

Consider the following curve:

$$y = e^{-x}\sin x, \quad 0 \le x \le 2\pi$$

(a)

Domain:

The function $e^{-x}\sin x$ exists because the functions e^{-x} and $\sin x$ are defined at every point in Real numbers system.

So, domain is $D = \mathbb{R}$ domain of the funcation $y = e^{-x}\sin x$

(b)

Intercepts:

The *x*- intercept is a point in the equation where the *y*-value is zero.

$y=0$ for the *x*-intercept,

$$e^{-x}\sin x=0$$
$$x=0$$
$$x=(2n+1)\pi,\ \text{for}\ n\in Z$$

Therefore, the *x*-intercept is $(2n+1)\pi,$ for $n\in Z$.

The *y*- intercept is a point in the equation where the *x*-value is zero.

$x=0$ for the *y*-intercept,

$$\begin{aligned} y&=e^{0}\sin(0)\\ &=0\\ y&=0 \end{aligned}$$

Therefore, the *y*- intercept is 0.

(c)

Symmetry:

When the function $f(-x)=e^{-(-x)}\sin(-x)$
$$=-e^{x}\sin x$$
$$\neq f(x)$$

So, the function is neither even nor odd.

If $f(x+p)=f(x)$ for all x in D, where *p* is a positive constant, then *f* is called a periodic function.

Here, $f(x+\pi)=f(x)$ for all *x* and so *f* is periodic and has period π.

Therefore, the interval is $0\le x\le 2\pi$.

(e)

Interval of increase or decrease:

$$\begin{aligned} f'(x)&=e^{-x}\cos x-e^{-x}\sin x\\ &=e^{-x}(\cos x-\sin x) \end{aligned}$$

Since the function is increasing when $f'(x)>0$

$$e^{-x}(\cos x-\sin x)>0$$

Here, e^{x} is always positive, $e^{-x}>0$ and $\cos x-\sin x>0$ only on the interval $\left(-\frac{3}{4}\pi,\frac{1}{4}\pi\right)$

$$\begin{aligned} f'(x)&=e^{-x}\cos x-e^{-x}\sin x\\ &=e^{-x}(\cos x-\sin x) \end{aligned}$$

Since the function is decreasing when $f'(x)<0$

$$e^{-x}(\cos x-\sin x)<0$$

Here, the function is $\cos x-\sin x<0$ on the interval $\left(\frac{1}{4}\pi,\frac{5}{4}\pi\right)$

Therefore, $f(x)$ is decreasing on the interval $\left(\frac{1}{4}\pi,\frac{5}{4}\pi\right)$.

(f)

Local maximum and minimum values:

$f'(x)=0$

$e^{-x}(\cos x-\sin x)=0$

$f''(x)=-2e^{-x}\cos x \quad x=\frac{1}{4}\pi$

$$\begin{aligned} f''(x) &= e^{-x}(-\sin x-\cos x)-e^{-x}(\cos x-\sin x) \\ &= -2e^{-x}\cos x \end{aligned}$$

Substitute for $x=\frac{1}{4}\pi$ in $f''(x)$.

$$\begin{aligned} f''\left(\frac{1}{4}\pi\right) &= 2e^{-\frac{1}{4}\pi}\cos\left(\frac{1}{4}\pi\right) \\ &= 2e^{-\frac{1}{4}\pi}\cos\left(\frac{\pi}{4}\right) \\ &> 0 \end{aligned}$$

Therefore, $x=\frac{1}{4}\pi$ is a local minimum and $f\left(\frac{\pi}{4}\right)=0.3223$

(g)

Concavity and inflection point:

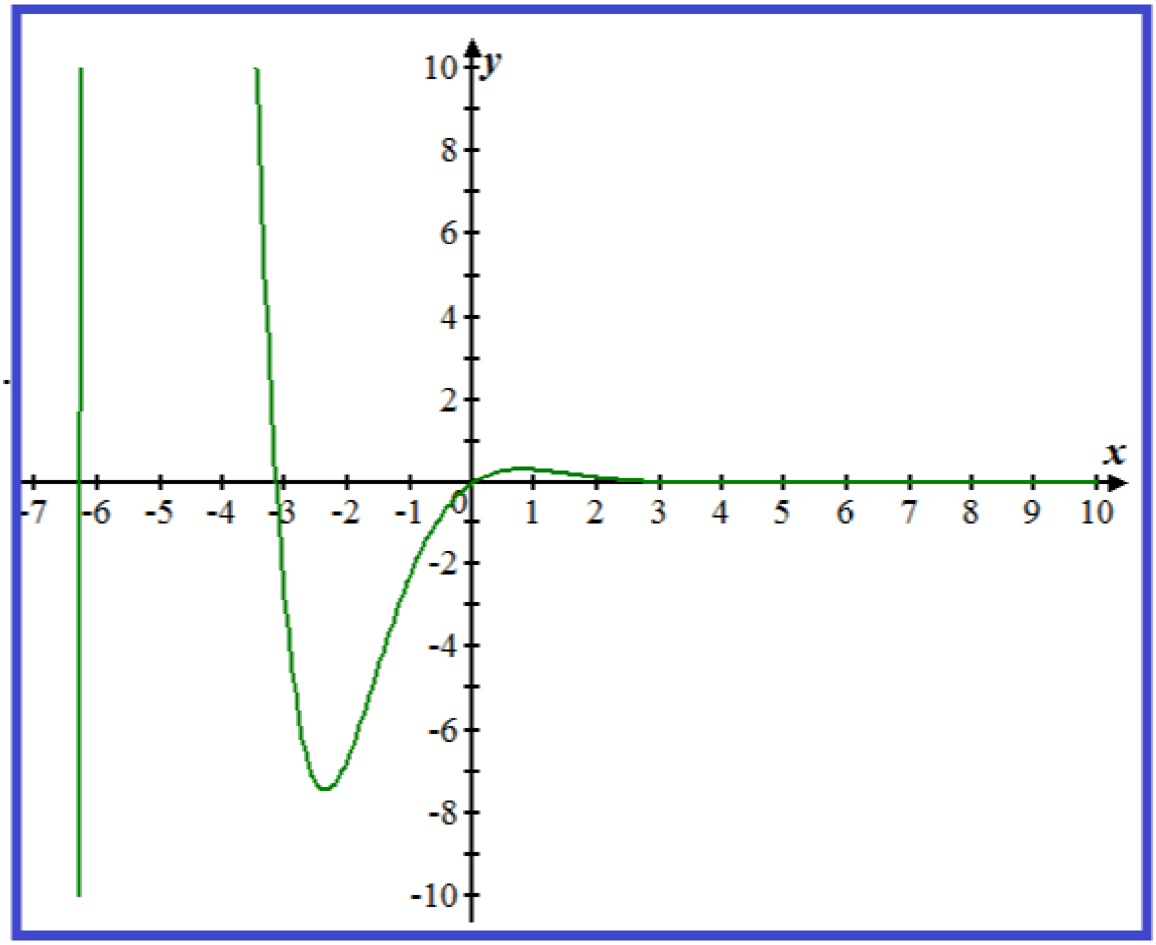
$$\begin{aligned} f''(x) &= -2e^{-x}\cos x \\ &= -2e^{-x}\cos x<0 \\ &= \left(\frac{-1}{2}\pi,\frac{1}{2}\pi\right) \end{aligned}$$

Concave down on $\left(\frac{-1}{2}\pi,\frac{1}{2}\pi\right)$.

$$\begin{aligned} f''(x) &= -2e^{-x}\cos x \\ &= -2e^{-x}\cos x>0 \\ &= \left(\frac{1}{2}\pi,\frac{3}{2}\pi\right) \end{aligned}$$

Concave up on $\left(\frac{1}{2}\pi,\frac{3}{2}\pi\right)$.

Sketch for the given curve is



Answer 73E.

Consider the following curve:

$$y=\frac{1}{\left(1+e^{-x}\right)}$$

(a)

Domain:

The function $y=\frac{1}{\left(1+e^{-x}\right)}$ $y=\frac{1}{\left(1+e^{-x}\right)}$

$$=\frac{1}{1+\frac{1}{e^x}}$$

$$=\frac{e^x}{e^x+1}$$

$$y=1-\frac{1}{e^x+1}$$

The exponential function exists because e^x is always positive and e^x is defined at point in the Real number system.

So, domain is $D=\mathbb{R}$ domain of the function $y=\frac{1}{\left(1+e^{-x}\right)}$.

(b)

Intercepts:

The *x*- intercept is a point in the equation where the *y*- value is zero.

y=0 for the *x*- intercept,

$$\frac{1}{\left(1+e^{-x}\right)}=0$$
$$1=0$$

Therefore, there is no *x*- intercept.

The *y*- intercept is a point in the equation where the *x*- value is zero.

x=0 for the *y*- intercept,

$$\begin{aligned} y &= \frac{1}{1+e^{-0}} \\ &= \frac{1}{2} \\ &= 0.5 \\ y &= 0.5 \end{aligned}$$

Therefore, the *y*- intercept is 0.5.

(c)

Symmetry:

$$\text{When the function } f(-x) = \frac{1}{1+e^{-(-x)}} = \frac{1}{1+e^{x}} \neq f(x)$$

So, the function is neither even nor odd.

If $f(x+p)=f(x)$ for all x in D, where *p* is a positive constant, then *f* is called a periodic function.

Here, $f(x+2i\pi)=f(x)$ for all *x* and *f* is periodic in *x* with period $2i\pi$.

(d)

Asymptote:

The function is either $\lim_{x\to\infty} f(x)=L$ or $\lim_{x\to-\infty} f(x)=L$, then the line $y=L$ is a horizontal asymptote of the curve $y=f(x)$.

$$\begin{aligned} \lim_{x\to\infty}\frac{1}{1+e^{-x}} &= \frac{1}{1+\lim_{x\to\infty} e^{-x}} \\ &= \frac{1}{1+0} \\ &= 1 \end{aligned}$$

use $t=-x,\ t\to-\infty$ as $x\to\infty$

$$\lim_{x\to\infty}\frac{1}{1+e^{-x}}=0$$

Therefore, $y=0, y=1$ is a horizontal asymptote.

(e)

Interval of increase or decrease:

$$f'(x)=\frac{(1+e^{-x})(0)-1(-e^{-x})}{(1+e^{-x})^2}$$

$$=\frac{e^{-x}}{(1+e^{-x})^2}$$

Since the function is increasing when $f'(x)>0$

$$\frac{e^{-x}}{(1+e^{-x})^2}>0$$

Exponential function is always positive and the function $\frac{e^{-x}}{(1+e^{-x})^2}$ is increasing on $\mathbb{R}$.

(f)

Local maximum and minimum values:

$$f'(x)=0$$

$$\frac{e^{-x}}{(1+e^{-x})^2}=0$$

$$e^{-x}=0$$

Therefore, there is no local maximum and local minimum.

(g)

Concavity and inflection point:

$$f''(x)=\frac{e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}$$

$$=\frac{e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}>0$$

$$=(-\infty,0)$$

Concave up on $(-\infty,0)$.

$$f''(x)=\frac{e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}$$

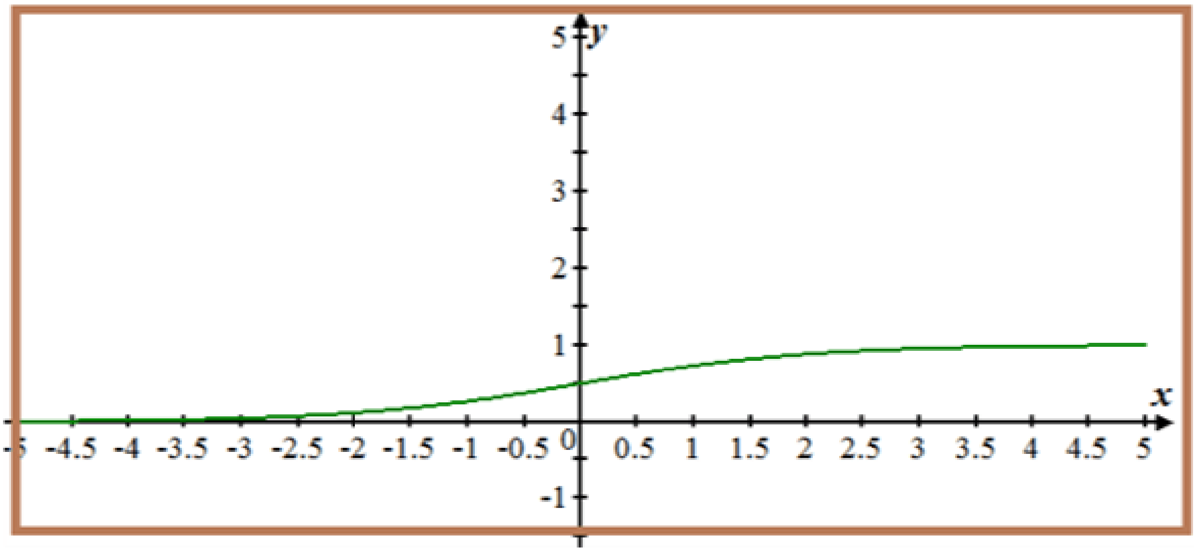
$$=\frac{e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}<0$$

$$=(0,\infty)$$

Concave down on $(0,\infty)$.

(h)

Sketch the curve is



Answer 74E.

Consider the following functions:

$g(x) = e^{cx} + f(x)$ and $h(x) = e^{kx} f(x)$

The objective is to find the functions $g'(0)$ and $g''(0)$ in terms of c.

(a)

Differentiate the function $g(x)$ with respect to *x*.

$$g'(x) = ce^{cx} + f'(x)$$

Substitute 0 for *x* and

$$\begin{aligned} g'(0) &= ce^{0\cdot x} + f'(0) \\ &= c + 5 \end{aligned}$$

Differentiate the function $g'(x)$ with respect to *x*.

$$g''(x) = c^2 e^{cx} + f''(x)$$

Substitute 0 for *x* and

$$\begin{aligned} g''(0) &= c^2 e^{0\cdot x} + f''(0) \\ &= c^2 - 2 \end{aligned}$$

Therefore, $g'(0)$ and $g''(0)$ are in terms of *c*.

(b)

The following function:

$h(x)=e^{kx}f(x)$.

The objective is to find the tangent line equation at the point $x=0$

Find slope:

Differentiate the function with respect to *x*.

$h'(x)=e^{kx}f'(x)+f(x)\cdot ke^{kx}$

Substitute 0 for *x*

$$\begin{aligned}h'(0)&=e^{k\cdot 0}f'(0)+f(0)\cdot ke^{k\cdot 0}\\&=1\cdot 5+3\cdot k\cdot 1\\&=5+3k\end{aligned}$$

$h'(0)=5+3k$

Therefore, Slope $m=h'(x)$

Find slope point at $x=0$.

$$\begin{aligned}m&=h'(0)\\&=5+3k\end{aligned}$$

$m=5+3k$

Find the point (x_1,y_1) at $x=0$ in $h(x)$.

$$\begin{aligned}h(0)&=e^{k\cdot 0}f(0)\\&=1\cdot 3\\&=3\end{aligned}$$

Therefore, the point is $(0,3)$

Therefore, the point is $(0,3)$

General equation of the tangent line passing through the point (x_1,y_1) and slope intercept form is $y-y_1=m(x-x_1)$.

The tangent line is passing through the point $(0,3)$ and slope $m=5+3k$

$$\begin{aligned}(y-3)&=(5+3k)(x-0)\\y-3&=5x+3kx\\y&=(5+3k)x+3\end{aligned}$$

Hence the equation is in terms of k.

Therefore, the equation of the tangent line to the point at $x=0$ is $\boxed{y=(5+3k)x+3}$

Answer 75E.

Consider the function defined for the surge function

$S(t)=At^pe^{-kt}$

For a particular drug, $A=0.01, p=4, k=0.07$

For that particular drug, surge function is

$S(t)=0.01t^4e^{-0.07t}$

The inflection of this curve is the points at which the drug level changes.

Inflection points are obtained by the differentiating the equation twice with respect to *t*.

Find the first derivative of the function by the use of constant multiple rule and product rule and then use power rule.

State them as follows.

Constant multiple rule is

If f is differentiable function of x and c is a real number then

$$\frac{d}{dx}\left[cf(x)\right]=cf'(x),\ \ c \text{ is a constant}$$

Product rule:

$$\frac{d}{dx}\left[f(x)g(x)\right]=g(x)f'(x)+f(x)g'(x)$$

Power rule:

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$S'(t)=\frac{d}{dt}\left(0.01t^4e^{-0.07t}\right)$$

$$=0.01\left(\frac{d}{dt}\left(t^4\right)e^{-0.07t}+t^4\frac{d}{dt}\left(e^{-0.07t}\right)\right)$$

Apply constant multiple rule and product rule

$$=0.01\left(4t^3e^{-0.07t}+t^4(-0.07)e^{-0.07t}\right)$$ Use power rule and $\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$

$$=\left(0.04t^3-0.0007t^4\right)e^{-0.07t}$$

Find the second derivative of the function.

$$S''(t)=\frac{d}{dt}\left(\left(0.04t^3-0.0007t^4\right)e^{-0.07t}\right)$$

$$=\left(-0.0007t^4+0.04t^3\right)\cdot\frac{d}{dt}e^{-0.07t}+\frac{d}{dt}\left(-0.0007t^4+0.04t^3\right)\cdot e^{-0.07t}$$

Use product rule

$$=\left(-0.0007t^4+0.04t^3\right)\left(-0.07e^{-0.07t}\right)+\left(-0.0028t^3+0.12t^2\right)e^{-0.07t}$$

Use $\frac{d}{dx}\left(e^{ax}\right)=ae^{ax}$

$$=\left(0.000049t^4-0.0028t^3\right)e^{-0.07t}+\left(-0.0028t^3+0.12t^2\right)e^{-0.07t}$$

$$=\left(0.000049t^4-0.0056t^3+0.12t^2\right)e^{-0.07t}$$ Simplify ...... (1)

Thus, there are inflection points when

$$0.000049t^4-0.0056t^3+0.12t^2=0$$

$$0.000049t^2-0.0056t+0.12=0$$

This is a quadratic equation in t. solve this equation by the use of quadratic formula.

From the Quadratic Formula, there are inflections points (besides $t=0$) at

$$t=\frac{0.0056\pm\sqrt{0.00003136-0.00002352}}{0.000098}$$

$$\approx 28.57,\ 85.71$$

Substitute the value $t = 28.57$ in (1).

$$S''(t) = \left(0.000049t^4 - 0.0056t^3 + 0.12t^2\right)e^{-0.07t}$$

$$= \left(0.000049(28.57)^4 - 0.0056(28.57)^3 + 0.12(28.57)^2\right)e^{-0.07(28.57)}$$

$$= 0.000441925$$

$$S''(t) = \left(0.000049t^4 - 0.0056t^3 + 0.12t^2\right)e^{-0.07t}$$

$$= \left(0.000049(85.71)^4 - 0.0056(85.71)^3 + 0.12(85.71)^2\right)e^{-0.07(85.71)}$$

$$= -0.000218563$$

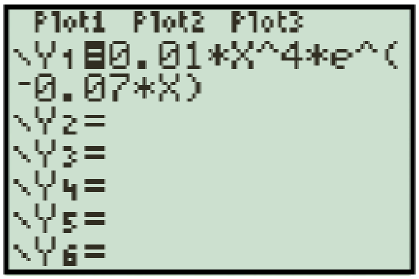
For the first inflection point, $t \approx 28.57$, the function is increasing, so after about 28.57 minutes, the rate of increase of the drug level in the bloodstream is greatest.

For the second inflection point, $t \approx 85.71$, the function is decreasing, so after about 85.71 minutes, the rate of decrease of the drug level in the bloodstream is greatest.

This can be shown by means of a graph as follows.

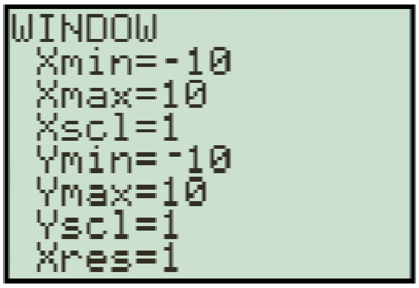
Sketch the graph of this function using graphing utility.

First enter the function in $\boxed{\text{Y=}}$



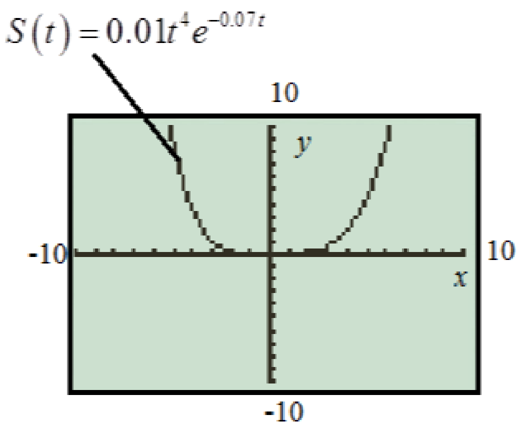
Adjust the window so as to view the graph clearly.

To enter click on $\boxed{\text{WINDOW}}$ button



Hit the $\boxed{\text{GRAPH}}$ button to view the graph.

The graph of the function is as follows.



Answer 76E.

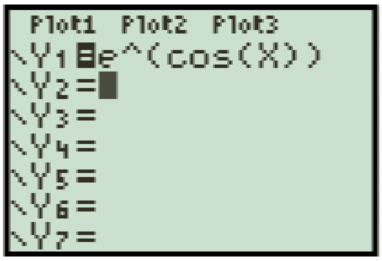
Consider the function

$f(x) = e^{\cos x}$ ...... (1)

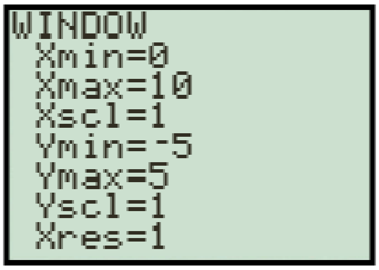
Draw a graph of f by use of graphing utility.

First enter the function in the in Y screen. To do click on $\boxed{\text{Y=}}$.

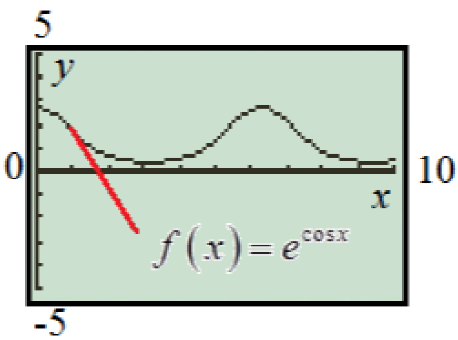
Enter the function as Y= e^ (cos(x))



Adjust the scale by clicking on $\boxed{\text{WINDOW}}$.



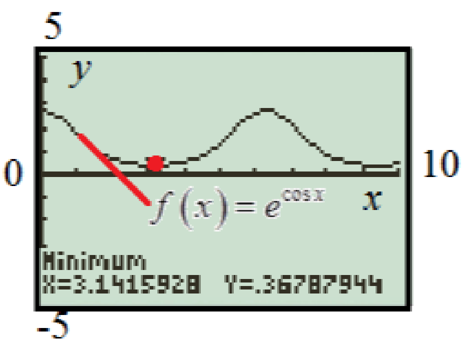
Hit the $\boxed{\text{GRAPH}}$ button to view the graph.

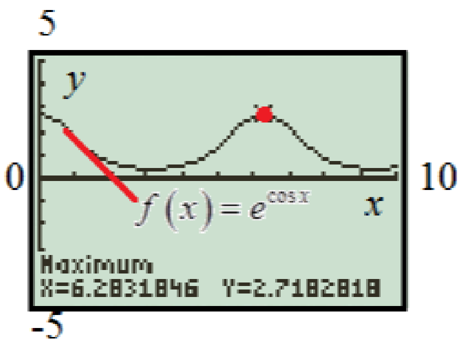


Find the local minimum and maximum.

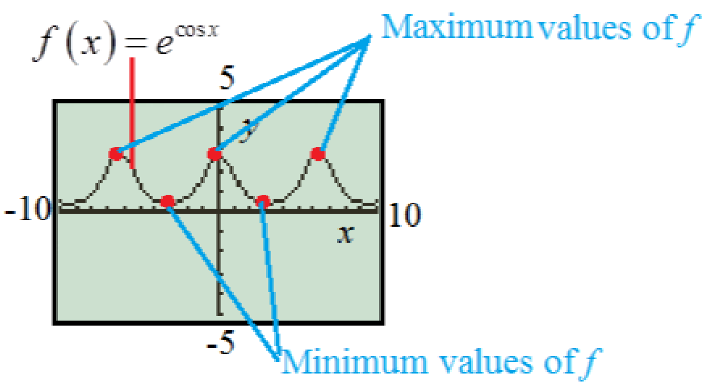
For this, hit $\boxed{\text{2nd}} + \boxed{\text{TRACE}}$ and then select, $3:\text{minimum}$ to calculate local minimum, $4:\text{maximum}$ to calculate local maximum.

Maximum and minimum are shown in graph as follows.





More visually, the graph of the function,



The minimum exists for the same value of *y* for different values of *x*.

From the graph, the local maximum seems to be about $x = 2\pi n$.

The local minimum seems to be about $x = \pi + 2\pi n$.

To find the local minimum and maximum numerically, find $f'(x)$ and equate it to zero.

$f'(x) = e^{\cos x}(-\sin x)$

$= -\sin x e^{\cos x}$

Solve the equation $f'(x) = 0$

$-\sin x e^{\cos x} = 0$

The term $e^{\cos x}$ never be zero, so $\sin x = 0$.

The set of all values of *x* satisfying $\sin x = 0$ are $n\pi$.

Therefore, the local maximum or minimum of (1) are $\boxed{n\pi, n = 0,1,2,...}$

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The derivative is 0 when $x = \pi n$.

When $x = 2\pi n$, $f(x) = e.$

So, this value of $f(x)$ is a maximum.

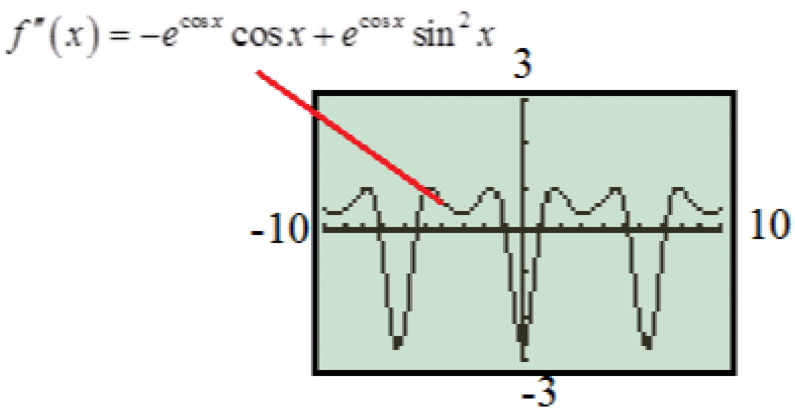
When $x = \pi + 2\pi n$, $f(x) = e^{-1}.$

So, this value of $f(x)$ is a minimum.

Use the Product Rule to find the second derivative.

$$f''(x) = -e^{\cos x} \cdot \frac{d}{dx}(\sin x) - \frac{d}{dx}\left(e^{\cos x}\right) \cdot \sin x$$
$$= -e^{\cos x} \cos x + e^{\cos x} \sin^2 x$$

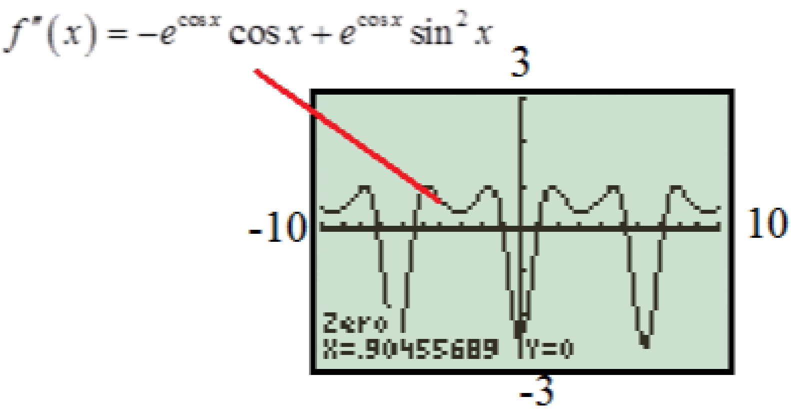
Graph this function.



The zeros of the graph of the function are inflection points.

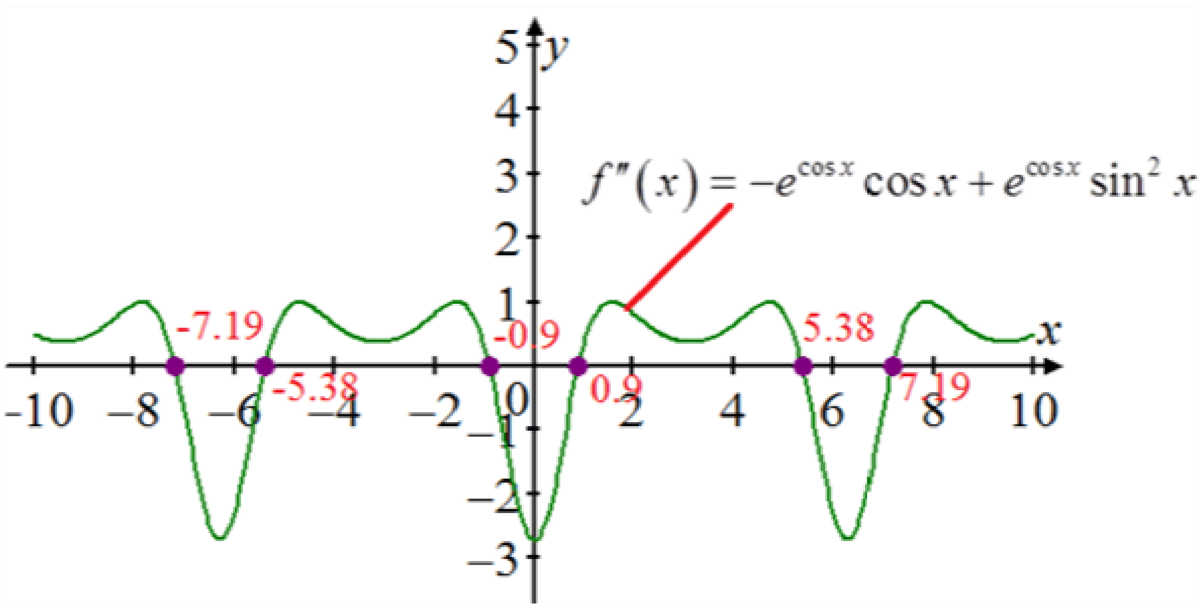
Hit [2nd] + [TRACE] and select 2: zeros to find zeros of the function.

One of the zero by graphing calculator is as follows.



Similarly, the remaining zeros are obtained.

Noting down the zeros, the graph is clearly as follows.



In the graph, the inflection point seems to be about $\boxed{x \approx -7.2,\ -5.5,\ -1,\ 1,\ 5.5, \text{ and } 7.2}$.

Answer 77E.

Consider the function $f(x)=e^{x^3-x}$ ...... (1)

Estimate the local minimum and local maximum of (1)

Find the derivative of (1)

$$f(x)=e^{x^3-x}$$

$$f'(x)=e^{x^3-x}\left(3x^2-1\right)$$

If f has a local maximum or local minimum at c, and if $f'(c)$ exists, then $f'(c)=0$

$$f'(x)=e^{x^3-x}\left(3x^2-1\right)$$

$$3x^2-1=0 \qquad \left(\text{Since } e^{x^3-x}\neq 0\right)$$

$$3x^2=1$$

$$x^2=\frac{1}{3}$$

$$x=\pm\sqrt{\frac{1}{3}}$$

Thus, use the following intervals to test local extremes.

$$\left(-\infty,-\sqrt{\frac{1}{3}}\right),\left(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}}\right)\text{and}\left(\sqrt{\frac{1}{3}},\infty\right)$$

To test the interval $\left(-\infty,-\sqrt{\frac{1}{3}}\right)$, choose a number belonging to that interval.

Let $x=-1$ in $\left(-\infty,-\sqrt{\frac{1}{3}}\right)$, plug in $f'(x)$

$$f'(-1)=e^{-1-(-1)}\left(3(-1)^2-1\right)$$

$$=e^0(3-1)$$

$$=2$$

Therefore in that interval for every value of x, value of $f'(x)$ is positive.

Therefore the function is increasing in this interval.

To test the interval $\left(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}}\right)$, choose a number belonging to that interval.

Let $x=0$ in $\left(-\sqrt{\frac{1}{3}},\sqrt{\frac{1}{3}}\right)$, plug in $f'(x)$

$$f'(0)=e^0\left(3(0)^2-1\right)$$

$$=-1$$

Therefore in that interval for every value of x, value of $f'(x)$ is negative.

Therefore the function is decreasing in this interval.

To test the interval $\left(\sqrt{\frac{1}{3}},\infty\right)$, choose a number belonging to that interval.

Let $x=1$ in $\left(\sqrt{\frac{1}{3}},\infty\right)$, plug in $f'(x)$

$$f'(1)=e^{1-(1)}\left(3(1)^2-1\right)$$

$$=e^0(3-1)$$

$$=2$$

Therefore in that interval for every value of x, value of $f'(x)$ is positive.

Therefore the function is increasing in this interval.

If $f'(x)$ changes from negative to positive at c , then f has a relative minimum at $(c, f(c))$

Sign of $f'(x)$ changes negative to positive at $\frac{1}{\sqrt{3}}$

There local minimum would be at $\left(\frac{1}{\sqrt{3}}, f\left(\frac{1}{\sqrt{3}}\right)\right)$

Local minimum is

$$f(x) = e^{x^3 - x}$$

$$f\left(\frac{1}{\sqrt{3}}\right) = e^{\left(\frac{1}{\sqrt{3}}\right)^3 - \left(\frac{1}{\sqrt{3}}\right)}$$

$$\approx \boxed{0.635}$$

If $f'(x)$ changes from positive to negative at c , then f has a relative maximum at $(c, f(c))$

Sign of $f'(x)$ changes positive to negative at $-\frac{1}{\sqrt{3}}$

There local maximum would be at $\left(-\frac{1}{\sqrt{3}}, -f\left(\frac{1}{\sqrt{3}}\right)\right)$

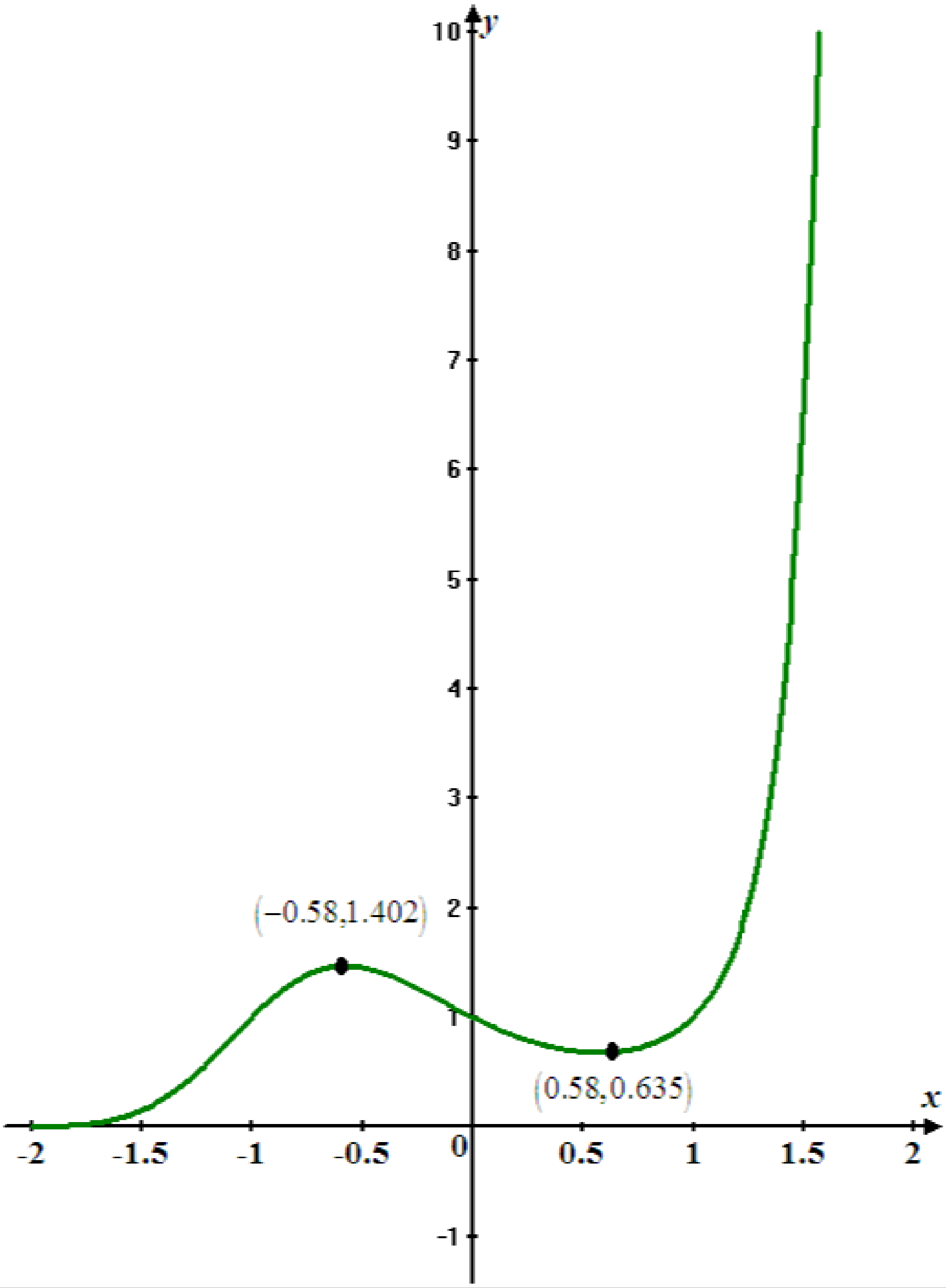
Local maximum is

$$f(x) = e^{x^3 - x}$$

$$f\left(-\frac{1}{\sqrt{3}}\right) = e^{\left(-\frac{1}{\sqrt{3}}\right)^3 - \left(-\frac{1}{\sqrt{3}}\right)}$$

$$\approx \boxed{1.402}$$

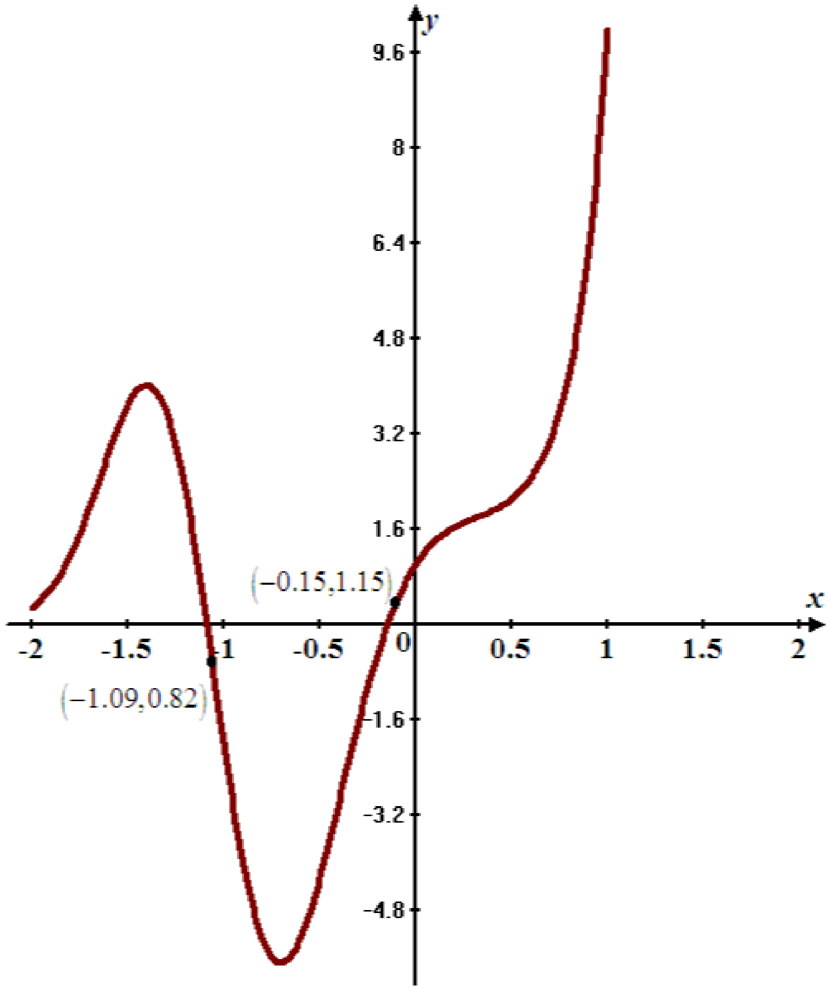
The graph of the function $f(x) = e^{x^3 - x}$ is shown below:



To estimate inflection points, sketch the graph of $f''(x)$.

$$f'(x) = e^{x^3-x}\left(3x^2-1\right)$$
$$f''(x) = e^{x^3-x}(6x) + \left(3x^2-1\right)e^{x^3-x}\left(3x^2-1\right)$$
$$= e^{x^3-x}\left(6x + \left(3x^2-1\right)^2\right).$$

A point $x = c$ is said to be inflection point of the function, if it is continues and the concavity of the graph changes at the point.



From the graph, inflection points are $\boxed{(-0.15,1.15),(-1.09,0.82)}$

Answer 78E.

Consider the normal density function $y=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-u)^2}{2\sigma^2}}$

(a)

As x 8 or 8, the exponent $-x^2/(2\sigma^2)$ goes to 8, so the expression $e^{-x^2/(2\sigma^2)}$ goes to 0. Thus, the function has horizontal asymptotes at $y=0$.

To find the maximum value, find the first derivative of *f* using $\frac{d}{dx}(e^u)=e^u\frac{du}{dx}$.

$$
\begin{aligned}
f'(x) &= \frac{d}{dx}\left[e^{-x^2/(2\sigma^2)}\right] \\
&= e^{-x^2/(2\sigma^2)}\cdot\frac{d}{dx}\left(-\frac{x^2}{2\sigma^2}\right) \\
&= -\frac{x}{\sigma^2}e^{-x^2/(2\sigma^2)}
\end{aligned}
$$

The derivative is 0 when $x=0$.

Since the derivative goes from positive to negative at this value, $x=0$ is a maximum of the function.

The maximum value is

$$
\begin{aligned}
f(0) &= e^{-(0)^2/(2\sigma^2)} \\
&= e^0 \\
&= 1
\end{aligned}
$$

To find the inflection points of *f*, find the second derivative.

$$f''(x) = -\frac{x}{\sigma^2}\cdot\frac{d}{dx}\left[e^{-x^2/(2\sigma^2)}\right] - \frac{d}{dx}\left(\frac{x}{\sigma^2}\right)\cdot e^{-x^2/(2\sigma^2)}$$
$$= -\frac{x}{\sigma^2}\left[-\frac{x}{\sigma^2}e^{-x^2/(2\sigma^2)}\right] - \frac{1}{\sigma^2}e^{-x^2/(2\sigma^2)}$$
$$= \frac{x^2}{\sigma^4}e^{-x^2/(2\sigma^2)} - \frac{\sigma^2}{\sigma^4}e^{-x^2/(2\sigma^2)}$$
$$= \frac{x^2-\sigma^2}{\sigma^4}e^{-x^2/(2\sigma^2)}$$

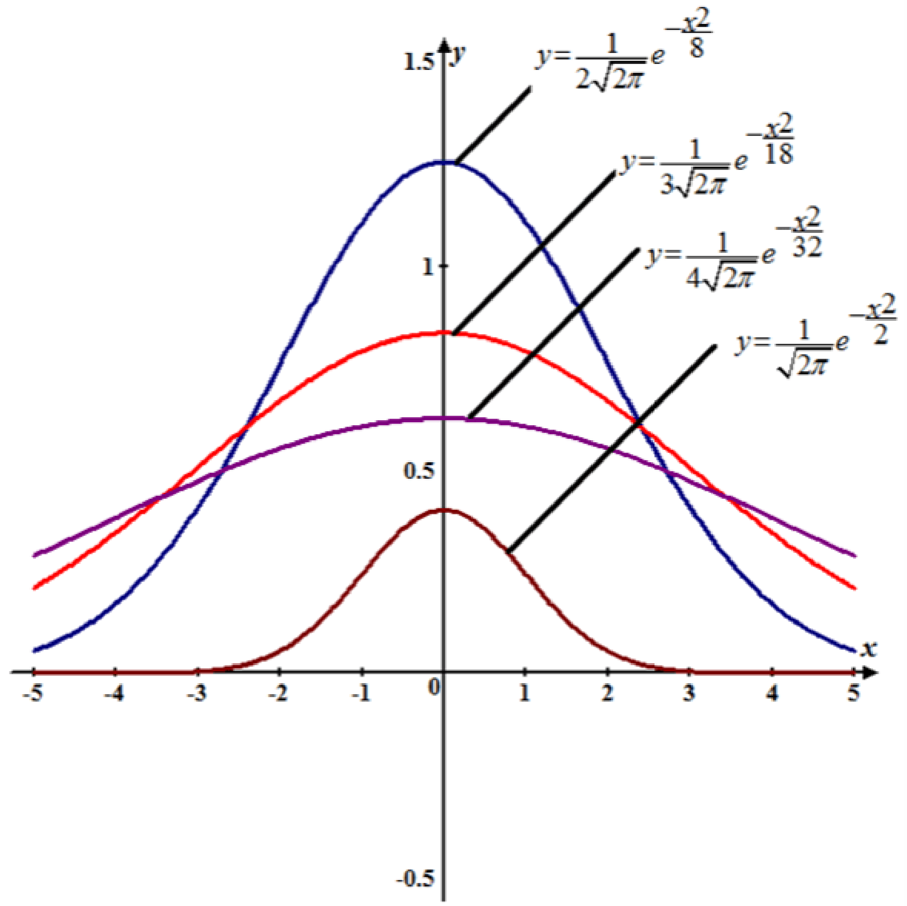
The second derivative is 0 when $x = \pm\sigma$. Therefore, *f* has inflection points at $x = \pm\sigma$.

(b)

Since $x = \pm\sigma$ are inflection points, the positive constant *is the point at which the curvature changes.*

(c)

Graph the function with $\sigma = 1, 2, 3, \text{ and } 4$.



Answer 79E.

Consider $\int_0^1 \left(x^e + e^x\right)dx$

$$= \frac{x^{e+1}}{e+1} + e^x\Big|_0^1$$
$$= \left(\frac{1}{1+e} + e\right) - 1$$
$$= \frac{1}{e+1} + e - 1$$

Therefore $\boxed{\int_0^1 \left(x^e + e^x\right)dx = \frac{1}{1+e} + e - 1}$

Answer 80E.

Consider $\int_{-5}^{5} e dx = e\int_{-5}^{5} dx$

$$= e(x)_{-5}^{5}$$
$$= e(5+5)$$
$$= 10e$$

Therefore $\boxed{\int_{-5}^{5} e\,dx = 10e}$

Answer 81E.

Consider the following integral:

$$\int_0^2 \frac{dx}{e^{\pi x}}$$

Evaluate the above integral.

Rewrite the above integral as shown below:

$$\int_0^2 e^{-\pi x} dx \qquad \text{......(1)}$$

Use the U-substitution to evaluate the integral.

Let $u = -\pi x,$ then $du = -\pi dx$ or $-\frac{1}{\pi}du = dx.$

Thus, the above integral becomes $\int_0^2 e^{-\pi x} dx = -\frac{1}{\pi}\int_0^2 e^u du.$

Change the limit of the integration.

When $x = 0,$ then $u = -\pi(0)(=0).$

When $x = 2,$ then $u = -\pi(2)(=-2\pi).$

Substitute the values in the integral and simplify as shown below:

$$-\frac{1}{\pi}\int_0^2 e^u du = -\frac{1}{\pi}\int_0^{-2\pi} e^u du$$
$$= -\frac{1}{\pi}\left[e^u\right]_0^{-2\pi}$$
$$= -\frac{1}{\pi}\left[e^{-2\pi} - e^0\right]$$
$$= \frac{1}{\pi}\left[1 - e^{-2\pi}\right]$$

$\boxed{\approx 0.3177}$.

Answer 82E.

Consider $\int x^2 e^{x^3} dx$

Put $\quad x^3 = t$ then $3x^2 dx = dt$

Therefore $\int x^2 e^{x^3} dx = \int e^t \cdot \frac{dt}{3}$

$$= \frac{e^t}{3} + c$$
$$= \frac{e^{x^3}}{3} + c$$

Therefore $\boxed{\int x^2 e^{x^3} dx = \frac{e^{x^3}}{3} + c}$

Answer 83E.

The given integral is $\int e^x\sqrt{1+e^x}\,dx$

Put $1+e^x=t$

Differentiating both sides, we get,

$$\frac{d}{dx}\left(1+e^x\right)=dt$$
$$\Rightarrow \quad e^x dx=dt$$

Therefore, $\int e^x\sqrt{1+e^x}\,dx=\int\sqrt{t}\,dt$

$$=\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$
$$=\frac{2}{3}t^{3/2}+C$$
$$=\frac{2}{3}\left(1+e^x\right)^{3/2}+C$$

Hence,

$$\boxed{\int e^x\sqrt{1+e^x}\,dx=\frac{2}{3}\left(1+e^x\right)^{3/2}+C}$$

Answer 84E.

1679-6.2-84E AID: 760 | 11/09/2013

Consider the integral $\int\frac{\left(1+e^x\right)^2}{e^x}dx$ ------ (1)

Find the integral of (1)

$$\int\frac{\left(1+e^x\right)^2}{e^x}dx=\int\frac{1+2e^x+\left(e^x\right)^2}{e^x}dx \qquad \left(\left(a+b\right)^2=a^2+2ab+b^2\right)$$
$$=\int\frac{1+2e^x+\left(e^x\right)^2}{e^x}dx$$
$$=\int\left(\frac{1}{e^x}+\frac{2e^x}{e^x}+\frac{\left(e^x\right)^2}{e^x}\right)dx$$

Distribute the denominator
to each term in the in the numerator

$$=\int e^{-x}dx+\int 2dx+\int e^x dx$$
$$=-e^{-x}+2x+e^x+C$$

Therefore $\int\frac{\left(1+e^x\right)^2}{e^x}dx=\boxed{e^x-e^{-x}+2x+C}$

Answer 85E.

1679-6.2-85E AID: 760 | 11/09/2013

Consider the integral $\int \left(e^x + e^{-x}\right)^2 dx$ ...... (1)

Find the integral of (1)

$$\int \left(e^x + e^{-x}\right)^2 dx = \int \left[\left(e^x\right)^2 + 2e^x e^{-x} + \left(e^{-x}\right)^2\right] dx \qquad \left(\text{Since}\left(a+b\right)^2 = a^2 + 2ab + b^2\right)$$

$$= \int \left[e^{2x} + 2e^x e^{-x} + e^{-2x}\right] dx$$

$$= \int \left[e^{2x} + 2 + e^{-2x}\right] dx \qquad \left(\text{Since} e^x e^{-x} = 1\right)$$

$$= \int e^{2x} dx + \int 2dx + \int e^{-2x} dx$$

$$= \frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2} + C \qquad \left(\text{Since } \int e^{ax} dx = \frac{e^{ax}}{a}\right)$$ Therefore

$$= \frac{e^{2x} - e^{-2x}}{2} + 2x + C$$

$$\int \left(e^x + e^{-x}\right)^2 dx = \boxed{\frac{e^{2x} - e^{-2x}}{2} + 2x + C}$$

Answer 86E.

1679-6.2-86E RID: 760 | 11/09/2013

Consider the integral $\int e^x \left(4 + e^x\right)^5 dx$ ...... (1)

Find the integral of (1)

Let $4 + e^x = u$

$e^x dx = du$

u du

u du

$$\int e^x \left(4 + e^x\right)^5 dx = \int \left(4 + e^x\right)^5 e^x dx$$

$$= \int \left(u\right)^5 du$$

$$= \frac{u^6}{6} + C$$

$$= \frac{\left(4 + e^x\right)^6}{6} + C$$

Therefore $\int e^x \left(4 + e^x\right)^5 dx = \boxed{\frac{\left(4 + e^x\right)^6}{6} + C}$

Answer 87E.

Consider $\int e^{\tan x} \sec^2 x dx$

Put $\tan x = t$ then on differentiation

$\sec^2 x \; dx = dt$

Therefore $\int e^{\tan x} \sec^2 x dx = \int e^t dt$

$$= e^t + c$$

$$= e^{\tan x} + c$$

Therefore $\boxed{\int e^{\tan x} \sec^2 x \; dx = e^{\tan x} + c}$

Answer 88E.

Consider $\int e^x \left(\cos e^x\right) dx$

Put $e^x = t$ then on differentiation

$e^x dx = dt$

Therefore $\int e^x\left(\cos\left(e^x\right)\right)dx = \int \cos t\, dt$

$= \sin t + c$

$= \sin\left(e^x\right) + c$

Therefore $\boxed{\int e^x\left(\cos e^x\right)dx = \sin\left(e^x\right) + c}$

Answer 89E.

Consider the definite integral $\int_1^2 \frac{e^{1/x}}{x^2}dx$

Let $\frac{1}{x} = t$

Differentiating both sides, obtain that

$-\frac{1}{x^2}dx = dt$

Or, $\frac{1}{x^2}dx = -dt$

So when $x = 1$ then $t = 1$

And when $x = 2$ then $t = \frac{1}{2}$

Therefore,

$$\int_1^2 \frac{e^{1/x}}{x^2}dx = \int_1^{1/2} e^t(-dt)$$

$$= -\int_1^{1/2} e^t dt$$

$$= -\left(e^t\right)_1^{1/2}$$

$$= -\left(e^{1/2} - e^1\right)$$

$$= -\left(\sqrt{e} - e\right)$$

$$= \boxed{e - \sqrt{e}}$$

Answer 90E.

There are various techniques to evaluate the integral.

One of the techniques is the method of substitution.

In the method of substitution, an expression is substituted for one variable in order to simplify the integrand so that it is brought into a form that is easy to evaluate.

Consider the integral:

$$\int_0^1 \frac{\sqrt{1+e^{-x}}}{e^x}dx$$

Make the substitution as shown below:

$$e^{-x} = t$$
$$-e^{-x}dx = dt$$
$$e^{-x}dx = -dt$$

Determine the limits of the integral.

Take the value of $x = 0$:

$$t = e^{-0}$$
$$= 1$$

Take the value of $x = 1$:

$$t = e^{-1}$$
$$= \frac{1}{e}$$

Evaluate the integral as shown below:

$$\int_0^1 \frac{\sqrt{1+e^{-x}}}{e^x}dx = \int_{t=1}^{\frac{1}{e}} \sqrt{1+t}\,(-dt)$$
$$= \int_{\frac{1}{e}}^{1} (1+t)^{1/2}\,dt$$
$$= \left.\frac{(1+t)^{3/2}}{3/2}\right|_{t=\frac{1}{e}}^{1}$$

Substitute the limits to determine the value:

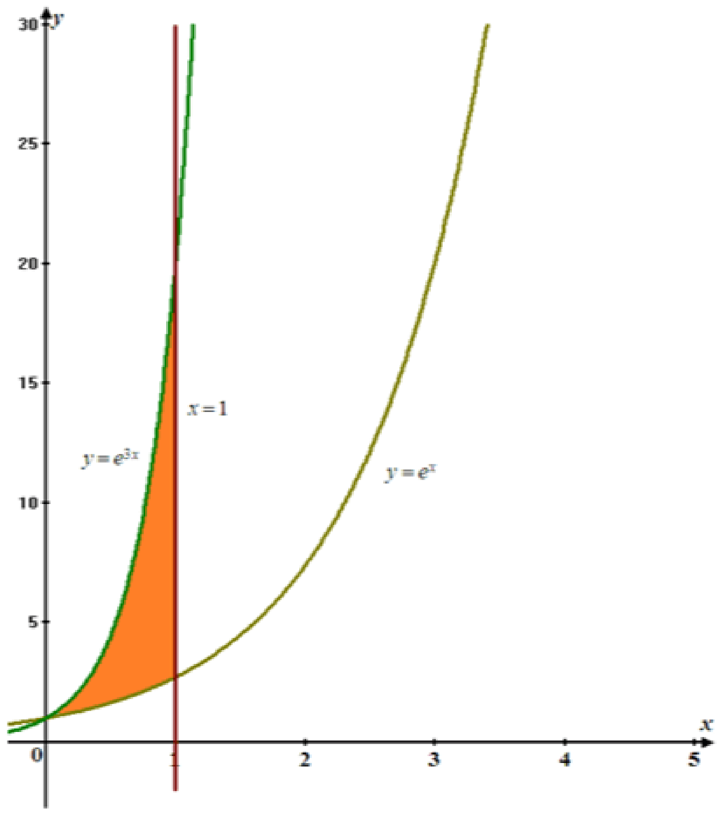
$$\int_0^1 \frac{\sqrt{1+e^{-x}}}{e^x}dx = \left.\frac{(1+t)^{3/2}}{3/2}\right|_{t=\frac{1}{e}}^{1}$$
$$= \frac{(1+1)^{3/2}}{3/2} - \frac{\left(1+\frac{1}{e}\right)^{3/2}}{3/2}$$
$$= \frac{2}{3}\left(2\sqrt{2} - \left(1+\frac{1}{e}\right)^{3/2}\right)$$
$$\approx 0.81907$$

Hence, the final value of the integral is $\boxed{0.81907}$.

Answer 91E.

Consider the curves $y = e^x, y = e^{3x}$, and $x = 1$

The region bounded by the curves $y = e^x, y = e^{3x}$, and $x = 1$ as shown below.



From the graph, limits of the bounded region are 0 to 1.

Since the graphs $y=e^x, y=e^{3x}$ intersects at $x=0$, so take $x=0$ as a lower limit and $x=1$ as a upper limit.

The graph of the $y=e^{3x}$ is above the graph of $y=e^x$.

Therefore area of the bounded region is $\int_0^1 (e^{3x}-e^x)dx$ ...... (1)

Now find the integral of (1)

$$\int_0^1 (e^{3x}-e^x)dx = \int_0^1 (e^{3x})dx - \int_0^1 (e^x)dx$$

$$= \left(\frac{e^{3x}}{3}\right)_0^1 - (e^x)_0^1$$

$$= \left(\frac{e^3}{3}-\frac{1}{3}\right)-(e-1)$$

$$= \frac{e^3}{3}-e+1-\frac{1}{3}$$

$$= \frac{e^3}{3}-e+\frac{2}{3}$$

$$= 4.643$$

Therefore $\int_0^1 (e^{3x}-e^x)dx = 4.643$

Therefore area of the bounded region is 4.643 square units.

Answer 92E.

Given $f''(x)=3e^x+5\sin x$

$\Rightarrow \quad \frac{d}{dx}f'(x)=3e^x+5\sin x$

$\Rightarrow \quad d\ f'(x)=(3e^x+5\sin x)dx$

Integrating both sides, we get,

$\int d\ f'(x)=\int(3e^x+5\sin x)\,dx$

$\Rightarrow \quad f'(x)=3e^x-5\cos x+c$ [where c is a constant.]

Given when $x=0$, $f'(0)=2$

Therefore, putting $x=0$ and $f'(0)=2$, we get,

$2=3e^0-5\cos 0+c$

$\Rightarrow \quad 2=3\times1-5\times1+c$

$\Rightarrow \quad 2=-2+c$

$\Rightarrow \quad c=4$

Thus $f'(x)=3e^x-5\cos x+4$

$\Rightarrow \quad \frac{d}{dx}f(x)=3e^x-5\cos x+4$

$\Rightarrow \quad d\,f(x)=(3e^x-5\cos x+4)\,dx$

Integrating both sides, we get,

$\int d\ f(x)=\int(3e^x-5\cos+4)\,dx$

$\Rightarrow \quad f(x)=3e^x-5\sin x+4x+c_1$

Where c_1 is a constant.

Given, when x = 0 , f(0) = 1.
So, putting x = 0 and f(0) = 1.
We get,

$$1 = 3e^0 - 5\sin 0 + 4\times 0 + c_1$$

$$\Rightarrow \quad 1 = 3\times 1 - 5\times 0 + 0 + c_1$$

$$\Rightarrow \quad c_1 = 1 - 3 = -2$$

Therefore,

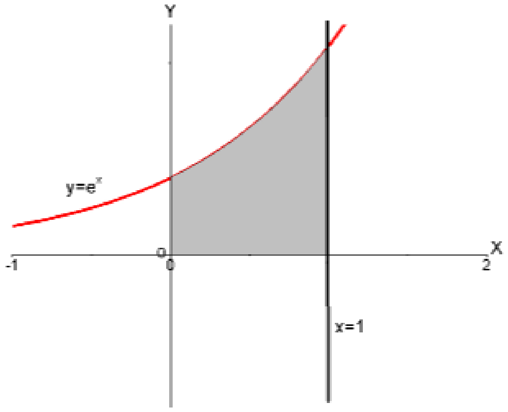
$$f(x) = 3e^x - 5\sin x + 4x - 2$$

Hence,

$$\boxed{f(x) = 3e^x - 5\sin x + 4x - 2}$$

Answer 93E.

First we sketch the region bounded by the curves $y = e^x, y = 0, x = 0,$ and $x = 1$



Now we consider a vertical strip with thickness Δx in this region. If we rotate this region about x-axis, then we get a typical disk of radius $y = e^x$

Then the area of the cross section is $A(x) = \pi(e^x)^2 = \pi e^{2x}$

And so the volume of approximating disk is $A(x)\Delta x = \pi e^{2x}\Delta x$

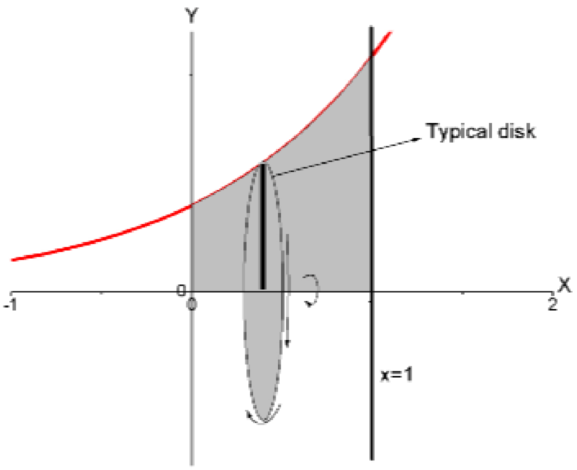


Fig.2

The solid lies between $x = 0$ and $x = 1$
Then the volume of the resulting solid obtained by rotating the region about x-axis is

$$\begin{aligned} V &= \int_0^1 A(x)dx \\ &= \int_0^1 \pi e^{2x} dx \\ &= \pi \int_0^1 e^{2x} dx \end{aligned}$$

Let $2x = u \Rightarrow \quad 2dx = du$

When $x = 0, u = 0$ and when $x = 1, u = 2$

Therefore

$$V = \frac{\pi}{2}\int_0^2 e^u du$$

$$= \frac{\pi}{2}\left[e^u\right]_0^2$$

$$= \frac{\pi}{2}\left[e^2 - e^0\right]$$

Or $$\boxed{V = \frac{\pi}{2}\left(e^2 - 1\right)}$$

Answer 94E.

First we sketch the region bounded by the curves

$$y = e^{-x^2}, y = 0, x = 0 \; and \; x = 1$$

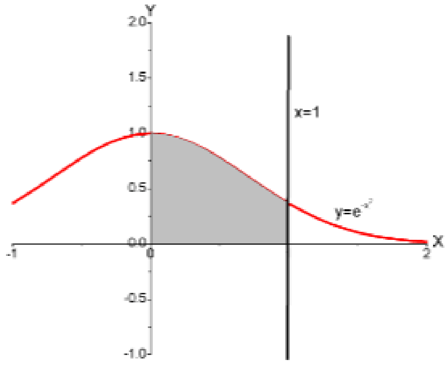


Fig.1

Now in this region, we consider vertical strip with thickness Δx at a distance x from the origin. If we rotate this region about y- axis, then we get a cylindrical shell of radius x. the height of the cylindrical shell $= e^{-x^2}$ the circumference of the shell is $= 2\pi x$

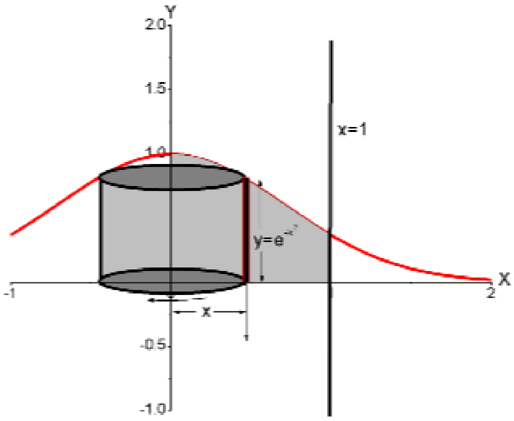


Fig.2

The region lies between $x = 0$ and $x = 1$

Then the volume of the resulting solid obtained by rotating the region about y-axis is

$$V = \int_0^1 (\text{circumference of the shell})(\text{height of the shell})dx$$

$$= \int_0^1 2\pi x e^{-x^2} dx$$

$$= \pi\int_0^1 e^{-x^2} .2x dx$$

Let $-x^2 = u \Rightarrow \quad -2xdx = du$

When $x = 0, u = 0$ and when $x = 1, u = -1$

Therefore

$$V = \pi\int_0^{-1} e^u(-du)$$
$$= \pi\int_{-1}^{0} e^u du$$
$$= \pi\left[e^u\right]_{-1}^{0}$$
$$= \pi\left[e^0 - e^{-1}\right]$$

Or $\boxed{V = \pi(1 - 1/e)}$

Answer 95E.

Given error function $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$

Hence $\int_0^x e^{-t^2}dt = \frac{\sqrt{\pi}}{2}erf(x)$

Consider $\int_a^b e^{-t^2}dt = \int_0^b e^{-t^2}dt - \int_0^a e^{-t^2}dt$

$$= \frac{\sqrt{\pi}}{2}erf(b) - \frac{\sqrt{\pi}}{2}erf(a)$$

Therefore $\boxed{\int_a^b e^{-t^2}dt = \frac{\sqrt{\pi}}{2}\left(erf(b) - erf(a)\right)}$

Answer 96E.

Given $y = e^{x^2}erf(x)$ and

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

On differentiation,

$$y' = \frac{dy}{dx}$$
$$= \frac{d}{dx}\left[e^{x^2}erf(x)\right]$$
$$= e^{x^2}\cdot\frac{d}{dx}\left(erf(x)\right) + \left[(erf)(x)\right]\frac{d}{dx}\left(e^{x^2}\right)$$
$$= e^{x^2}\frac{d}{dx}\left[\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt\right] + 2x\cdot y$$
$$= \frac{2}{\sqrt{\pi}} + 2xy$$

Therefore $\boxed{y' = 2xy + \frac{2}{\sqrt{\pi}}}$

Answer 97E.

Given that an oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate $r(t) = 100\,e^{-0.01t}$ lt per minute.

Amount of oil leaks out during the first hour

$$= \int_0^{60} r(t)dt$$
$$= \int_0^{60} 100e^{-0.01t}dt$$
$$= 100\left.\frac{e^{-0.01t}}{-0.01}\right|_0^{60}$$
$$= -10{,}000\left[e^{-0.6} - 1\right]$$
$$\approx 10{,}000\left(1 - e^{-0.6}\right)$$
$$\approx \boxed{4512L}$$

Answer 98E.

Initially bacteria population is 400 bacteria.

Population grows at a rate of $r(t) = (450.268)e^{1.12567t}$ bacteria per hour.

Now find the number of bacteria after three hours.

Population function is a constant multiple of the exponential function.

Here $r(t) = ae^{bt}$ with $a = 450.268$ and $b = 1.12567$

Let $n(t)$ is a total population after t hours. Since $r(t) = n'(t)$

$\int_0^3 r(t)\,dt = n(3) - n(0)$, is the total change in the population after three hours.

$n(3) = n(0) + \int_0^3 r(t)\,dt$,

Here $n(3)$ indicates total population after three years, $n(0)$ indicates the initial population of the bacteria, that is $n(0) = 400$

Here $n(3)$ indicates total population after three years, $n(0)$ indicates the initial population of the bacteria, that is $n(0) = 400$

$$
\begin{aligned}
n(3) &= n(0) + \int_0^3 r(t)\,dt \\
&= 400 + \int_0^3 r(t)\,dt \\
&= 400 + \int_0^3 ae^{bt}\,dt \\
&= 400 + \left(\frac{ae^{bt}}{b}\right)_0^3 \\
&= 400 + \frac{a}{b}\left(e^{3b} - e^{(0)b}\right) \\
&= 400 + \frac{a}{b}\left(e^{3b} - 1\right) \\
&= 400 + \frac{450.268}{1.12567}\left(e^{3(1.12567)} - 1\right) \quad (\text{Substitute the values of } a, b) \\
&= 400 + 400\left(e^{3(1.12567)} - 1\right) \\
&= 400\left(1 + e^{3(1.12567)} - 1\right) \\
&= 400e^{3(1.12567)} \\
&\approx 11{,}713
\end{aligned}
$$

Therefore the total population of bacteria after 3 hours is $\boxed{11{,}713}$

Answer 99E.

The given function is $f(x) = 3 + x + e^x$

Differentiating with respect to x, we get

$$
\begin{aligned}
f'(x) &= \frac{d}{dx}f(x) = \frac{d}{dx}\left(3 + x + e^x\right) \\
&= \frac{d}{dx}3 + \frac{d}{dx}x + \frac{d}{dx}e^x \\
&= 0 + 1 + e^x \\
&= 1 + e^x
\end{aligned}
$$

Since e^x is always positive.

So $1 + e^x$ i.e. $f'(x)$ is always positive

$\Rightarrow \quad f(x)$ is increasing.

Since $f(x)$ is increasing so it will be one – to – one.

Now we need to find $f^{-1}(4)$. For this we have $f(x) = 3 + x + e^x$

Therefore $f(0) = 3 + 0 + e^0$

$\Rightarrow \quad f(0) = 3 + 0 + 1$

$\Rightarrow \quad f(0) = 4$

$\Rightarrow \quad f^{-1}(4) = 0 \qquad$ Since f is one – to – one.

We know that if f is one – to – one differentiable function with inverse function f^{-1}

and $f'\left[f^{-1}(a)\right] \neq 0$ Then the inverse function is differentiable at a and

$$\left(f^{-1}\right)'(a) = \frac{1}{f'\left[f^{-1}(a)\right]}$$

Now, we have $f^{-1}(4) = 0$

Also $f'(x) = 1 + e^x$

So $f'\left[f^{-1}(4)\right] = f'(0) = 1 + e^0 = 1 + 1 = 2$

Therefore

$$\left(f^{-1}\right)'(4) = \frac{1}{f'\left[f^{-1}(4)\right]} = \frac{1}{2}$$

Hence

$$\boxed{\left(f^{-1}\right)'(4) = \frac{1}{2}}$$

Answer 100E.

We have to evaluate $\lim_{x \to \pi} \frac{e^{\sin x} - 1}{x - \pi}$

Multiply both the numerator and denominator by (sin x)

Then $\lim_{x \to \pi} \frac{e^{\sin x} - 1}{(x - \pi)} = \lim_{x \to \pi} \frac{\left(e^{\sin x} - 1\right) \cdot \sin x}{(x - \pi) \cdot \sin x}$

$$\Rightarrow \quad \lim_{x \to \pi} \frac{e^{\sin x} - 1}{(x - \pi)} = \left[\lim_{x \to \pi} \frac{e^{\sin x} - 1}{\sin x}\right] \cdot \left[\lim_{x \to \pi} \frac{\sin x}{(x - \pi)}\right] \qquad \text{----- (1) [By limit laws]}$$

First we evaluate $\lim_{x \to \pi} \frac{e^{\sin x} - 1}{\sin x}$

Let $\sin x = h$

Then $h \to 0$ as $x \to \pi$

So, $\lim_{x \to \pi} \frac{e^{\sin x} - 1}{\sin x} = \lim_{h \to 0} \frac{e^h - 1}{h}$

$= 1 \qquad$ [By the definition of the number e]

Now we evaluate $\lim_{x \to \pi} \frac{\sin x}{(x - \pi)}$

$$\lim_{x \to \pi} \frac{\sin x}{(x - \pi)} = \lim_{x \to \pi} \frac{\sin x}{-(\pi - x)}$$

$$= -\lim_{x \to \pi} \frac{\sin(\pi - x)}{(\pi - x)} \qquad [\text{Since } \sin(\pi - \theta) = \sin\theta]$$

Now let $u = \pi - x$

Then $u \to 0$ as $x \to \pi$

Then $\lim_{x \to \pi} \frac{\sin x}{\pi - x} = -\lim_{u \to 0} \frac{\sin u}{u}$

$= -1 \qquad \left[\text{Since } \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$

Therefore, from equation (1) we have

$$\lim_{x\to\pi}\frac{e^{\sin x}-1}{x-\pi}=1\cdot(-1)=-1$$

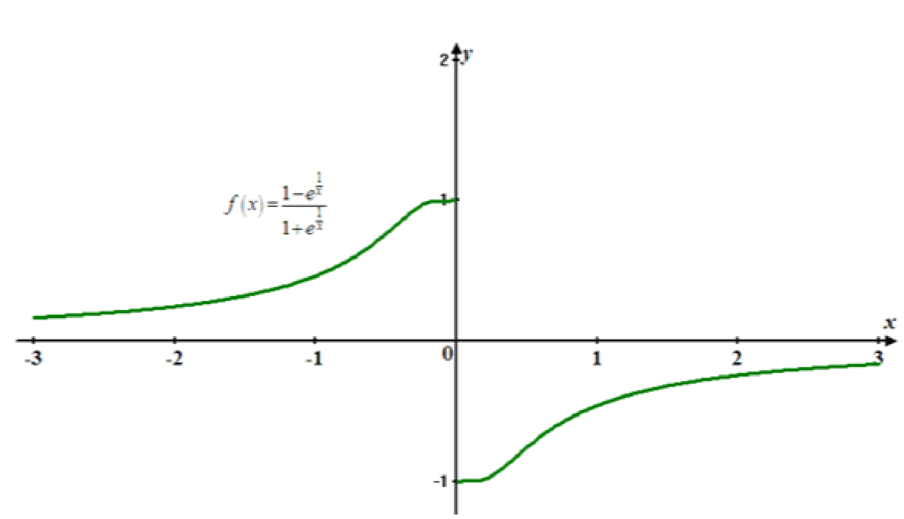
Thus,

$$\boxed{\lim_{x\to\pi}\frac{e^{\sin x}-1}{x-\pi}=-1}$$

Answer 101E.

Consider the function $f(x)=\frac{1-e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$ ...... (1)

Sketch the graph of the function (1)



If the function $f(x)$ is said to be an odd function, satisfy the condition $f(-x)=-f(x)$

Consider $f(-x)$

$$f(-x)=\frac{1-e^{\frac{1}{(-x)}}}{1+e^{\frac{1}{(-x)}}}$$

$$=\frac{1-e^{-\frac{1}{x}}}{1+e^{-\frac{1}{x}}}$$

$$=\frac{1-\frac{1}{e^{\frac{1}{x}}}}{1+\frac{1}{e^{\frac{1}{x}}}}$$

$$=\frac{\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}}}{\frac{e^{\frac{1}{x}}+1}{e^{\frac{1}{x}}}}$$

$$=\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$$

$$=-\left(\frac{1-e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}\right)$$

$$=-f(x)$$

Therefore $f(-x)=-f(x)$

Hence given function is an $\boxed{\text{odd function}}$.

Answer 102E.

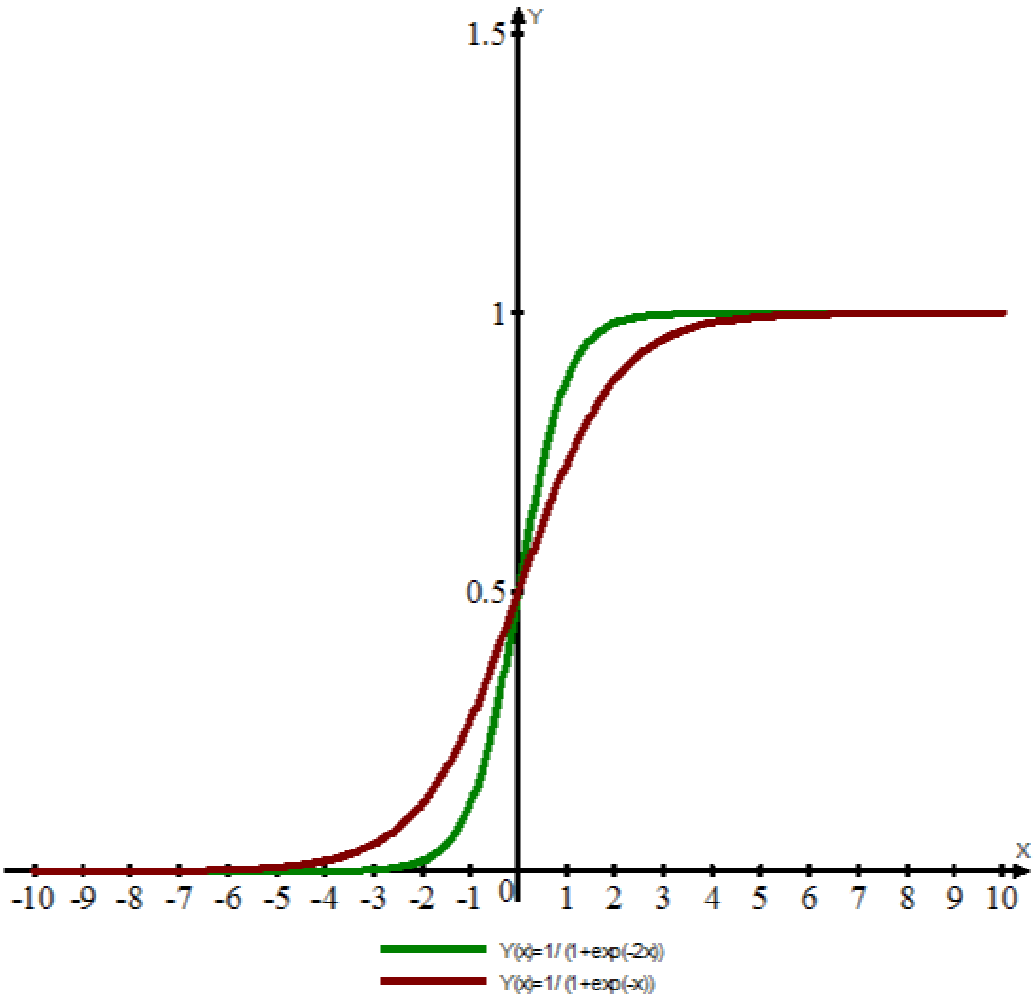
Consider the following function

$$f(x)=\frac{1}{1+ae^{bx}},a>0$$

Consider the different functions with fixed value $a=1$ and different values of b

$$f(x)=\frac{1}{1+e^{bx}}$$

For different values of $b=-2,-1$ and fixed value $a=1$ the graphs of functions is as shown below



From the graphs of functions,

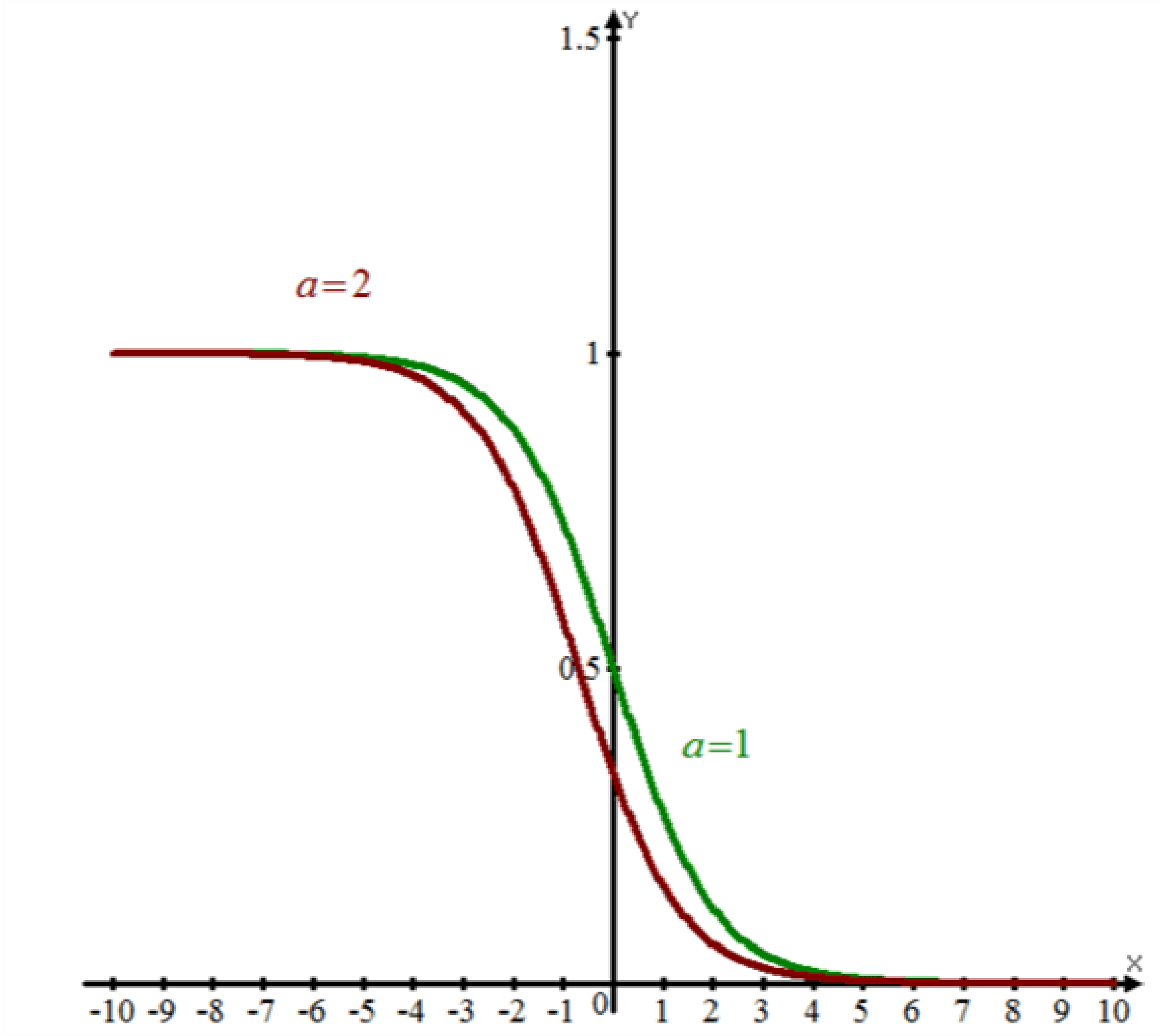
$$as\ x\to-\infty, f(x)\to 0$$
$$as\ x\to\infty, f(x)\to 1$$

In case of $b=-2$ approaches zero or one for large values of x compare with $b=-1$

Consider the different functions with fixed value $b=1$ and different values of a

$$f(x)=\frac{1}{1+e^{bx}}$$

For different values of $a=1,2$ and fixed value $a=1$ the graphs of functions is as shown below



From the above graphs of functions

$as\ x\to-\infty, \mathrm{f}(x)\to 1$

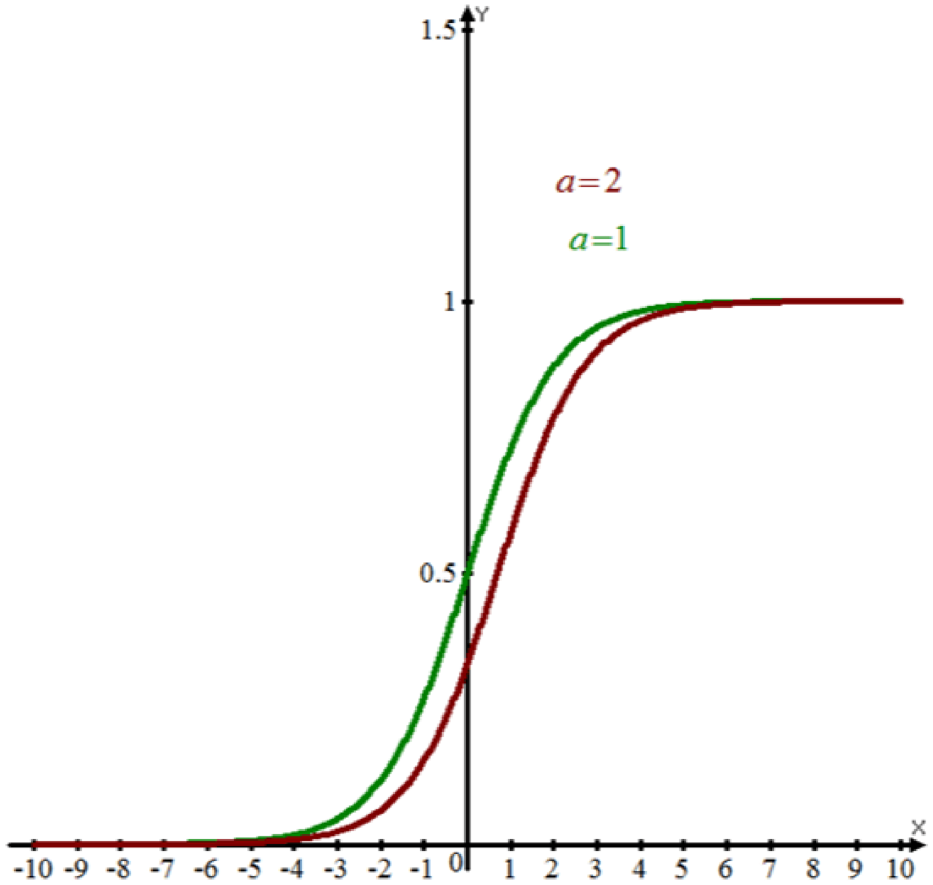
$as\ x\to\infty, \mathrm{f}(x)\to 0$

In case of $a=2$ approaches zero or one for large values of x compare with $a=1$

Consider the different functions with fixed value $b=-1$ and different values of a

$f(x)=\frac{1}{1+e^{bx}}$

For different values of $a=1,2$ and fixed value $a=1$ the graphs of functions is as shown below



From the above graphs of functions

$as\ x\to-\infty, \mathrm{f}(x)\to 0$

$as\ x\to\infty, \mathrm{f}(x)\to 1$

In case of $a=2$ approaches zero or one for large values of x compare with $a=1$

Answer 103E.

(a)

Consider the inequality $e^x\geq 1+x \text{ if } x\geq 0$ ...... (1)

Prove that the inequality (1)

Let $f(x)=e^x-(1+x)$

It is enough to show that $f(x)$ is increasing function.

That is show that $f(x)\geq 0$ for all $x\geq 0$

Now find $f'(x)=e^x-1$

Since $e^x\geq 1 \text{ for } x\geq 0$

$e^x-1\geq 0 \text{ for } x\geq 0$

Therefore $f(x)$ is increasing for $x\geq 0$

Hence $e^x\geq 1+x \text{ if } x\geq 0$

(b)

Consider $x^2 \le 1 \text{ for all } x \in [0,1]$

$e^{x^2} \le e^1$

$\int_0^1 e^{x^2} dx \le \int_0^1 e dx$ Taking integration on both sides.

Find $\int_0^1 e dx = e(x)_0^1$

$= e(1-0)$
$= e$

Therefore $\int_0^1 e^{x^2} dx \le e$ ...... (2)

Now define $g(x) = e^{x^2} - (1+x^2)$

$$g'(x) = 2xe^{x^2} - 2x$$
$$= 2x(e^{x^2} - 1)$$

Therefore $g'(x) \ge 0 \text{ if } x \ge 0$

So $g(x)$ is increasing function.

Hence $e^{x^2} \ge (1+x^2)$

$\int_0^1 e^{x^2} dx \ge \int_0^1 (1+x^2) dx$ Taking integration on both sides

Consider $\int_0^1 (1+x^2) dx$

$$\int_0^1 (1+x^2) dx = \left(x + \frac{x^3}{3}\right)_0^1$$
$$= \left(1+\frac{1}{3}\right) - \left(0 + \frac{(0)^3}{3}\right)$$
$$= \frac{4}{3}$$

Therefore $\int_0^1 e^{x^2} dx \ge \int_0^1 (1+x^2) dx = \frac{4}{3}$

$\int_0^1 e^{x^2} dx \ge \frac{4}{3}$ ...... (3)

From (2) and (3), deduce that $\boxed{\frac{4}{3} \le \int_0^1 e^{x^2} dx \le e}$

Answer 104E.

(a)

Consider the inequality $e^x \ge 1+x+\frac{x^2}{2}$ if $x \ge 0$ ...... (1)

Prove that the inequality (1)

Let $f(x)=e^x-\left(1+x+\frac{x^2}{2}\right)$

It is enough to show that $f(x)$ is increasing function.

That is show that $f(x)\ge 0$ for all $x \ge 0$

Now find $f'(x)=e^x-\left(1+\frac{2x}{2}\right)$

$=e^x-(1+x)$ ...... (2)

Now let $g(x)=f'(x)$

$g(x)=e^x-(1+x)$ From (2)

Now find $g'(x)=e^x-1$

Since $e^x \ge 1 \text{ for } x \ge 0$

$e^x-1 \ge 0 \text{ for } x \ge 0$

Therefore $g(x)$ is increasing for $x \ge 0$

Therefore $f(x)$ is increasing for $x \ge 0$

Hence $\boxed{e^x \ge 1+x+\frac{x^2}{2} \text{ if } x \ge 0}$

(b)

From part (a) $e^x \ge 1+x+\frac{x^2}{2}$ if $x \ge 0$

Consider $x^2 \le 1 \text{ for all } x \in [0,1]$

$e^{x^2} \le e^1$

$\int_0^1 e^{x^2}dx \le \int_0^1 e\,dx$ Taking integration on both sides.

Find $\int_0^1 e\,dx = e(x)_0^1$

$=e(1-0)$

$=e$

Therefore $\int_0^1 e^{x^2}dx \le e$ ...... (3)

Let $e^{x^2} \geq 1 + x^2 + \frac{(x^2)^2}{2}$ if $x \geq 0$ Substitute $x = x^2$ in (1)

$$\int_0^1 e^{x^2} dx \geq \int_0^1 \left(1 + x^2 + \frac{(x^2)^2}{2}\right) dx$$

$$\geq \int_0^1 \left(1 + x^2 + \frac{x^4}{2}\right) dx$$

$$\geq \left(x + \frac{x^3}{3} + \frac{x^5}{10}\right)_0^1$$

$$\geq \left(1 + \frac{(1)^3}{3} + \frac{(1)^5}{10}\right) - \left(0 + \frac{(0)^3}{3} + \frac{(0)^5}{10}\right)$$

$$\geq \left(1 + \frac{1}{3} + \frac{1}{10}\right)$$

$$\geq \frac{43}{30}$$

Therefore $\int_0^1 e^{x^2} dx \geq \frac{43}{30}$ ...... (4)

From (3) and (4) deduce that $\boxed{\frac{43}{30} \leq \int_0^1 e^{x^2} dx \leq e}$

Answer 105E.

(A) We have to prove $e^x \geq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + --- + \frac{x^n}{n!}$ by mathematical induction.

For, n = 1, we have $e^x \geq 1$

Which is true as for $x \geq 0$

Let us suppose that the given inequality is true for n =k. So we have

$$e^x \geq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + --- + \frac{x^k}{(k)!}$$

Now, let $f(x) = e^x - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^k}{k!} + \frac{x^{k+1}}{(k+1)!}\right)$

Differentiating with respect to x, we get,

$$f'(x) = \frac{d}{dx}\left[e^x - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ... + \frac{x^k}{k!} + \frac{x^{k+1}}{(k+1)!}\right)\right]$$

$$= \frac{d}{dx}e^x - \frac{d}{dx}\left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ... + \frac{x^k}{k!} + \frac{x^{k+1}}{(k+1)!}\right]$$

$$= e^x - \left[0 + 1 + \frac{2x}{2!} + \frac{3x^2}{3!} + \frac{4x^3}{4!} + ... + \frac{kx^{k-1}}{k!} + \frac{(k+1)x^k}{(k+1)!}\right]$$

$$= e^x - \left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^{k-1}}{(k-1)!} + \frac{x^k}{(k)!}\right]$$

≥ 0 Since, $e^x \geq 1 + x + \frac{x^2}{2!} + ... + \frac{x^k}{k!}$

So, f(x) is increasing for all x > 0.

Also, $f(0) = e^0 - \left[1 + 0 + \frac{0^2}{2!} + \frac{0^3}{3!} + ... + \frac{0^{k+1}}{(k+1)!}\right]$

$= 1 - 1$

$= 0$

And $f(x)$ is increasing for $x > 0$

So, $f(x) \geq 0$ for all $x \geq 0$

$$\Rightarrow \quad e^x - \left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^k}{k!} + \frac{x^{k+1}}{(k+1)!}\right] \geq 0$$

$$\Rightarrow \quad e^x \geq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots + \frac{x^{k+1}}{(k+1)!}$$

Thus, the inequality is true for $n = k+1$

Therefore, the inequality

$$e^x \geq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots + \frac{x^n}{n!} \text{ is true for all n.}$$

(B) We have to show that $e > 2.7$

From part (a) we have,

$$e^x \geq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots\dots + \frac{x^n}{n!}$$

Where $x \geq 0$

Putting x = 1, we get,

$$e \geq 1 + 1 + \frac{1^2}{2!} + \frac{1^3}{3!} + \frac{1^4}{4!} + \frac{1^5}{5!} + \dots\dots + \frac{1^n}{n!}$$

$$\geq 2 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \dots\dots + \frac{1}{n!}$$

$$\geq 2 + 0.5 + 0.16666 + 0.041666 + \dots\dots + \frac{1}{n!}$$

$$\geq 2.708332 + \dots\dots + \frac{1}{n!}$$

$$> 2.7$$

Hence,

$$\boxed{e > 2.7}$$

(C) From part (a) we have,

$$e^x \geq 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots + \frac{x^n}{n!} \text{ Where n is any positive integer.}$$

Since k is any given positive integer so we have,

$$\frac{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\dots + \frac{x^n}{n!}}{x^k}$$

$$= \frac{1}{x^k} + \frac{x}{x^k} + \frac{x^2}{x^k 2!} + \frac{x^3}{x^k 3!} + \dots\dots + \frac{x^k}{x^k k!} + \frac{x^{k+1}}{x^k (k+1)!} + \dots + \frac{x^n}{n! x^k}$$

$$= \frac{1}{x^k} + \frac{1}{x^{k-1}} + \frac{1}{x^{k-2} 2!} + \frac{1}{x^{k-3} 3!} + \dots\dots + \frac{1}{k!} + \frac{x}{(k+1)!} + \dots\dots + \frac{x^{n-k}}{n!}$$

So,

$$\lim_{x\to\infty}\frac{1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....+\frac{x^n}{n!}}{x^k}$$

$$=\lim_{x\to\infty}\left[\frac{1}{x^k}+\frac{1}{x^{k-1}}+\frac{1}{x^{k-2}2!}+.....+\frac{1}{k!}+\frac{x}{(k+1)!}+....+\frac{x^{n-k}}{n!}\right]$$

$$=\lim_{x\to\infty}\frac{1}{x^k}+\frac{1}{\lim_{x\to\infty}x^{k-1}}+\frac{1}{\lim_{x\to\infty}x^{k-2}2!}+.....+\frac{1}{k!}+\frac{\lim_{x\to\infty}x}{(k+1)!}.....+\frac{\lim_{x\to\infty}x^{n-k}}{n!}$$

$$=0+0+0\qquad+\frac{1}{k!}+\infty+\infty+.....+\infty$$

$$=\infty$$

Since $e^x\geq 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.......+\frac{x^n}{n!}$

$$\Rightarrow\qquad \frac{e^x}{x^k}\geq\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....+\frac{x^3}{n!}\right)$$

As we are finding $\lim_{x\to\infty}$ So ,x > 1.

$$\Rightarrow\qquad \lim_{x\to\infty}\frac{e^x}{x^k}\geq\lim_{x\to\infty}\frac{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....+\frac{x^n}{n!}\right)}{x^k}$$

$$\Rightarrow\qquad \lim_{x\to\infty}\frac{e^x}{x^k}\geq\infty$$

$$\Rightarrow\qquad \lim_{x\to\infty}\frac{e^x}{x^k}=\infty$$

Hence,

$$\boxed{\lim_{x\to\infty}\frac{e^x}{x^k}=\infty}$$