GUJCET-ME-2020

Test Booklet Set No. **07**

DO NOT open this Test Booklet until you are asked to do so.

Important Instructions :

1) The Mathematics test consists of 40 questions. Each question carries **1** mark. For each correct response, the candidate will get **1** mark. For each incorrect response, ¼ mark will be deducted. The maximum marks are **40**.
2) This Test is of **1 hour** duration.
3) Use **Black Ball Point Pen only** for writing particulars on OMR Answer Sheet and marking answers by darkening the circle '●'.
4) Rough work is to be done on the space provided for this purpose in the Test Booklet only.
5) **On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room / Hall. The candidates are allowed to take away this Test Booklet with them.**
6) The Set No. for this Booklet is 07 . Make sure that the Set No. printed on the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
7) The candidate should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet.
8) Do not write your Seat No. anywhere else, except in the specified space in the Test Booklet / Answer Sheet.
9) Use of White fluid for correction is not permissible on the Answer Sheet.
10) Each candidate must show on demand his / her Admission Card to the Invigilator.
11) No candidate, without special permission of the Superintendent or Invigilator, should leave his / her seat.
12) Use of Simple (Manual) Calculator is permissible.
13) The candidate should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and must sign the Attendance Sheet (Patrak - 01). Cases where a candidate has **not** signed the Attendance Sheet (Patrak - 01) will be deemed not to have handed over the Answer Sheet and will be dealt with as an unfair means case.
14) The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.
15) No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
16) The candidates will write the Correct Test Booklet Set No. as given in the Test Booklet / Answer Sheet in the Attendance Sheet (Patrak - 01).

MATHEMATICS

1) The tangent to the curve given by $x = e^{\theta}\cdot\cos\theta$, $y = e^{\theta}\cdot\sin\theta$ at $\theta = \frac{\pi}{4}$ makes an angle with X-axis is _____.

(A) $\frac{\pi}{2}$ (B) 0

(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

2) The minimum value of $f(x) = x \log x$ is _____.

(A) 0 (B) $-\frac{1}{e}$

(C) $\frac{1}{e}$ (D) e

3) If $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx = \frac{x^3}{3} + f(x) + C$, then $f(1) =$ _____.

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{1}{2}$

(Space for Rough Work)

4) $\int \frac{x+100}{(x+101)^2} e^x \, dx =$ _____ + C.

(A) $\frac{1}{x+101} e^x$

(B) $\frac{x}{x+101} e^x$

(C) $\frac{1}{x+100} e^x$

(D) $(x + 101)e^x$

5) $\int \frac{\sqrt{\cot x}}{\cos x \sin x} dx =$ _____ + C.

(A) $-2\sqrt{\cot x}$

(B) $-2\sqrt{\tan x}$

(C) $2\sqrt{\cot x}$

(D) $\frac{1}{\sqrt{\cot x}}$

6) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \log\left(\frac{2019-x}{2019+x}\right) dx =$ _____.

(A) π

(B) 0

(C) $\frac{\pi}{2}$

(D) 1

(Space for Rough Work)

7) $\int_4^9 \frac{\sqrt{x}}{\left(30 - x^{3/2}\right)^2} dx =$ _____.

(A) $\frac{19}{66}$ (B) $\frac{19}{33}$

(C) $\frac{38}{99}$ (D) $\frac{19}{99}$

8) If $f(a+b-x) = f(x)$ then $\int_a^b x \cdot f(x) dx$ is equal to _____.

(A) $\frac{a+b}{2}\int_a^b f(x) dx$ (B) $\frac{a+b}{2}\int_a^b f(b+x) dx$

(C) $\frac{a+b}{2}\int_a^b f(b-x) dx$ (D) $\frac{b-a}{2}\int_a^b f(x) dx$

9) The area of the parabola $y^2 = 4ax$ bounded by its latus rectum is _____.

(A) $\frac{16}{3}a^2$ (B) $\frac{4}{3}a^2$

(C) $\frac{8}{3}a^2$ (D) $4a^2$

(Space for Rough Work)

10) The area enclosed by the curve $x = 4\cos\theta$, $y = 3\sin\theta$ is _____.

(A) 4π (B) 6π

(C) 8π (D) 12π

11) The smallest area enclosed by circle $x^2 + y^2 = 4$ and line $x + y = 2$ is _____.

(A) $\pi + 2$ (B) $\pi - 2$

(C) π (D) 2π

12) The order and degree of differential equation $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2} = \frac{d^2y}{dx^2}$ are p and q respectively then $p + q =$ _____.

(A) 6 (B) 4

(C) 2 (D) 5

13) Integrating factor of differential equation $(\tan^{-1}y - x)dy = (1 + y^2)dx$ is _____

(A) e^{1+y^2} (B) e^{y}

(C) $e^{\tan^{-1}x}$ (D) $e^{\tan^{-1}y}$

(Space for Rough Work)

14) The differential equation $y\frac{dy}{dx}+x=k$ represents _____.

(A) circles
(B) hyperbolas
(C) parabolas
(D) ellipses

15) If $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}-2\hat{k}$, $\vec{c}=\hat{i}+3\hat{j}-\hat{k}$, if $\vec{a}$ is perpendicular to $\lambda\vec{b}+\vec{c}$, then the value of λ is _____.

(A) 0
(B) 2
(C) –2
(D) 3

16) For three vectors $\vec{a}, \vec{b}, \vec{c}$ satisfies $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=4$, $|\vec{c}|=2$ then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ = _____.

(A) 29
(B) $\frac{29}{2}$
(C) $-\frac{9}{2}$
(D) $-\frac{29}{2}$

17) If $|\vec{a}|=3$ then value of $|\vec{a}\times\hat{i}|^2+|\vec{a}\times\hat{j}|^2+|\vec{a}\times\hat{k}|^2$ = _____.

(A) 9
(B) 18
(C) 27
(D) 36

(Space for Rough Work)

18) The co-ordinates of the foot of perpendicular drawn from origin to the plane $2x - 3y + 4z - 6 = 0$ is _____.

(A) $\left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$ (B) $\left(\frac{12}{29}, -\frac{18}{29}, -\frac{24}{29}\right)$

(C) $\left(\frac{12}{29}, \frac{18}{29}, \frac{24}{29}\right)$ (D) $\left(-\frac{12}{29}, -\frac{18}{29}, -\frac{24}{29}\right)$

19) The angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$ is _____.

(A) $\cos^{-1}\frac{8}{21}$ (B) $\tan^{-1}\frac{8}{\sqrt{377}}$

(C) $\sin^{-1}\frac{8}{\sqrt{377}}$ (D) $\sin^{-1}\left(\frac{21}{8}\right)$

20) If the points $(1, 1, p)$ and $(-3, 0, 1)$ be equidistant from the plane $\vec{r}\cdot\left(3\hat{i}+4\hat{j}-12\hat{k}\right)+13=0$ then the values of p are _____.

(A) $1, \frac{7}{3}$ (B) $1, \frac{4}{3}$

(C) $2, \frac{4}{3}$ (D) $\frac{7}{3}, 2$

(Space for Rough Work)

21) The maximum value of $Z = 3x + 4y$ subject to constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ is _____.

(A) 16 (B) 12

(C) 0 (D) not possible

22) If A and B are independent events such that $P(A) = p$, $P(B) = 2p$ and P(Exactly one of A and B) $= \frac{5}{9}$ then $p =$ _____.

(A) $\frac{1}{3}, \frac{5}{12}$ (B) $\frac{1}{2}, \frac{3}{4}$

(C) $\frac{1}{12}, \frac{5}{3}$ (D) $\frac{2}{15}, \frac{5}{12}$

23) For the probability distribution

X	1	2	3	4
P(X)	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$

$E(X^2) =$ _____.

(A) 7 (B) 5

(C) 3 (D) 10

(Space for Rough Work)

24) If A and B are any two events such that P(A) + P(B) – P(A ∩ B) = P(A) then _____.

(A) $P\left(\frac{B}{A}\right)=1$ (B) $P\left(\frac{A}{B}\right)=0$

(C) $P\left(\frac{B}{A}\right)=0$ (D) $P\left(\frac{A}{B}\right)=1$

25) Let $f: R \to R$ be defined by $f(x)=2x^2-5$ and $g: R \to R$ by $g(x)=\frac{x}{x^2+1}$ then gof is _____.

(A) $\frac{2x^2-5}{4x^4+20x^2+26}$ (B) $\frac{2x^2-5}{4x^4-20x^2+26}$

(C) $\frac{2x^2}{x^4+2x^2-4}$ (D) $\frac{2x^2}{4x^4-20x^2+26}$

26) Let $f:[2,\infty)\to R$ be the function defined by $f(x)=x^2-4x+5$. Then the range of f is _____.

(A) $[1,\infty)$ (B) $[4,\infty)$

(C) R (D) $[5,\infty)$

(Space for Rough Work)

27) On R, binary operation $*$ is defined by $a * b = a + b + ab$ then identity and inverse of $*$ are _____ respectively.

(A) $0, \frac{a}{1-a}$

(B) $1, \frac{a}{1+a}$

(C) $0, -\frac{a}{1+a}$

(D) $1, \frac{a}{1-a}$

28) $\sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{8}{17}\right) =$ _____.

(A) $\cos^{-1}\left(\frac{84}{85}\right)$

(B) $\cos^{-1}\left(\frac{24}{85}\right)$

(C) $\sin^{-1}\left(\frac{24}{85}\right)$

(D) $\sin^{-1}\left(\frac{84}{85}\right)$

29) $\tan^2\left(\sec^{-1}3\right) + \operatorname{cosec}^2\left(\cot^{-1}2\right) + \cos^2\left(\cos^{-1}\frac{2}{3} + \sin^{-1}\frac{2}{3}\right) =$ _____.

(A) 15

(B) 16

(C) 14

(D) 13

(Space for Rough Work)

30) If $A=\begin{bmatrix} a & b \\ c & -a \end{bmatrix}$ is such that $A^2 = I$ then _____.

(A) $1 - a^2 + bc = 0$ (B) $1 + a^2 + bc = 0$

(C) $1 + a^2 - bc = 0$ (D) $1 - a^2 - bc = 0$

31) If A is a square matrix such that $A^2 = I$ then $(A - I)^3 + (A + I)^3 - 7A$ is equal to _____.

(A) I+A (B) I−A

(C) A (D) 3A

32) If $A=\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ and inverse of A is $\frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -8 & 6 & -2 \\ x & -3 & 1 \end{bmatrix}$ then $x =$ _____.

(A) 5 (B) 3

(C) 2 (D) 4

33) Let $f(t)=\begin{vmatrix} \cos t & t & 1 \\ 2\tan t & t & 2t \\ \tan t & t & t \end{vmatrix}$. Then $\lim_{t\to 0} \frac{f(t)}{t^2}$ is equal to _____.

(A) 3 (B) 1

(C) −1 (D) 0

(Space for Rough Work)

34) If $x, y \in R$ and $\begin{vmatrix} (a^x + a^{-x})^2 & (a^x - a^{-x})^2 & 1 \\ (b^x + b^{-x})^2 & (b^x - b^{-x})^2 & 1 \\ (c^x + c^{-x})^2 & (c^x - c^{-x})^2 & 1 \end{vmatrix} = 2y + 6$ then $y =$ _____.

(A) 0
(B) 3
(C) −3
(D) 6

35) For ΔABC, the value of $\begin{vmatrix} 0 & \sin A & \tan B \\ -\sin(B+C) & 0 & \cos C \\ \tan(A+C) & -\cos C & 0 \end{vmatrix}$ = _____.

(A) −1
(B) 0
(C) 1
(D) $\sin A \cos C$

36) If function $f(\alpha) = \begin{cases} \dfrac{1-\cos 6\alpha}{36\alpha^2} & \text{if } \alpha \neq 0 \\ k & \text{if } \alpha = 0 \end{cases}$ is continuous at $\alpha = 0$ then $k =$ _____.

(A) $-\dfrac{1}{2}$
(B) 1
(C) $\dfrac{1}{2}$
(D) 0

(Space for Rough Work)

37) If $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ and $\frac{dy}{dx} = \frac{2^{x+1}\log 2}{f(x)}$ then $f(0) =$ _____.

(A) 0 (B) -2

(C) 2 (D) $2 \log 2$

38) For function $f(x) = x + \frac{1}{x}$, $x \in [1, 2]$, the value of C for mean value theorem is _____.

(A) 2 (B) $\sqrt{2}$

(C) 1 (D) $\sqrt{3}$

39) The interval in which $y = x^2 e^{-x}$ is increasing is _____.

(A) $(0, 2)$ (B) $(2, \infty)$

(C) $(-\infty, \infty)$ (D) $(-2, 0)$

40) The rate of change of volume of sphere with respect to its radius r at $r = 2$ is _____.

(A) 24π (B) 32π

(C) 16π (D) 8π

(Space for Rough Work)