CBSE Test Paper 02

Chapter 2 Inverse Trigonometric Functions

1. $\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}$ is equal to

   a. None of these
   b. $\frac{\pi}{2}$
   c. $\frac{\pi}{4}$
   d. $\frac{3\pi}{4}$

2. $\sin\left\{\sin^{-1}(\frac{\sqrt{3}}{5})+\cos^{-1}(\frac{\sqrt{3}}{5})\right\}$ is equal to

   a. 1
   b. $\frac{\pi}{2}$
   c. None of these
   d. 0

3. $\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)=\frac{\pi}{2}$ holds true for

   a. all $x \in R - \{0\}$
   b. all x>0
   c. all x > 1
   d. all $x \in R$

4. $\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$ is equal to

   a. $\frac{3\pi}{4}$
   b. None of these
   c. $\frac{5\pi}{4}$
   d. $-\frac{\pi}{4}$

5. If $\sin^{-1}x=\frac{\pi}{5}$, then $\cos^{-1}x$ is equal to

   a. $\frac{5\pi}{4}$
   b. $\frac{7\pi}{4}$
   c. $\frac{3\pi}{10}$

d. $\frac{\pi}{10}$

6. The principle value branch of cos-1x is ________.
7. If cos(tan^{-1}x + cot$^{-1}\sqrt{3}$) = 0, then value of x is ________.
8. The set of values of sec$^{-1}\left(\frac{1}{2}\right)$ is ________.
9. Find the value of $\cos\left(\sec^{-1}x + \cos ec^{-1}x\right)$. **(1)**
10. Find the value of $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$. **(1)**
11. Using the principal values, evaluate $\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right)$.
12. If $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\cos ec\theta)$ then show that $\theta = \frac{\pi}{4}$, where n is any integer. **(2)**
13. Evaluate: $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.
14. Evaluate: $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$. **(2)**
15. Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$.
16. $\sin(\tan^{-1}x) =$. **(4)**
17. Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3}$. **(4)**
18. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$. Prove that $\frac{x^2}{a^2} + \frac{y^2}{b^2} - 2\frac{xy}{ab}\cos\alpha = \sin^2\alpha$.

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Solution

1. c. $\frac{\pi}{4}$

   Explanation: $\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{3}{4} \Rightarrow \tan^{-1}\frac{1}{7}+\tan^{-1}\frac{2.\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}$

   $=\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{3}{4} \Rightarrow \tan^{-1}\frac{\frac{1}{7}+\frac{3}{4}}{1-\frac{1}{7}.\frac{3}{4}} \Rightarrow \tan^{-1}(1)=\frac{\pi}{4}$

2. a. 1

   Explanation: $\sin\left\{\sin^{-1}(\frac{\sqrt{3}}{5})+\cos^{-1}(\frac{\sqrt{3}}{5})\right\}$ = sin($\pi/2$) = 1.[$\sin^{-1}(x)+\cos^{-1}(x)=\frac{\pi}{2}$]

3. b. all x>0

   Explanation: $\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)=\frac{\pi}{2}$

   $\tan^{-1}\left(\frac{1}{x}\right)=\frac{\pi}{2}-\tan^{-1}x=\cot^{-1}x\ \forall x\in R$

   $\therefore \tan^{-1}\left(\frac{1}{x}\right)=\cot^{-1}x$

   $\Rightarrow$ x > 0

   Since, we know that $\tan^{-1}\left(\frac{1}{x}\right)=\cot^{-1}x$ when x>0

4. a. $\frac{3\pi}{4}$

   Explanation: We know that principle value branch of $\cos^{-1}$ is $[0,\pi]$ and $\frac{3\pi}{4}\notin[0,\pi]$ but $\left(2\pi-\frac{5\pi}{4}\right)\in[0,\pi]$

   $\therefore \cos^{-1}\left(\cos\frac{5\pi}{4}\right)=\cos^{-1}\left(\cos\left(2\pi-\frac{5\pi}{4}\right)\right)=\cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right)=\frac{3\pi}{4}$

5. c. $\frac{3\pi}{10}$

   Explanation: $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$

   $\cos^{-1}x=\frac{\pi}{2}-\sin^{-1}x=\frac{\pi}{2}-\frac{\pi}{5}=\frac{3\pi}{10}$

6. $[0,\pi]$
7. $\sqrt{3}$
8. ϕ
9. $\cos(\sec^{-1}x+\cos ec^{-1}x)$

   $\cos\frac{\pi}{2}=0$
10. $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$

$= \tan^{-1}\sqrt{3} - (\pi - \cot^{-1}\sqrt{3})$ $[\because \cot^{-1}(-x) = \pi - \cot^{-1}x]$

$= \tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$

$= (\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}) - \pi$ $\left[\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right]$

$= \frac{\pi}{2} - \frac{\pi}{1} = \frac{-\pi}{2}$

11. using the principal values, we have to evaluate $\tan^{-1}(1) + \sin^{-1}\left(-\frac{1}{2}\right)$.

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$

$\left[\because \tan\frac{\pi}{4} = 1 \text{ and } \sin\frac{\pi}{6} = \frac{1}{2}\right]$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \sin^{-1}\left[\sin\left(-\frac{\pi}{6}\right)\right]$

$[\therefore -\sin\theta = \sin(-\theta)]$

$= \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$

$\left[\because \tan^{-1}(\tan\theta) = \theta; \forall\theta\epsilon\left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \text{ and } \sin^{-1}(\sin\theta) = \theta; \forall\theta\epsilon\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right]$

12. We have, $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\cos ec\theta)$,

$\Rightarrow \tan^{-1}\left(\frac{2\cos\theta}{1-\cos^2\theta}\right) = \tan^{-1}(2\cos ec\theta)$

$\left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right]$

$\Rightarrow \left(\frac{2\cos\theta}{\sin^2\theta}\right) = (2\cos ec\theta)$

$\Rightarrow (\cot\theta.2\cos ec\theta) = (2\cos ec\theta) \Rightarrow \cot\theta = 1$

$\Rightarrow \cot\theta = \cot\frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$

13. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \tan^{-1}\sqrt{3} - \left[\pi - \sec^{-1}2\right]$

$= \frac{\pi}{3} - \pi + \cos^{-1}\left(\frac{1}{2}\right)$

$= -\frac{2\pi}{3} + \frac{\pi}{3} = -\frac{\pi}{3}$

14. Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$

$\Rightarrow \sec y = \frac{2}{\sqrt{3}}$

$\Rightarrow \sec y = \sec\frac{\pi}{6}$

Since, the principal value branch of $\sec^{-1}$ is $[0, \pi]$.

Therefore, Principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

15. Let $\tan^{-1}\frac{2}{3} = x$ and $\tan^{-1}\sqrt{3} = y$

so that $\tan x = \frac{2}{3}$ and $\tan y = \sqrt{3}$

Therefore,

$\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$

= sin(2x) + cos y

$= \frac{2\tan x}{1+\tan^2 x} + \frac{1}{\sqrt{1+\tan^2 y}}$

$= \frac{2.\frac{2}{3}}{1+\frac{4}{9}} + \frac{1}{\sqrt{1+(\sqrt{3})^2}}$

$= \frac{12}{13} + \frac{1}{2} = \frac{37}{26}$

16. Let $\tan^{-1}x = \theta$

$= \frac{x}{1} = \tan\theta$

$\Rightarrow \sin\theta = \frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \theta = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$\Rightarrow \sin(\tan^{-1}x) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$

$= \frac{x}{\sqrt{1+x^2}}$

17. **To prove**, $\tan^{-1}(\frac{1}{4}) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)$

$\Rightarrow 2\left[\tan^{-1}(\frac{1}{4}) + \tan^{-1}\left(\frac{2}{9}\right)\right] = \tan^{-1}\left(\frac{4}{3}\right)$

LHS $= 2\left[\tan^{-1}(\frac{1}{4}) + \tan^{-1}\left(\frac{2}{9}\right)\right]$

$= 2\left[\tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\times\frac{2}{9}}\right)\right] [\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right); xy < 1\,]$

$= 2\tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right)$

$= 2\tan^{-1}\left(\frac{17}{34}\right)$

$= 2\tan^{-1}(\frac{1}{2})$

$= tan^{-1}\left(\frac{2\times(\frac{1}{2})}{1-(\frac{1}{2})^2}\right) \left[\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right); -1 < x < 1\right]$

$= \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right)$

$= \tan^{-1}\left(\frac{4}{3}\right)$ = **RHS**

$\therefore \tan^{-1}(\frac{1}{4}) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)$

Hence proved.

18. $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$

$\left[\because \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)\right]$

$$\cos^{-1}\left[\frac{x}{a}\cdot\frac{y}{b}-\sqrt{1-\frac{x^2}{a^2}}\cdot\sqrt{1-\frac{y^2}{b^2}}\right]=\alpha$$

$$\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}\cdot\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha$$

$$\frac{xy}{ab}-\cos\alpha=\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$$

Squaring both sides,

$$\left(\frac{xy}{ab}-\cos\alpha\right)^2=\left(\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right)^2$$

$$\frac{x^2y^2}{a^2b^2}+\cos^2\alpha-2.\frac{xy}{ab}.\cos\alpha=\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$$

$$\frac{x^2y^2}{a^2b^2}+\cos^2\alpha-2.\frac{xy}{ab}.\cos\alpha=1-\frac{y^2}{b^2}-\frac{x^2}{a^2}+\frac{x^2y^2}{a^2b^2}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-2\frac{xy}{ab}\cos\alpha=1-\cos^2\alpha$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-2\frac{xy}{ab}\cos\alpha=\sin^2\alpha$$