

CLASS: X
SUBJECT: MATHEMATICS

TOPIC: REAL NUMBERS

1. If $7 \times 5 \times 3 \times 2 + 3$ is composite number? Justify your answer
2. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$ where q is a positive integer
3. Show that 8^n cannot end with the digit zero for any natural number n
4. Prove that $\frac{3\sqrt{2}}{5}$ is irrational
5. Prove that $\sqrt{2} + \sqrt{5}$ is irrational
6. Prove that $5 - 2\sqrt{3}$ is an irrational number
7. Prove that $\sqrt{2}$ is irrational
8. Use Euclid's Division Algorithms to find the H.C.F of
 - a) 135 and 225 (45)
 - b) 4052 and 12576 (4)
 - c) 270, 405 and 315 (45)
9. Using Euclid's division algorithm, check whether the pair of numbers 50 and 20 are co-prime or not.
10. Find the HCF and LCM of 26 and 91 and verify that $\text{LCM} \times \text{HCF} = \text{Product of two numbers}$ (13,182)
11. Explain why $\frac{29}{2^3 \times 5^3}$ is a terminating decimal expansion
12. $\frac{163}{150}$ will have a terminating decimal expansion. State true or false .Justify your answer.
13. Find HCF of 96 and 404 by prime factorization method. Hence, find their LCM. (4, 9696)
14. Using prime factorization method find the HCF and LCM of 72, 126 and 168 (6, 504)
15. If $\text{HCF}(6, a) = 2$ and $\text{LCM}(6, a) = 60$ then find a (20)
16. given that $\text{LCM}(77, 99) = 693$, find the $\text{HCF}(77, 99)$ (11)
17. Find the greatest number which exactly divides 280 and 1245 leaving remainder 4 and 3 (138)
18. The LCM of two numbers is 64699, their HCF is 97 and one of the numbers is 2231. Find the other (2813)
19. Two numbers are in the ratio 15: 11. If their HCF is 13 and LCM is 2145 then find the numbers (195,143)
20. Express 0.363636..... in the form a/b (4/11)
21. Write the HCF of smallest composite number and smallest prime number
22. Write whether $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$ on simplification give a rational or an irrational number (6)
23. State whether 10.064 is rational or not. If rational, express in p/q form
24. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$
25. State the fundamental theorem of arithmetic
26. The decimal expansion of the rational number $\frac{74}{2^3 \cdot 5^4}$ will terminate after Places

1. Find the zeroes of the polynomial and verify the relationship between the zeroes and the coefficient
 - a) $4x^2 + 1 - 4x$
 - b) $x^2 - 3$
 - c) $4x^2 - 7$
 - d) $\sqrt{3}x^2 - 8x + 4\sqrt{3}$
2. Find the polynomial, whose zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$ $(x^2 - 4x + 1)$
3. Form a quadratic polynomial, one of whose zero is $2 + \sqrt{5}$ and the sum of zeroes is 4 $(x^2 - 4x - 1)$
4. Find a quadratic polynomial whose sum and product of the zeroes are $21/8$ and $5/16$ $(16x^2 - 42x + 5)$
5. If α and β are zeroes of the polynomial $x^2 - 2x - 15$, then form a quadratic polynomial whose zeroes are 2α and 2β
6. If α and β are the zeroes of the polynomial $x^2 + 3x - 4$, find a quadratic polynomial whose zeroes are $1/\alpha$ and $1/\beta$
7. Write a quadratic polynomial, the sum and product of whose zeroes are 3 and -2 $(x^2 - 3x - 2)$
8. Find the sum and the product of the zeroes of cubic polynomial $2x^3 - 5x^2 - 14x + 8$ $(5/2, -7, -4)$
9. Find the sum and product of the zeroes of quadratic polynomial $x^2 - 3$ $(0, -3)$
10. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a , $a + b$, find a and b $(1, \pm\sqrt{2})$
11. If α and β are the zeroes of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$ (-1)
12. If α and β are the zeroes of the polynomial $f(x) = 6x^2 + x - 2$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$ $(5/6)$
13. If α and β are the zeroes of the quadratic polynomial $2x^2 + 3x - 5$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ $(3/5)$
14. If α and β are the zeroes of the polynomial $f(x) = x^2 - 8x + k$ such that $\alpha^2 + \beta^2 = 40$, find k (12)
15. If α, β are the zeroes of quadratic polynomial $2x^2 + 5x + k$, find the value of k such that $(\alpha + \beta)^2 - \alpha\beta = 24$ $(-71/2)$
16. If α and β are zeroes of $x^2 + 5x + 5$, find the value of $\alpha^{-1} + \beta^{-1}$ (-1)
17. α, β are the zeroes of the quadratic polynomial $x^2 - (k+6)x + 2(2k - 1)$. Find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$ (7)
18. If α, β are the zeroes of the quadratic polynomial $x^2 - 7x + 10$, find the value of $\alpha^3 + \beta^3$ (133)
19. m, n are zeroes of $ax^2 - 12x + c$. Find the value of a and c if $m + n = m n = 3$ $(4, 12)$
20. If 1 is a zero of polynomial $ax^2 - 3(a-1) - 1$, then find the value of a (1)
21. If the product of zeroes of the polynomial $ax^2 - 6x - 6$ is 4, find the value of a $(-3/2)$
22. If one root of the polynomial $5x^3 + 13x + k$ is reciprocal of the other, then find the value of k ? $(k = 5)$
23. If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the value of a (3)
24. Show that $x^2 - 3$ is a factor of $2x^4 + 3x^3 - 2x^2 - 9x - 12$
25. Divide $(6 + 19x + x^2 - 6x^3)$ by $(2 + 5x - 3x^2)$ and verify the division algorithm
26. What must be subtracted from $2x^4 - 11x^3 + 29x^2 - 40x + 29$, so that the resulting polynomial is exactly divisible By $x^2 - 3x + 4$ $(-2x + 5)$
27. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$ $(x^2 - x + 1)$
28. Find other zeroes of the polynomial $p(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$ if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$ $(3/2, -5)$
29. Find all the zeroes of the polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{5}/3$ and $-\sqrt{5}/3$ $(-1, -1)$
30. Find all the zeroes of $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$ $(1, -1/2)$
31. Find all the zeroes of polynomial $4x^4 - 20x^3 + 23x^2 + 5x - 6$ if two of its zeroes are 2 and 3 $(1/2, -1/2)$
32. When a polynomial $f(x)$ is divided by $x^2 - 5$ the quotient is $x^2 - 2x - 3$ and remainder is zero. Find the polynomial and all its zeroes $(3, -1, \sqrt{5}, -\sqrt{5})$
33. Given $p(x) = 2x^4 + 5x^3 - 5x - 2$ and $g(x) = 2x^2 + 3x + 1$. Check whether $g(x)$ is a factor of $p(x)$ by applying division algorithm. If yes, Find all the factors of $p(x)$
34. If the polynomial $f(x) = x^4 - 6x^3 + 16x^2 - 25x + 10$, is divided by another polynomial $x^2 - 2x + k$ the remainder Comes out to be $x + a$, Find k and a $(k = 5, a = -5)$
35. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b $(-20, -25)$
36. Find the values of m and n so that $x^4 + mx^3 + nx^2 - 3x + n$ is divisible by $x^2 - 1$ $(m = 3, n = -1/2)$

1. If $\cot\theta = 15/8$, evaluate $\frac{(2 + 2\sin\theta)(1 - \sin\theta)}{(1 + \cos\theta)(2 - 2\cos\theta)}$ (225/64)
2. If $\operatorname{cosec}\theta = 2$, show that $\cot\theta + \frac{\sin\theta}{1 + \cos\theta} = 2$
3. If $\tan A = 2$. Evaluate $\sec A \sin A + \tan^2 A - \operatorname{cosec} A$
4. In a $\triangle ABC$, right angled at A, if $\tan C = \sqrt{3}$, find the value of $\sin B \cos C + \cos B \sin C$ (1)
5. In $\triangle PQR$, right angled at Q, $QR = 6$ cm, $\angle QPR = 60^\circ$. Find the length of PQ and PR
6. If $7 \sin^2\theta + 3 \cos^2\theta = 4$, show that $\tan\theta = 1/\sqrt{3}$
7. If $\sec\theta - \tan\theta = 4$, then prove that $\cos\theta = 8/17$
8. If $\cos\theta - \sin\theta = \sqrt{2} \sin\theta$, prove that $\cos\theta + \sin\theta = \sqrt{2} \cos\theta$
9. If $\sqrt{3} \tan\theta = 3 \sin\theta$, find the value of $\sin^2\theta - \cos^2\theta$
10. Evaluate: $\sqrt{2} \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$ ($\sqrt{2}$)
11. Evaluate: $\tan^2 60^\circ - 2 \cos^2 60^\circ - \frac{3}{4} \sin^2 45^\circ - 4 \sin^2 30^\circ$ (9/8)
12. Evaluate: $(\sin 90^\circ + \cos 45^\circ + \cos 60^\circ)(\cos 0^\circ - \sin 45^\circ + \sin 30^\circ)$ (7/4)
13. If $\sin 2x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$, find x (15)
14. If $A = B = 30^\circ$, verify that:
 $\sin(A + B) = \sin A \cos B + \cos A \sin B$
15. If $\sec^2\theta (1 + \sin\theta) (1 - \sin\theta) = k$, find the value of k ($k = 1$)
16. Evaluate: $\frac{\sec^2 54^\circ - \cot^2 36^\circ}{\operatorname{Cosec}^2 57^\circ - \tan^2 33^\circ} + 2 \sin^2 38^\circ \sec^2 52^\circ - \sin^2 45^\circ$ (5/2)
17. Evaluate: $\sec(90 - \theta) \operatorname{cosec}\theta - \tan(90 - \theta) \cot\theta + \frac{\cos^2 35^\circ + \cos^2 55^\circ}{\tan 5^\circ \tan 15^\circ \tan 45^\circ \tan 75^\circ \tan 85^\circ}$ (2)
18. Find the value of:
 $\frac{2 \sin 68^\circ}{\cos 22^\circ} - \frac{2 \cot 15^\circ}{5 \tan 75^\circ} - \frac{3 \tan 45^\circ \tan 20^\circ \tan 40^\circ \tan 50^\circ \tan 70^\circ}{5}$ (1)
19. If $\cos(40^\circ + x) = \sin 30^\circ$, find the value of x (20°)
20. $\sin 4A = \cos(A - 20^\circ)$, where $4A$ is an acute angle, find the value of A (22°)
21. Find the value of θ in $2 \cos 3\theta = 1$ (20°)
22. Solve for θ : $2 \sin^2\theta = \frac{1}{2}$ (30°)
23. If $\sin\theta + \cos\theta = \sqrt{2} \cos(90^\circ - \theta)$, determine $\cot\theta$ ($\sqrt{2} - 1$)
24. Find the acute angles A and B, $A > B$, if $\sin(A + 2B) = \sqrt{3}/2$ and $\cos(A + 4B) = 0$ ($30^\circ, 15^\circ$)
25. If $\tan(A + B) = \sqrt{3}$, $\tan(A - B) = 1$, $0^\circ < A + B \leq 90^\circ$, $A > B$, then find A and B (52.5, 7.5)
26. If $\sin(A + B) = 1$, $\cos(A - B) = 1$, find A and B ($45^\circ, 45^\circ$)
27. If $\sin A - \cos B = 0$, prove that $A + B = 90^\circ$
28. What is the maximum value of $1/\sec\theta$
29. Express $\cos 56^\circ + \cot 56^\circ$ in terms of 0° and 45°
30. Express $\cos A$ in terms of $\tan A$
31. Find the value of $\tan 60^\circ$ geometrically
32. If A, B and C are interior angles of triangle ABC, show that $\cos \left\{ \frac{B+C}{2} \right\} = \sin \frac{A}{2}$
33. If $x = a \sin\theta$, $y = b \tan\theta$. Prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$
34. Prove that: $\frac{1}{1 + \sin\theta} + \frac{1}{1 - \sin\theta} = 2 \sec^2 \theta$
35. Prove that: $\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = 2 \operatorname{cosec}\theta$

36. Prove: $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{\cos A}{1 - \sin A}$

37. Prove that $\sin(90^\circ - \theta) \cos(90^\circ - \theta) = \frac{\tan \theta}{1 + \tan^2 \theta}$

38. If $x = a \sec \theta + b \tan \theta$ and $y = a \tan \theta + b \sec \theta$ prove that $x^2 - y^2 = a^2 - b^2$

39. Show that $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$

40. Prove that $\sec^2 \theta + \operatorname{cosec}^2 \theta = \sec^2 \theta \cdot \operatorname{cosec}^2 \theta$

41. Prove that $\frac{\cot \theta}{1 + \tan \theta} = \frac{\cot \theta - 1}{2 - \sec^2 \theta}$

42. Prove that $\frac{1 - \sin \theta}{1 + \sin \theta} = (\sec \theta - \tan \theta)^2$

43. Prove that: $\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cdot \cos^2 B}$

44. Prove that: $(\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$

45. Prove that $(\operatorname{cosec} \theta - \cot \theta)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$

46. Prove that $\frac{1}{(\sec \theta - \tan \theta)} - \frac{1}{\cos \theta} = \frac{1}{\cos \theta} - \frac{1}{(\sec \theta + \tan \theta)}$

47. Prove that

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

48. Prove that $\sec^4 \theta - \tan^4 \theta = 1 + 2 \tan^2 \theta$

49. Show that $\frac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$

50. If $\sec \theta + \tan \theta = p$, prove that $\sin \theta = \frac{p^2 - 1}{p^2 + 1}$

51. Prove that $\frac{\tan \theta + \sin \theta}{\tan \theta - \sin \theta} = \frac{\sec \theta + 1}{\sec \theta - 1}$