Chapter 1

Fluid Properties and Manometry

CHAPTER HIGHLIGHTS

- Shear and Normal Stresses
- Fluid Properties
- Variation of Viscosity of Fluids with Temperature
- Velocity Gradient
- Aliter
- Classification of Fluids
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- Real Fluid
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Introduction

Fluid Mechanics is defined as the science that deals with a fluid's behaviour, when it is at rest for in motion, and the fluid's interaction with other fluids or solids at the boundaries. *Fluid Statics* deals with fluids at rest while *Fluid Dynamics* deals with fluids in motion. Fluid statics is generally referred to as *hydrostatics* when the fluid is a liquid.

What is a Fluid?

Matter can be primarily classified as: (a) Solids (b) Liquids and (c) Gases

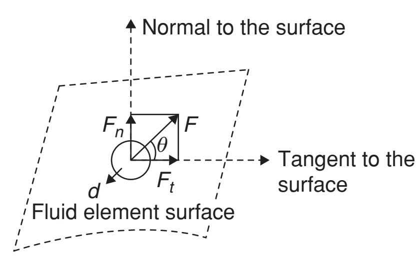
Matter	Inter-molecular	
	Space	Cohesive forces
Solids	Small	Large
Liquids	Large	Small
Gases	Very large	Very small

Liquids and gases (including vapours) are commonly referred to as fluids. A fluid is defined as a substance that deforms continuously under the influence of a shear stress of any magnitude, i.e., when subjected to an external shear force, of any magnitude, a fluid will deform continuously as long as the force is applied. A fluid has negligible shear resistance, i.e., it offers negligible resistance towards an applied shear (or tangential) stress that tends to change the shape of the fluid body.

Shear and Normal Stresses

Stress is defined as force per unit area (area upon which the force acts). Let us consider a small area dA, on the surface of a fluid element, on which a force F acts as shown in the figure below.

If the tangential and normal components of the force F are respectively F_t and F_n, then



Shear stress (τ) at the surface of the fluid element $= \dfrac{F_t}{dA} = \dfrac{F\cos\theta}{dA}$

Normal stress at the surface of the fluid element $= \dfrac{F_n}{dA} = \dfrac{F\sin\theta}{dA}$

> Normal stress and shear stress are vector quantities

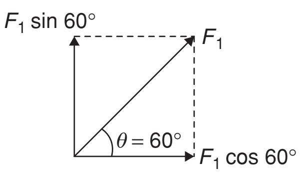
> For a static fluid body, i.e., a body of fluid that is at rest or has zero velocity, the shear stress is always zero. Also for static fluids, the normal stress is always positive.

Solved Examples

Example 1: A force F_1 (=20 N) is applied on an area A_1 (=0.1 cm^2) at the surface of a fluid element in the outward direction. The force F_1 acts at an angle of 60° from the tangential plane at the point of application of the force. Another force F_2 (=60 N) is applied, in the same manner as the force F_1, on an another area A_2 (=0.2 cm^2) at the surface of the same fluid element. The ratio of the normal stress at area A_1 to the shear stress at area A_2 is

(A) $2:3$ (B) $2:3\sqrt{3}$
(C) $2:\sqrt{3}$ (D) $1:\sqrt{3}$

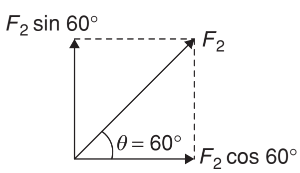
Solution:
Area A_1:



Normal stress acting on area A_1

$$= \frac{F_1 \sin 60°}{A_1} = \frac{20}{0.1\times10^{-4}} \times \frac{\sqrt{3}}{2} = \sqrt{3}\times10^6\ \frac{\text{N}}{\text{m}^2}$$

Area A_2:



Shear stress acting on area A_2

$$= \frac{F_2 \cos 60°}{A_2} = \frac{60\times1}{0.2\times10^{-4}\times2} = 1.5\times10^6\ \frac{\text{N}}{\text{m}^2}$$

Ratio of the normal stress at area A_1 to the shear stress at area A_2

$$= \frac{\sqrt{3}\times10^6}{1.5\times10^6} = \frac{2}{\sqrt{3}} \quad \text{or} \quad = 2:\sqrt{3}$$

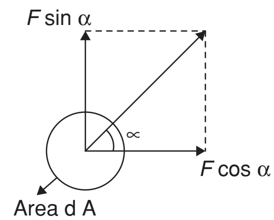
Example 2: An example for a normal stress is
(A) Volume (B) shear stress
(C) Pressure (D) temperature

Solution:
Pressure is an example for a normal stress. In static fluids, the pressure at a given position is equal to the normal stress at that position.

Example 3: On an area of 0.1 cm^2 at the surface of a static fluid element, a force of 40 N is observed to act in the outward direction. If the force acts at an angle α from the tangential plane at the point of application of the force, and the fluid still remains Static then the value of α is
(A) 0° (B) 30°
(C) 45° (D) 90°

Solution:

Shear stress acting on the given area $= \dfrac{F\cos\alpha}{A}$



For a static fluid element, Shear stress = 0

$$\Rightarrow \frac{F\cos\alpha}{A} = 0$$

or $\cos\alpha = 0$ ($\because F \neq 0, A \neq \infty$)
$\therefore \alpha = 90°$

Fluid Properties

Density (Mass Density or Specific Mass)

Density is defined as mass per unit volume. If m is the mass of a fluid body having a volume V, then the density of the fluid, denoted by ρ, is $\boxed{\rho = \frac{\text{m}}{\text{V}}}$. The S.*I*., unit of density is $\dfrac{\text{kg}}{\text{m}^3}$.

For practical calculations, the density of water is taken to be the density of water at 4°C which is $1000\dfrac{\text{kg}}{\text{m}^3}$ or $1\dfrac{\text{g}}{\text{cm}^3}$ or $1\dfrac{\text{Kg}}{\text{L}}$. For most gases, density is inversely proportional to the temperature and proportional to pressure. For liquids, variations in pressure and temperature induce a small (negligible) variation in the density.

Example 4: A gas behaves like a real gas at temperature T_1 and pressure P_1. The gas can be made to behave approximately like an ideal gas by either changing the temperature from T_1 to T_2 or by changing the pressure from P_1 to P_2. One may then conclude that

(A) $T_2 > T_1$ and $P_2 < P_1$ (B) $T_2 < T_1$ and $P_2 < P_1$
(C) $T_2 > T_1$ and $P_2 > P_1$ (D) $T_2 < T_1$ and $P_2 > P_1$

Solution:
Real gases have been experimentally observed to behave like ideal gases at low densities.
The density of most gases can be reduced by increasing the temperature $\left(\text{as } \rho \alpha \frac{1}{T}\right)$ or by decreasing the pressure (as $\rho\, \alpha P$).
$\therefore\ T_2 > T_1$ and $P_2 < P_1$

Specific Volume

Specific volume is defined as volume per unit mass. The reciprocal of a fluid's density (ρ) is its specific volume (v), i.e., $\boxed{v = \frac{1}{\rho} = \frac{\text{V}}{\text{m}}}$. The S.*I* unit of specific volume is $\frac{\text{M}^3}{\text{kg}}$.

Specific Weight (weight Density)

Specific weight is defined as weight per unit volume. The specific weight of a fluid, $\boxed{\omega = \frac{\text{W}}{\text{V}} = \frac{\text{mg}}{\text{V}} = \rho g}$, where g is the acceleration due to gravity and W, V, m and ρ are respectively the weight, volume, mass and density of the fluid. The S.*I.* unit of specific weight is $\frac{\text{kg}}{\text{m}^2\text{s}^2}$ or $\frac{\text{N}}{\text{m}^3}$. For practical calculations, the specific weight of water is taken to be $9.81\frac{\text{kN}}{\text{m}^3}$.

Specific Gravity (Relative Density)

Specific gravity of a fluid is the ratio of the density of the fluid to the density of a standard fluid. The standard fluid is taken to be pure water at 4ϒC. Sometimes for gases, the standard fluid is taken to be air at standard temperature and pressure.
Specific gravity of a fluid,

$$\boxed{SG_{\text{fluid}} = \frac{\rho_{\text{fluid}}}{\rho_{\text{standard fluid}}} = \frac{\omega_{\text{fluid}}}{\omega_{\text{standard fluid}}}}$$

Where ω is the specific weight? Specific gravity is a dimensionless quantity, i.e., it has no units. For practical calculations, the specific gravities of water and mercury are taken to be 1 and 13.6 respectively.

Example 5: The specific weight of a body of fluid A is twelve times that of a body of fluid B. The acceleration due to gravity acting on the fluid A is four times that acting on the fluid B. If the specific gravity of fluid B is 1.2, then the density of fluid A: $\left(\text{in } \frac{\text{g}}{\text{cm}^3}\right)$ is

(A) 57.6 (B) 3.6 (C) 14.4 (D) 0.4

Solution:

$$\frac{\text{Specific weight of fluid } A}{\text{Specific weight of fluid } B} = \frac{\omega_A}{\omega_B}$$

$$= \frac{\rho_A g_A}{\rho_B g_B} \qquad (1)$$

It is given that $\frac{\omega_A}{\omega_B} = \frac{12}{1}$ and $\frac{g_A}{g_B} = \frac{4}{1}$

From equation (1), we have $\frac{\rho_A}{\rho_B} = \frac{3}{1}$

Specific gravity of fluid A

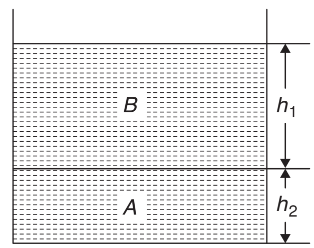
$= \text{specific gravity of fluid } B \times \left(\frac{\rho_A}{\rho_B}\right)$

$= 1.2 \times 3 = 3.6.$

Density of fluid A = (specific gravity of fluid A) ×(density of pure water at 4ϒC)

$$= 3.6 \times 1 = 3.6\frac{\text{g}}{\text{cm}^3}.$$

Example 6: When two immiscible liquids A and B are poured into a cylindrical container, then these separate out into two distinct layers of different heights as shown in the following figure. The specific gravity of liquid A is thrice that of the liquid B. If the ratio $h_1 : h_2$ is $2:1$, then the ratio of the mass of the liquid A to the mass of the liquid B in the container is



(A) 1:6 (B) 2:3 (C) 6:1 (D) 3:2

Solution:
If m_A and m_B are the masses of the liquids A and B respectively in the container, then $\frac{m_A}{m_B} = \frac{SG_A V_A}{SG_B V_B}$, where SG is the fluid's specific gravity and V is the volume of the fluid. Since the specific gravity of liquid A is greater than that of liquid B ($SG_A = 3 \times SG_B$), liquid A is denser. Hence, the height h_2 corresponds to the liquid A, i.e., $V_A = h_2 \times a$, where a is the area of the container base and $V_B = h_1 \times a$

$$\therefore\ \frac{m_A}{m_B} = \frac{SG_A h_2}{SG_B h_1} = \frac{3}{2}.$$

Viscosity

Viscosity is the property of the fluid by virtue of which it resists fluid flow, i.e., viscosity represents the internal resistance (fluid friction) of a fluid to motion (or the fluidity) or to shearing stresses. The S.*I.* unit of viscosity is $\frac{\text{kg}}{\text{ms}}$ or $\frac{\text{Ns}}{\text{m}^2}$ or Pa.s. Another unit (in C.G.S. units) for viscosity is poise.

$$\boxed{1\,\text{poise} = 0.1\frac{\text{Ns}}{\text{m}^2}}$$

Viscosity of water, for practical calculations, is taken to be 1 centipoise or 0.01 poise. The device that measures viscosity is called a viscometer.

Variation of Viscosity of Fluids with Temperature

The cohesive forces and molecular momentum transfer result in viscous forces in fluids.

Since temperature affects the cohesive forces and molecular momentum transfer, viscosity of fluids are affected by variations in temperature.

For Liquids

As liquids have a closely packed molecular structure (compared to gases), cohesive forces dominate over the molecular momentum transfer. With increase in temperature, the cohesive forces decrease in liquids, which in turn decreases the viscosity?

Hence viscosity of liquids decreases with increase in temperature and vice versa.

The relation between viscosity and temperature in liquids is:

$$\mu = \mu_0\left[\frac{1}{1+\alpha t+\beta t^2}\right],$$

where

μ = viscosity of liquid at t°C, in poise

μ_0 = viscosity of liquid at 0°C, in poise

α, β = constants for the liquid

The viscosity of water at 1°C is 1 centipoise.

Liquids with increasing order of viscosity are gasoline, water, crude oil, castor oil etc.

Gases

In the case of gases, the molecular momentum transfer dominates over the cohesive forces. As the temperature increases, molecular momentum transfer also increases.

Hence the viscosity of gases increases with increase in temperature and vice versa.

The relation between viscosity and temperature for gases is:

$$\boxed{\mu = \mu_0 + \alpha t - \beta t^2}$$

where

μ = viscosity of gas at t°C, in poise

μ_0 = viscosity of gas at 0°C, in poise

α, β = constants for the gas

The relation between absolute temperature (T) and dynamic viscosity of an ideal gas is given by Sutherland equation,

Which is $\boxed{\dfrac{\mu}{\mu_0} = \left(\dfrac{T}{T_0}\right)^{\frac{3}{2}}\dfrac{(T_0+S)}{(T+S)}}$,

where

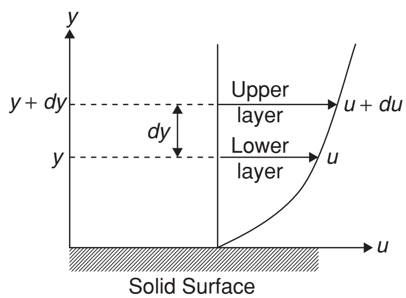
μ = viscosity at absolute temperature T

μ_0 = viscosity at absolute temperature T_0

S = Sutherland temperature of the gas (in Kelvin)

Velocity Gradient

Consider the flow of a fluid over a solid surface as shown in the figure below. Consider in this fluid flow, two fluid layers which are at a distance 'dy' apart. The upper fluid layer (at $y + dy$) is assumed to move at a velocity of $u + du$ while the lower fluid layer (at y) is assumed to move at a velocity of u.



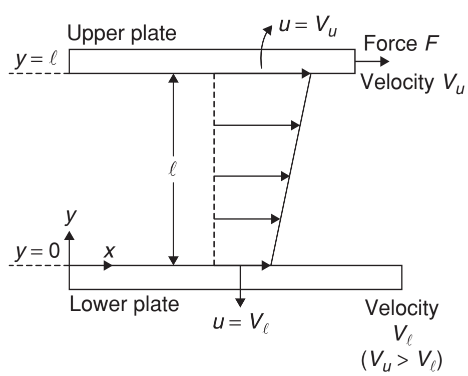
Then, the velocity gradient

$$= \frac{(u+du)-u}{(y+dy)-y} = \frac{du}{dy}$$

$$\boxed{\frac{du}{dy} \approx \frac{\Delta u}{\Delta y} = \frac{u_{y=y_2} - u_{y=y_1}}{y_2 - y_1}}$$

This equation is valid when y_2 is very close to y_1 or for a linear velocity profile.

Now consider a fluid layer between two very large parallel plates, separated by a distance ℓ, as shown in the following figure.



Let a constant parallel force F be applied to the upper plate which would move it at a constant speed V_u, after the initial dynamics. This force would move the fluid layer in contact with the upper plate at the same speed V_u in the direction of motion of the upper plate (due to no-slip condition). Similarly, if the lower plate moves with a velocity V_ℓ) the fluid in contact with the lower plate would move with the same velocity V_ℓ in the direction of motion of the lower plate.

If the fluid flow between the plates is steady and laminar, then a linear velocity profile is seen to develop in the fluid layer. That is, the fluid velocity between the plates varies linearly between V_ℓ and V_u.

For the linear velocity profile, the velocity gradient $\boxed{\frac{du}{dy} = \frac{V_u - V_\ell}{\ell - 0} = \frac{V_u - V_\ell}{\ell}}$

The linear velocity profile is given by $\boxed{u(y) = \frac{y}{\ell}(V_u - V_\ell)}$

Case 1: When the lower plate is held fixed

In this case, $V_\ell = 0$. Therefore, the velocity gradient

$$\boxed{\frac{du}{dy} = \frac{V_u}{\ell}}$$

Case 2: When the lower plate moves in the direction opposite to that of the upper plate motion

In this case, velocity gradient

$$\boxed{\frac{du}{dy} = \frac{V_u - (-V_\ell)}{\ell} = \frac{V_u + V_\ell}{\ell}}$$

For a fluid element, it can be shown that the velocity gradient is equivalent to the rate of deformation or the rate of angular displacement or the rate of shear strain.

Newton's Law of Viscosity

When two fluid layers move relative to each other, the viscosity and the relative velocity causes a shear stress to act between the fluid layers. The top fluid layer causes a shear stress on the adjacent lower layer while the lower fluid layer causes a shear stress on the adjacent top layer. Newton's law of viscosity states that the shear stress acting on a fluid layer is directly proportional to the rate of deformation or the velocity gradient, i.e.,

$$\boxed{\tau \alpha \frac{du}{dy}} \quad \text{or} \quad \boxed{\tau = \mu \frac{du}{dy}}$$

where μ is known as the coefficient of viscosity or the dynamic viscosity or the absolute viscosity or simply as viscosity. Fluids which follow this law are generally referred to as Newtonian fluids.

For most fluids, shear stress is directly proportional to the velocity gradient or the rate of deformation or the rate of angular displacement or the rate of shear strain.

Direction for questions 7 and 8: A fluid flowing over a flat solid surface develops a parabolic velocity distribution. The vertex of the parabolic distribution is situated 10 cm away from the solid surface, where the fluid velocity is 1.5 m/s. The shear stress at a point 5 cm from the solid surface is determined to be $30\frac{\text{N}}{\text{m}^2}$. The fluid follows Newton's law of viscosity.

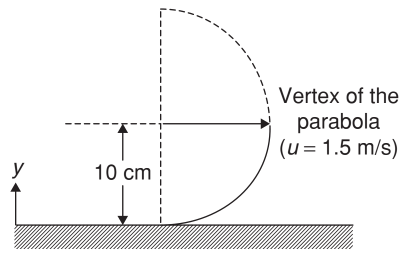
Example 7: The viscosity of the fluid is

(A) 0.2 poise (B) 2 poise
(C) 0 poise (D) 0.1 poise

Solution:

Let the parabolic velocity distribution be

$$u(y) = ay^2 + by + c \quad (1)$$



At $y = 0$, $u = 0$ (no slip condition)

∴ From equation (1), we have $c = 0$

$$\therefore u(y) = ay^2 + by \quad (2)$$

At $y = 0.1$ m(10 cm), $u = 1.5\frac{\text{m}}{\text{s}}$

∴ From equation (2), we have:

$$150 = a + 10\text{ b} \quad (3)$$

At the vertex of the parabolic velocity distribution, i.e., at $y = 0.1$ m (10 cm), we have, $\frac{du}{dy} = 0$

Hence, from equation (2), we have,

$$2a + 10b = 0 \quad (4)$$

Solving equations (3) and (4), we get a = –150 and $b = 30$

$$\therefore u(y) = -150y^2 + 30y \quad (5)$$

At $y = 0.05$ m (5 cm),

$$\tau = 30\frac{\text{N}}{\text{m}^2}$$

$$\text{i.e., } 30 = \mu\left(\frac{du}{dy}\right)_{y=0.05} \quad (6)$$

∵ Fluid follows Newton's law of viscosity.

Inserting the differential of equation (5) in equation (6) and substituting the value of y by 0.05, we get

$$\mu = 2\frac{\text{Ns}}{\text{m}^2} = 0.2 \text{ poise}$$

Example 8: The shear stress at the solid surface is

(A) $30\frac{\text{N}}{\text{m}^2}$ (B) $10\frac{\text{N}}{\text{m}^2}$

(C) $60\frac{\text{N}}{\text{m}^2}$ (D) $0\frac{\text{N}}{\text{m}^2}$

Solution:

Now, shear stress $\tau = \mu\frac{du}{dy}$

From equation (5), $\frac{du}{dy} = -300y + 30$

At the solid surface, $y = 0$

$\therefore$ Shear stress at the wall

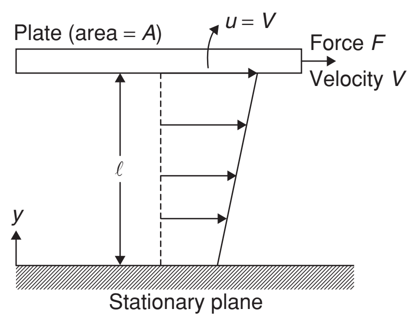
$$= \mu\left(\frac{du}{dy}\right)_{y=0} = 2 \times 30 = 60\frac{\text{N}}{\text{m}^2}.$$

Example 9: A square thin plate, of length 80 cm and mass 30 kg, slides parallel to a solid plane surface inclined at an angle of 60Υ to the horizontal. A Newtonian fluid layer of thickness 2 mm is present in between the plate and the plane surface. Had the plane been horizontal, a constant force of 192 N would have been required to move the plate at a constant velocity of 3 m/s. If the fluid's velocity profile can be assumed to be linear, then the constant force to be applied, parallel to the inclined plane, on the plate to make it slide at a instant velocity of 6 m/s is

(A) 254.87 N (B) 129.13 N
(C) 384 N (D) 89.7 N

Solution:

When the plane is horizontal



Here, shear stress $\tau = \frac{F}{A} = \mu\frac{du}{dy}$

$\because$ Fluid is Newtonian

Since the velocity profile is linear, $\frac{du}{dy} = \frac{V}{\ell}$

$$\therefore \quad F = \frac{\mu AV}{\ell} \qquad (1)$$

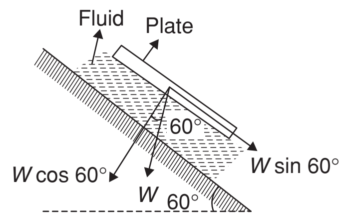
Given $F = 192$ N, $V = 3$ m/s,

$A = 0.8 \times 0.8$ m^2 and $\ell = 0.002$ m. Substituting these values in equation (1), we get $\mu = 0.2\frac{\text{Ns}}{\text{m}^2}$.

When the plane is inclined: Constant force to be applied on the plate to make it slide down with a constant velocity of 6 m/s,

$$F = \frac{\mu AV}{\ell} = \frac{0.2 \times 0.8 \times 0.8 \times 6}{0.002} = 384 \text{ N}$$

Part of this constant force to be applied will be taken care of by the component of the weight of the plate in the downward direction parallel to the inclined plane surface, i.e., by $W \sin 60°$



$\therefore$ Constant force to be applied

$$= 384 - W \sin 60°$$

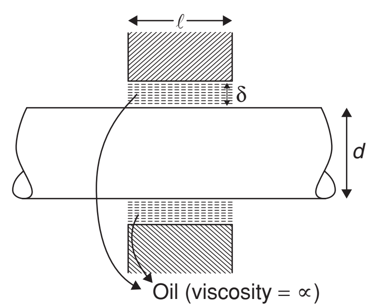
$$= 384 - 30 \times 9.81 \times \frac{\sqrt{3}}{2} = 129.13 \text{ N}$$

Example 10: In a journal bearing of length 500 mm, a 200 mm diameter shaft is rotating at 1000 r.p.m. The uniform space between the shaft and the journal bearing is completely filled with an oil (Newtonian fluid) having a viscosity of 900 centipoise. If energy is being dissipated as heat at the rate of $15.5\frac{\text{kJ}}{\text{sec}}$, while overcoming friction and the velocity profile in the oil is linear, then the thickness of the oil layer between the shaft and the bearing is

(A) 5 mm (B) 1 mm
(C) 2 mm (D) 3 mm

Solution:

The rate of energy dissipation as heat, while overcoming friction, can be considered to be the power dissipated as heat or the power utilized (or lost) to overcome the resistance imparted by the fluid viscosity.



If the shaft is rotating at N rpm., then the tangential velocity of the shaft, $u = \frac{\pi dN}{60}$, where d is the diameter of the shaft $\therefore$

$$u = \frac{\pi \times 0.2 \times 1000}{60} = 10.472\frac{\text{m}}{\text{s}}$$

We have $F = \mu A\frac{d\vartheta}{dy}$

$$0.9 \times 0.2 \times 0.5\left(\frac{10.472}{\delta}\right) \qquad (1)$$

But $F \times u = P = 15500$

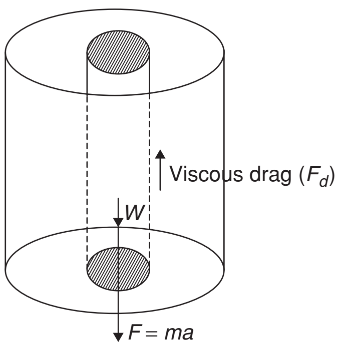
$$F = \frac{15500}{10.472} = 1480.14,$$

$\therefore$ From (1) $\delta = 2$ mm.

Example 11: A solid cylinder of diameter d, length ℓ and density ρ_c falls due to gravity inside a pipe of diameter D. The clearance between the solid cylinder and the pipe is filled with a Newtonian fluid of density ρ and μ. For this clearance fluid, the terminal velocity of the cylinder is determined to be V, assuming a linear velocity profile. However, if the clearance fluid was changed to a Newtonian fluid of density 2ρ and viscosity 2μ, then for an assumed linear velocity profile, the terminal velocity of the cylinder was determined to be V_1. From the results of these experiments, one may write that

(A) $V_1 = V$ (B) $V = 2V_1$
(C) $2V = V_1$ (D) $V = 4V_1$

Solution:
Resolving the forces acting on the cylinder, $F = W - F_d$ or ma $= W - F_d$,



where m, W and a are the mass, weight and acceleration respectively of the solid cylinder.

When the cylinder attains terminal velocity, $a = 0$

$$\therefore W - F_d = 0 \quad (1)$$

Now $F_d = \tau A$

Since the fluid is Newtonian,

$$F_d = \frac{\mu V}{\frac{D-d}{2}} \times \pi dl \quad (2)$$

for the first experiment

Now the weight of the cylinder,

$$W = \rho_c \times g \times \pi \times \frac{d^2}{4}\ell \quad (3)$$

Substituting equations (2) and (3) in equation (1) and rearranging, we get:

$$V = \frac{\rho_c g \times d(D-d)}{8\mu}$$

∴ The terminal velocity of the cylinder does not depend on the density of the fluid.

Hence $\frac{V_1}{V} = \frac{\mu}{2\mu}$ or $V = 2V_1$.

Aliter: At the condition of terminal velocity force of the drag is the weight. Force of drag

$F = 6\pi a\mu\, v$

Where μ = the Coeff of viscosity

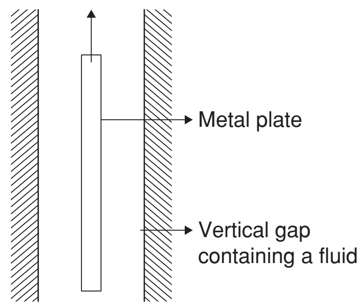
$\therefore F_D \, \alpha \, \mu v$

$\therefore \mu\, v_1 = \mu_2 v_2$

$$v_2 = \frac{\mu_1 v_1}{\mu_2} = \frac{\mu_1 v_1}{2\mu_1} = \frac{v_1}{2}$$

$$v_2 = \frac{v_1}{2}.$$

Example 12: A vertical gap, of width 5 cm and of an infinite extent, contains a Newtonian fluid of viscosity $3\frac{\text{Ns}}{\text{m}^2}$ and specific gravity 0.5. A metal plate (1.5 m × 1.5 m × 0.5 cm) with a weight of 50 N is to be lifted with a constant velocity of 0.5 m/s as shown in the following figure.



If the plate is lifted such that the plate is parallely apart from the left side of the gap by a distance of 2 cm always, then the force required to pull the plate, neglecting buoyancy effects and assuming linear velocity profiles, is

(A) 468.81 N (B) 929 N
(C) 353.75 N (D) 390.25 N

Solution:
The shear force acting on the left side of the metal plate, $F_\ell = A \times \mu \times \left(\frac{V-0}{d_\ell}\right)$, where A is the surface area of the plate, μ is the fluid viscosity, V is the constant velocity with which the plate moves and $d\ell$ is the distance of the plate from the left side of the vertical gap.

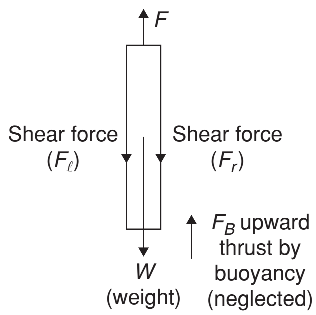
$$\therefore \quad F\ell = 1.5 \times 1.5 \times 3 \times \frac{0.5}{0.02} = 168.75 \text{ N}$$

The shear force acting on the right side of the metal plate, $F_r = A \times \mu \times \left(\frac{V-0}{d_r}\right)$, where d_r is the distance of the plate from the right side of the vertical gap.

Here, $d_r = 0.05 - 0.02 - 0.005 = 0.025$ m

$$\therefore \quad F_r = 1.5 \times 1.5 \times 3 \times \frac{0.5}{0.025} = 135 \text{ N}.$$

If buoyancy effects were not neglected, then an upward thrust experienced by the metal plate due to buoyancy should be accounted for in the calculations to follow.



∴ Force required to lift the plate

$= F\ell + F_r + W - F_B$

$= 168.75 + 135 + 50$ (∵ F_B is neglected)

$= 353.75$ N.

Classification of Fluids

Fluids can be classified into the following types.

1. Ideal fluid (hypothetical fluid) or perfect fluid.
2. Real fluid (practical fluids).
3. Newtonian fluid.
4. Non-Newtonian fluid.

These an explained as follows:

1. **Ideal Fluid or Perfect fluid:** These fluids have zero viscosity (i.e., inviscid) and are incompressible (ie constant density). These fluids do not offer shear resistance when the fluid is set in motion. Though ideal fluids are hypothetical (i.e., they do not exist in reality), this concept is used in mathematical analysis of flow problems.
2. **Real Fluid:** Real fluids have non-zero viscosity and hence they offer resistance when set in motion. Real fluids have variable density and hence they have some compressibility. There is surface tension also for real fluids.
3. **Newtonian Fluid**: These are real fluids. These fluids obey Newton's law of viscosity i.e., the shear stress in the fluid is directly proportional to the rate of shear strain (which is also known as velocity gradient). For such fluids, the graph of shear stress versus velocity gradient is a straight line passing through the origin (point of zero shear stress and zero velocity gradients). The slope of the graph is constant and represents the constant viscosity of the fluid at a given temperature.

   Air, water, light oils, gasoline etc are examples of Newtonian fluids.

   $\boxed{\tau = \mu \frac{du}{dy}}$ for Newtonian fluids, where:

   τ = fluid shear stress
   μ = viscosity of fluid and
   $\frac{du}{dy}$ = velocity gradient (or rate of shear strain)

   The density of Newtonian fluids can be constant or variable (i.e., they can be compressible or incompressible).
4. **Non-Newtonian Fluid:** These are real fluids in which the shear stress is not equal to rate of shear strain i.e., these fluids do not obey the Newton's law of viscosity.

   i.e., $\tau \neq \mu \frac{du}{dy}$ for non-Newtonian fluids.

   The relation between shear stress and velocity gradient for non-Newtonian fluid is $\tau = A\left(\frac{du}{dy}\right)^n + B$, where A and B are constants that depend upon type of fluid and condition of flow.

   The non-Newtonian fluids can further be classified as shown below:

Time independent non-Newtonian fluids These are of two types. The first type of fluids start flowing as soon as a shear stress is applied and do not require any minimum shear stress to cause flow. **Dilatent fluids** and **Pseudoplastic fluids** belong to this category.

For Dilatent fluids, $n > 1$, $A = \mu$ and $B = 0$

For example, Butter, Quick sand

For Pseudoplastic fluids, $n < 1$, $A = \mu$ and $B = 0$

For example, Lipsticks, paints, blood, paper pulp, rubber solution, polymeric solutions etc.

The second type of time independent non-Newtonian fluids are called **Ideal plastics** or **Bingham plastic fluids**. For these fluids, the flow occurs only when the shear stress exceeds the yield stress. Once this yield stress is exceeded, increase in shear stresses is proportional to the velocity gradient. Hence for Bingham plastic fluids, $n = 1$, $A = \mu$ and $B \neq 0$ but independent of time.

E.g., Tooth paste and gel, drilling mud, sewage sludge etc.

Time dependent non-Newtonian fluids For these fluids, flow occurs only when the shear stress exceeds the yield stress.

For **Thixotropic fluids**, $n < 1$, $A = \mu$ and $B \neq 0$. Also B is a function of time (t). Hence, shear stress is of the form $\tau = \mu\left(\frac{du}{dy}\right)^n + f(t)$.

E.g., Printer ink, enamels.

Viscosity increases with time for such fluids.

For **Rheopectic fluids**, $n > 1$, $A = \mu$ and $B \neq 0$ and B is a function of time (t).

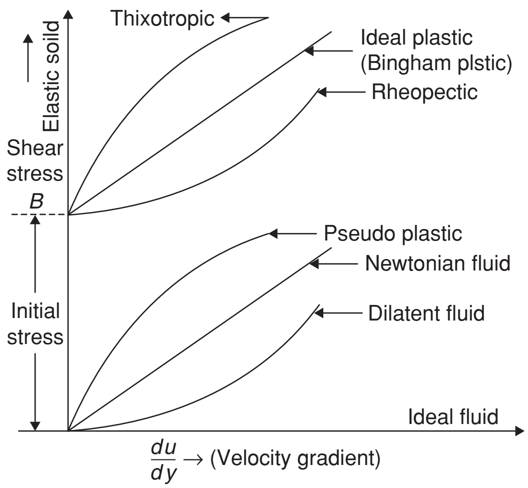
$$\therefore \quad \tau = \mu\left(\frac{du}{dy}\right)^n + f(t)$$

Viscosity decreases with time for such fluids.

For example, Gypsum solution in water, Bentonite solution

For non-Newtonian fluids also, the density may be constant or variable, hence non-Newtonian fluids can be incompressible or compressible.

The variation of shear stress with velocity gradient for various types of fluids is shown below.



Apparent Viscosity The slope of the shear stress versus velocity gradient curve at a point is the apparent viscosity of the respective fluid at that point.

Kinematic Viscosity Kinematic viscosity(γ)of a fluid is the ratio of the dynamic viscosity (μ) to the density (ρ) of the fluid, i.e., $\boxed{\gamma = \frac{\mu}{\rho}}$. The S.I. unit of kinematic viscosity is $\frac{\text{m}^2}{\text{s}}$. Another unit (in C.G.S. units) for kinematic viscosity is stoke $\boxed{1\,\text{stoke} = \frac{1\,\text{cm}^2}{\text{s}} = 10^{-4}\,\frac{\text{m}^2}{\text{s}}}$

Example 13: The kinematic viscosity of air at 70°C is $2.11 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}$. If the Sutherland temperature for air is 110.4° K, then the kinematic viscosity of air at 50°C is

(A) $2.11 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}$. (B) $1.9 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}$

(C) $1.5 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}$ (D) $3 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}$

Solution:
Sutherland equation relating absolute temperature and the dynamic viscosity of an ideal gas is

$$\boxed{\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^{\frac{3}{2}} \left(\frac{T_o + S}{T + S}\right)}$$

$\mu \rightarrow$ Viscosity at absolute temperature T
$\mu_o \rightarrow$ viscosity at absolute temperature T_o
$S \rightarrow$ Sutherland temperature.

For air, $\frac{\rho}{\rho_0} = \frac{T_o}{T}$ ($\because$ air is assumed as an ideal gas at constant pressure.)

$$\text{Now } \frac{\gamma}{\gamma_o} = \frac{\mu\rho_o}{\rho\mu_o} = \frac{\mu}{\mu_o}\left(\frac{T}{T_o}\right)$$

$$\therefore \quad \frac{\gamma}{\gamma_o} = \left(\frac{T}{T_o}\right)^{\frac{5}{2}} \left(\frac{T_o + S}{T + S}\right),$$

Where $S = 110.4$°K, $T = 323.15$°K,

$T_o = 343.15$°K, $\gamma_o = 2.11 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}$ and γ is the kinematic viscosity.

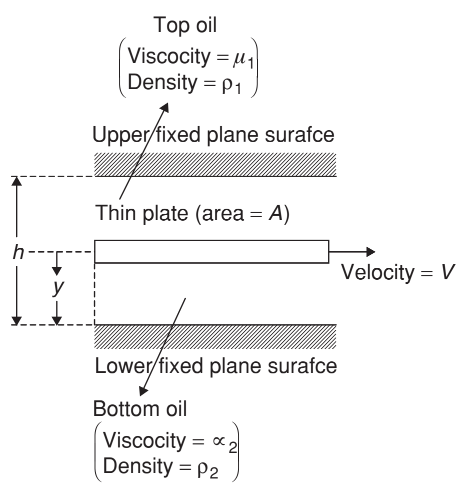
$\therefore$ Kinematic viscosity of air at

$50°\text{C} = \gamma = 1.8996 \times 10^{-5}\,\frac{\text{m}^2}{\text{s}}$.

Example 14: Between two large fixed parallel plane surfaces, a thin plate is pulled, parallel to the lower plane surface, with a constant force. The space between the plate and the plane surface is filled with two types of oil where the top oil (oil at the top side of the plate) and the bottom oil (oil at the bottom side of the plate) have different kinematic viscosities. The distance between the plate and the lower plane surface is one fourth the distance between the two plane surfaces. In this horizontal position, the force required to drag the plate is the minimum compared to that required for any other horizontal positions. If the ratio of the specific mass of the top oil to that of the bottom oil is 1: 3, then the corresponding ratio of their kinematic viscosities, should be

(A) 27 : 1 (B) 9 : 1
(C) 3 : 1 (D) 1 : 3

Solution:
For a thin plate, it can be assumed that the plate thickness is negligible



Given $\frac{\rho_1}{\rho_2} = \frac{1}{3}$ and $\frac{y}{h} = \frac{1}{4}$.

The oils are assumed to be Newtonian fluids. A linear velocity profile is assumed to be present in the oils.

Shear force on the top side of the plate,

$$F_t = A\mu\frac{du}{dy} = A\mu_1\frac{V}{h-y}$$

Similarly shear force on the bottom side of the plate, $F_b = A\mu_2\frac{v}{y}$

Total force required to drag the plate,

$$= F_t + F_b = AV\left[\frac{\mu_1}{h-y} + \frac{\mu_2}{y}\right]$$

For the required force to be minimum for a given horizontal position of the plate , $\frac{\partial F}{\partial y} = 0$

$$\Rightarrow \quad \frac{\mu_1}{(h-y)^2} - \frac{-\mu_2}{y^2} = 0$$

$$\therefore \quad \frac{\mu_1}{\mu_2} = \frac{\left(\frac{3}{4}\right)^2}{\left(\frac{1}{4}\right)^2} = 9$$

Ratio of Kinematic viscosities

$$\frac{r_1}{r_2} = \frac{\mu_1}{\rho_1}\frac{\rho_2}{\mu_2} = 9\times 3 = 27$$

Since $\frac{y}{h} = \frac{1}{4}$, from equation (2) we get $\frac{\mu_1}{\mu_2} = 9$

$$\frac{\text{kinematic viscosity of the top oil}}{\text{kinematic viscosity of the bottom oil}}$$

$$= \frac{\mu_1}{\rho_1}\times\frac{\rho_2}{\mu_2} = 9\times 3 = 27 \quad \text{or} \quad = 27:1.$$

Vapour Pressure

Vapour pressure of a liquid, at a particular temperature, is the pressure exerted by its vapour in phase equilibrium (when the vapour is saturated) with the liquid at that temperature. As the temperature increases, vapour pressure also increases. When the vapour pressure of a liquid is equal to the external environmental pressure, the liquid will start to boil.

This property plays a role in the phenomenon called cavitation. Cavitation, which is highly undesirable due to its destructive properties, is the formation and collapse of vapour bubbles in liquid flow systems. Vapour bubbles are formed at locations where the pressure in the liquid flow system is below the vapour pressure of the liquid.

Difference Between Vaporisation and Boiling

The translational momentum of some surface molecules of the liquid enable them to overcome the molecular attractive force and these molecules escape into the free space above the liquid surface to become vapour. This process is vaporisation and it can occur at all temperatures. Vaporisation can be minimized by increasing the pressure over the free surface of liquid.

When the pressure above the liquid free surface is less than or equal to the vapour pressure of the liquid at that temperature, there is continuous escape of liquid molecules from the free surface into the space above the liquid surface. This process is called boiling.

Bulk Modulus (K)

It is also known as bulk modulus of elasticity, coefficient of compressibility or bulk modulus of compressibility.

$$\boxed{K = -V\left(\frac{\partial P}{\partial V}\right) = \rho\left(\frac{\partial P}{\partial \rho}\right)}$$

The SI unit of the bulk modulus is $\frac{\text{N}}{\text{m}^2}$ or Pascal. It is also defined as the ratio of the compressive stress to the volumetric strain. Bulk modulus increases for gases as pressure and temperature increases. As temperature increases bulk modulus decreases for liquids.

Lower the value of the bulk modulus of a fluid, more compressible is the fluid considered to be. For a truly incompressible fluid (i.e., fluid whose volume cannot be changed), K = infinity. Liquids are usually considered to be incompressible, i.e., they have a large value of bulk modulus.

The reciprocal of the bulk modulus is called as the compressibility (α), i.e., $\boxed{\alpha = \frac{1}{k}}$

Gases are usually considered to be compressible, i.e., they have a large value of compressibility.

Isothermal bulk modulus,

$$K_T = V\left(\frac{\partial P}{\partial V}\right)_T \text{ (i.e., at constant temperature } T)$$

Adiabatic bulk modulus

$$K_s = -V\left(\frac{\partial P}{\partial V}\right)_S \text{ (i.e., at constant entropy } S).$$

Isothermal Compressibility,

$$\alpha_T = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_T \text{ (i.e., at constant temperature } T)$$

Adiabatic Compressibility,

$$\alpha_s = \frac{-1}{V}\left(\frac{\partial V}{\partial P}\right)_S \text{ (i.e., at constant entropy } S)$$

Example 15: In a piston cylinder arrangement containing gas A, it is found that to reduce is othermally the volume of the gas to 75% of its original volume, an additional pressure of 2 atm is required. In another piston cylinder arrangement containing gas B $\left(\text{density} = 1.5\frac{\text{kg}}{\text{m}^3}\right)$, it is found that the density of the gas can be increased by $1.5\frac{\text{kg}}{\text{m}^3}$ at a constant temperature, if a pressure change of 6 bar is provided. From these observations, one can state that

(A) Gas A and gas B have equal isothermal compressibility.
(B) Gas A is 1.2 times more isothermally compressible than gas B.
(C) Gas B is 1.35 times more isothermally compressible than gas A.
(D) Enough information is not available for the comparison of the isothermal compressibility of the two gases.

Solution:
For gas A, let V_1 and V_2 be the original volume and the volume of the gas after compression respectively.
Given, $V_2 = 0.75\ V_1$

$$\Rightarrow \frac{\Delta V}{V} = \frac{V_2 - V_1}{V_1} = -0.25$$

$\Delta P = 2$ at $m = 2 \times 1.01325$ bar

$$K_{TA} = -V\left(\frac{\partial P}{\partial V}\right)_T$$

$$\cong -\left(\frac{\Delta P}{\Delta \frac{V}{V}}\right)_T$$

$$\cong -\frac{2\times 1.01325}{-0.25} \cong 8.106 \text{ bar}$$

For gas B,

$$\rho = 1.5\frac{\text{kg}}{\text{m}^3}$$

$$\Delta P = 1.5\frac{\text{kg}}{\text{m}^3}$$

$$\frac{\Delta\rho}{\rho} = 1$$

$\Delta P = 6$ bar

$$\therefore \quad K_{TB} = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \cong \frac{6}{1} \cong 6 \text{ bar}$$

$$\therefore \quad \frac{K_{TA}}{K_{TB}} = \frac{8.106}{6} = 1.35$$

$\therefore$ gas B is 1.35 times more isothermally compressible than gas A.

Coefficient of Volume Expansion (β)

It is also known as volume expansivity.

$\boxed{\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P}$. The S.I unit of the coefficient of volume expansion is $\left(\frac{1}{^\circ K}\right)$.

Example 16: If the isothermal compressibility and volume expansivity of a fluid are α_T and β respectively, then the fractional change in the volume $\left(\frac{dV}{V}\right)$ of the fluid for a change in temperature (dT) and change in pressure (dP) is equal to.
(A) $\alpha_T dT - \beta\, dP$
(B) $\beta dT - \alpha_T dP$
(C) $\alpha_T dT + \beta\, dP$
(D) $\alpha_T dP + \beta dT$

Solution:
The volume of the fluid (V) is a function of temperature (T) and pressure (P). This can be written as $V = V(T, P)$
Differentiating, we get

$$dV = \left(\frac{\partial V}{\partial T}\right)_P dT + \left(\frac{\partial V}{\partial P}\right)_T dP \quad (1)$$

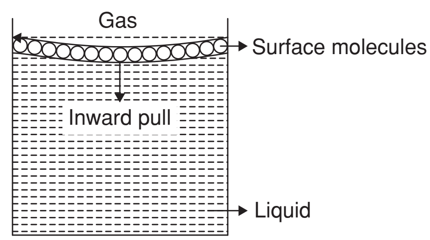
Now $\alpha_T = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$ and $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$

Substituting the above relations for α_T and β in equation (1) and rearranging, we get

$$\frac{dV}{V} = \beta dT - \alpha_T dP$$

Surface Tension

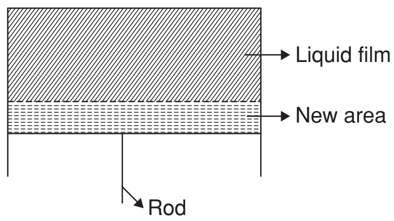
The layer of molecules at the surface of a liquid, in contact with a gas (or another immiscible liquid), tends to behave like a stretched membrane (membrane on which a tensile force is exerted).



This behaviour is a result of the inward pull, arising due to the cohesive forces (intermolecular forces of attraction between molecules of the same liquid), experienced by the liquid's surface molecules.

At the liquid surface, the tensile force dF acting parallely to the plane of the surface (or tangentially to the surface) over a surface length $d\ell$ is given by the equation, $\boxed{df = \sigma d\ell}$, where σ is called as the (coefficient of) surface tension of the liquid. Hence, surface tension is equal to the magnitude of the (tensile) force acting tangentially at the surface per unit length of the surface. The S.I. unit of surface tension is $\frac{\text{N}}{\text{m}}$.

Imagine a metallic frame in which a liquid film is maintained as shown in the following figure.



When the rod is slightly pulled down, the liquid film gets stretched over a larger area. The work done for creating the new area is the surface energy.

$$\frac{\text{Surface energy}}{\text{New area created}} = \text{Surface tension}$$

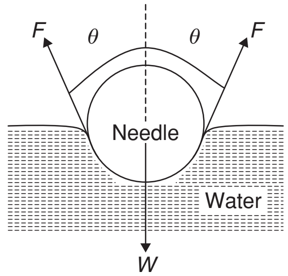
$\therefore$ Surface Energy per unit area = surface tension

Surface tension $\left(\text{in } \frac{\text{N}}{\text{m}} \text{ or } \frac{\text{J}}{\text{m}^2}\right)$ thus also represents the amount of (stretching) work required to increment the surface area by an unit amount. Surface tension of a liquid decreases with temperature and becomes zero at the critical point. The effect of pressure on the surface tension of a liquid can be considered to be negligible. Surface tension of a liquid can be increased or decreased by adding impurities. For example, surface tension of water can be decreased or increased by adding surfactants or NaCl respectively.

Example 17: A solid cylindrical needle $\left(\text{density} = 7.8\frac{\text{g}}{\text{cm}^3}\right)$ of length 5 cm is placed very gently on the surface of a body of water (surface tension = 73 dynes/cm) such that it floats on the water surface. Neglect buoyancy effects and surface tension effects at the circular faces of the needle. The maximum diameter that the needle can have, such that it will still be able to float on the water surface, is

(A) 1.56 mm (B) 4.88 mm
(C) 5.26 mm (D) 1.31 mm

Solution:



Let F be the force, due to surface tension of water, acting along the length of the needle on either side as shown in the above figure. Let W be the weight of the needle.

Now $F = \sigma L$, where σ is the surface tension of water and L is the length of the needle.

If θ is the angle that the force F makes with the vertical, then writing a force balance on the needle gives:

$$W = F\cos\theta + F\cos\theta$$
$$= 2\,\sigma L\cos\theta \qquad (1)$$

If d and ρ are the diameter and density of the needle, then from equation (1) we can write

$$\pi\frac{d^2}{4}L\rho g = 2\sigma L\cos\theta$$

$$d = \sqrt{\frac{8\sigma\cos\theta}{\pi\rho g}}$$

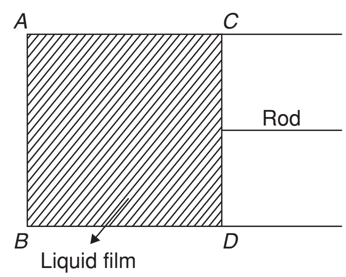
The maximum value of d (d_{max}) is obtained when $\theta = 0°$ (provided all other parameters are fixed).

$$\therefore\quad d_{max} = \sqrt{\frac{8\sigma}{\pi\rho g}}$$

$$= \sqrt{\frac{8\times 0.073}{3.14\times 7800\times 9.81}}\ \boxed{1\,\text{dyne} = 10^{-5}\,\text{N}}$$

$= 1.56$ mm.

Example 18: A liquid film, exposed to the atmosphere on both sides, is present in the area $ABCD$ of the metallic frame work shown in the following figure.



The side CD, of length 10 cm, is movable and can be pulled with the help of a rod. The work done to increase the length of side BD by 1 mm, still maintaining the liquid film (surface tension = 0.073 N/m) in the area $ABCD$, is

(A) $7.3\times10^{-6} J$ (B) $1.46\times10^{-5} J$
(C) $1.46\times10^{-4} J$ (D) $7.3\times10^{-5} J$

Solution:

Let L be the length of the side CD. Then, $L = 10$ cm $= 0.1$ m

At the side CD, there are two lengths on which surface tension acts since the film of liquid is exposed to the atmosphere on both sides. Hence the length along which the surface tension acts at the side $CD = 2L$.

$\therefore$ Work done $= \sigma 2L\,\Delta x$, where $\sigma 2L$ represents the force due to surface tension acting at the side CD.

Here $\Delta x = 1$ mm $= 1\times10^{-3}$ m

$\sigma = 0.073$ N/m

Work done $= 0.073\times2\times0.1\times1\times10^{-3}$
$= 1.46\times10^{-5} J$.

Effects of Surface Tension

1. A falling rain drop attaining a spherical shape.
2. Sap rising in a tree.
3. Birds being able to drink water from ponds.
4. Capillary rise.
5. Dust particles collecting on the surface of a liquid.
6. Liquid jets breaking up.

Excess Pressure

In liquid droplets, gas bubbles, soap bubbles and liquid jets, an amount of pressure in excess to the external pressure is present due to surface tension for maintaining the shape.

Liquid droplet or gas bubble

$$\boxed{P_i - P_o = \Delta P = \frac{4\sigma}{d}},$$

where P_i is the pressure inside the liquid droplet or gas bubble, P_o is the pressure outside the liquid droplet or gas bubble, d is the diameter of the (spherical) liquid droplet or gas bubble and ΔP is the excess pressure.

Soap or liquid bubble A soap or liquid bubble has air both inside and outside it and hence it has two free surfaces on which surface tension acts.

$$\boxed{P_i - P_o = \Delta P = \frac{8\sigma}{d}},$$

Where d is the outer diameter of the soap or liquid bubble.

Cylindrical liquid jet

$$\boxed{P_i - P_o = \Delta P = \frac{2\sigma}{d}}$$

Where d is the diameter of the cylindrical jet.

Example 19: The pressures inside and outside of a water bubble and water drop are found to be the same. If d is the diameter of the water bubble and if the bubble and drop are at the same temperature, then the diameter of the water drop is

(A) d (B) $3d$
(C) $2d$ (D) $d/2$

Solution:
Since the inside and outside pressures of the water drop are equal to that of the water bubble, we have Excess pressure inside the water drop = Excess pressure inside the water bubble.

i.e., $\frac{4\sigma}{d_d} = \frac{8\sigma}{d_b}$, where d_d and d_b are the diameters of the water drop and water bubble respectively.

$$\therefore \quad d_d = \frac{d_b}{2} = \frac{d}{2}.$$

Example 20: Two cylindrical liquid jets A and B have the surface tensions σ_A and σ_B respectively such that $\sigma_A = 2\sigma_B$. The jets A and B are exposed to the respective external pressures P_A and P_B, such that $P_B - P_A = \frac{2\sigma_B}{d_B}$, where d_B is the diameter of the cylindrical jet B. If the two jets have the same inside pressure, then the diameter of the cylindrical jet A is

(A) d_B (B) $2\,d_B$
(C) $0.5\,d_B$ (D) $4\,d_B$

Solution:
Given $\sigma_A = 2\,\sigma_B$ and

$$P_A - P_B = \frac{2\sigma_B}{d_B} \qquad (1)$$

Jets A and B have the same inside pressure, hence

$$\frac{2\sigma_A}{d_A} + P_A = \frac{2\sigma_B}{d_B} + P_B, \qquad (2)$$

where d_A is the diameter of the cylindrical jet A.

$$P_B - P_A = \frac{2\sigma_A}{dA} - 2\sigma_B$$

But $$P_B - P_A = \frac{2\sigma_B}{dB}$$

Equating,

$$\therefore \quad \frac{2\sigma_B}{dB} = \frac{2\sigma A}{dA} - \frac{2\sigma_B}{dB}$$

$$\frac{4\sigma_B}{dB} = \frac{2\sigma_A}{dA}$$

$$\therefore dA = dB.$$

Capillarity

When a small diameter tube is inserted into a body of liquid, the liquid rises or falls in the tube giving rise to the phenomenon known as capillarity. Capillarity is due to the forces of cohesion (attraction between the same molecules) between the liquid molecules and the forces of adhesion (attraction between different molecules) between the liquid and solid (constituting the tube) molecules.

The rise of the liquid is called as the capillary rise while the fall is called as the capillary drop or capillary depression. Capillarity or capillary effect can be termed to be a consequence of surface tension.

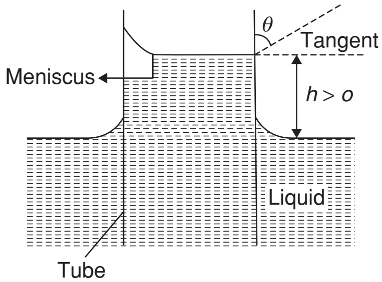
The strength of capillarity (or capillary effect) is quantified by a parameter called as the contact (or wetting) angle (θ). The contact angle is defined as the angle between the solid surface and the tangent to the liquid surface at the point of contact between the two surfaces. The surface tension force acts along the tangent towards the solid surface. The magnitude of the capillary rise of a liquid (surface tension = σ, density = ρ) having a contact angle θ with a tube of constant diameter d is given by

$$h = \frac{4\sigma\cos\theta}{\rho g d}$$

The contact angle of water with clean glass is nearly zero, i.e., $\theta \approx 0^0$. (If θ^0, then it is called complete or perfect wetting.)

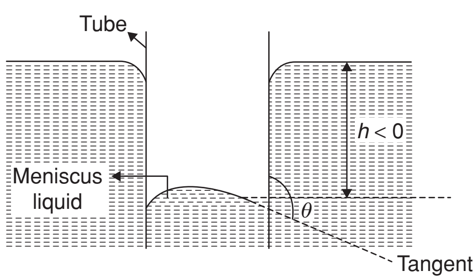
For glass tubes with diameters greater than 1 cm the capillarity effect of water is negligible.

Liquid wets solid surface



1. Contact angle θ is greater t
2. Contact angle θ is less than 90°.
3. When a small diameter tube made of the solid is dipped in the liquid, capillary rise occurs.
4. Magnitude of cohesive forces < magnitude of adhesive forces.
5. For example, water – glass.
6. Capillary drop = h.

Liquid does not wet solid surface



1. Contact angle θ is greater than 90°.
2. When a small diameter tube made of the solid is dipped in the liquid, capillary drop occurs.
3. Magnitude of adhesive forces < magnitude of cohesive forces.
4. Liquid is termed as a non-wetting liquid.
5. For example, mercury–glass
6. Capillary drop = $|h|$

Example 21: When tube A is dipped into the body of a liquid, the liquid makes a contact angle of 30° with the tube. When tube B of different material having twice the diameter of tube A, is dipped into the same liquid body, the liquid makes a contact angle of 120° with the tube. The ratio of the capillary rise seen in one of the tubes to the capillary drop seen in the other is;

(A) 0.28 (B) 1.73 (C) 3.46 (D) 0.58

Solution:

Let d_A and θ_A be the diameter and contact angle for tube A.
Let d_B and θ_B be the diameter and contact angle for tube B.
Given $d_B = 2d_A$, $\theta_A = 30°$ and $\theta_B = 120°$
Since $\theta_A < 90°$, capillary rise (h_r) will be seen when tube A is dipped.

$$\therefore \quad h_r = \frac{4\sigma\cos\theta_A}{\rho g d_A} \qquad (1)$$

Since $\theta_B > 90°$, capillary drop (h_d) will be seen when tube B is dipped.

$$\therefore \quad h_r = \frac{-4\sigma\cos\theta_B}{\rho g d_B}$$

(Negative sign is introduced since h_d is already referred to as capillary drop)
From equations (1) and (2), we have

$$\therefore \quad \frac{h_r}{h_d} = \frac{-\cos\theta_A \times d_B}{\cos\theta_B \times d_A}$$

$$= \frac{-\cos 30° \times 2d_A}{\cos 120° \times d_A} = 3.46.$$

Example 22: The maximum diameter that a capillary tube can have to ensure that a capillary rise of at least 6 mm is achieved when the tube is dipped into a body of liquid with surface tension $= 0.08\ \frac{\text{N}}{\text{m}}$ and density $= 900\ \frac{\text{kg}}{\text{m}^3}$, is

(A) 3 mm (B) 6 mm
(C) 5 m (D) 8 mm

Solution:

The capillary rise $h = \frac{4\sigma\cos\theta}{\rho g d}$, where σ, θ, ρ, g and d have their usual meanings.

$\therefore$ diameter of the capillary tube $d = \frac{4\sigma\cos\theta}{\rho g h}$

Here, θ is taken to be 0°. The diameter d gets the maximum value ($d_{\max}$) when h is minimum (i.e., $h = h_{\min}$)
Given $h_{\min} = 6$ mm

$$\therefore \quad d_{\max} = \frac{4\sigma}{\rho g h_{\min}} = \frac{4 \times 0.08}{900 \times 9.81 \times 0.006}$$

$$= 6 \text{ mm.}$$

PRESSURE

Pressure is defined as a normal force exerted by a fluid per unit area. The normal stress on any plane through a fluid element of rest is equal to the fluid pressure. The S.I. unit of pressure is Pascal (Pa) or $\frac{\text{N}}{\text{m}^2}$.

$$1\text{Pa} = 1\frac{\text{N}}{\text{m}^2}$$

Other units for pressure are atm (1 atm = 101325 Pa), psi (1atm = 14.696 psi) and bar (1 bar = 10^5 Pa). Pressure is a scalar quantity. At a point on a surface which is in contact with a fluid, the pressure force exerted by the fluid is normal to the surface.

Atmospheric, Absolute and Gauge Pressure

Atmospheric pressure (P_{atm}) is the pressure exerted on a surface by a planet's atmosphere (e.g., the Earth's atmosphere) present above the surface.

Absolute pressure (P_{abs}) is the pressure measured relative to an absolute vacuum (where $P_{abs} = 0$). At any given position, the actual pressure is the absolute pressure.

Gauge pressure (P_{gauge}) is the pressure indicated by a pressure – measuring device (or pressure gauge) relative to the local atmospheric pressure. This is stated with the assumption that the pressure gauge is cali berated with the local atmospheric pressure as reference.

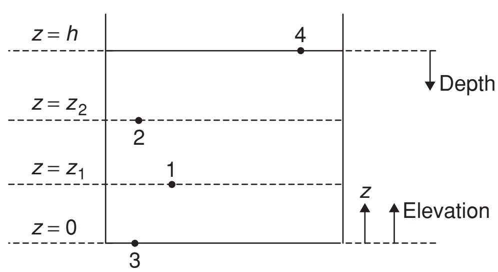
$$\boxed{P_{gauge} = P_{abs} - P_{atm}}$$

If $P_{abs} < P_{atm}$, then P_{gauge} is negative and the negative of the gauge pressure is called as the vacuum pressure (P_{vac}). Pressure gauges measuring vacuum pressures are called as vacuum gauges.

$$\boxed{P_{vac} = P_{atm} - P_{abs}}$$

Pressure Varying with Elevation or Depth (for Static Fluids)

Consider a static body of liquid (density = ρ, specific weight = ω) of height h present in a container as shown in the following figure.



The variation of pressure P in the liquid with respect to the elevation z is given by

$$\boxed{\frac{dP}{dz} = -\rho g = -\omega} \quad (1)$$

Equation (1), called as the hydrostatic (differential) equation, corresponds to the hydrostatic law which states that "The rate of increase of pressure in a vertically downward direction must be equal to the specific weight of the fluid.

Conventionally at $z = 0$, elevation = 0 and depth = h, while at $z = h$, elevation = h and depth = 0. If P_1 and P_2 are the pressures at points 1 ($z = z_1$) and 2 ($z = z_2$), from equation (1) we have

$$\boxed{P_2 - P_1 = \Delta P = -\int_{z=z_1}^{z=z_2} \rho g dz} \quad (2)$$

For liquids, usually the density is considered to be constant upto certain large depths. If the acceleration due to gravity (g) is also constant with respect to the elevation z, then

$$\boxed{P_2 - P_1 = \rho g (z_1 - z_2) = -\rho g \Delta_z} \quad (3)$$

where Δz ($= z_2 - z_1$) is sometimes called as the *pressure head* and is interpreted as the height of a column of liquid of density ρ required to provide a pressure difference of $P_1 - P_2$.

If the surface of the liquid in the container is exposed to the atmosphere and ρ and g are assumed to be constant with respect to z, then

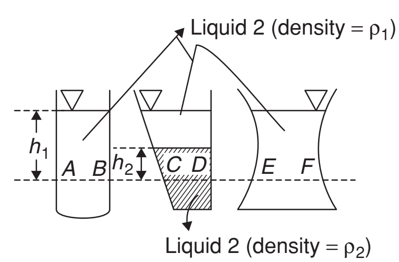
P_{abs} at point 4 = P_{atm}
P_{abs} at point 2 = $P_{atm} + \rho g (h - z_2)$
P_{gauge} at point 1 = $\rho g (h - z_1)$
P_{abs} at point 3 = $P_{atm} + \rho gh$

Equation (1) is also applicable for gases. However, as gases have a low density, the variation of pressure with height (for small to moderate heights) can be considered to be negligible for a gas.

Pressure Varying Horizontally (for Static Fluids)

For a fluid resting inside a container, pressure does not depend on the shape or cross-section of the container. Also, the pressure is the same at all points on any horizontal plane considered in the fluid present in the container.

Consider three containers, open to the atmosphere, of different shapes where the free surface of the liquids in them are at the same level as shown in the following figure.



The points A, B, C, D, E and F all lie on the same horizontal plane .Here,

$P_A = P_B = P_E = P_F$ and $P_C = P_D$

Since $\rho_2 > \rho_1$, it can be seen that $P_c > P_B$ and hence $P_C \neq P_B$.

Pascal's Law

Pascal's law states that the pressure at a point in a static fluid has the same magnitude in all directions. This is also true for non-static fluids which have no shear stress, for example, for fluids which move like rigid bodies where there is no relative motion between the fluid elements.

Another version of Pascal's law states that when there is an increase in pressure at any point in a confined fluid, there is an equal increase in the pressure at every other point in the confined fluid. Pascal's law forms the underlying principle of the hydraulic jack and hydraulic press.

Example 23: A hydraulic press has a plunger of 5 cm diameter. If the weight lifted by the hydraulic press is twice the force applied at the plunger, then the diameter of the ram of the hydraulic press is;

(A) 5 cm (B) 10 cm

(C) $5\sqrt{2}$ cm (D) $10\sqrt{2}$ cm

Solution:

Let the force applied at the plunger be F. Then weight lifted by the hydraulic press, $W = 2F$. (1)

Let d and D be the diameters of the plunger and ram respectively and let a and A be their respective areas.

$$\therefore \quad a = \frac{\pi d^2}{4} \text{ and } A = \frac{\pi D^2}{4} \qquad (2)$$

From Pascal's law, $\frac{F}{a} = \frac{W}{A}$ (3)

Substituting equations (1) and (2) in equation (3), we get

$$D = \sqrt{2}d$$

Given $d = 5$ cm $\therefore \quad D = 5\sqrt{2}$ cm.

NOTE

When the plunger and the Ram are of circular Cross section and 'F' is the load applied at the plunger, Load lifted at the ram is

$$= \frac{F}{\frac{Ad^2}{4}} \times \frac{\pi D^2}{4} = F\frac{D^2}{d^2}$$

Here, $F\frac{D^2}{d^2} = 2F$

$\therefore \quad D = \sqrt{2}d.$

Example 24: Oil weight density = 8.5 kN/m^3 is present in a tank up to a depth of 6 m. It is observed that an immiscible liquid, with a depth of 2 m, is present in the tank below the oil. The reading on the pressure gauge connected to the tank's bottom is 70 kPa. The specific gravity of the immiscible liquid is:

(A) 0.982 (B) 0.968

(C) 0.873 (D) 0.893

Solution:

Let the weight density of the immiscible liquid and the oil is ω_L and ω_O respectively.

Pressure at the bottom of the tank,

$P_b = 6 \times \omega_O + 2 \times \omega_L$

Given $P_b = 70kPa$ and $\omega_O = 8.5k\frac{\text{N}}{\text{m}^3}$

$$\therefore \quad \omega_L = \frac{70\times10^3 - 6\times8.5\times10^3}{2} = 9500\frac{\text{N}}{\text{m}^3}$$

Specific gravity of the liquid, $SG_L = \frac{\omega_L}{\rho_\omega \times g}$,

Where $\rho_\omega \left(= 1000\frac{\text{kg}}{\text{m}^3}\right)$ is the density of pure water at 4°C.

$$\therefore \quad SG_L = \frac{9500}{1000\times9.81} = 0.968.$$

Manometry (Some Cases to Measure the Gauge Pressure)

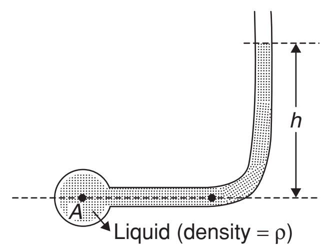
Manometers are pressure measuring devices which employ liquid columns in vertical or inclined tubes to measure pressure. Manometers are classified as (i) simple manometers and (ii) differential manometers.

Simple Manometers

A simple manometer consists of a tube whose one end is connected to a point where the pressure is to be measured and the other end is open to the atmosphere. The common types of simple manometers are (*a*) piezometer (*b*) u-tube manometer and (*c*) single column manometer.

For the following discussion, consider P_1 and P_A to be the pressures at points 1 and A respectively.

Piezometer



Analysis : $P_1 - P_{\text{atm}} + h\rho g$

$P_A = P_1$, since the points A and 1 are at the same elevation and in the same liquid.

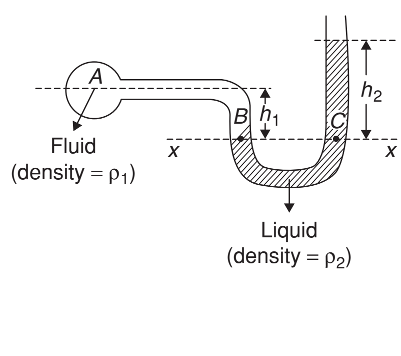
$\therefore \quad \boxed{P_A = P_{atm} + hpg}$

NOTE

It is implicitly assumed here that surface tension effects (capillary rise) are negligible.

U-tube manometer

I.

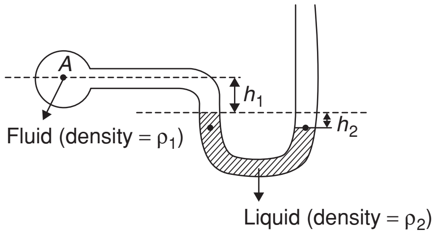


Analysis: Along the section XX,
Pressure at point B = pressure at point C
($\because$ points B and C are at the same elevation and in the same liquid)

i.e., $P_A + h_1\rho_1 g = P_{\text{atm}} + h_2\rho_2 g$

$$\boxed{P_A = P_{atm} + (h_2\rho_2 - h_1\rho_1)g}$$

II.

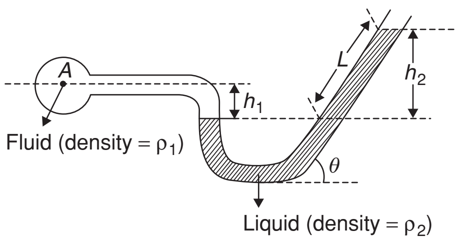


$$\boxed{P_A = P_{atm} - (h_1\rho_1 + h_2\rho_2)g}$$

NOTE

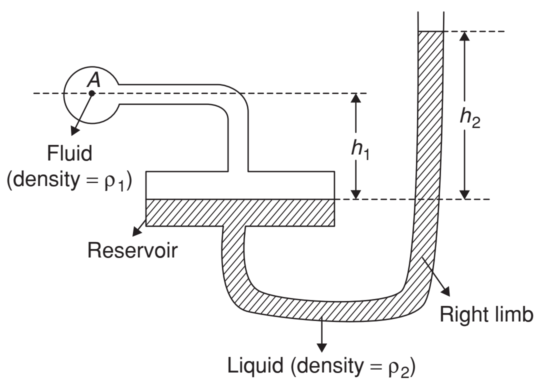
P_A is vacuum pressure

III.



$$\boxed{\begin{aligned} P_A &= P_{atm} + g(h_2\rho_2 - h_1\rho_1) \\ &= P_{atm} + g(L\sin\theta\rho_2 - h_1\rho_1) \end{aligned}}$$

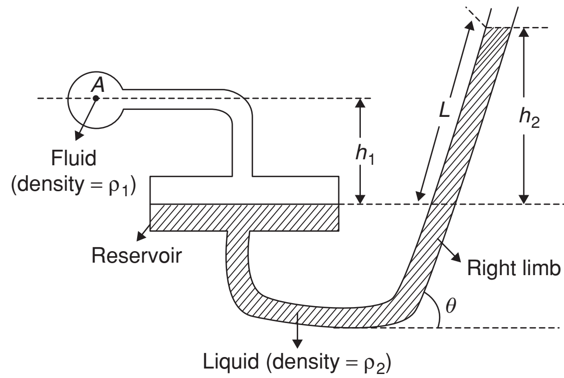
Vertical Single Column Manometer



$$\boxed{\begin{aligned} P_A = P_{atm} + \frac{a \times h_2}{A}(\rho_2 g - \rho_1 g) \\ + h_2\rho_2 g \end{aligned}}$$

Where A and a are the cross – sectional areas of the reservoir and the right limb respectively.

Inclined Single Column Manometer



$$\boxed{\begin{aligned} P_A &= P_{atm} + \frac{a \times h_2}{A}(\rho_2 g - \rho_1 g) \\ &\quad + h_2\rho_2 g - h_1\rho_1 g \\ P_A &= P_{atm} + \frac{a \times L\ \sin\theta}{A}(\rho_2 g - \rho_1 g) \\ &\quad + L\ \sin\theta\rho_2 g - h_1\rho_1 g \end{aligned}}$$

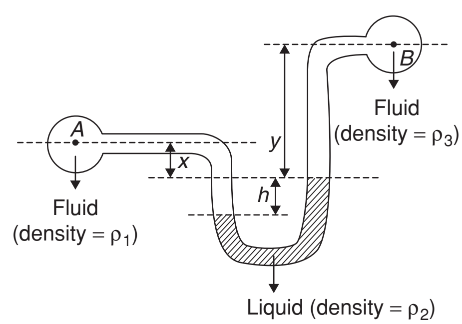
Where A and a are the cross-sectional areas of the reservoir and the right limb respectively.

Differential Manometers

Differential manometers are the devices used for measuring the difference between the pressure at a given point in a fluid and the pressure at some other point in the same or different fluid. A differential manometer consists of a u-tube, in which a heavy liquid is present, where two ends are connected to points whose pressure difference is to be measured. Most common types of differential manometers are (i) u-tube differential manometer and (ii) inverted u-tube differential manometer.

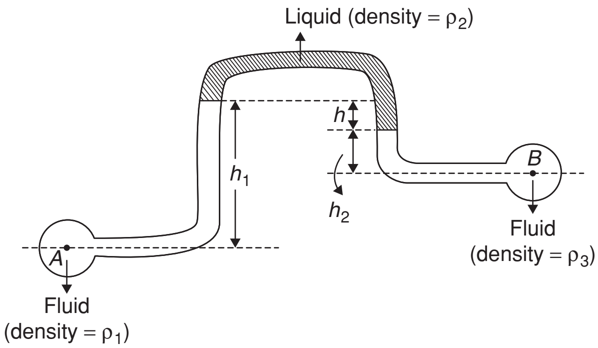
For the following discussion, consider P_A and P_B to be the pressures at the points A and B respectively.

U-tube differential Manometer



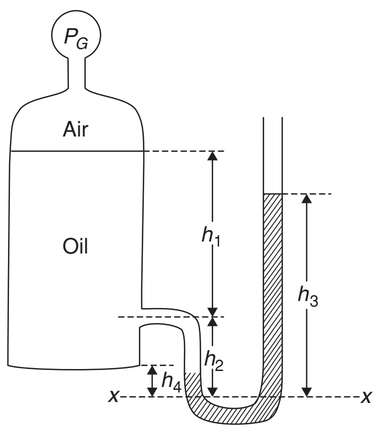
$$\boxed{P_A - P_B - h(\rho_2 - \rho_1)g + y\rho_3 g - x\rho_1 g}$$

Inverted u-tube Manometer



$$\boxed{P_A - P_B = h_1\rho_1 g - h_2\rho_3 g - h\rho_2 g}$$

Example 25: A closed tank consists of oil (density = ρ_1) and compressed air as shown in the following figure. A u-tube



manometer using a liquid with density = ρ_2, is connected to the tank. The variation of pressure with height is negligible in the tank volume occupied by air. If the pressure reading in the pressure gauge connected to the top of the tank is P_G, then an expression for the height of oil in the tank can be

(A) $h_3\left(\frac{\rho_1}{\rho_2}\right) - \frac{P_G}{\rho_1 g} - h_4$

(B) $h_3\left(\frac{\rho_2}{\rho_1}\right) - \frac{P_g}{\rho_1 g} - h_4$

(C) $h_3\left(\frac{\rho_2}{\rho_1}\right) - \frac{P_G}{\rho_1 g} - h_2$

(D) $h_3\left(\frac{\rho_1}{\rho_2}\right) - \frac{P_G}{\rho_2 g} - h_4$

Solution:
Equating pressures at a point in the left limb and at a point in the right limb, where both the points lie on a horizontal plane passing through the meniscus of the liquid (density = ρ_2) in the left limb of the u-tube manometer, gives

$P_{air} + (h_1 + h_2)\,\rho_1\,g = P_{atm} + h_3\,\rho_2\,g$ (1)

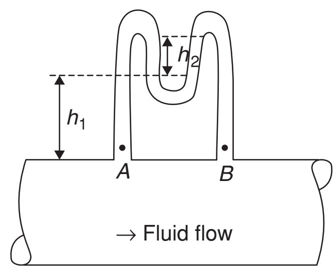
Now $P_G = P_{air} - P_{atm}$ (2)

From the figure in the question it can be shown that the height of the oil in the tank, $h = h_1 + (h_2 - h_4)$ (3)

Substituting equations (2) and (3) in equation (1) and rearranging, we get

$$h = h_3\left(\frac{\rho_2}{\rho_1}\right) - \frac{P_G}{\rho_1 g} - h_4$$

Example 26: A fluid (weight density = ω_1) flows through a pipe as shown in the following figure. A differential u-tube manometer, with a liquid of weight density = ω_2, is fitted to the pipe in order to determine the pressure difference ($P_A - P_B$) where P_A and P_B are the pressures at the respective points A and B on the pipe.



From the set of variables {h_1, h_2, ω_1, ω_2}, the set of the least number of variables whose values are to be known in order to determine the required pressure difference ($P_A - P_B$) is

(A) {h_1, h_2, ω_1, ω_2}
(B) {h_1, ω_1, ω_2}
(C) {h_2 , ω_2}
(D) {h_2, ω_1, ω_2)

Solution:
Equating pressures at a point in the left limb and at a point in the right limb, where both points lie on a horizontal plane passing through the meniscus of the liquid (weight density = ω_2 in the left limb of the differential u-tube manometer, gives

$P_A - h_1\,\omega_1 = P_B - (h_1 + h_2)\,\omega_1 + h_2\,\omega_2$

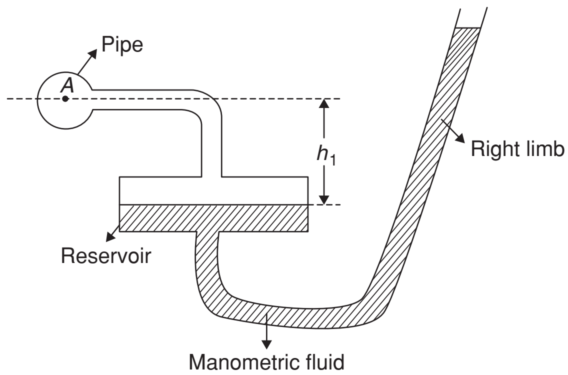
or $P_A - P_B = h_2\,(\omega_2 - \omega_1)$

∴ The set of variables whose values are to be known = {h_2, ω_1, ω_2}

Example 27: An inclined single column manometer is connected to a pipe transporting a liquid of specific weight $(\omega_1) = 9.81\frac{\text{kN}}{\text{m}^3}$, as shown in the following figure. The area of the reservoir is very large compared to the area of the right limb of the manometer. The specific weight (ω_2) of the manometric fluid is $13.6\frac{\text{kN}}{\text{m}^3}$.. The length ($L$) of the manometric fluid in the right limb, above the manometric fluid's surface in the reservoir, is 100 cm. The gauge pressure

(P) at the point A in the pipe is 3.857 kPa. If the value of h is 30 cm, then the right limb of the manometer is inclined to the horizontal at an angle of

(A) 45° (B) 60°
(C) 30° (D) 15°



Solution:
Let θ be the angle at which the right limb is inclined to the horizontal.
If a and A are the respective cross-sectional areas of the right limb and the reservoir, then p is very small and negligible ($\because A >>> a$).
For the inclined column manometer, one can write

$$P = \frac{a}{A} \times L \times \sin\theta(\omega_2 - \omega_1) + L\sin\theta\omega_2 - h\omega_1$$

Since $\frac{a}{A}$ is negligible,
$P = L \sin \theta\, \omega_2 - h\omega\omega_1$

$$\therefore \quad \sin\theta = \frac{P + h\omega_1}{L\omega_2}$$

$$= \frac{3.857 \times 10^3 + 0.3 \times 9.81 \times 10^3}{1 \times 13.6 \times 10^3}$$

i.e., $\theta = 30°$.

Exercises

Practice Problems I

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. A flat thin disk (diameter = 100 cm) is rotated at 1200 r.p.m. at a distance of 2 mm from a flat horizontal stationary surface. If the gap between the horizontal disk and the surface is filled with a Newtonian fluid of 4 poise viscosities, then the torque required to rotate the disk is
   (A) 1.52 KN-m (B) 1.87 KN-m
   (C) 2.47 KN-m (D) 3.94 KN-m
2. Three thin plates are oriented parallel to each other with the lowest plate being fixed. The top plate, located at a distance of x meters above the fixed plate, is towed with a speed of V_1 m/s. The middle plate is located at a distance of y meters above the fixed plate. The viscosity of the Newtonian fluid in between the fixed plate and the middle plate is twice that of the Newtonian fluid between the middle plate and the top plate. If the middle plate moves with a constant speed of V_2 m/s, then the fraction $\frac{V_1}{V_2}$ is equal to:
   (A) $3 - \frac{2x}{y}$ (B) $2 + \frac{2x}{y}$
   (C) $2 - \frac{x}{y}$ (D) $-2 + \frac{2x}{y}$
3. A 10 kg block is sliding down a plane in clined at an angle of angle of 30° to the horizontal. The block is separated from the plane by a 1 mm thick layer of oil (Newtonian) of viscosity 2 poise. It is to be assumed that the velocity distribution in the oil is linear and that the block has already reached the terminal velocity. The area of the block in contact with the oil is 0.1 m^2. The present velocity of the block is:
   (A) 2.4525 m/s (B) 0.24525 m/s
   (C) 4.905 m/s (D) 0.4905 m/s
4. For an ideal gas (density = ρ) at pressure P and temperature T, the isothermal compressibility is equal to:
   (A) P (B) T (C) $\frac{I}{P}$ (D) $\frac{I}{T}$

Direction for questions 5 and 6: A set of n identical spherical drops of radius r of a liquid (surface tension = σ) combine to form a single large spherical drop of radius R.

5. An expression for R is:
   (A) $R = rn^{1/2}$ (B) $R = rn^{1/3}$
   (C) $R = nr$ (D) $R = n^2r$
6. The energy released during the combination process is equal to:
   (A) $4\pi\sigma r^2(1-n^{-1/3})$ (B) $4\pi\sigma r^2(n^{-1/3}-1)$
   (C) $4\pi\sigma r^2 n(1-n^{-1/3})$ (D) $4\pi\sigma r^2(n^{2/3}-n)$
7. The work done in blowing a soap bubble of 5 cm diameter, where the surface tension of the soap solution is 40×10^{-3} N/m, is:
   (A) 3.14×10^{-4} J (B) 0.00785 J
   (C) 0.0157 J (D) 6.28×10^{-4} J
8. A stream of bubbles is generated by introducing air through a nozzle into a tank of water. The ratio of the maximum diameter to the minimum diameter of the bubbles generated is 2 : 1. The pressure of the water surrounding the nozzle remains constant and is denoted by P_o. If P_{min} is the minimum air pressure at the nozzle, then the maximum air pressure at the nozzle is equal to
   (A) $\frac{P_{min} + 3P_0}{2}$ (B) $\frac{2P_{min} + P_0}{2}$
   (C) $\frac{2(P_{min} + P_0)}{2}$ (D) $\frac{3}{2}P_{min} + P_0$

9. The maximum diameter of a metallic (density = ρ) spherical ball that can float in a constant temperature liquid (surface tension = σ) bath is proportional to

(A) $\sqrt{\sigma\rho}$ (B) $\sqrt{\frac{1}{\sigma\rho}}$

(C) $\sqrt{\frac{\rho}{\sigma}}$ (D) $\sqrt{\frac{\sigma}{\rho}}$

10. Two parallel glass plates, each of width W and negligible thickness, are dipped vertically into a body of liquid (surface tension = σ, density = ρ). If the distance between the plates is t and the contact angle is θ, then the capillary rise of the liquid between the plates is given by:

(A) $\frac{2\sigma\cos\theta}{W\rho g}$ (B) $\frac{2\sigma\cos\theta}{t\rho g}$

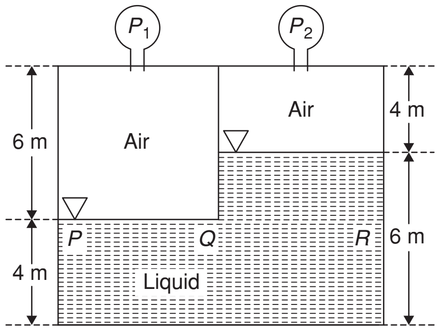
(C) $\frac{4\sigma\cos\theta}{t\rho g}$ (D) $\frac{\sigma\cos\theta}{t\rho g}$

11. The weight density ω(in N/m^3) of a liquid in a large open container varies with the depth h (in m) as: $\omega = 70 + 0.3$ h. The pressure at a depth of 5 m is:*t*

(A) 101325 Pa
(B) 353.75 Pa
(C) 101678.75 Pa
(D) 101501.88 Pa

12. Air (density = 1.2 kg/m^3) and a liquid (density = 900 kg/m^3) is present in a closed tank as shown in the following figure. The pressure gauge $P1$ reads 5 k Pa. Person A calculates the pressure reading in the gauge P_2 to be $P_{2,A}$. Person B considers the specific weight of air to be negligible and calculates the pressure reading in the gauge P_2 to be $P_{2,B}$. The difference between $P_{2,A}$ and $P_{2,B}$ is

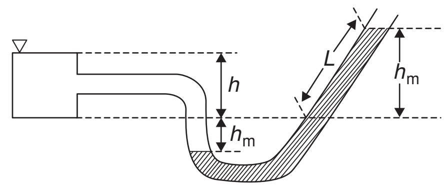
(A) 23.54 Pa (B) 0 Pa
(C) –23.54 Pa (D) 47.08 Pa



13. An inclined u-tube manometer, using a manometric liquid of density $\rho_{m,}$ is connected to an open tank containing a liquid of density ρ_{w}, as shown in the following figure. If the ratio $\rho_m : \rho_w$ is 2 : 1, then the right limb of the u-tube manometer is inclined to the horizontal by an angle of

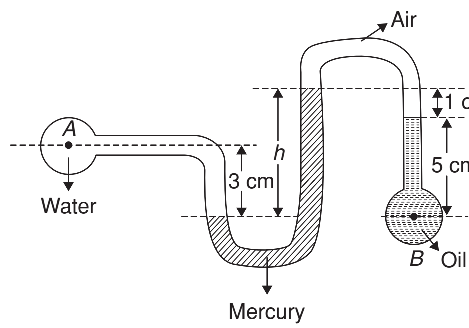
(A) $\sin^{-1}\left(\frac{3h}{L}\right)$ (B) $\sin^{-1}\left(\frac{h}{L}\right)$

(C) $\sin^{-1}\left(\frac{h}{3L}\right)$ (D) $\sin^{-1}\left(\frac{L}{h}\right)$



14. A manometer connects two pipelines, one containing an oil (specific gravity = 0.86) and the other containing water as shown in the figure. The manometric readings are shown in the figure. If the density of air is taken to be 1.2 kg/m^3 and the difference of pressures ($P_A - P_B$) is 10 kPa then the value of h (in cm) is

(A) 10 (B) 6
(C) 7 (D) 8



15. Two spherical soap bubbles, one having a smaller diameter than the other, are present at the two ends of a hollow horizontal cylindrical tube. A restriction at the centre of the tube prevents the flow of air between the two bubbles. If the restriction is removed, then which one of the following is the ONLY possible consequence?

(A) Smaller bubble grows in size.
(B) Both the bubbles do not change in size.
(C) Larger bubble grows in size.
(D) Larger bubble could grow or shrink in size.

Direction for questions 16 and 17: A 500 mm diameter shaft is rotating at 300 r.p.m. in a bearing of length 150 mm. The thickness of the lubricant (Newtonian fluid) film is 2 mm. The torque required to overcome the friction in the bearing is 647.7 Nm. A linear velocity profile is approximately developed in the lubricant.

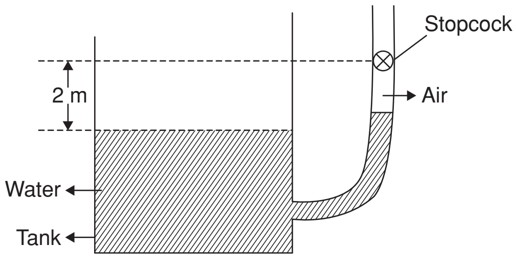
16. The viscosity of the lubricant is

(A) 0.8 Ns$_{/m}^{2}$ (B) 1.8 Ns/m^2
(C) 3.8 Ns$_{/m}^{2}$ (D) 2.8 Ns/m^2

17. The power utilized in overcoming the viscous resistance is

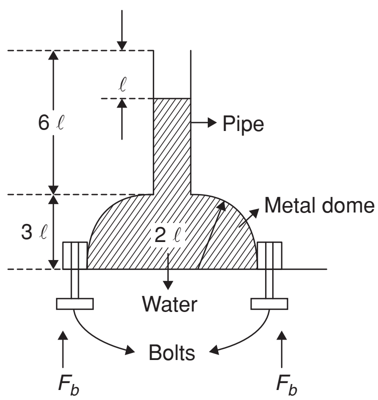
(A) 10.21 kW (B) 20.35 kW
(C) 30.68 kW (D) 15.29 kW

18. A tank to which a manometer is attached contains water as shown in the following figure. A stopcock is present 2 m away from the water surface in the manometer. The stopcock is closed and water is added to the tank up to the level of the stopcock. If the air trapped in the manometer (due to the closing of the stopcock) is compressed isothermally, then the increase in the elevation of water in the manometer is:



(A) 0.2848 m (B) 0 m
(C) 5.234 m (D) 0.1172 m

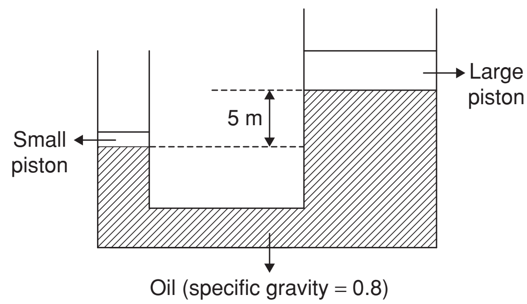
19. A metal dome with a pipe on top is filled with water as shown in the following figure. The metal dome and the pipe weighs 7 kN. The radius of the hemispherical metal dome is 2 ℓ while the diameter of the pipe is 0.2 ℓ. If the value of ℓ is 100 cm, then the force (F_b) that must be exerted through the bolts to hold the dome in place is:



(A) 957121.8 N
(B) 647923.6 N
(C) –647923.6 N
(D) –957121.8 N

20. In the setup shown in the figure below, the weight of the small and large piston is 1000 N and 1500 N respectively. If the force applied to the small piston (diameter = 5 cm) is 100 N, then the magnitude of the force that can be resisted by the large piston (diameter = 10 cm) is:

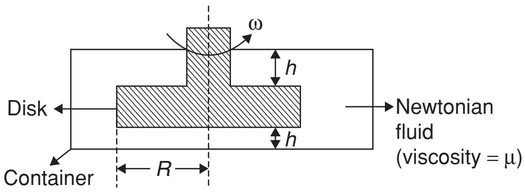
(A) 100 N
(B) 2006.9 N
(C) 2473.2 N
(D) 2591.8 N



Practice Problems 2

Direction for questions 1 to 20: Select the correct alternative from the given choices.

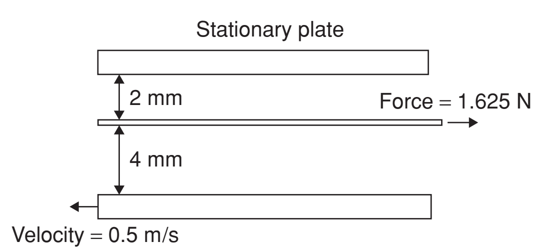
1. The viscous torque on a disk of radius R_1, rotating at an angular velocity of ω_1 inside a container containing a Newtonian fluid of viscosity μ as shown in the figure below, is determined to be T_1. To determine the viscous torque, a linear velocity profile is assumed and the shear on the outer disk edges is neglected. For another disk of radius R_2 rotating at an angular velocity of ω_2 inside the same container containing the same fluid, the viscous torque on the disk is determined to be T_2. If the clearance of the disk surfaces from the container edges are the same in both cases, $\omega_2 = 8\omega_1$, and $R_2 = 0.5R_1$, then:



T_2 is equal to:

(A) 2 T_1 (B) 0.25 T_1
(C) 0.5 T_1 (D) 4 T_1

2. A thin square plate (10 cm ×10 cm) is pulled with a force of 1.625 N horizontally through a 6 mm thick layer of Newtonian fluid (viscosity = 1poise) between two plates, where the top plate is stationary and the bottom plate is moving with a velocity of 0.5 m/s, as shown in the following figure. If a linear velocity profile is assumed, then the minimum distance from the bottom plate, at which the velocity of the fluid is zero, is



(A) 6 mm (B) 5 mm
(C) 2 mm (D) 0.8 mm

3. A block (ℓ meters × b meters × h meters) weighing W Newtons is moved at a constant velocity of V m/s up a plane, inclined at an angle of 30° to the horizontal, by a force F applied in the horizontal direction. If an oil (Newtonian fluid of viscosity μ poise) film of thickness t mm, separates the block and the inclined surface, then W is equal to:

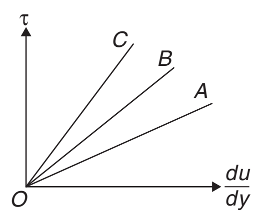
(A) $\sqrt{3}F - \dfrac{2\mu\ell bv}{t}$ (B) $\sqrt{3}F - \dfrac{200\mu\ell bv}{t}$

(C) $F - \dfrac{2\mu\ell bv}{t}$ (D) $F - \dfrac{200\mu\ell bv}{t}$

4. On the free surface of a body of liquid resting inside an open container, a constant shear force is applied. Which one of the following events is most unlikely to follow afterwards?
(A) The liquid deforms continuously.
(B) A liquid flow pattern develops inside the container.
(C) The liquid changes its shape.
(D) The liquid remains at rest.

5. If the straight line plots, between shear stress (τ) and rate of deformation $\left(\dfrac{du}{dy}\right)$, for three fluids

A (viscosity = μ_A), B (viscosity = μ_B) and C (viscosity = μ_C are as given in the below figure, then



(A) $\mu C > \mu_A$ (B) $\mu_C < \mu_B < \mu_A$
(C) $\mu_C > \mu_B < \mu_A$ (D) $\mu_C < \mu_B > \mu_A$

6. For which of the following fluids, the apparent viscosity can be considered to be independent of the rate of shear strain and equal to the fluid's viscosity?
(A) Ketchup (B) Water
(C) Cornstarch solution (D) Blood

7. For an ideal gas (density = ρ) at pressure P and temperature T, the coefficient of volume expansion is equal to
(A) T (B) P
(C) $\dfrac{1}{T}$ (D) $\dfrac{1}{P}$

Direction for questions 8 and 9: A spherical drop of liquid (surface tension = σ) of radius 10 cm is split into small identical spherical drops of radius 2 cm under isothermal conditions.

8. The number of the small spherical drops formed is
(A) 25 (B) 15
(C) 35 (D) 125

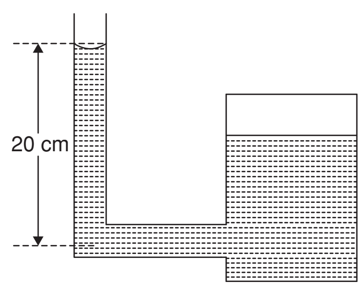
9. The volume of the liquid of the large drop still unconverted to small spherical drops is
(A) 4189 cm^3 (B) 33.51 cm^3
(C) 0 cm^3 (D) 3351 cm^3

10. If 5×10^{-4} J of energy is expended in blowing up a soap bubble, using a soap solution having a surface tension of 50×10^{-3} N/m, then the diameter of the bubble is
(A) 4 cm (B) 2 cm
(C) 8 cm (D) 3 cm

11. Small liquid droplets, at 20°C, of constant diameter are sprayed using a spray nozzle into the atmosphere. The average diameter of the droplets is 100 μm. If the surface tension of the liquid at 20°C is 2.69×10^{-2} N/m, then the pressure inside the droplets is
(A) 1076 Pa
(B) 102401 Pa
(C) 101325 Pa
(D) 101863 Pa

12. The maximum diameter of a metallic (density = ρ) spherical ball that can float in a constant temperature liquid (surface tension = σ) bath is d_1. If the density ρ is made eightfold and the surface tension σ is doubled, then the maximum diameter becomes d_2. Then, one can write that
(A) $d_2 = 2d_1$ (B) $4d_2 = d_1$
(C) $2d_2 = d_1$ (D) $d_2 = 4d_1$

13. A glass tube, of diameter 2 mm, is used to measure the pressure in a water tank as shown in the following figure.

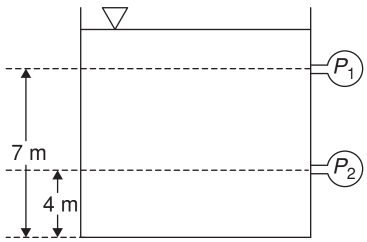


If the surface tension of water is 0.073 N/m, then the height of water in the tube to be used to determine the pressure in the tank, when surface tension effects in the tube are not negligible,
(A) 18.51 cm (B) 20 cm
(C) 19.25 cm (D) 10 cm

14. A hydraulic jack has a large piston of diameter 15 cm and a small piston of 5 cm diameter. The small piston is above the large piston by a height h. If a force of 100 N applied on the small piston lifts a load of 990 N placed on the large piston, then the value of h (in cm) is:

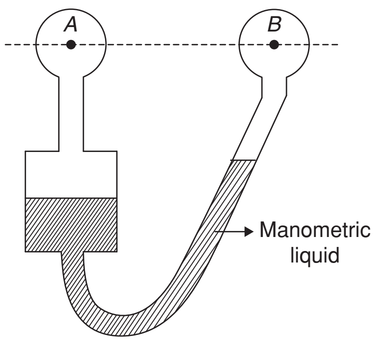
(A) 14 (B) 67
(C) 40 (D) 52

15. A liquid is present in a tank fitted with two pressure gauges as shown in the following figure. If the readings in the pressure gauges are $P_1 = 60$ kPa and $P_2 = 80$ KPa, then the density of the liquid (in kg/m^3) is



(A) 679.58 (B) 701.28
(C) 504.32 (D) 462.95

16. The inclined differential manometer, with the right limb inclined at an angle of θ to the horizontal, contains a manometric liquid of specific weight ω_m. The manometer is fitted to the two pipes as shown in the following figure.

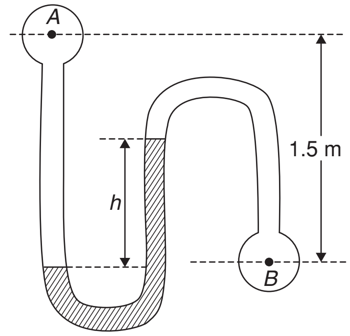


The pressure differential between the two points A and B in the respective pipes, which both contain the same liquid of specific weight ω_b, is zero. It is observed that $\omega_m = 2\omega_b$. When a pressure differential of P $(= P_A - P_B)$ occurs, then the manometer gives a differential reading of Δh (measured along the inclined tube). If all the variables are in the S.I units, then Δh is equal to

(A) $\dfrac{P}{\omega_m \sin\theta}$ (B) $\dfrac{P}{\omega_b \sin\theta}$

(C) $\dfrac{3P}{\omega_m \sin\theta}$ (D) $\dfrac{P}{2\omega_b \sin\theta}$

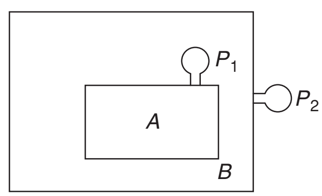
17. Two pipes transporting water are connected by a manometer as shown in the figure. The specific gravity of the manometric fluid is 2. If the difference in pressures at the points B and A is 10 kP$_a$, then the value of h is



(A) 52 cm (B) 89 cm
(C) 48 cm (D) 62 cm

18. Two open containers contain liquids in them such that the free surface of the liquid in contact with the atmosphere is at the same elevation in both the containers. The pressure at a point in the first container and the pressure at a point in the second container can be
(A) Equal only if the two points are at the same elevation.
(B) Equal or unequal.
(C) Equal only if the two points are at the same depth.
(D) Equal only if the two points are in the same liquid.

19. Two tanks A and B are present in the configuration as shown in the following figure. The pressure readings at the Bourdon pressure gauges P_1 and P_2 are 3 atm and 2 atm respectively. If the atmospheric pressure outside tank B is 1 at m, then the absolute pressure in tank A is



(A) 6 atm (B) 4 atm
(C) 2 atm (D) 3 atm

20. Which one of the following statements is NOT correct?
(A) Liquids wetting a solid surface have acute contact angles.
(B) Non-wetting liquids have obtuse contact angles.
(C) For hydrophilic surfaces, water has a large contact angle.
(D) For hydrophobic surfaces, water has a large contact angle.

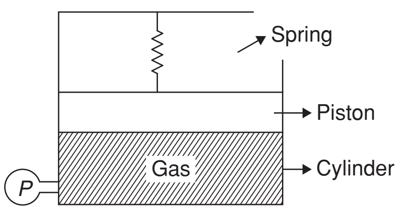
21. A cylindrical tank is filled with water upto a height h. An air bubble of diameter d is present at the bottom of the tank. If d_n is be diameter of the bubble after it has traveled a distance of $h/2$ while rising to the surface, then
(A) $d_n > d$ (B) $d_n = d/2$
(C) $d_n = d$ (D) $d_n < d$

Direction for questions 22 and 23: A shaft of diameter d is rotating at a speed of N r.p.m. in a bearing of length ℓ. The thickness of the lubricant (Newtonian fluid) film is t where the viscosity of the lubricant is μ. The torque and power required to rotate the shaft was determined to be T and P. After doubling the length of the bearing and reducing the speed of the shaft to half its value, the torque and power required to rotate the shaft was determined to be T_1 and P_1. A linear velocity profile could always be assumed in the lubricant.

22. The relationship between the required torques T and T_1 is
(A) $2T_1 = T$ (B) $T = T_1$
(C) $T_1 = 2T$ (D) $T_1 = 8T$

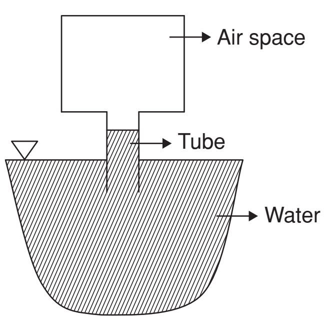
23. The relationship between the required powers P and P_1 is
(A) $P = P_1$ (B) $P_1 = 2P$
(C) $2P_1 = P$ (D) $P_1 = 3P$

24. A frictionless piston-cylinder device, shown in the following figure, has a piston of mass 5 kg and a cross-section at area of 50 cm^2. A compressed spring above the piston exerts a force of 50 N on the piston. If the atmospheric pressure is 100 kPa, the reading on the pressure gauge attached at the bottom of the cylinder is



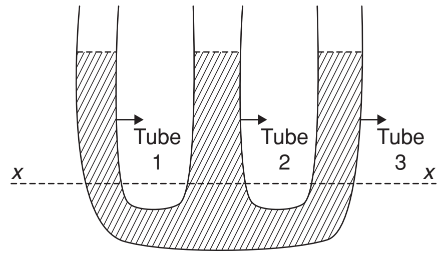
(A) –80190 Pa (B) 19810 Pa
(C) 100000 Pa (D) 10000 Pa

25. The air space above a tube, shown in the following figure, is pressurized to 70 kPa vaccum. Water from a large reservoir fills the tube to a certain height. If the air space pressure is reduced to 50 kPa, then the height of the water in the tube will



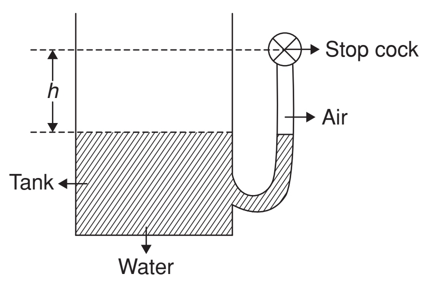
(A) Not change.
(B) Increase by one metre.
(C) decrease by 2.82 metres.
(D) decrease by 2.04 metres.

26.



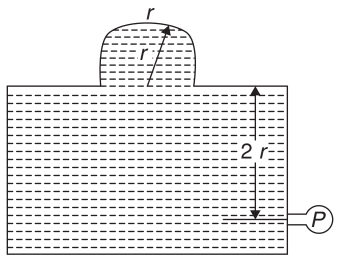
In the three capillary tube structure shown above, water is present in all the three tubes. The height of the free surface of water in tubes 1, 2 and 3, from the horizontal line x x, are h_1, h_2 and h_3 respectively. If the diameter of the tubes 1, 2 and 3 are d_1, d_2 and d_3 respectively such that $d_3 > d_2 > d_1$, then
(A) $h_3 > h_2 > h_1$
(B) $h_1 < h_3 < h_2$
(C) $h_3 < h_2 < h_1$
(D) $h_1 > h_2 < h_3$

27. A tank to which a manometer is attached contains water as shown in the following figure. A stopcock is present h metres away from the surface of the water in the manometer. The stopcock is closed and water is added to the tank upto the level of the stopcock. If the trapped air in the manometer (due to the closing of the stopcock) is compressed isothermally and that the increase in the elevation of water in the manometer is 0.3 m, then the value of h is equal to
(A) 1.06 m
(B) 4.06 m
(C) 1.03 m
(D) 2.06 m



28. In the following figure, the hemispherical dome has a radius, $r = 100$ cm. The dome weighs about 1000 N.

The specific gravity of the liquid inside the closed structure is 1. If the reading on the pressure gauge is 29.62 kPa, then it could be stated that the metal at the base of the hemispherical dome
(A) is in tension.
(B) is in compression.
(C) is neither in tension nor in compression.
(D) can be in tension or in compression.



29. In a hydraulic jack configuration, a load W placed on the large piston (area = A) is balanced by a force F applied on the small piston (area = a) such that the bases of both the pistons (whose weights can be assumed to be negligible) are at the same horizontal level. The specific weight of the liquid used in the jack is ω. If the load on the large piston is doubled, then the vertical distance between the bases of the two pistons will be equal to

(A) $\dfrac{W}{a\omega}$ (B) $\dfrac{F}{A\omega}$

(C) $\dfrac{W}{A\omega}$ (D) $\dfrac{F^2}{Wa\omega}$

30. Two open containers, having the same base area, are filled with the same liquid such that the elevation of the free surface of the liquid from the base is same for both the containers. However, the volume of the liquid in the second container is twice the volume of the liquid in the first container. The ratio of the pressure force exerted at the base of the first container to that exerted at the base of the second container is:

(A) 2:1
(B) 1:1
(C) 1:2
(D) Not possible to be determined with the given conditions.

Previous Years' Questions

1. An incompressible fluid (kinematic viscosiy, 7.4×10^{-7} m^2/s, specific gravity 0.88) is held between two parallel plates. If the top plate is moved with a velocity of 0.5 m/s while the bottom one is held stationary, the fluid attains a linear velocity profile in the gap of 0.5 mm between these plates; the shear stress in Pascals on the surface of top plate is **[2004]**

(A) 0.651×10^{-3} (B) 0.651
(C) 6.51 (D) 0.651×10^3

2. For a Newtonian fluid **[2006]**

(A) Shear stress is proportional to shear strain
(B) Rate of shear stress is proportional to shear strain
(C) Shear stress is proportional to rate of shear strain
(D) Rate of shear stress is proportional to rate of shear strain

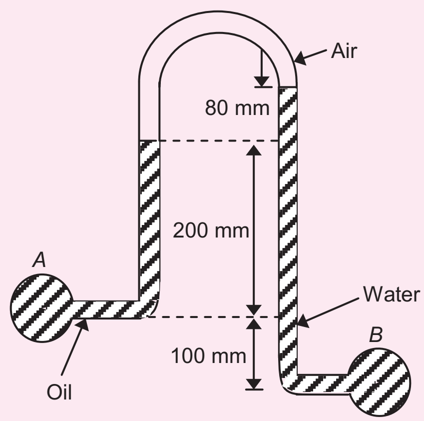
3. For an incompressible flow field, $\vec{V}$, which one of the following conditions must be satisfied? **[2014]**

(A) $\nabla \cdot \vec{V} = 0$ (B) $\nabla \times \vec{V} = 0$

(C) $(\vec{V} \cdot \nabla)\vec{V} = 0$ (D) $\dfrac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla)\vec{V} = 0$

4. An inverted U-tube manometer is used to measure the pressure difference between two pipes A and B, as shown in the figure. Pipe A is carrying oil (specific gravity = 0.8) and pipe B is carrying water. The densities of air and water are 1.16 kg/m^3, respectively. The pressure difference between pipes A and B is ________ kPa. **[2016]**

Acceleration due to gravity g = 10 m/s^2.



Answer Keys

Exercises

Practice Problems 1

1. C	**2.** A	**3.** A	**4.** C	**5.** B	**6.** C	**7.** D	**8.** A	**9.** D	**10.** B
11. C	**12.** A	**13.** C	**14.** D	**15.** C	**16.** D	**17.** B	**18.** A	**19.** D	**20.** D

Practice Problems 2

1. C	**2.** D	**3.** B	**4.** D	**5.** A	**6.** B	**7.** C	**8.** A	**9.** D	**10.** A
11. B	**12.** C	**13.** A	**14.** D	**15.** A	**16.** B	**17.** C	**18.** B	**19.** A	**20.** A
21. A	**22.** B	**23.** C	**24.** B	**25.** D	**26.** C	**27.** D	**28.** A	**29.** C	**30.** B

Previous Years' Questions

1. B	**2.** C	**3.** A	**4.** –2.2 kPa