

Unit 9 (Circles)

Exercise 9.1 Multiple Choice Questions (MCQs)

Question 1:

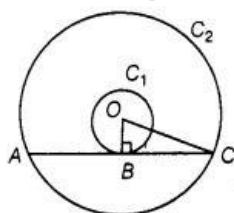
If radii of two concentric circles are 4 cm and 5 cm, then length of each chord of one circle which is tangent to the other circle, is

- (a) 3 cm (b) 6 cm (c) 9 cm (d) 1 cm

Solution:

(b) Let O be the centre of two concentric circles C_1 and C_2 , whose radii are $r_1 = 4$ cm and $r_2 = 5$ cm. Now, we draw a chord AC of circle C_2 , which touches the circle C_1 at B .

Also, join OB , which is perpendicular to AC . [Tangent at any point of circle is perpendicular to radius through the point of contact]



Now, in right angled $\triangle OBC$, by using Pythagoras theorem,

$$OC^2 = BC^2 + BO^2$$

$$[\because (\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2]$$

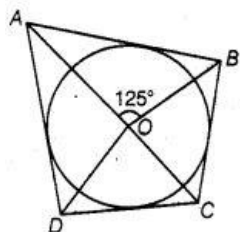
$$\Rightarrow 5^2 = BC^2 + 4^2$$

$$\Rightarrow BC^2 = 25 - 16 = 9 \Rightarrow BC = 3 \text{ cm}$$

$$\therefore \text{Length of chord } AC = 2 BC = 2 \times 3 = 6 \text{ cm}$$

Question 2:

In figure, if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to



- (a) 62.5° (b) 45° (c) 35° (d) 55°

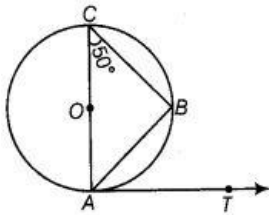
Solution:

(d) We know that, the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

$$\begin{aligned}
 \text{i.e.,} \quad \angle AOB + \angle COD &= 180^\circ \\
 \Rightarrow \quad \angle COD &= 180^\circ - \angle AOB \\
 &= 180^\circ - 125^\circ = 55^\circ
 \end{aligned}$$

Question 3:

In figure, AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to



- (a) 45° (b) 60° (c) 50° (d) 55°

Solution:

(c) In figure, AOC is a diameter of the circle. We know that, diameter subtends an angle 90° at the circle.

$$\begin{aligned}
 \text{So,} \quad \angle ABC &= 90^\circ \\
 \text{In } \triangle ACB, \quad \angle A + \angle B + \angle C &= 180^\circ \quad [\text{since, sum of all angles of a triangle is } 180^\circ] \\
 \Rightarrow \quad \angle A + 90^\circ + 50^\circ &= 180^\circ \\
 \Rightarrow \quad \angle A + 140^\circ &= 180^\circ \\
 \Rightarrow \quad \angle A = 180^\circ - 140^\circ &= 40^\circ \\
 \angle A \text{ or } \angle OAB &= 40^\circ
 \end{aligned}$$

Now, AT is the tangent to the circle at point A. So, OA is perpendicular to AT.

$$\therefore \angle OAT = 90^\circ \quad [\text{from figure}]$$

$$\Rightarrow \angle OAB + \angle BAT = 90^\circ$$

On putting $\angle OAB = 40^\circ$, we get

$$\Rightarrow \angle BAT = 90^\circ - 40^\circ = 50^\circ$$

Hence, the value of $\angle BAT$ is 50° .

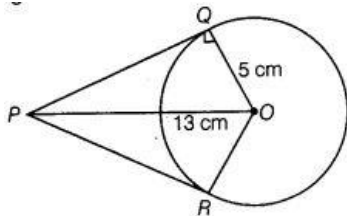
Question 4:

From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle is drawn. Then, the area of the quadrilateral PQOR is

- (a) 60 cm^2 (b) 65 cm^2 (c) 30 cm^2 (d) 32.5 cm^2

Solution:

(a) Firstly, draw a circle of radius 5 cm having centre O. P is a point at a distance of 13 cm from O. A pair of tangents PQ and PR are drawn.



Thus, quadrilateral POQR is formed.

$$\begin{aligned}
 \therefore \quad OQ &\perp QP \quad [\text{since, AP is a tangent line}] \\
 \text{In right angled } \triangle PQO, \quad OP^2 &= OQ^2 + QP^2 \\
 \Rightarrow \quad 13^2 &= 5^2 + QP^2 \\
 \Rightarrow \quad QP^2 &= 169 - 25 = 144 \\
 \Rightarrow \quad QP &= 12 \text{ cm} \\
 \text{Now,} \quad \text{area of } \triangle OQP &= \frac{1}{2} \times QP \times OQ \\
 &= \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2 \\
 \therefore \quad \text{Area of quadrilateral QORP} &= 2 \triangle OQP \\
 &= 2 \times 30 = 60 \text{ cm}^2
 \end{aligned}$$

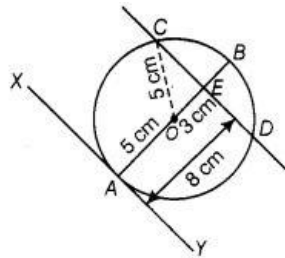
Question 5:

At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A, is

- (a) 4 cm (b) 5 cm
(c) 6 cm (d) 8 cm

Solution:

(d) First, draw a circle of radius 5 cm having centre O. A tangent XY is drawn at point A.



A chord CD is drawn which is parallel to XY and at a distance of 8 cm from A.

Now, $\angle OAY = 90^\circ$

[Tangent and any point of a circle is perpendicular to the radius through the point of contact]

$$\angle OAY + \angle OED = 180^\circ \quad [\because \text{sum of co-interior is } 180^\circ]$$

$$\Rightarrow \angle OED = 180^\circ$$

Also, $AE = 8 \text{ cm}$. Join OC

$$\text{Now, in right angled } \triangle OEC, \quad OC^2 = OE^2 + EC^2$$

$$\Rightarrow EC^2 = OC^2 - OE^2 \quad [\text{by Pythagoras theorem}]$$

$$= 5^2 - 3^2$$

$$[\because OC = \text{radius} = 5 \text{ cm}, OE = AE - AO = 8 - 5 = 3 \text{ cm}]$$

$$= 25 - 9 = 16$$

$$\Rightarrow EC = 4 \text{ cm}$$

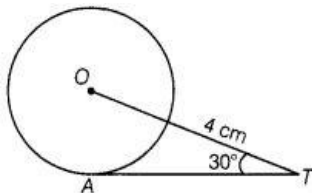
$$\text{Hence, length of chord } CD = 2 EC = 2 \times 4 = 8 \text{ cm}$$

[since, perpendicular from centre to the chord bisects the chord]

Question 6:

In figure, AT is a tangent to the circle with centre O such that OT = 4 cm and $\angle OTA = 30^\circ$.

Then, AT is equal to



- (a) 4 cm (b) 2 cm (c) $2\sqrt{3} \text{ cm}$ (d) $4\sqrt{3} \text{ cm}$

Solution:

(c) Join OA

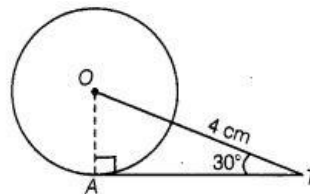
We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\therefore \angle OAT = 90^\circ$$

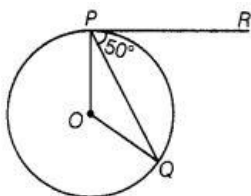
$$\text{In } \triangle OAT, \quad \cos 30^\circ = \frac{AT}{OT}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AT}{4}$$

$$\Rightarrow AT = 2\sqrt{3} \text{ cm}$$

**Question 7:**

In figure, if O is the centre of a circle, PQ is a chord and the tangent PR at P, ; makes an angle of 50° with PQ, then $\angle POQ$ is equal to



- (a) 100° (b) 80° (c) 90° (d) 75°

Solution:

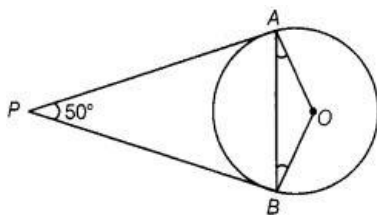
(a) Given, $\angle QPR = 50^\circ$

We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\begin{aligned}
 \therefore \quad & \angle OPR = 90^\circ \\
 \Rightarrow \quad & \angle OPQ + \angle QPR = 90^\circ \quad [\text{from figure}] \\
 \Rightarrow \quad & \angle OPQ = 90^\circ - 50^\circ = 40^\circ \quad [\because \angle QPR = 50^\circ] \\
 \text{Now,} \quad & OP = OQ = \text{Radius of circle} \\
 \therefore \quad & \angle OQP = \angle OPQ = 40^\circ \quad [\text{since, angles opposite to equal sides are equal}] \\
 \text{In } \triangle OPQ, \quad & \angle O + \angle P + \angle Q = 180^\circ \quad [\text{since, sum of angles of a triangle} = 180^\circ] \\
 \Rightarrow \quad & \angle O = 180^\circ - (40^\circ + 40^\circ) \quad [\because \angle P = 40^\circ = \angle Q] \\
 & = 180^\circ - 80^\circ = 100^\circ
 \end{aligned}$$

Question 8:

In figure, if PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$, then $\angle OAB$ is equal to



- (a) 25° (b) 30° (c) 40° (d) 50°

Solution:

(a) Given, PA and PB are tangent lines.

$$\begin{aligned}
 \therefore \quad & PA = PB \quad [\text{since, the length of tangents drawn from an external point to a circle is equal}] \\
 \Rightarrow \quad & \angle PBA = \angle PAB = \theta \quad [\text{say}] \\
 \text{In } \triangle PAB, \quad & \angle P + \angle A + \angle B = 180^\circ \quad [\text{since, sum of angles of a triangle} = 180^\circ] \\
 \Rightarrow \quad & 50^\circ + \theta + \theta = 180^\circ \\
 \Rightarrow \quad & 2\theta = 180^\circ - 50^\circ = 130^\circ \\
 \Rightarrow \quad & \theta = 65^\circ \\
 \text{Also,} \quad & OA \perp PA \quad [\text{since, tangent at any point of a circle is perpendicular to the radius through the point of contact}] \\
 \therefore \quad & \angle PAO = 90^\circ \\
 \Rightarrow \quad & \angle PAB + \angle BAO = 90^\circ \\
 \Rightarrow \quad & 65^\circ + \angle BAO = 90^\circ \\
 \Rightarrow \quad & \angle BAO = 90^\circ - 65^\circ = 25^\circ
 \end{aligned}$$

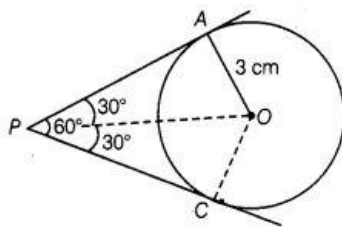
Question 9:

If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then the length of each tangent is

- (a) $\frac{3}{2}\sqrt{3}$ cm (b) 6 cm (c) 3 cm (d) $3\sqrt{3}$ cm

Solution:

(d) Let P be an external point and a pair of tangents is drawn from point P and angle between these two tangents is 60° .



Join OA and OP.

Also, OP is a bisector line of $\angle APC$.

$$\therefore \angle APO = \angle CPO = 30^\circ$$

Also, $OA \perp AP$

Tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\text{In right angled } \triangle OAP, \quad \tan 30^\circ = \frac{OA}{AP} = \frac{3}{AP}$$

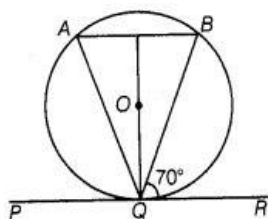
$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AP}$$

$$\Rightarrow AP = 3\sqrt{3} \text{ cm}$$

Hence, the length of each tangent is $3\sqrt{3}$ cm.

Question 10:

In figure, if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70^\circ$, then $\angle AQB$ is equal to



(a) 20°

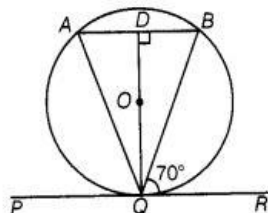
(b) 40°

(c) 35°

(d) 45°

Solution:

(b) Given, $AB \parallel PR$



$$\angle ABQ = \angle BQR = 70^\circ$$

[alternate angles]

Also, QD is perpendicular to AB and QD bisects AB.

In $\triangle QDA$ and $\triangle QDB$,

$$\angle QDA = \angle QDB$$

[each 90°]

$$AD = BD$$

$$QD = QD$$

[common side]

$$\triangle ADQ \cong \triangle BDQ$$

[by SAS similarity criterion]

$$\angle QAD = \angle QBD$$

[CPCT] ... (i)

$\therefore$

Then

Also

$$\angle ABQ = \angle BQR$$

[alternate interior angle]

$\therefore$

$$\angle ABQ = 70^\circ$$

$$[\because \angle BQR = 70^\circ]$$

Hence,

$$\angle QAB = 70^\circ$$

[from Eq. (i)]

Now, in $\triangle ABQ$,

$$\angle A + \angle B + \angle Q = 180^\circ$$

$\Rightarrow$

$$\angle Q = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$$

Exercise 9.2 Very Short Answer Type Questions

Question 1:

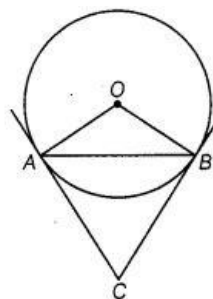
If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60° .

Solution:

False

Since a chord AB subtends an angle of 60° at the centre of a circle.

i.e., $\angle AOB = 60^\circ$
 As $OA = OB = \text{Radius of the circle}$
 $\therefore \angle OAB = \angle OBA = 60^\circ$
 The tangent at points A and B is drawn, which intersect at C.
 We know, $OA \perp AC$ and $OB \perp BC$.
 $\therefore \angle OAC = 90^\circ, \angle OBC = 90^\circ$
 $\Rightarrow \angle OAB + \angle BAC = 90^\circ$
 and $\angle OBA + \angle ABC = 90^\circ$
 $\Rightarrow \angle BAC = 90^\circ - 60^\circ = 30^\circ$
 and $\angle ABC = 90^\circ - 60^\circ = 30^\circ$
 In $\triangle ABC$, $\angle BAC + \angle CBA + \angle ACB = 180^\circ$
 [since, sum of all interior angles of a triangle is 180°]
 $\Rightarrow \angle ACB = 180^\circ - (30^\circ + 30^\circ) = 120^\circ$



Question 2:

The length of tangent from an external point P on a circle is always greater than the radius of the circle.

Solution:

False

Because the length of tangent from an external point P on a circle may or may not be greater than the radius of the circle.

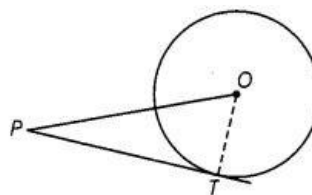
Question 3:

The length of tangent from an external point P on a circle with centre O is always less than OP.

Solution:

True

PT is a tangent drawn from external point P. Join OT .
 $\therefore OT \perp PT$
 So, $\triangle OPT$ is a right angled triangle formed.
 In right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.
 $\therefore OP > PT$
 or $PT < OP$



Question 4:

The angle between two tangents to a circle may be 0° .

Solution:

True

'This may be possible only when both tangent lines coincide or are parallel to each other.

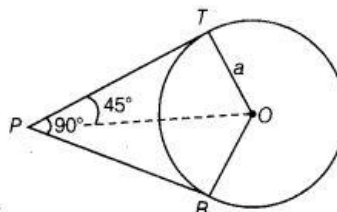
Question 5:

If angle between two tangents drawn from a point P to a circle of radius a and centre O is 90° , then $OP = a\sqrt{2}$.

Solution:

True

From point P, two tangents are drawn.
 Given, $OT = a$
 Also, line OP bisects the $\angle RPT$.
 $\therefore \angle TPO = \angle RPO = 45^\circ$
 Also, $OT \perp PT$
 In right angled $\triangle OTP$, $\sin 45^\circ = \frac{OT}{OP}$
 $\Rightarrow \frac{1}{\sqrt{2}} = \frac{a}{OP} \Rightarrow OP = a\sqrt{2}$



Question 6:

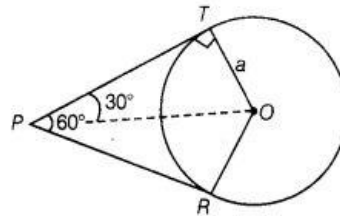
If angle between two tangents drawn from a point P to a circle of radius a and centre O is 60° , then $OP = a\sqrt{3}$.

Solution:

True

From point P , two tangents are drawn.

Given, $OT = a$
 Also, line OP bisects the $\angle RPT$.
 $\therefore \angle TPO = \angle RPO = 30^\circ$
 Also, $OT \perp PT$
 In right angled $\triangle OTP$,
 $\sin 30^\circ = \frac{OT}{OP}$
 $\Rightarrow \frac{1}{2} = \frac{a}{OP}$
 $\Rightarrow OP = 2a$



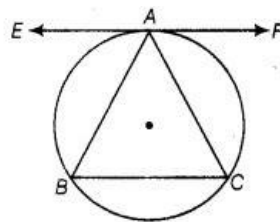
Question 7:

The tangent to the circumcircle of an isosceles $\triangle ABC$ at A , in which $AB = AC$, is parallel to BC .

Solution:

True

Let EAF be tangent to the circumcircle of $\triangle ABC$.



To prove

$$EAF \parallel BC$$

$$\angle EAB = \angle ABC$$

Here,

$$AB = AC$$

$\Rightarrow$

$$\angle ABC = \angle ACB$$

... (i)

[angle between tangent and chord equal to angle made by chord in the alternate segment]

$\therefore$ Also,

$$\angle EAB = \angle BCA$$

... (ii)

From Eqs. (i) and (ii), we get

$$\angle EAB = \angle ABC$$

$\Rightarrow$

$$EAF \parallel BC$$

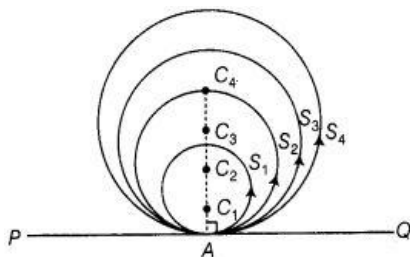
Question 8:

If a number of circles touch a given line segment PQ at a point A , then their centres lie on the perpendicular bisector of PQ .

Solution:

False

Given that PQ is any line segment and $S_1, S_2, S_3, S_4, \dots$ circles are touch a line segment PQ at a point A . Let the centres of the circles $S_1, S_2, S_3, S_4, \dots$ be $C_1, C_2, C_3, C_4, \dots$ respectively.



To prove centres of these circles lie on the perpendicular bisector PQ

Now, joining each centre of the circles to the point A on the line segment PQ by a line segment i.e., $C_1A, C_2A, C_3A, C_4A, \dots$ so on.

We know that, if we draw a line from the centre of a circle to its tangent line, then the line is always perpendicular to the tangent line. But it not bisect the line segment PQ .

So,

$$C_1A \perp PQ$$

[for S_1]

$$C_2A \perp PQ$$

[for S_2]

$$C_3A \perp PQ$$

[for S_3]

$$C_4A \perp PQ$$

[for S_4]

... so on.

Since, each circle is passing through a point A. Therefore, all the line segments

$C_1A, C_2A, C_3A, C_4A, \dots$ so on are coincident.

So, centre of each circle lies on the perpendicular line of PQ but they do not lie on the perpendicular bisector of PQ.

Hence, a number of circles touch a given line segment PQ at a point A, then their centres lie

Question 9:

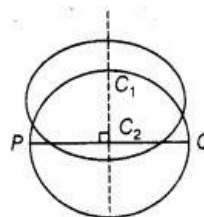
If a number of circles pass through the end points P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

Solution:

True

We draw two circle with centres C_1 and C_2 passing through the end points P and Q of a line segment PQ. We know, that perpendicular bisectors of a chord of a circle always passes through the centre of circle.

Thus, perpendicular bisector of PQ passes through C_1 and C_2 . Similarly, all the circle passing through PQ will have their centre on perpendicular bisectors of PQ.



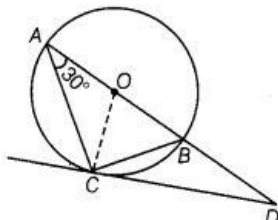
Question 10:

AB is a diameter of a circle and AC is its chord such that $\angle BAC = 30^\circ$. If the tangent at C intersects AB extended at D, then $BC = BD$.

Solution:

True

To Prove, $BC = BD$



Join BC and OC.

Given,

$$\angle BAC = 30^\circ$$

$\Rightarrow$

$$\angle BCD = 30^\circ$$

[angle between tangent and chord is equal to angle made by chord in the alternate segment]

$\therefore$

$$\angle ACD = \angle ACO + \angle OCD = 30^\circ + 90^\circ = 120^\circ$$

$$[\because OC \perp CD \text{ and } OA = OC = \text{radius} \Rightarrow \angle OAC = \angle OCA = 30^\circ]$$

In $\triangle ACD$,

$$\angle CAD + \angle ACD + \angle ADC = 180^\circ$$

[since, sum of all interior angles of a triangle is 180°]

$\Rightarrow$

$$30^\circ + 120^\circ + \angle ADC = 180^\circ$$

$\Rightarrow$

$$\angle ADC = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$$

Now, in $\triangle BCD$

$$\angle BCD = \angle BDC = 30^\circ$$

$\Rightarrow$

$$BC = BD$$

[since, sides opposite to equal angles are equal]

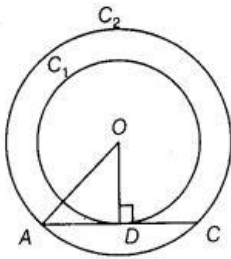
Exercise 9.3 Short Answer Type Questions

Question 1:

Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

Solution:

Let C_1 and C_2 be the two circles having same centre O. AC is a chord which touches the C_2 at point D.



Join OD .

Also,

$$OD \perp AC$$

$$AD = DC = 4 \text{ cm}$$

[perpendicular line OD bisects the chord]

$\therefore$ In right angled $\triangle AOD$,

$$OA^2 = AD^2 + DO^2$$

[by Pythagoras theorem, i.e., (hypotenuse)² = (base)² + (perpendicular)²]

$\Rightarrow$

$$DO^2 = 5^2 - 4^2$$

$$= 25 - 16 = 9$$

$\Rightarrow$

$$DO = 3 \text{ cm}$$

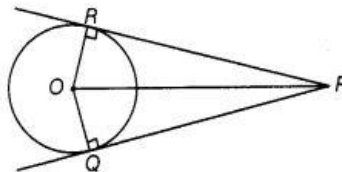
$\therefore$ Radius of the inner circle $OD = 3 \text{ cm}$

Question 2:

Two tangents PQ and PR are drawn from an external point to a circle with centre O . Prove that $QORP$ is a cyclic quadrilateral.

Solution:

Given Two tangents PQ and PR are drawn from an external point to a circle with centre O .



To prove $QORP$ is a cyclic quadrilateral.

proof Since, PR and PQ are tangents.

So,

$$OR \perp PR \text{ and } OQ \perp PQ$$

[since, if we draw a line from centre of a circle to its tangent line. Then, the line always perpendicular to the tangent line]

$\therefore$

$$\angle ORP = \angle OQP = 90^\circ$$

Hence,

$$\angle ORP + \angle OQP = 180^\circ$$

So, $QOPR$ is cyclic quadrilateral.

[If sum of opposite angles is quadrilateral in 180° , then the quadrilateral is cyclic]

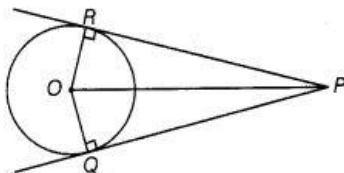
Hence proved.

Question 3:

Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

Solution:

Given Two tangents PQ and PR are drawn from an external point P to a circle with centre O .



To prove Centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

In $\angle RPQ$.

Construction Join OR , and OQ .

In $\triangle POP$ and $\triangle POO$

$$\angle PRO = \angle PQO = 90^\circ$$

[tangent at any point of a circle is perpendicular to the radius through the point of contact]

$$OR = OQ$$

[radii of some circle]

Since OP is common

$\therefore$

Hence,

$$\begin{aligned}\Delta PRO &\equiv \Delta PQO \\ \angle RPO &= \angle QPO\end{aligned}$$

[RHS]
[by CPCT]

Thus, O lies on angle bisector of PR and PQ.

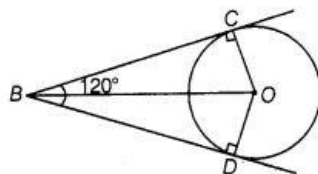
Hence proved.

Question 4:

If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120^\circ$, prove that $BC + BD = BO$ i.e., $BO = 2 BC$.

Solution:

Two tangents BD and BC are drawn from an external point B.



$$\begin{aligned}BO &= 2BC \\ \angle DBC &= 120^\circ\end{aligned}$$

To prove

Given,

Join OC, OD and BO.

Since, BC and BD are tangents.

$\therefore$ $OC \perp BC$ and $OD \perp BD$

We know, OB is a angle bisector of $\angle DBC$.

$\therefore$ $\angle OBC = \angle ODB = 60^\circ$

In right angled $\triangle OBC$,

$$\cos 60^\circ = \frac{BC}{OB}$$

$\Rightarrow$

$$\frac{1}{2} = \frac{BC}{OB}$$

$\Rightarrow$

$$OB = 2 BC$$

Also,

$$BC = BD$$

[tangents drawn from an external point to a circle are equal]

$\therefore$

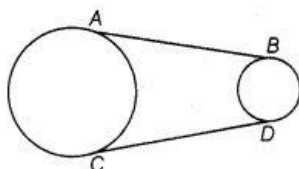
$$OB = BC + BD$$

$\Rightarrow$

$$OB = BC + BD$$

Question 5:

In figure, AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$

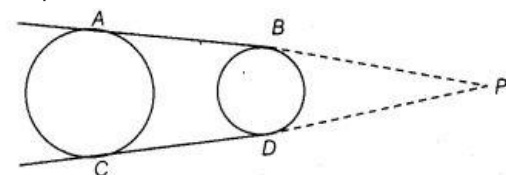


Solution:

Given AS and CD are common tangents to two circles of unequal radius

To prove

$$AB = CD$$



Construction Produce AB and CD, to intersect at P.

Proof

$$PA = PC$$

[the lengths of tangents drawn from an external point to a circle are equal]

Also,

$$PB = PD$$

[the lengths of tangents drawn from an external point to a circle are equal]

$\therefore$

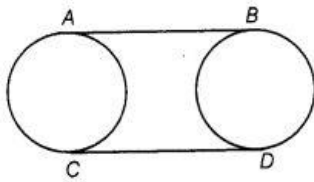
$$PA - PB = PC - PD$$

$$AB = CD$$

Hence proved.

Question 6:

In figure, AB and CD are common tangents to two circles of equal radii. Prove that $AB = CD$.

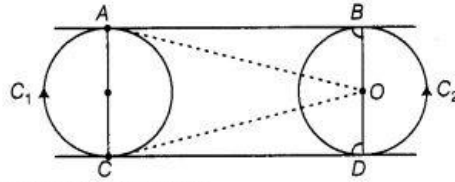


Solution:

Given AB and CD are tangents to two circles of equal radii.

To prove

$$AB = CD$$



Construction Join $OA, OC, O'B$ and $O'D$

Proof Now, $\angle OAB = 90^\circ$

[tangent at any point of a circle is perpendicular to radius through the point of contact]

Thus, AC is a straight line.

Also,

$$\angle OAB + \angle OCD = 180^\circ$$

$\therefore$

$$AB \parallel CD$$

Similarly, BD is a straight line

and

$$\angle O'BA = \angle O'DC = 90^\circ$$

Also,

$$AC = BD$$

[radii of two circles are equal]

In quadrilateral ABCD,

$$\angle A = \angle B = \angle C = \angle D = 90^\circ$$

and

$$AC = BD$$

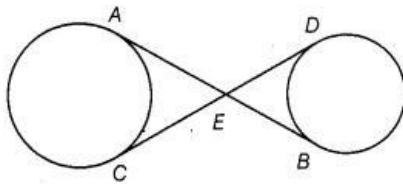
ABCD is a rectangle

Hence,

$$AB = CD \quad [\text{opposite sides of rectangle are equal}]$$

Question 7:

In figure, common tangents AB and CD to two circles intersect at E. Prove that $AB = CD$.

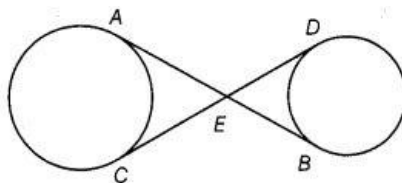


Solution:

Given Common tangents AB and CD to two circles intersecting at E.

To prove

$$AB = CD$$



Proof

$$EA = EC$$

...(i)

[the lengths of tangents drawn from an internal point to a circle are equal]

$$EB = ED$$

...(ii)

On adding Eqs. (i) and (ii), we get

$$EA + EB = EC + ED$$

$\Rightarrow$

$$AB = CD$$

Hence proved.

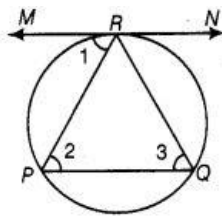
Question 8:

A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

Solution:

Given Chord PQ is parallel to tangent at R.

To prove R bisects the arc PRQ



Proof

$$\angle 1 = \angle 2$$

[alternate interior angles]

$$\angle 1 = \angle 3$$

[angle between tangent and chord is equal to angle made by chord in alternate segment]

$$\therefore \angle 2 = \angle 3$$

$\Rightarrow PR = QR$ [sides opposite to equal angles are equal]

$\Rightarrow PR = QR$

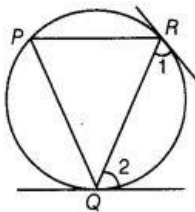
So, R bisects PQ.

Question 9:

Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Solution:

To prove $\angle 1 = \angle 2$, let PQ be a chord of the circle. Tangents are drawn at the points R and Q.



Let P be another point on the circle, then, join PQ and PR.

Since, at point Q, there is a tangent.

$\therefore \angle 2 = \angle P$ [angles in alternate segments are equal]

Since, at point R, there is a tangent.

$\therefore \angle 1 = \angle P$ [angles in alternate segments are equal]

$\therefore \angle 1 = \angle 2 = \angle P$ **Hence proved.**

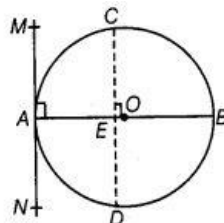
Question 10:

Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

Solution:

Given, AB is a diameter of the circle.

A tangent is drawn from point A. Draw a chord CD parallel to the tangent MAN.



So, CD is a chord of the circle and OA is a radius of the circle.

$$\angle MAO = 90^\circ$$

[tangent at any point of a circle is perpendicular to the radius through the point of contact]

$$\angle CEO = \angle MAO$$

[corresponding angles]

$\therefore \angle CEO = 90^\circ$

Thus, OE bisects CD, [perpendicular from centre of circle to chord bisects the chord]

Similarly, the diameter AB bisects all. Chords which are parallel to the tangent at the point A.

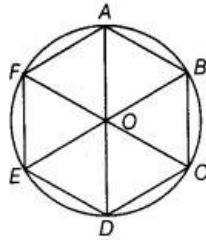
Exercise 9.4 Long Answer Type Questions

Question 1:

If a hexagon ABCDEF circumscribe a circle, prove that
 $AB + CD + EF = BC + DE + FA$

Solution:

Given A hexagon ABCDEF circumscribe a circle.



To prove $AB + CD + EF = BC + DE + FA$

Proof $AB + CD + EF = (AQ + QB) + (CS + SD) + (EU + UF)$
 $= AP + BR + CR + DT + ET + FP$
 $= (AP + FP) + (BR + CR) + (DT + ET)$

$$AB + CD + EF = AF + BC + DE$$

$$AQ = AP$$

$$QB = BR$$

$$CS = CR$$

$$DS = DT$$

$$EU = ET$$

[tangents drawn from an external point to a circle are equal]

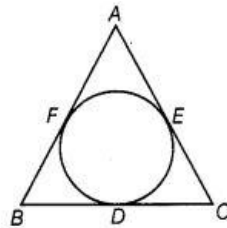
Hence proved.

Question 2:

Let s denotes the semi-perimeter of a ΔABC in which $BC = a$, $CA = b$ and $AB = c$. If a circle touches the sides BC , CA , AB at D , E , F , respectively. Prove that $BD = s - b$.

Solution:

A circle is inscribed in the ΔABC , which touches the BC , CA and AB .



Given, $BC = a$, $CA = b$ and $AB = c$

By using the property, tangents are drawn from an external point to the circle are equal in length.

$$\therefore BD = BF = x \quad [\text{say}]$$

$$DC = CE = y \quad [\text{say}]$$

$$AE = AF = z \quad [\text{say}]$$

and

$$\text{Now, } BC + CA + AB = a + b + c$$

$$\Rightarrow (BD + DC) + (CE + EA) + (AF + FB) = a + b + c$$

$$\Rightarrow (x + y) + (y + z) + (z + x) = a + b + c$$

$$\Rightarrow 2(x + y + z) = 2s$$

$$[\because 2s = a + b + c = \text{perimeter of } \Delta ABC]$$

$$\Rightarrow s = x + y + z$$

$$\Rightarrow x = s - (y + z)$$

$$\Rightarrow BD = s - b$$

$$[\because b = AE + EC = z + y]$$

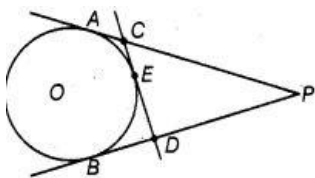
Hence proved.

Question 3:

From an external point P , two tangents, PA and PB are drawn to a circle with centre O . At one point E on the circle tangent is drawn which intersects PA and PB at C and D , respectively. If $PA = 10$ cm, find the perimeter of the triangle PCD .

Solution:

Two tangents PA and PB are drawn to a circle with centre O from an external point P

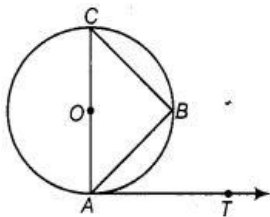


$$\begin{aligned}
 \text{Perimeter of } \triangle PCD &= PC + CD + PD \\
 &= PC + CE + ED + PD \\
 &= PC + CA + DB + PD \\
 &= PA + PB \\
 &= 2PA = 2(10) \\
 &= 20 \text{ cm}
 \end{aligned}$$

[$\because CE = CA, DE = DB, PA = PB$ tangents from internal point to a circle are equal]

Question 4:

If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in figure. Prove that $\angle BAT = \angle ACB$.



Solution:

Since, AC is a diameter line, so angle in semi-circle makes an angle 90° .

$$\therefore \angle ABC = 90^\circ \quad [\text{by property}]$$

$$\text{In } \triangle ABC, \quad \angle CAB + \angle ABC + \angle ACB = 180^\circ$$

[$\because$ sum of all interior angles of any triangle is 180°]

$$\Rightarrow \angle CAB + \angle ACB = 180^\circ - 90^\circ = 90^\circ \quad \dots (i)$$

Since, diameter of a circle is perpendicular to the tangent.

$$\text{i.e.} \quad CA \perp AT$$

$$\therefore \angle CAT = 90^\circ$$

$$\Rightarrow \angle CAB + \angle BAT = 90^\circ \quad \dots (ii)$$

From Eqs. (i) and (ii),

$$\angle CAB + \angle ACB = \angle CAB + \angle BAT$$

$$\Rightarrow \angle ACB = \angle BAT$$

Hence proved.

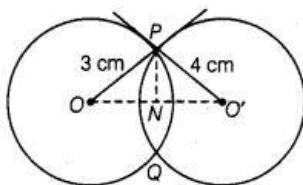
Question 5:

Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

Solution:

Here, two circles are of radii $OP = 3 \text{ cm}$ and $PO' = 4 \text{ cm}$

These two circles intersect at P and Q.



Here, OP and PO' are two tangents drawn at point P.

$$\angle OPO' = 90^\circ$$

[tangent at any point of circle is perpendicular to radius through the point of contact]

Join OO' and PN .

In right angled $\triangle OPO'$,

$$(OO')^2 = (OP)^2 + (PO')^2 \quad [\text{by Pythagoras theorem}]$$

i.e.,

$$\begin{aligned} (\text{Hypotenuse})^2 &= (\text{Base})^2 + (\text{Perpendicular})^2 \\ &= (3)^2 + (4)^2 = 25 \end{aligned}$$

$\Rightarrow$

$$OO' = 5 \text{ cm}$$

Also,

$$PN \perp OO'$$

Let $ON = x$, then $NO' = 5 - x$

In right angled $\triangle OPN$,

$$(OP)^2 = (ON)^2 + (NP)^2 \quad [\text{by Pythagoras theorem}]$$

$\Rightarrow$

$$(NP)^2 = 3^2 - x^2 = 9 - x^2 \quad \dots(i)$$

and in right angled $\triangle PNO'$,

$$(PO')^2 = (PN)^2 + (NO')^2 \quad [\text{by Pythagoras theorem}]$$

$\Rightarrow$

$$(4)^2 = (PN)^2 + (5 - x)^2$$

$\Rightarrow$

$$(PN)^2 = 16 - (5 - x)^2 \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$9 - x^2 = 16 - (5 - x)^2$$

$\Rightarrow$

$$7 + x^2 - (25 + x^2 - 10x) = 0$$

$\Rightarrow$

$$10x = 18$$

$\therefore$

$$x = 1.8$$

Again, in right angled $\triangle OPN$,

$$OP^2 = (ON)^2 + (NP)^2 \quad [\text{by Pythagoras theorem}]$$

$\Rightarrow$

$$3^2 = (1.8)^2 + (NP)^2$$

$\Rightarrow$

$$(NP)^2 = 9 - 3.24 = 5.76$$

$\therefore$

$$(NP) = 2.4$$

$\therefore$ Length of common chord,

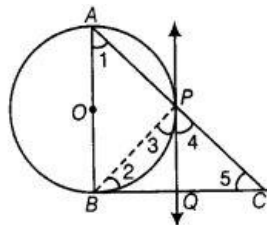
$$PQ = 2 PN = 2 \times 2.4 = 4.8 \text{ cm}$$

Question 6:

In a right angle $\triangle ABC$ in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P . Prove that the tangent to the circle at PQ bisects BC .

Solution:

Let O be the centre of the given circle. Suppose, the tangent at P meets BC at Q . Join BP .



To prove
Proof

$$BQ = QC$$

[angles in alternate segment]

$$\angle ABC = 90^\circ$$

[tangent at any point of circle is perpendicular to radius through the point of contact]

$\therefore$ In $\triangle ABC$,

$$\angle 1 + \angle 5 = 90^\circ \quad [\text{angle sum property, } \angle ABC = 90^\circ]$$

$$\angle 3 = \angle 1$$

[angle between tangent and the chord equals angle made by the chord in alternate segment]

$\therefore$

$$\angle 3 + \angle 5 = 90^\circ$$

$\dots(i)$

Also,

$$\angle APB = 90^\circ$$

[angle in semi-circle]

$\Rightarrow$

$$\angle 3 + \angle 4 = 90^\circ$$

[$\angle APB + \angle BPC = 180^\circ$, linear pair]

From Eqs. (i) and (ii), we get

$$\angle 3 + \angle 5 = \angle 3 + \angle 4$$

$\Rightarrow$

$$\angle 5 = \angle 4$$

$\Rightarrow$

$$PQ = QC$$

[sides opposite to equal angles are equal]

Also,

$$QP = QB$$

[tangents drawn from an internal point to a circle are equal]

$\Rightarrow$

$$QB = QC$$

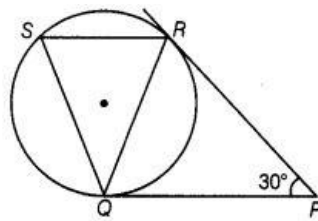
Hence proved.

Question 7:

In figure, tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ . Find the $\angle RQS$.

Solution:

PQ and PR are two tangents drawn from an external point P .



$\therefore PQ = PR$
 [the lengths of tangents drawn from an external point to a circle are equal]
 $\Rightarrow \angle PQR = \angle QRP$
 [angles opposite to equal sides are equal]
 Now, in $\triangle PQR$ $\angle PQR + \angle QRP + \angle RPQ = 180^\circ$
 [sum of all interior angles of any triangle is 180°]
 $\Rightarrow \angle PQR + \angle PQR + 30^\circ = 180^\circ$
 $\Rightarrow 2 \angle PQR = 180^\circ - 30^\circ$
 $\Rightarrow \angle PQR = \frac{180^\circ - 30^\circ}{2} = 75^\circ$
 Since, $SR \parallel QP$
 $\therefore \angle SRQ = \angle RQP = 75^\circ$ [alternate interior angles]
 Also, $\angle PQR = \angle QSR = 75^\circ$ [by alternate segment theorem]
 In $\triangle QRS$, $\angle Q + \angle R + \angle S = 180^\circ$
 [sum of all interior angles of any triangle is 180°]
 $\Rightarrow \angle Q = 180^\circ - (75^\circ + 75^\circ)$
 $= 30^\circ$
 $\therefore \angle RQS = 30^\circ$

Question 8:

AB is a diameter and AC is a chord of a circle with centre O such that $\angle BAC = 30^\circ$. The tangent at C intersects extended AB at a point D. Prove that $BC = BD$.

Solution:

A circle is drawn with centre O and AB is a diameter.

AC is a chord such that $\angle BAC = 30^\circ$.

Given AB is a diameter and AC is a chord of circle with centre O, $\angle BAC = 30^\circ$.

Question 9:

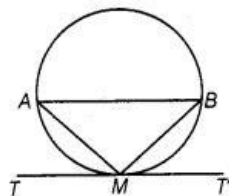
Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Solution:

Let mid-point of an arc AMB be M and TMT' be the tangent to the circle.

Join AB, AM and MB.

Since, arc $AM = \text{arc } MB$
 $\Rightarrow$ Chord $AM = \text{Chord } MB$
 $AM = MB$
 In $\triangle AMB$, $\angle MAB = \angle MBA$
 $\Rightarrow$ [equal sides corresponding to the equal angle] ... (i)



Since, TMT' is a tangent line.

$\therefore \angle AMT = \angle MBA$
 [angles in alternate segments are equal]
 $= \angle MAB$ [from Eq. (i)]

But $\angle AMT$ and $\angle MAB$ are alternate angles, which is possible only when

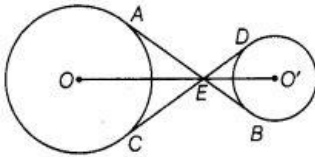
$AB \parallel TMT'$

Hence, the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc

Hence proved.

Question 10:

In a figure the common tangents, AB and CD to two circles with centres O and O' intersect at E. Prove that the points O, E and O' are collinear.

**Solution:**

Join AO, OC and O'D, O'B.

Now, in $\triangle EO'D$ and $\triangle EO'B$,

$$O'D = O'B$$

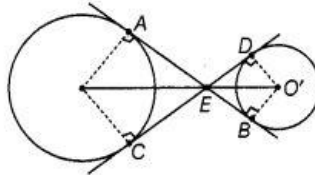
[radius]

$$O'E = O'E$$

[common side]

$$ED = EB$$

[since, tangents drawn from an external point to the circle are equal in length]



$$\therefore \triangle EO'D \cong \triangle EO'B \quad \text{[by SSS congruence rule]}$$

$\Rightarrow$

$$\angle O'ED = \angle O'EB$$

O'E is the angle bisector of $\angle DEB$ (i)

Similarly, OE is the angle bisector of $\angle AEC$.

Now, in quadrilateral DEBO',

$$\angle O'DE = \angle O'BE = 90^\circ$$

[since, CED is a tangent to the circle and O'D is the radius, i.e., $O'D \perp CED$]

$$\Rightarrow \angle O'DE + \angle O'BE = 180^\circ$$

$$\therefore \angle DEB + \angle DO'B = 180^\circ \quad \text{[since, DEBO' is cyclic quadrilateral]} \quad \dots (ii)$$

Since, AB is a straight line,

$$\therefore \angle AED + \angle DEB = 180^\circ$$

$$\Rightarrow \angle AED + 180^\circ - \angle DO'B = 180^\circ \quad \text{[from Eq. (ii)]}$$

$$\Rightarrow \angle AED = \angle DO'B \quad \dots (iii)$$

$$\text{Similarly, } \angle AED = \angle AOC \quad \dots (iv)$$

$$\text{Again from Eq. (ii), } \angle DEB = 180^\circ - \angle DO'B$$

Divided by 2 on both sides, we get

$$\frac{1}{2} \angle DEB = 90^\circ - \frac{1}{2} \angle DO'B$$

$$\Rightarrow \angle DEO' = 90^\circ - \frac{1}{2} \angle DO'B \quad \dots (v)$$

$$\text{[since, O'E is the angle bisector of } \angle DEB \text{ i.e., } \frac{1}{2} \angle DEB = \angle DEO']$$

$$\text{Similarly, } \angle AEC = 180^\circ - \angle AOC$$

Divided by 2 on both sides, we get

$$\frac{1}{2} \angle AEC = 90^\circ - \frac{1}{2} \angle AOC$$

$$\Rightarrow \angle AEO = 90^\circ - \frac{1}{2} \angle AOC \quad \dots (vi)$$

$$\text{[since, OE is the angle bisector of } \angle AEC \text{ i.e., } \frac{1}{2} \angle AEC = \angle AEO]$$

$$\begin{aligned} \text{Now, } \angle AED + \angle DEO' + \angle AEO &= \angle AED + \left(90^\circ - \frac{1}{2} \angle DO'B\right) + \left(90^\circ - \frac{1}{2} \angle AOC\right) \\ &= \angle AED + 180^\circ - \frac{1}{2} (\angle DO'B + \angle AOC) \\ &= \angle AED + 180^\circ - \frac{1}{2} (\angle AED + \angle AED) \quad \text{[from Eqs. (iii) and (iv)]} \\ &= \angle AED + 180^\circ - \frac{1}{2} (2 \times \angle AED) \\ &= \angle AED + 180^\circ - \angle AED = 180^\circ \end{aligned}$$

$$\therefore \angle AEO + \angle AED + \angle DEO' = 180^\circ$$

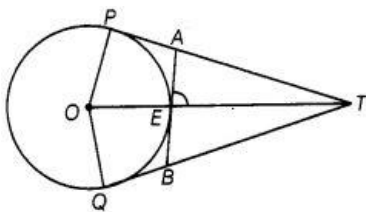
So, OEO' is straight line.

Hence, O, E and O' are collinear.

Hence proved.

Question 11:

In figure, O is the centre of a circle of radius 5 cm, T is a point such that OT = 13 and OT intersects the circle at E, if AB is the tangent to the circle at E, find the length of AB.



Solution:

Given, $OT = 13$ cm and $OP = 5$ cm

Since, if we draw a line from the centre to the tangent of the circle. It is always perpendicular to the tangent i.e., $OP \perp PT$.

In right angled $\triangle OPT$, $OT^2 = OP^2 + PT^2$
 [by Pythagoras theorem, (hypotenuse)² = (base)² + (perpendicular)²]

$$\Rightarrow PT^2 = (13)^2 - (5)^2 = 169 - 25 = 144$$

$$\Rightarrow PT = 12 \text{ cm}$$

Since, the length of pair of tangents from an external point T is equal.

$$\therefore QT = 12 \text{ cm}$$

$$\text{Now, } TA = PT - PA$$

$$\Rightarrow TA = 12 - PA \quad \dots(i)$$

$$\text{and } TB = QT - QB$$

$$\Rightarrow TB = 12 - QB \quad \dots(ii)$$

Again, using the property, length of pair of tangents from an external point is equal.

$$\therefore PA = AE \text{ and } QB = EB \quad \dots(iii)$$

$$\therefore OT = 13 \text{ cm}$$

$$\therefore ET = OT - OE \quad [\because OE = 5 \text{ cm} = \text{radius}]$$

$$\Rightarrow ET = 13 - 5$$

$$\Rightarrow ET = 8 \text{ cm}$$

Since, AB is a tangent and OE is the radius.

$$\therefore OE \perp AB$$

$$\Rightarrow \angle OEA = 90^\circ$$

$$\therefore \angle AET = 180^\circ - \angle OEA \quad [\text{linear pair}]$$

$$\Rightarrow \angle AET = 90^\circ$$

Now, in right angled $\triangle AET$,

$$(AT)^2 = (AE)^2 + (ET)^2 \quad [\text{by Pythagoras theorem}]$$

$$\Rightarrow (PT - PA)^2 = (AE)^2 + (8)^2$$

$$\Rightarrow (12 - PA)^2 = (PA)^2 + (8)^2 \quad [\text{from Eq. (iii)}]$$

$$\Rightarrow 144 + (PA)^2 - 24 \cdot PA = (PA)^2 + 64$$

$$\Rightarrow 24 \cdot PA = 80$$

$$\Rightarrow PA = \frac{10}{3} \text{ cm}$$

$$\therefore AE = \frac{10}{3} \text{ cm} \quad [\text{from Eq. (iii)}]$$

Join OQ .

$$\text{Similarly } BE = \frac{10}{3} \text{ cm}$$

$$\begin{aligned} \text{Hence, } AB &= AE + EB \\ &= \frac{10}{3} + \frac{10}{3} \\ &= \frac{20}{3} \text{ cm} \end{aligned}$$

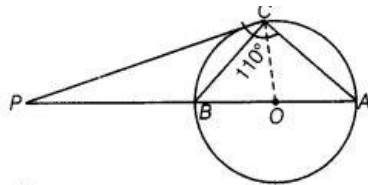
Hence, the required length AB is $\frac{20}{3}$ cm.

Question 12:

The tangent at a point C of a circle and a diameter AB when extended intersect at P . If $\angle PCA = 110^\circ$, find $\angle CBA$.

Solution:

Here, AB is a diameter of the circle from point C and a tangent is drawn which meets at a point P .



Join OC. Here, OC is radius.

Since, tangent at any point of a circle is perpendicular to the radius through point of contact circle.

$$\begin{aligned} \therefore & OC \perp PC \\ \text{Now,} & \angle PCA = 110^\circ \quad [\text{given}] \\ \Rightarrow & \angle PCO + \angle OCA = 110^\circ \\ \Rightarrow & 90^\circ + \angle OCA = 110^\circ \\ \Rightarrow & \angle OCA = 20^\circ \\ \therefore & OC = OA = \text{Radius of circle} \\ \Rightarrow & \angle OCA = \angle OAC = 20^\circ \\ & [\text{since, two sides are equal, then their opposite angles are equal}] \end{aligned}$$

Since, PC is a tangent, so $\angle BCP = \angle CAB = 20^\circ$
[angles in an alternate segment are equal]

$$\begin{aligned} \text{In } \triangle PBC, \quad \angle P + \angle C + \angle A &= 180^\circ \\ \angle P &= 180^\circ - (\angle C + \angle A) \\ &= 180^\circ - (110^\circ + 20^\circ) \\ &= 180^\circ - 130^\circ = 50^\circ \end{aligned}$$

$$\begin{aligned} \text{In } \triangle PBC, \quad \angle BPC + \angle PCB + \angle PBC &= 180^\circ \\ & [\text{sum of all interior angles of any triangle is } 180^\circ] \end{aligned}$$

$$\begin{aligned} \Rightarrow & 50^\circ + 20^\circ + \angle PBC = 180^\circ \\ \Rightarrow & \angle PBC = 180^\circ - 70^\circ \\ \Rightarrow & \angle PBC = 110^\circ \end{aligned}$$

Since, APB is a straight line.

$$\begin{aligned} \therefore & \angle PBC + \angle CBA = 180^\circ \\ \Rightarrow & \angle CBA = 180^\circ - 110^\circ = 70^\circ \end{aligned}$$

Question 13:

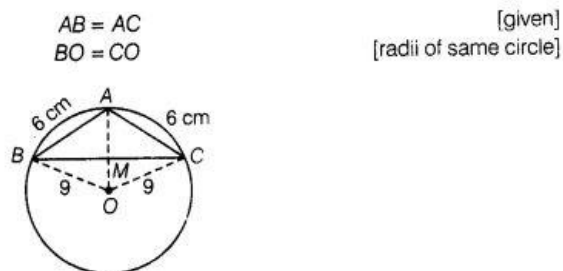
If an isosceles $\triangle ABC$ in which $AB = AC = 6$ cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.

Solution:

In a circle, $\triangle ABC$ is inscribed.

Join OB, OC and OA.

Consider $\triangle ABO$ and $\triangle ACO$



AO is common.

$$\begin{aligned} \therefore & \triangle ABO \cong \triangle ACO \quad [\text{by SSS congruence rule}] \\ \Rightarrow & \angle 1 = \angle 2 \quad [\text{CPOT}] \end{aligned}$$

Now, in $\triangle ABM$ and $\triangle ACM$,

$$\begin{aligned} AB &= AC \quad [\text{given}] \\ \angle 1 &= \angle 2 \quad [\text{proved above}] \end{aligned}$$

AM is common.

$$\therefore \triangle AMB \cong \triangle AMC \quad [\text{by SAS congruence rule}]$$

$$\Rightarrow \angle AMB = \angle AMC \quad [\text{CPCT}]$$

$$\text{Also, } \angle AMB + \angle AMC = 180^\circ \quad [\text{linear pair}]$$

$$\Rightarrow \angle AMB = 90^\circ$$

We know that a perpendicular from centre of circle bisects the chord.

So, OA is perpendicular bisector of BC.

Let $AM = x$, then $OM = 9 - x$

In right angled $\triangle AMC$, $AC^2 = AM^2 + MC^2$ [by Pythagoras theorem]

$$\text{i.e., } (\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$$

$$\Rightarrow MC^2 = 6^2 - x^2 \quad \dots(i)$$

$$\text{and in right } \triangle OMC, \quad OC^2 = OM^2 + MC^2 \quad [\text{by Pythagoras theorem}]$$

$$\Rightarrow MC^2 = 9^2 - (9 - x)^2 \quad \dots(ii)$$

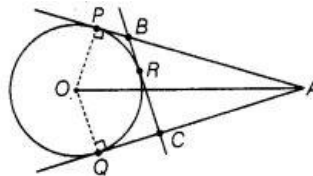
From Eqs. (i) and (ii),
 $\Rightarrow 6^2 - x^2 = 9^2 - (9 - x)^2$
 $\Rightarrow 36 - x^2 = 81 - (81 + x^2 - 18x)$
 $\Rightarrow 36 = 18x \Rightarrow x = 2$
 $\therefore AM = x = 2$
 In right angled $\triangle ABM$,
 $AB^2 = BM^2 + AM^2$ [by Pythagoras theorem]
 $6^2 = BM^2 + 2^2$
 $\Rightarrow BM^2 = 36 - 4 = 32$
 $\Rightarrow BM = 4\sqrt{2}$
 $\therefore BC = 2 BM = 8\sqrt{2} \text{ cm}$
 $\therefore$ Area of $\triangle ABC = \frac{1}{2} \times \text{Base} \times \text{Height}$
 $= \frac{1}{2} \times BC \times AM$
 $= \frac{1}{2} \times 8\sqrt{2} \times 2 = 8\sqrt{2} \text{ cm}^2$
 Hence, the required area of $\triangle ABC$ is $8\sqrt{2} \text{ cm}^2$.

Question 14:

A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the $\triangle ABC$.

Solution:

Given Two tangents are drawn from an external point A to the circle with centre O,



$$OA = 13 \text{ cm}$$

Tangent BC is drawn at a point R. radius of circle equals 5cm.

To find perimeter of $\triangle ABC$.

Proof

$$\angle OPA = 90^\circ$$

[tangent at any point of a circle is perpendicular to the radius through the point of contact]

$$\therefore OA^2 = OP^2 + PA^2 \quad [\text{by Pythagoras theorem}]$$

$$(13)^2 = 5^2 + PA^2$$

$$\Rightarrow PA^2 = 144 = 12^2$$

$$\Rightarrow PA = 12 \text{ cm}$$

Now,

$$\text{perimeter of } \triangle ABC = AB + BC + CA$$

$$= (AB + BR) + (RC + CA)$$

$$= AB + BP + CQ + CA$$

$$[\because BR = BP, RC = CQ \text{ tangents from internal point to a circle are equal}]$$

$$= AP + AQ$$

$$= 2 AP$$

$$= 2(12)$$

$$= 24 \text{ cm}$$

$$[AP = AQ \text{ tangent from internal point to a circle are equal}]$$

Hence, the

$$\text{perimeter of } \triangle ABC = 24 \text{ cm.}$$