# HOTS (Higher Order Thinking Skills)

**Que 1. Prove that:** $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\,\mathrm{cosec}\theta=1+\tan\theta+\cot\theta.$

**Sol.** LHS = $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=\frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}}+\frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$=\frac{\sin\theta\times\sin\theta}{\cos\theta\,(\sin\theta-\cos\theta)}+\frac{\cos\theta}{\sin\theta}\times\frac{\cos\theta}{(\cos\theta-\sin\theta)}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}+\frac{\cos^2\theta}{\sin\theta\{-(\sin\theta-\cos\theta)\}}$

$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}=\frac{\sin^3\theta-\cos^3\theta}{\cos\theta(\sin\theta-\cos\theta)\sin\theta}$

$=\frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{\cos\theta\sin\theta\,(\sin\theta-\cos\theta)}=\frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$=\frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}=\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}+1$ …..(i)

$=\sec\theta\,\mathrm{cosec}\theta+1$ ……(ii)

**For second part**

Now from (i), we have

LHS $=\frac{1}{\sin\theta\cos\theta}+1$ [Putting 1 = $\sin^2\theta+\cos^2\theta$]

$=\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}+1=\frac{\sin^2\theta}{\sin\theta\cos\theta}+\frac{\cos^2\theta}{\cos\theta\sin\theta}+1$

$=\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}+1=\tan\theta+\cot\theta+1$

**Que 2. If tan A = n tan B and sin A = m sin B, prove that** $\cos^2 A=\frac{m^2-1}{n^2-1}$**.**

**Sol.** We have to find $\cos^2 A$ in terms of m and n. This means that the angle B is to be eliminated from the given relations.

Now, tan A = n tan B

$\Rightarrow\quad \tan B=\frac{1}{n}\tan A \qquad \Rightarrow \qquad \cot B=\frac{n}{\tan A}$

And $\sin A=m\sin B$

$\Rightarrow\quad \sin B=\frac{1}{m}\sin A \qquad \Rightarrow \qquad \mathrm{cosec}\,B=\frac{m}{\sin A}$

Substituting the value of cot B and cosec B in $\mathrm{cosec}^2 B-\cot^2 B=1$, we get

$\frac{m^2}{\sin^2 A}-\frac{n^2}{\tan^2 A}=1 \qquad \Rightarrow \qquad \frac{m^2}{\sin^2 A}-\frac{n^2\cos^2 A}{\sin^2 A}=1$

$\Rightarrow \quad \frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1 \qquad \Rightarrow \quad \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow \quad \frac{m^2 - n^2\cos^2 A}{\sin^2 A} = 1 \qquad \Rightarrow \quad m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow \quad m^2 - n^2\cos^2 A = 1 - \cos^2 A \qquad \Rightarrow \quad m^2 - 1 = n^2\cos^2 A - \cos^2 A$

$\Rightarrow \quad m^2 - 1 = (n^2 - 1)\cos^2 A \qquad \Rightarrow \quad \frac{m^2-1}{n^2-1} = \cos^2 A.$

**Que 3. Prove the following identity, where the angle involved is acute angle for which the expressions are defined.**

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \boldsymbol{cosec\,A + cot\,A},$$

**Using the identity cosec$^2$ A = 1 + cot$^2$ A.**

**Sol.** LHS = $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \frac{\frac{\sin A}{}}{\frac{\cos A + \sin A - 1}{\sin A}} = \frac{cot\,A - 1 + cosec\,A}{cot\,A + 1 - cosec\,A}$

= $\frac{(cot\,A + cosec\,A) - (cosec^2 - cot^2\,A)}{cot\,A - cosec\,A + 1}$ $\quad [\because\ cosec^2 A - cot^2\,A = 1]$

= $\frac{(cot\,A + cosec\,A) - [(cosec\,A + cot\,A)(cosec\,A - cot\,A)]}{cot\,A - cosec\,A + 1}$

= $\frac{(cosec\,A + cot\,A)(1 - cosec\,A + cot\,A)}{(cot\,A - cosec\,A + 1)}$

= cosec A + cot A = RHS.

**Que 4. If x sin$^3$ $\boldsymbol{\theta + ycos^3\theta = sin\theta cos\theta}$ and sin$\boldsymbol{\theta}$ = y cos$\boldsymbol{\theta}$, prove x$^2$ + y$^2$ = 1.**

**Sol.** We have, $\quad x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$

$\Rightarrow \quad (x\sin\theta)\sin^2\theta + (y\cos\theta)\cos^2\theta = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta\,(\sin^2\theta) + (x\sin\theta)\cos^2\theta = \sin\theta\cos\theta \qquad [\because\ x\sin\theta = y\cos\theta]$

$\Rightarrow \quad x\sin\theta\,(\sin^2\theta + \cos^2\theta) = \sin\theta\cos\theta$

$\Rightarrow \quad x\sin\theta = \sin\theta\cos\theta \quad \Rightarrow \quad x = \cos\vartheta$

Now, we have $x$ sin $\theta$ = y cos $\theta$

$\Rightarrow \quad \cos\theta\sin\theta = y\,cos\theta \qquad [\because\ x = \cos\theta]$

$\Rightarrow \quad$ y = sin$\theta$

Hence, $x^2 + y^2 = \cos^2\theta + \sin^2\theta = 1.$

**Que 5. If tan$\boldsymbol{\theta}$ + sin$\boldsymbol{\theta}$ = m and tan$\boldsymbol{\theta}$ – sin$\boldsymbol{\theta}$ = n, show that (m$^2$ – n$^2$) = 4 $\boldsymbol{\sqrt{mn}}$.**

**Sol.** we have given tan$\theta$ + sin$\theta$ = m, and tan$\theta$ – sin$\theta$ = n, then

LHS = (m$^2$ – n$^2$) = (tan$\theta$ + sin$\theta$)$^2$– (tan$\theta$ – sin$\theta$)$^2$

= tan$^2\theta$ + sin$^2\theta$ + 2 tan$\theta$ sin$\theta$ – tan$^2\theta$ – sin$^2\theta$ + 2 tan$\theta$ sin$\theta$

= 4 tan$\theta$ sin$\theta$ = $4\sqrt{\tan^2\theta\,\sin^2\theta}$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}(1 - \cos^2\theta)} = 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{(\tan\theta - \sin\theta)(\tan\theta + \sin\theta)} = 4\sqrt{mn} = RHS$$