

CBSE Test Paper 01
Chapter 4 Determinants

1. The roots of the equation $\det. \begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0$ are

- a. None of these
- b. 2 and 3
- c. 1, 2 and 3
- d. 1 and 3

2. If A' is the transpose of a square matrix A , then

- a. $|A| + |A'| = 0$
- b. $|A| = |A'|$
- c. $|A| \neq |A'|$
- d. None of these

3. If $f(x) = \begin{vmatrix} 2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$ then, $f(\frac{\pi}{3}) =$.

- a. 0
- b. 1
- c. -1
- d. 2

4. The roots of the equation $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$ are

- a. -1, -2
- b. -1, 2
- c. 1, -2
- d. 1, 2

5. If A and B are any 2×2 matrices, then $\det. (A+B) = 0$ implies

- a. $\det A + \det B = 0$
- b. $\det A = 0$ or $\det B = 0$
- c. None of these

d. $\det A = 0$ and $\det B = 0$

6. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then x is _____.
7. Multiplying a determinant by k means multiplying the elements of only one row (or one column) by _____.
8. If elements of a row (or a column) in a determinant can be expressed as the sum of two or more elements, then the given determinant can be expressed as the _____ of two or more determinants.
9. Find adj A for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$.
10. $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular or not.
11. Evaluate $2 \begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}$.
12. Evaluate: $\begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$.
13. Find the area of Δ whose vertices are (3, 8) (-4, 2) and (5, 1).
14. Find the equation of the line joining A (1, 3) and B (0, 0) using det. Find K if D (K, 0) is a point such that area of ΔABD is 3 square unit.
15. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then find $(A')^{-1}$.
16. If $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, find matrix B such that $AB = I$.
17. Using properties of determinants, prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$.
18. Given $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$. find AB and use this result in solving the following system of equation.
 $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$

CBSE Test Paper 01
Chapter 4 Determinants

Solution

1. c. 1, 2 and 3

Explanation: Expanding along C_1

$$\begin{vmatrix} 1-x & 2 & 3 \\ 0 & 2-x & 0 \\ 0 & 2 & 3-x \end{vmatrix} = 0 \Rightarrow (1-x)(2-x)(3-x) = 0 \Rightarrow x = 1, 2, 3.$$

2. b. $|A| = |A'|$

Explanation: The determinant of a matrix A and its transpose always same.

Because if we interchange the rows into column in a determinant the value of determinant remains unaltered.

3. c. -1

Explanation: $\begin{vmatrix} 2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$

Put $x = \frac{\pi}{3}$, $\begin{vmatrix} 2\cos \frac{\pi}{3} & 1 & 0 \\ 1 & 2\cos \frac{\pi}{3} & 1 \\ 0 & 1 & 2\cos \frac{\pi}{3} \end{vmatrix}$

$$\Rightarrow \begin{vmatrix} 2 \cdot \frac{1}{2} & 1 & 0 \\ 1 & 2 \cdot \frac{1}{2} & 1 \\ 0 & 1 & 2 \cdot \frac{1}{2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{vmatrix} \Rightarrow 1(0) - 1(1) = -1$$

4. b. -1, 2

Explanation: $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

Apply, $R_3 \rightarrow R_3 - R_1$, $R_2 \rightarrow R_2 - R_1$,

$$\Rightarrow \begin{vmatrix} 1 & 4 & 20 \\ 0 & -6 & -15 \\ 0 & 2x-4 & 5x^2-20 \end{vmatrix} = 0$$

$$\Rightarrow -6(5x^2 - 20) + 15(2x - 4) = 0$$

$$\Rightarrow (x - 2)(x + 1) = 0 \Rightarrow x = 2, -1.$$

5. c. None of these

Explanation: If $\det(A+B)=0$ implies that $A+B$ a Singular matrix.

6. $x = \pm 3$

7. k

8. sum

$$9. \text{adj}A = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$\left[\begin{array}{c} \because A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \\ \text{change sign} \quad \text{inter-change} \end{array} \right]$$

$$10. |A| = \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix}$$

$$= 8 - 8$$

$$= 0$$

Hence A is singular

$$11. \text{ According to the question, we have to evaluate } 2 \begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix}.$$

$$\text{Now, } 2 \begin{vmatrix} 7 & -2 \\ -10 & 5 \end{vmatrix} = 2[35 - (20)]$$

$$= 2 \times 15 = 30$$

$$12. \text{ Let } \Delta = \begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cos \beta & \sin \alpha \sin \beta & \cos \alpha \end{vmatrix}$$

Expanding along first row,

$$= \cos \alpha \cos \beta (\cos \alpha \cos \beta - 0) - \cos \alpha \sin \beta (-\cos \alpha \sin \beta - 0)$$

$$- \sin \alpha (-\sin \alpha \sin^2 \beta - \sin \alpha \cos^2 \beta)$$

$$= \cos^2 \alpha \cos^2 \beta + \cos^2 \alpha \sin^2 \beta + \sin^2 \alpha (\sin^2 \beta + \cos^2 \beta)$$

$$= \cos^2 \alpha (\cos^2 \beta + \sin^2 \beta) + \sin^2 \alpha (\sin^2 \beta + \cos^2 \beta)$$

$$= \cos^2 \alpha + \sin^2 \alpha$$

$$= 1$$

$$\begin{aligned}
 13. \Delta &= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \\
 &= \frac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix} \\
 &= \frac{1}{2} [3(2-1) - 8(-4-5) + 1(-4+10)] \\
 &= \frac{1}{2} [3 + 72 - 14] = \frac{61}{2}
 \end{aligned}$$

14. Let P (x, y) be any point on AB. Then the equation of line AB is,

$$\frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 1 & 3 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$y = 3x$$

Area $\Delta ABD = 3$ square unit

$$\frac{1}{2} \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ K & 0 & 1 \end{vmatrix} = \pm 3$$

$$k = \pm 2$$

15. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, then we have to find $(A')^{-1}$.

$$\text{Now, } A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix} \text{ Therefore, we have, } |A| = \begin{vmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{vmatrix}$$

$$= 1(-1-8) + 2(0+8) + 3(0-2)$$

[expanding along R_1]

$$= -9 + 16 - 6 = 1 \neq 0$$

Therefore, A is non-singular matrix and hence its inverse exists.

Cofactors of an element of $|A|$ are given by

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 4 \\ 2 & 1 \end{vmatrix} = (-1-8) = -9$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 4 \\ -2 & 1 \end{vmatrix} = -(0+8) = -8$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & -1 \\ -2 & 2 \end{vmatrix} = (0 - 2) = -2$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ 2 & 1 \end{vmatrix} = -(-2 - 6) = 8$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = (1 + 6) = 7$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -(2 - 4) = 2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ -1 & 4 \end{vmatrix} = (-8 + 3) = -5$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -(4 - 0) = -4$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} = (-1 - 0) = -1$$

$$\text{Thus, } \text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{1} \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}$$

$$\text{Now, } (A')^{-1} = (A^{-1})' = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}' = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

16. $|A| = 2 \neq 0$

Therefore A^{-1} exists

$$AB = I$$

$$A^{-1}AB = A^{-1}I$$

$$B = A^{-1}$$

$$\text{adj } A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

$$= \frac{1}{2} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

$$\text{Hence } B = \begin{bmatrix} 1 & 2 \\ \frac{1}{2} & \frac{3}{2} \end{bmatrix}$$

17. According to the question, we have to use properties of determinants to prove that,

$$\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$$

$$\text{Let LHS} = \begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$$

Therefore, on applying $C_1 \rightarrow C_1 + C_2 + C_3$ we get,

$$\Delta = \begin{vmatrix} 2(a+b+c) & c+a & a+b \\ 2(p+q+r) & r+p & p+q \\ 2(x+y+z) & z+x & x+y \end{vmatrix}$$

on taking 2 common from C_1 , we get,

$$\Delta = 2 \begin{vmatrix} a+b+c & c+a & a+b \\ p+q+r & r+p & p+q \\ x+y+z & z+x & x+y \end{vmatrix}$$

On applying $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$,

we get

$$\Delta = 2 \begin{vmatrix} a+b+c & -b & -c \\ p+q+r & -q & -r \\ x+y+z & -y & -z \end{vmatrix}$$

on applying $C_1 \rightarrow C_1 + C_2 + C_3$, we get,

$$\Delta = 2 \begin{vmatrix} a & -b & -c \\ p & -q & -r \\ x & -y & -z \end{vmatrix}$$

$$\therefore \Delta = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} \text{ [taking } (-1) \text{ common from both } C_2 \text{ and } C_3]$$

= RHS

$$18. \quad x - y + z = 4$$

$$x - 2y - 2z = 9$$

$$2x + y + 3z = 1$$

$$\text{Let } A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$AX = C$$

$$AB = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$AB = 8I$$

$$\begin{aligned} A^{-1} &= \frac{1}{8}B \begin{bmatrix} \because A^{-1}AB = 8A^{-1}I \\ B = 8A^{-1} \end{bmatrix} \\ &= \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \end{aligned}$$

$$X = A^{-1}C$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

$$x = 3, y = -2, z = -1$$