CBSE Test Paper 02

Chapter 2 Inverse Trigonometric Functions

1.
$$\tan^{-1}\frac{1}{7}+2\tan^{-1}\frac{1}{3}$$
 is equal to

- a. None of these

2.
$$\sin\Bigl\{\sin^{-1}\bigl(\frac{\sqrt{3}}{5}\bigr)+\cos^{-1}\bigl(\frac{\sqrt{3}}{5}\bigr)\Bigr\}$$
 is equal to

- a. 1
- b. $\frac{\pi}{2}$
- c. None of these

3.
$$an^{-1}x+ an^{-1}\left(rac{1}{x}
ight)=rac{\pi}{2}$$
 holds true for

- a. all $x \in R \{0\}$
- b. all x>0
- c. all x > 1
- d. all $x \in R$

4.
$$\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$$
 is equal to

- a. $\frac{3\pi}{4}$
- b. None of these
- c. $\frac{5\pi}{4}$ d. $-\frac{\pi}{4}$

5. If
$$\sin^{-1}x = \frac{\pi}{5}$$
, then $\cos^{-1}x$ is equal to

- d. $\frac{\pi}{10}$
- 6. The principle value branch of cos-1x is _____.
- 7. If $\cos(\tan^{-1}x + \cot^{-1}\sqrt{3}) = 0$, then value of x is _____.
- 8. The set of values of $\sec^{-1}\left(\frac{1}{2}\right)$ is _____.
- 9. Find the value of $\cos(\sec^{-1}x+\cos e \, c^{-1}x)$. (1)
- 10. Find the value of $an^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$. (1)
- 11. Using the principal values, evaluate $\tan^{-1}(1) + \sin^{-1}(-\frac{1}{2})$.
- 12. If $2 an^{-1}(\cos\theta)= an^{-1}(2\cos ec\theta)$ then show that $\theta=\frac{\pi}{4}$, where n is any integer. (2)
- 13. Evaluate: $\tan^{-1}\sqrt{3} \sec^{-1}(-2)$.
- 14. Evaluate: $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$. (2)
- 15. Find the value of $\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$.
- 16. $\sin(\tan^{-1}x) = .$ (4)
- 17. Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3}$. (4)
- 18. If $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=lpha$. Prove that $\frac{x^2}{a^2}+\frac{y^2}{b^2}-2\frac{xy}{ab}\coslpha=\sin^2lpha$.

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Solution

1. c.
$$\frac{\pi}{4}$$

Explanation:
$$\tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{3}{4} \Rightarrow \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{2 \cdot \frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2}$$

$$= \tan^{-1}\frac{1}{7} + 2\tan^{-1}\frac{3}{4} \Rightarrow \tan^{-1}\frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \cdot \frac{3}{4}} \Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$$

Explanation:
$$\sin \left\{ \sin^{-1}(\frac{\sqrt{3}}{5}) + \cos^{-1}(\frac{\sqrt{3}}{5}) \right\} = \sin(\pi/2) = 1.[$$
 $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}]$

Explanation:
$$\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$$

$$\tan^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2} - \tan^{-1}x = \cot^{-1}x \ \forall x \in R$$

$$\therefore \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x$$

$$\Rightarrow x > 0$$

Since, we know that
$$\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x$$
 when x>0

4. a.
$$\frac{3\pi}{4}$$

Explanation: We know that principle value branch of \cos^{-1} is $[0,\pi]$

and
$$\frac{3\pi}{4} \not\in [0,\pi]$$
 but $\left(2\pi - \frac{5\pi}{4}\right) \in [0,\pi]$
 $\therefore \cos^{-1}\left(\cos\frac{5\pi}{4}\right) = \cos^{-1}\left(\cos\left(2\pi - \frac{5\pi}{4}\right)\right) = \cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right) = \frac{3\pi}{4}$

5. c.
$$\frac{3\pi}{10}$$

Explanation:
$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

 $\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$

6.
$$[0, \pi]$$

7.
$$\sqrt{3}$$

9.
$$\cos(\sec^{-1}x + \cos ec^{-1}x)$$

 $\cos\frac{\pi}{2} = 0$

10.
$$\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$$

$$= \tan^{-1}\sqrt{3} - (\pi - \cot^{-1}\sqrt{3}) \ [\because \cot^{-1}(-x) = \pi - \cot^{-1}x]$$

$$= \tan^{-1}\sqrt{3} - \pi + \cot^{-1}\sqrt{3}$$

$$= (\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}) - \pi \ [\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}]$$

$$= \frac{\pi}{2} - \frac{\pi}{1} = \frac{-\pi}{2}$$

11. using the principal values, we have to evaluate $\tan^{-1}(1) + \sin^{-1}(-\frac{1}{2})$.

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right) + \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$$

$$egin{aligned} \left[\because anrac{\pi}{4} = 1 ext{ and } \sinrac{\pi}{6} = rac{1}{2}
ight] \ &= an^{-1} \left(anrac{\pi}{4}
ight) + \sin^{-1} \left[\sin\left(-rac{\pi}{6}
ight)
ight] \end{aligned}$$

$$[\because -\sin\theta = \sin(-\theta)]$$

$$=\frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{12}$$

$$\left[\because \tan^{-1}(\tan\theta) = \theta; \forall \theta \epsilon \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \text{ and } \sin^{-1}(\sin\theta) = \theta; \forall \theta \epsilon \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\right]$$

12. We have, $2\tan^{-1}(\cos\theta) = \tan^{-1}(2\cos ec\theta)$,

$$\Rightarrow an^{-1}\left(rac{2\cos heta}{1-\cos^2 heta}
ight) = an^{-1}(2\cos ec heta)$$

$$\left[\because 2 an^{-1}x = an^{-1}\left(rac{2x}{1-x^2}
ight)
ight]$$

$$\Rightarrow \left(rac{2\cos heta}{\sin^2 heta}
ight) = \left(2\cos ec heta
ight)$$

$$\Rightarrow (\cot \theta.2 \cos ec\theta) = (2 \cos ec\theta) \Rightarrow \cot \theta = 1$$

$$\Rightarrow \cot \theta = \cot \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4}$$

13.
$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \tan^{-1}\sqrt{3} - \left[\pi - \sec^{-1}2\right]$$

 $= \frac{\pi}{2} - \pi + \cos^{-1}\left(\frac{1}{2}\right)$

$$= \frac{\pi}{3} - \pi + \cos^{-1}\left(\frac{1}{2}\right)$$
$$= -\frac{2\pi}{3} + \frac{\pi}{3} = -\frac{\pi}{3}$$

14. Let
$$\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$$

$$\Rightarrow \sec y = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \sec y = \sec \frac{\pi}{6}$$

Since, the principal value branch of sec⁻¹ is $[0, \pi]$.

Therefore, Principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

15. Let $\tan^{-1}\frac{2}{3}=x$ and $\tan^{-1}\sqrt{3}=y$

so that
$$an x = rac{2}{3}$$
 and $an y = \sqrt{3}$

Therefore,

$$\sin\!\left(2 an^{-1}rac{2}{3}
ight)+\cos\!\left(an^{-1}\sqrt{3}
ight)$$

$$= \sin(2x) + \cos y$$

$$= \frac{2 \tan x}{1 + \tan^2 x} + \frac{1}{\sqrt{1 + \tan^2 y}}$$

$$= \frac{2 \cdot \frac{2}{3}}{1 + \frac{4}{9}} + \frac{1}{\sqrt{1 + (\sqrt{3})^2}}$$

$$= \frac{12}{13} + \frac{1}{2} = \frac{37}{26}$$

$$16. \text{ Let } \tan^{-1} x = \theta$$

$$= \frac{x}{1} = \tan \theta$$

$$\Rightarrow \sin \theta = \frac{x}{\sqrt{1 + x^2}}$$

$$\Rightarrow \theta = \sin^{-1} \frac{x}{\sqrt{1 + x^2}}$$

$$\Rightarrow \theta = \sin^{-1} \frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow \sin(an^{-1}x) = \sin\left(\sin^{-1}rac{x}{\sqrt{1+x^2}}
ight)$$

$$=rac{x}{\sqrt{1+x^2}}$$

17. **To prove**,
$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)$$
$$\Rightarrow 2\left[\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)\right] = \tan^{-1}\left(\frac{4}{3}\right)$$

LHS =
$$2\left[\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)\right]$$

$$a=2\left[an^{-1}\left(rac{rac{1}{4}+rac{2}{9}}{1-rac{1}{4} imesrac{2}{9}}
ight)
ight]\left[\because an^{-1}x+ an^{-1}y= an^{-1}\left(rac{x+y}{1-xy}
ight);xy<1
ight]$$

$$=2 an^{-1} \left(rac{rac{9+8}{36}}{rac{36-2}{36}}
ight)$$

$$=2\tan^{-1}\left(rac{17}{34}
ight)$$

$$=2\tan^{-1}\left(\frac{1}{2}\right)$$

$$a = tan^{-1}\left(rac{2 imes(rac{1}{2})}{1-(rac{1}{2})^2}
ight)\left[\because 2 an^{-1}x = an^{-1}\Big(rac{2x}{1-x^2}\Big); -1 < x < 1
ight]$$

$$= \tan^{-1} \left(\frac{1}{1 - \frac{1}{4}} \right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right)$$
 = RHS

$$\therefore \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right)$$

Hence proved.

18.
$$\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$$

$$\left[\because \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left(xy - \sqrt{1 - x^2}\sqrt{1 - y^2}\right) \right]$$

$$egin{aligned} \cos^{-1}\left[rac{x}{a}.rac{y}{b}-\sqrt{1-rac{x^2}{a^2}}.\sqrt{1-rac{y^2}{b^2}}
ight] = lpha \ rac{xy}{ab}-\sqrt{1-rac{x^2}{a^2}}.\sqrt{1-rac{y^2}{b^2}} = \coslpha \ rac{xy}{ab}-\coslpha = \sqrt{1-rac{x^2}{a^2}}\sqrt{1-rac{y^2}{b^2}} \end{aligned}$$

Squaring both sides,

$$egin{aligned} \left(rac{xy}{ab} - \coslpha
ight)^2 &= \left(\sqrt{1 - rac{x^2}{a^2}}\sqrt{1 - rac{y^2}{b^2}}
ight)^2 \ rac{x^2y^2}{a^2b^2} + \cos^2lpha - 2.rac{xy}{ab}.\coslpha &= \left(1 - rac{x^2}{a^2}
ight)\left(1 - rac{y^2}{b^2}
ight) \ rac{x^2y^2}{a^2b^2} + \cos^2lpha - 2.rac{xy}{ab}.\coslpha &= 1 - rac{y^2}{b^2} - rac{x^2}{a^2} + rac{x^2y^2}{a^2b^2} \ rac{x^2}{a^2} + rac{y^2}{b^2} - 2rac{xy}{ab}\coslpha &= 1 - \cos^2lpha \ rac{x^2}{a^2} + rac{y^2}{b^2} - 2rac{xy}{ab}\coslpha &= \sin^2lpha \end{aligned}$$