0293

A

Total No. of Questions—24

Total No. of Printed Pages—4

Regd. No. 1 9 6 0 3 2 0 2 3 5

Part III

MATHEMATICS, Paper - II(B)

(English Version)

Time : 3 Hours] ***[Max. Marks : 75***

Note :— This question paper consists of **THREE** sections A, B and C.

SECTION A

10×2=20

I. Very short answer type questions :

(*i*) Answer **ALL** questions.

(*ii*) Each question carries **TWO** marks.

1. Write the parametric equations of the circle $2x^2 + 2y^2 = 7$.

2. Find the value of k if the points (1, 3) and (2, k) are conjugated with respect to the circle $x^2 + y^2 = 35$.

3. Find the equation of radical axis of the circles $x^2 + y^2 + 4x + 6y - 7 = 0$, $4(x^2 + y^2) + 8x + 12y - 9 = 0$.

4. Find the equation of the normal to the parabola $y^2 = 4x$ which is parallel to $y - 2x + 5 = 0$.

5. If the eccentricity of the hyperbola is $\frac{5}{4}$, then find the eccentricity of its conjugate hyperbola.

6. Evaluate :

$$\int \frac{1 + \cos^2 x}{1 - \cos 2x} dx, \text{ on } I \subset R \setminus \{n\pi : n \in z\}.$$

7. Evaluate :

$$\int \frac{1}{x \log x[\log (\log x)]} dx, \text{ on } (1, \infty)$$

8. Evaluate :

$$\int_0^a (\sqrt{a} - \sqrt{x})^2 \, dx.$$

9. Find :

$$\int_0^{\pi/2} \cos^{11} x \, dx.$$

10. Find the general solution of :

$$\frac{dy}{dx} = \frac{2y}{x}.$$

SECTION B

5×4=20

II. Short answer type questions :

(*i*) Answer **ANY FIVE** questions.

(*ii*) Each question carries **FOUR** marks.

11. Find the equation of the circle which cuts orthogonally the circle $x^2 + y^2 - 4x + 2y - 7 = 0$ and having the centre at (2, 3).

12. The line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ intersect at A and B. If AB = 2λ, then show that :

$$c^2 = (1 + m^2)(a^2 - \lambda^2).$$

13. Find the equation of the ellipse with focus at (1, –1), $e = \frac{2}{3}$ and directrix as $x + y + 2 = 0$.

14. Find the equations of tangents to the ellipse $2x^2 + 3y^2 = 11$ at the points whose ordinate is 1.

15. Find the foci, eccentricity, equations of the directrix, length of latus rectum of the hyperbola $x^2 - 4y^2 = 4$.

16. Find :

$$\int_0^{2\pi} \sin^4 x \cos^6 x \, dx.$$

17. Solve the differential equation :

$$\cos x \frac{dy}{dx} + y \sin x = \sec^2 x.$$

SECTION C

5×7=35

III. Long answer type questions :

(*i*) Answer **ANY FIVE** questions.

(*ii*) Each question carries **SEVEN** marks.

18. Show that the points (9, 1), (7, 9), (–2, 12), (6, 10) are concyclic and find the equation of the circle on which they lie.

19. Show that, four common tangents can be drawn for the circles given by $x^2 + y^2 - 14x + 6y + 33 = 0$ and $x^2 + y^2 + 30x - 2y + 1 = 0$ and find the internal and external centres of similitude.

20. From an external point 'P' tangents are drawn to the parabola $y^2 = 4ax$ and these tangents make angles θ_1, θ_2 with its axis, such that $\cot \theta_1 + \cot \theta_2$ is a constant '*d*'. Then show that all such P lie on a horizontal line.

21. Evaluate :

$$\int e^{ax} \sin (bx + c)dx ; \quad (a, b, c \in \mathrm{R}; b \neq 0) \text{ on R.}$$

22. Obtain reduction formula for $\mathrm{I}_n = \int \cot^n x \, dx$; n being a positive integer; $n \geq 2$ and deduce the value of $\int \cot^4 x \, dx$.

23. Find :

$$\int_0^{\pi} x \sin^7 x \cos^6 x \, dx .$$

24. Solve the differential equation :

$$\frac{dy}{dx} = \frac{2y + x + 1}{2x + 4y + 3} .$$