CBSE Test Paper 02

Chapter 4 Determinants

1. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is
   a. a + b + c
   b. 0
   c. None of these
   d. 1 + a + b + c
2. The only integral root of the equation det. $\begin{vmatrix} 2-y & 2 & 3 \\ 2 & 5-y & 6 \\ 3 & 4 & 10-y \end{vmatrix} = 0$ is
   a. 2
   b. 1
   c. 3
   d. 4
3. Find the area of triangle with vertices (1, 1), (2, 2) and (3, 3).
   a. 1
   b. 3
   c. 0
   d. 2
4. The value of the determinant of a skew symmetric matrix of even order is
   a. A non zero perfect square
   b. None of these
   c. 0
   d. Negative
5. If the matrix AB = O , then
   a. A = O or B = O
   b. A = O and B = O
   c. It is not necessary that either A = O or B = O
   d. None of these

6. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then the determinant of A is ________.

7. If A is invertible matrix of order 3×3, then $|A^{-1}|$ = ________.

8. If we multiply each element of a row (or a column) of a determinant by constant k, then value of the determinant is ________ by k.

9. Find values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$.

10. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$ then show that $|2A| = 4|A|$.

11. Find value of x, if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$.

12. Show that $\begin{vmatrix} \sin 10^0 & -\cos 10^0 \\ \sin 80^0 & \cos 80 \end{vmatrix} = 1$.

13. In the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ Verify that $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$.

14. Find the area of the triangle with vertices at the points given (1, 0), (6, 0), (4, 3).

15. Prove that $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$.

16. Using properties of determinants, prove that

$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$ = xyz(x - y)(y - z)(z - x).

17. Using properties of determinants, prove that

$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$

18. Verify A (adj. A) = (adj. A) A = |A|I for following matrix:

$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

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Solution

1. b. 0

   Explanation: $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$

   Apply, $C_2 \to C_2 + C_3$,

   $\begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & c+a \\ 1 & a+b+c & a+b \end{vmatrix}$

   $\Rightarrow (a + b + c) \begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & c+a \\ 1 & 1 & a+b \end{vmatrix}$

   = 0 ($C_1 = C_2$)

   Since, C_1 and C_2 are identical

   $=(a+b+c)\times 0 = 0$

2. b. 1

   Explanation: The value of determinant is 0 if any two rows or column are identical and Clearly, y = 1 satisfies it.

   if we take common as 3 from C_3.Then, C_1 And C_3 Becomes identical after putting y=1.

3. c. 0

   Explanation: AREA OF TRIANGLE=

   $\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$ (Since C_1 and C_2 are identical)

   So, value of determinant = 0

   Hence, area of triangle = 0

4. a. A non zero perfect square

   Explanation: The determinant of a skew symmetric matrix of even order is A non zero perfect square and odd order is equal to 0.

5. c. It is not necessary that either A = O or B = O

Explanation: If the matrix AB = O , then , matrix A can be a non zero matrix as well as matrix B can be a non zero matrix because for the multiplication of two matrics to be equal to 0 the matrices need not to be equal to 0. So, it is not necessary that either A=0 or B=0.

6. ad - bc
7. $\frac{1}{|A|}$
8. multiplied
9. $(3-x)^2 = 3-8$

   $3-x^2 = 3-8$

   $-x^2 = -8$

   $x = \pm\sqrt{8}$

   $x = \pm 2\sqrt{2}$
10. $2A = 2\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix} = \begin{bmatrix}2 & 4\\ 8 & 4\end{bmatrix}$

    RHS = $4|A| = 4\times(2-8) = 4\times(-6) = -24$

    L.H.S = $|2A| = 8-32$= - 24

    Hence Proved
11. (2 – 20) = ($2x^2$ – 24)

    -18 = $2x^2$– 24

    -$2x^2$ = -24 + 18

    -$2x^2$ = -6

    $2x^2$ = 6

    x^2 = 3

    $x = \pm\sqrt{3}$
12. L.H.S$= \sin 10^\circ \cos 80^\circ + \cos 10^\circ \sin 80^\circ$

    $= \sin(10^\circ + 80^\circ)$

    $[\because \sin A.\cos B + \cos A.\sin B = \sin(A+B)]$

    = sin 90°

    = 1
13. a_{11} = 2, a_{12} = -3, a_{13} = 5

$A_{31} = -12 - 5 \times 0 = -12 - 0 = -12$

A_{32} = -(8 - 30) = -(-22) = 22

$A_{33} = 2 \times 10 - (-18) = 0 + 18 = 18$

L.H.S = $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

= 2 (-12) + (-3) (22) +5 (18)

= -24 - 66 + 90

= -90 + 90

= 0 Hence proved.

14. Area of triangle = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$

$= \frac{1}{2}$ [-3(1-6)]= $\frac{1}{2}$ (-3)(-5)= $\frac{15}{2}$ sq.units

15. $L.H.S = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$

$[C_1 \rightarrow C_1 - bC_3 \ and \ C_2 \rightarrow C_2 + aC_3]$

$= \begin{vmatrix} 1+a^2+b^2 & 0 & -2b \\ 0 & 1+a^2+b^2 & 2a \\ b\left(1+a^2+b^2\right) & -a\left(1+a^2+b^2\right) & 1-a^2-b^2 \end{vmatrix}$

Taking 1+a^2+b^2 common from each $C_1 \ and \ C_2$

$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ b & -a & 1-a^2-b^2 \end{vmatrix}$

$[R_3 \rightarrow R_3 - bR_1]$

$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 0 & -2b \\ 0 & 1 & 2a \\ 0 & -a & 1-a^2+b^2 \end{vmatrix}$

$= \left(1+a^2+b^2\right)^2 \begin{vmatrix} 1 & 2a \\ -a & 1-a^2+b^2 \end{vmatrix}$

$= \left(1+a^2+b^2\right)^2 \left(1-a^2+b^2+2a^2\right)$

$= \left(1+a^2+b^2\right)^3$ = R.H.S.

16. According to the question,We have to prove that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz (x - y) (y - z) (z - x)$$

We shall make use of the properties of determinants to prove the required result.

Let LHS = $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = xyz\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ [taking x, y and z common from C_1, C_2 and C_3, respectively]

On applying $C_1 \rightarrow C_1 - C_2$ and then $C_2 \rightarrow C_2 - C_3$,

We get

LHS = xyz$\begin{vmatrix} 0 & 0 & 1 \\ x-y & y-z & z \\ x^2-y^2 & y^2-z^2 & z^2 \end{vmatrix}$

On expanding along R_1, we get

LHS = xyz $\begin{vmatrix} x-y & y-z \\ x^2-y^2 & y^2-z^2 \end{vmatrix}$

= xyz$\begin{vmatrix} x-y & y-z \\ (x-y)(x+y) & (y-z)(y+z) \end{vmatrix}$

On taking (x - y) common from C_1 and (y- z) from C_2, we get

LHS = xyz (x- y) (y - z) $\begin{vmatrix} 1 & 1 \\ x+y & y+z \end{vmatrix}$

= xyz (x - y)(y- z)[(y + z - (x + y)]

= xyz (x - y) (y - z) (z - x)

= RHS

17. Applying $R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 - R_3$,we get,

$$\Delta = \begin{vmatrix} (x-z) & \left(x^2-z^2\right) & yz-xy \\ y-z & y^2-z^2 & zx-xy \\ z & z^2 & xy \end{vmatrix}$$

$$= \begin{vmatrix} x-z & (x-z)(x+z) & -y(x-z) \\ y-z & (y-z)(y+z) & -x(y-z) \\ z & z^2 & xy \end{vmatrix}$$

Taking (x-z) common from R_1 and (y-z) common from R_2 ,we have

$$\Delta = (x-z)(y-z)\begin{vmatrix} 1 & x+y & -(y) \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$,we get,

$$\Delta = (x-z)(y-z)\begin{vmatrix} 0 & x-y & x-y \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$$

Taking (x-y) common from R_1 ,we get,

$$\Delta = (x-z)(y-z)(x-y)\begin{vmatrix} 0 & 1 & 1 \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$$

Expanding along R_1 ,we get,

$$\Delta = (x-y)(y-z)(x-z)\left[-1(xy+zx)+1\left(z^2-yz-z^2\right)\right]$$
$$= (x-y)(y-z)(x-z)[-xy-zx-yz]$$
$$= (x-y)(y-z)(z-x)[xy+zx+yz]$$

18. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

$$\Rightarrow |A| = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\therefore A_{11} = +\begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = +0+0 = 0, A_{12} = -\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -(9+2) = -11$$

$$A_{13} = +\begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = +(0-0) = 0, A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} - (-3-0) = 3$$

$$A_{22} = +\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 3-2 = 1, A_{23} = -\begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(0+1) = -1$$

$$A_{31} = +\begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2-0 = 2,\ A_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -(-2-6) = 8$$

$$A_{33} = +\begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 3+0 = 3$$

$$\therefore adj.\, A = \begin{vmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{vmatrix}$$

$$\therefore A.(adj.A) = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0-0-0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \text{...(i)}$$

Again (adj. A). A $= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0-0+0 & 0+2+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \text{....(ii)}$$

And $|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$

$$= 1(0-0) - (-1)(9+2) + 2(0-0) = 0 + 11 + 0 = 11$$

Also $|A|\, I = |A|\, I_3 = 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$ ...(iii)

$\therefore$ From eq. (i), (ii) and (iii) A. (adj. A) = (adj. A). A = |A|