# 8 QUADRILATERALS

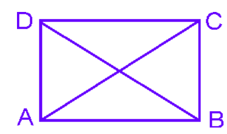
## EXERCISE 8.1

**Q.1.** *If the diagonals of a parallelogram are equal, then show that it is a rectangle.*

**Sol.** **Given :** ABCD is a parallelogram in which AC = BD.

**To Prove :** ABCD is a rectangle.

**Proof :** In $\Delta ABC$ and $\Delta ABD$




$AB = AB$ [Common]

$BC = AD$ [Opposite sides of a parallelogram]

$AC = BD$ [Given]

$\therefore \Delta ABC \cong \Delta BAD$ [SSS congruence]

$\angle ABC = \angle BAD$ ...(i) [CPCT]

Since, ABCD is a parallelogram, thus,

$\angle ABC + \angle BAD = 180°$ ...(ii) [Consecutive interior angles]

$\angle ABC + \angle ABC = 180°$

$\therefore \quad 2\angle ABC = 180°$ [From (i) and (ii)]

$\Rightarrow \quad \angle ABC = \angle BAD = 90°$

This shows that ABCD is a parallelogram one of whose angle is 90°.

Hence, ABCD is a rectangle. **Proved.**

**Q.2.** *Show that the diagonals of a square are equal and bisect each other at right angles.*

**Sol.** **Given :** ABCD is a square in which AC and BD are diagonals.

**To Prove :** AC = BD and AC bisects BD at right angles, i.e. $AC \perp BD$.

AO = OC, OB = OD

**Proof :** In $\Delta ABC$ and $\Delta BAD$,

$AB = AB$ [Common]

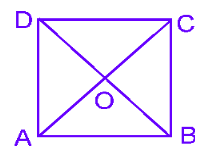
$BC = AD$ [Sides of a square]

$\angle ABC = \angle BAD = 90°$ [Angles of a square]

$\therefore \Delta ABC \cong \Delta BAD$ [SAS congruence]

$\Rightarrow AC = BD$ [CPCT]

Now in $\Delta AOB$ and $\Delta COD$,




$AB = DC$ [Sides of a square]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\angle OAB = \angle OCD$ [Alternate angles]

$\therefore \Delta AOB \cong \Delta COD$ [AAS congruence]

$\angle AO = \angle OC$ [CPCT]

Similarly by taking $\Delta AOD$ and $\Delta BOC$, we can show that OB = OD.

In $\Delta ABC$, $\angle BAC + \angle BCA = 90°$ [$\because \angle B = 90°$]

$\Rightarrow 2\angle BAC = 90°$ [$\angle BAC = \angle BCA$, as BC = AD]

$\Rightarrow \angle BCA = 45°$ or $\angle BCO = 45°$

Similarly $\angle CBO = 45°$

In $\Delta BCO$.

$\angle BCO + \angle CBO + \angle BOC = 180°$

$\Rightarrow 90° + \angle BOC = 180°$

$\Rightarrow \angle BOC = 90°$

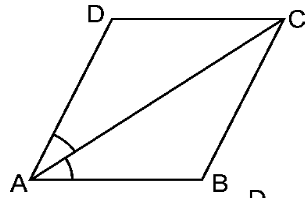
$\Rightarrow BO \perp OC \Rightarrow BO \perp AC$

Hence, $AC = BD$, $AC \perp BD$, $AO = OC$ and $OB = OD$. **Proved.**

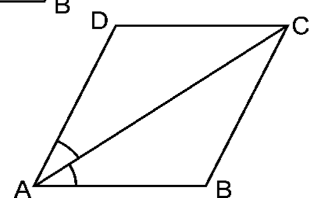
**Q.3.** *Diagonal AC of a parallelogram ABCD bisects ∠A (see Fig.). Show that*

*(i) it bisects ∠C also,*

*(ii) ABCD is a rhombus.*




**Given :** A parallelogram ABCD, in which diagonal AC bisects ∠A, i.e., ∠DAC = ∠BAC.




**To Prove :** (i) Diagonal AC bisects ∠C i.e., ∠DCA = ∠BCA

(ii) ABCD is a rhomhus.

**Proof :** (i)

∠DAC = ∠BCA [Alternate angles]

∠BAC = ∠DCA [Alternate angles]

But, ∠DAC = ∠BAC [Given]

∴ ∠BCA = ∠DCA

Hence, AC bisects ∠DCB

Or, AC bisects ∠C **Proved.**

(ii) In ΔABC and ΔCDA

AC = AC [Common]

∠BAC = ∠DAC [Given]

and ∠BCA = ∠DAC [Proved above]

∴ ΔABC ≅ ΔADC [ASA congruence]

∴ BC = DC [CPCT]

But AB = DC [Given]

∴ AB = BC = DC = AD

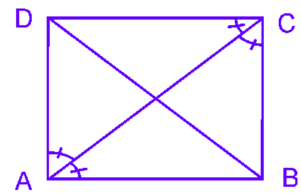
Hence, ABCD is a rhombus **Proved.**

[∵ opposite angles are equal]

**Q.4.** *ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that :*

*(i) ABCD is a square (ii) diagonal BD bisects ∠B as well as ∠D.*

**Sol. Given :** ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C.




**To Prove :** (i) ABCD is a square.

(ii) Diagonal BD bisects ∠B as well as ∠D.

**Proof :** (i) In ΔABC and ΔADC, we have

∠BAC = ∠DAC [Given]

∠BCA = ∠DCA [Given]

AC = AC

∴ ΔABC ≅ ΔADC [ASA congruence]

∴ AB = AD and CB = CD [CPCT]

But, in a rectangle opposite sides are equal,

i.e., AB = DC and BC = AD

∴ AB = BC = CD = DA

Hence, ABCD is a square **Proved.**

(ii) In ΔABD and ΔCDB, we have

$$\left.\begin{aligned} AD &= CD \\ AB &= CD \end{aligned}\right\} \text{[Sides of a square]}$$

BD = BD [Common]

∴ ΔABD ≅ ΔCBD [SSS congruence]

$$\left.\begin{aligned} \text{So, } \angle ABD &= \angle CBD \\ \angle ADB &= \angle CDB \end{aligned}\right\} \text{[CPCT]}$$

Hence, diagonal BD bisects ∠B as well as ∠D **Proved.**

**Q.5.** *In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see Fig.). Show that :*

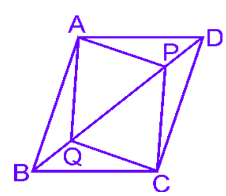
*(i) Δ APD ≅ ΔCQB*

*(ii) AP = CQ*

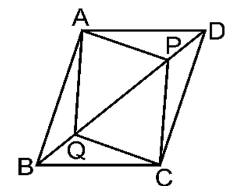
*(iii) Δ AQB ≅ ΔCPD*

*(iv) AQ = CP*

*(v) APCQ is a parallelogram*



**Sol.** **Given :** ABCD is a parallelogram and P and Q are points on diagonal BD such that DP = BQ.




**To Prove :** (i) $\Delta APD \cong \Delta CQB$

(ii) AP = CQ

(iii) $\Delta AQB \cong \Delta CPD$

(iv) AQ = CP

(v) APCQ is a parallelogram

**Proof :** (i) In $\Delta APD$ and $\Delta CQB$, we have

AD = BC [Opposite sides of a ||gm]

DP = BQ [Given]

$\angle ADP = \angle CBQ$ [Alternate angles]

$\therefore$ $\Delta APD \cong \Delta CQB$ [SAS congruence]

(ii) $\therefore$ AP = CQ [CPCT]

(iii) In $\Delta AQB$ and $\Delta CPD$, we have

AB = CD [Opposite sides of a ||gm]

DP = BQ [Given]

$\angle ABQ = \angle CDP$ [Alternate angles]

$\therefore$ $\Delta AQB \cong \Delta CPD$ [SAS congruence]

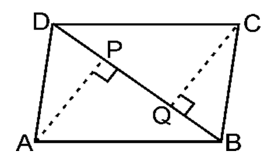
(iv) $\therefore$ AQ = CP [CPCT]

(v) Since in APCQ, opposite sides are equal, therefore it is a parallelogram. **Proved.**

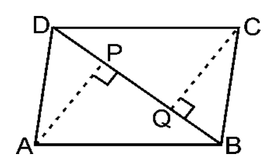
**Q.6.** *ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see Fig.). Show that*

*(i) $\Delta APB \cong \Delta CQD$*

*(ii) AP = CQ*




**Sol.** **Given :** ABCD is a parallelogram and AP and CQ are perpendiculars from vertices A and C on BD.




**To Prove :** (i) $\Delta APB \cong \Delta CQD$

(ii) AP = CQ

**Proof :** (i) In $\Delta APB$ and $\Delta CQD$, we have

$\angle ABP = \angle CDQ$ [Alternate angles]

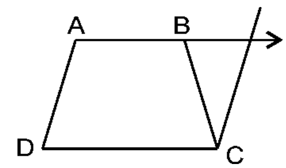
AB = CD [Opposite sides of a parallelogram]

$\angle APB = \angle CQD$ [Each = 90°]

$\therefore$ $\Delta APB \cong \Delta CQD$ [ASA congruence]

(ii) So, AP = CQ [CPCT] **Proved.**

**Q.7.** *ABCD is a trapezium in which AB || CD and AD = BC (see Fig.). Show that*




*(i)* $\angle A = \angle B$

*(ii)* $\angle C = \angle D$

*(iii)* $\Delta ABC \cong \Delta BAD$

*(iv)* *diagonal AC = diagonal BD*

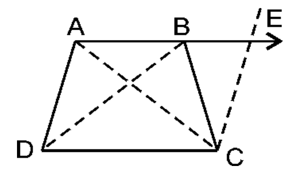
**Sol. Given :** In trapezium ABCD, AB || CD and AD = BC.

**To Prove :** (i) $\angle A = \angle B$

(ii) $\angle C = \angle D$

(iii) $\Delta ABC \cong \Delta BAD$

(iv) diagonal AC = diagonal BD




**Constructions :** Join AC and BD. Extend AB and draw a line through C parallel to DA meeting AB produced at E.

**Proof :** (i) Since AB || DC

$\Rightarrow$ AE || DC ...(i)

and AD || CE ...(ii) [Construction]

$\Rightarrow$ ADCE is a parallelogram [Opposite pairs of sides are parallel

$\angle A + \angle E = 180°$ ...(iii) [Consecutive interior angles]

$\angle B + \angle CBE = 180°$ ...(iv) [Linear pair]

AD = CE ...(v) [Opposite sides of a $||^{gm}$]

AD = BC ...(vi) [Given]

$\Rightarrow$ BC = CE [From (v) and (vi)]

$\Rightarrow$ $\angle E = \angle CBE$ ...(vii) [Angles opposite to equal sides]

$\therefore$ $\angle B + \angle E = 180°$ ...(viii) [From (iv) and (vii)

Now from (iii) and (viii) we have

$\angle A + \angle E = \angle B + \angle E$

$\Rightarrow$ $\angle A = \angle B$ **Proved.**

(ii) $\left.\begin{aligned}\angle A + \angle D &= 180^\circ \\ \angle B + \angle C &= 180^\circ\end{aligned}\right\}$ [Consecutive interior angles]

$\Rightarrow \angle A + \angle D = \angle B + \angle C$ $\quad [\because \angle A = \angle B]$

$\Rightarrow \angle D = \angle C$

Or $\angle C = \angle D$ **Proved.**

(iii) In $\Delta ABC$ and $\Delta BAD$, we have

$AD = BC$ [Given]

$\angle A = \angle B$ [Proved]

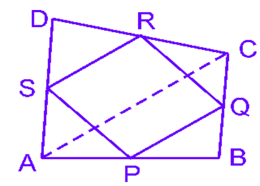
$AB = CD$ [Common]

$\therefore \Delta ABC \cong \Delta BAD$ [ASA congruence]

(iv) diagonal $AC$ = diagonal $BD$ [CPCT] **Proved.**

## EXERCISE 8.2

**Q.1.** *ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. (see Fig.). AC is a diagonal. Show that :*

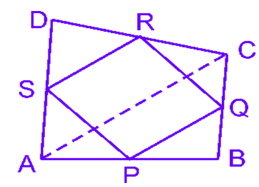



*(i)* $SR \parallel AC$ *and* $SR = \frac{1}{2}AC$

*(ii)* $PQ = SR$

*(iii) PQRS is a parallelogram.*

**Given :** ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA. AC is a diagonal.




**To Prove :** (i) $SR \parallel AC$ and $SR = \frac{1}{2}AC$

(ii) PQ = SR

(iii) PQRS is a parallelogram

**Proof :** (i) In $\Delta ABC$, P is the mid-point of AB and Q is the mid-point of BC.

$\therefore PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ ...(1)

[Mid-point theorem]

In $\Delta ADC$, R is the mid-point of CD and S is the mid-point of AD

$\therefore SR \parallel AC$ and $SR = \frac{1}{2}AC$ ...(2)

[Mid-point theorem]

(ii) From (1) and (2), we get

$PQ \parallel SR$ and $PQ = SR$

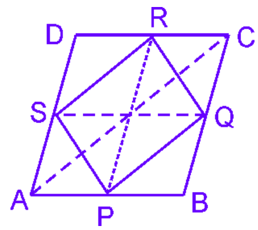
(iii) Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is equal and parallel.

$\therefore$ PQRS is a parallelogram. **Proved.**

**Q.2.** *ABCD is a rhombus and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rectangle.*

**Sol.** **Given :** ABCD is a rhombus in which P, Q, R and S are mid points of sides AB, BC, CD and DA respectively :

**To Prove :** PQRS is a rectangle.




**Construction :** Join AC, PR and SQ.

**Proof :** In $\Delta$ABC

P is mid point of AB [Given]

Q is mid point of BC [Given]

$\Rightarrow$ PQ || AC and PQ = $\frac{1}{2}$AC ...(i) [Mid point theorem]

Similarly, in $\Delta$DAC,

SR || AC and SR = $\frac{1}{2}$AC ...(ii)

From (i) and (ii), we have PQ||SR and PQ = SR

$\Rightarrow$ PQRS is a parallelogram

[One pair of opposite sides is parallel and equal]

Since ABQS is a parallelogram

$\Rightarrow$ AB = SQ [Opposite sides of a || gm]

Similarly, since PBCR is a parallelogram.

$\Rightarrow$ BC = PR

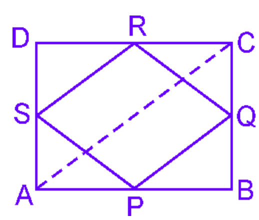
Thus, SQ = PR [AB = BC]

Since SQ and PR are diagonals of parallelogram PQRS, which are equal.

$\Rightarrow$ PQRS is a rectangle. **Proved.**

**Q.3.** *ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilataral PQRS is a rhombus.*

**Sol.** **Given :** A rectangle ABCD in which P, Q, R, S are the mid-points of AB, BC, CD and DA respectively, PQ, QR, RS and SP are joined.




**To Prove :** PQRS is a rhombus.

**Construction :** Join AC

**Proof :** In $\Delta ABC$, P and Q are the mid-points of the sides AB and BC.

$\therefore$ PQ || AC and PQ = $\frac{1}{2}$AC ...(i) [Mid point theorem]

Similarly, in $\Delta ADC$,

SR || AC and SR = $\frac{1}{2}$AC ...(ii)

From (i) and (ii), we get

PQ || SR and PQ = SR ...(iii)

Now in quadrilateral PQRS, its one pair of opposite sides PQ and SR is parallel and equal [From (iii)]

$\therefore$PQRS is a parallelogram.

Now AD = BC ...(iv)

[Opposite sides of a rectangle ABCD]

$\therefore \quad \frac{1}{2}AD = \frac{1}{2}BC$

$\Rightarrow \quad AS = BQ$

In $\Delta APS$ and $\Delta BPQ$

AP = BP [$\because$ P is the mid-point of AB]

AS = BQ [Proved above]

$\angle PAS = \angle PBQ$ [Each = 90°]

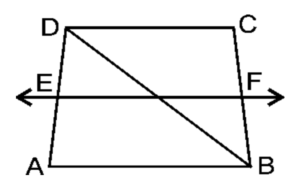
$\Delta APS \cong \Delta BPQ$ [SAS axiom]

$\therefore$ PS = PQ ...(v)

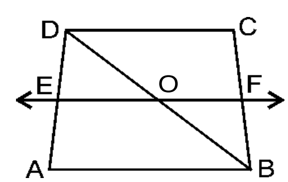
From (iii) and (v), we have

PQRS is a rhombus **Proved.**

**Q.4.** *ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see Fig.). Show that F is the mid-point of BC.*




**Sol. Given :** A trapezium ABCD with AB || DC, E is the mid-point of AD and EF || AB.




**To Prove :** F is the mid-point of BC.

**Proof :** AB || DC and EF || AB

$\Rightarrow$ AB, EF and DC are parallel.

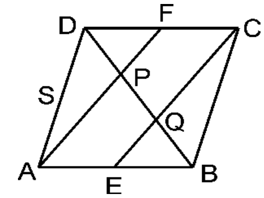
Intercepts made by parallel lines AB, EF and DC on transversal AD are equal.

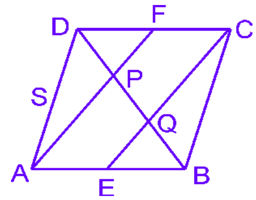
$\therefore$ Intercepts made by those parallel lines on transversal BC are also equal.

i.e., BF = FC

$\Rightarrow$ F is the mid-point of BC.

**Q.5.** *In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see Fig.). Show that the line segments AF and EC trisect the diagonal BD.*






**Sol.** **Given :** A parallelogram ABCD, in which E and F are mid-points of sides AB and DC respectively.

**To Prove :** DP = PQ = QB

**Proof :** Since E and F are mid-points of AB and DC respectively.

$\Rightarrow$ AE = $\frac{1}{2}$AB and CF = $\frac{1}{2}$DC ...(i)

But, AB = DC and AB || DC ...(ii)

[Opposite sides of a parallelogram]

$\therefore$ AE = CF and AE || CF.

$\Rightarrow$ AECF is a parallelogram.

[One pair of opposite sides is parallel and equal]

In $\Delta$BAP,

E is the mid-point of AB

EQ || AP

$\Rightarrow$ Q is mid-point of PB [Converse of mid-point theorem]

$\Rightarrow$ PQ = QB ...(iii)

Similarly, in $\Delta$DQC,

P is the mid-point of DQ

DP = PQ ...(iv)

From (iii) and (iv), we have

DP = PQ = QB

or line segments AF and EC trisect the diagonal BD. **Proved.**

**Q.6.** *ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that*

*(i) D is the mid-point of AC.*

*(ii) MD $\perp$ AC*

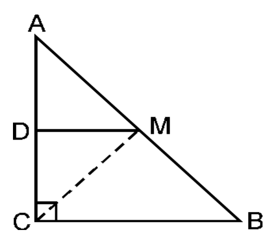
*(iii) CM = MA = $\frac{1}{2}$AB*

**Sol.** **Given :** A triangle ABC, in which $\angle$C = 90° and M is the mid-point of AB and BC || DM.

**To Prove :** (i) D is the mid-point of AC [Given]

(ii) DM $\perp$ BC

(iii) CM = MA = $\frac{1}{2}$AB



**Construction :** Join CM.

**Proof :** (i) In $\Delta ABC$,

M is the mid-point of AB. [Given]

$BC \parallel DM$ [Given]

D is the mid-point of AC

[Converse of mid-point theorem] **Proved.**

(ii) $\angle ADM = \angle ACB$ [$\because$ Coresponding angles]

But $\angle ACB = 90°$ [Given]

$\therefore \quad \angle ADM = 90°$

But $\angle ADM + \angle CDM = 180°$ [Linear pair]

$\therefore \quad \angle CDM = 90°$

Hence, $MD \perp AC$ **Proved.**

(iii) $AD = DC$ ...(1) [$\because$ D is the mid-point of AC]

Now, in $\Delta ADM$ and $\Delta CMD$, we have

$\angle ADM = \angle CDM$ [Each = 90°]

$AD = DC$ [From (1)]

$DM = DM$ [Common]

$\therefore \quad \Delta ADM \cong \Delta CMD$ [SAS congruence]

$\Rightarrow \quad CM = MA$ ...(2) [CPCT]

Since M is mid-point of AB,

$$\therefore \quad MA = \frac{1}{2}AB \quad ...(3)$$

Hence, $CM = MA = \frac{1}{2}AB$ **Proved.** [From (2) and (3)]