CBSE Test Paper 05

Chapter 10 Vector Algebra

1. Unit Vector is
    a. A vector whose direction angle β is unity
    b. A vector whose direction angle γ is unity
    c. A vector whose direction angle α is unity
    d. A vector whose magnitude is unity
2. If a vector $\vec{r}$ is expressed in component form as $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then the magnitude of the vector $\vec{r}$ is
    a. $\sqrt{2x^2 + y^2 + z^2}$
    b. $\sqrt{x^2 + y^2 + z^2}$
    c. $\sqrt{x^2 + 2y^2 + z^2}$
    d. $\sqrt{x^2 + y^2 + 2z^2}$
3. Find a vector in the direction of the vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has a magnitude of 8 units
    a. $\frac{40}{\sqrt{30}}\hat{i} + \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$
    b. $-\frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$
    c. $\frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$
    d. $\frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} - \frac{16}{\sqrt{30}}\hat{k}$
4. Find the value of x for which $x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector
    a. $\pm\frac{1}{\sqrt{2}}$
    b. $\pm\frac{1}{\sqrt{3}}$
    c. $\pm\frac{1}{\sqrt{7}}$
    d. $\pm\frac{1}{\sqrt{5}}$
5. Magnitude of the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ is

a. $\sqrt{3}$
b. 1-$\sqrt{3}$
c. $\sqrt{2}$
d. 1+$\sqrt{3}$

6. If $|\vec{a}|$ = 4 and -3 $\leq \lambda \leq$ 2, then the range of $|\lambda\vec{a}|$ is ________.
7. If vectors $\vec{a}$ and $\vec{b}$ represent the adjacent sides of a triangle, then its area is given by ________.
8. The unit vector in the direction of $\vec{a}$ is given by ________.
9. Write the angle between vectors a and b with magnitudes $\sqrt{3}$ and 2 respectively, having $\vec{a} \cdot \vec{b} = \sqrt{6}$.
10. Write the direction ratios of the vector $3\vec{a} + 2\vec{b}$, where $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$.
11. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.
12. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + \hat{2}j + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is $\perp$ to $\vec{c}$ is then find the value of λ.
13. Find a vector of magnitude 11 in the direction opposite to that of $\overrightarrow{PQ}$, where P and Q are the points (1, 3, 2) and (-1, 0, 8) respectively.
14. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.
15. Dot product of a vector with vectors $\hat{i} - \hat{j} + \hat{k}, 2\hat{i} + \hat{j} - 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ are respectively 4, 0 and 2. Find the vector.
16. Find the sine of the angle between the vectors. $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}, \vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$.
17. Prove that $[\vec{a}.\ \vec{b} + \vec{c}.\ \vec{d}\ ]$ = $\left[\vec{a} \quad \vec{b} \quad \vec{d}\right] + \left[\vec{a} \quad \vec{c} \quad \vec{d}\right]$.
18. Find the area of the Δ with vertices A (1, 1, 2) B (2, 3, 5) and C (1, 5, 5).

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Solution

1. d. A vector whose magnitude is unity

   Explanation: The vector whose magnitude is always 1 or unity is called a Unit Vector.

2. b. $\sqrt{x^2 + y^2 + z^2}$

   Explanation: If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then , $\left|\overrightarrow{r}\right| = \sqrt{x^2 + y^2 + z^2}$

3. c. $\frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

   Explanation: Let $\overrightarrow{a} = 5\hat{i} - \hat{j} + 2\hat{k}$

   ,then, $8\hat{a} = 8\frac{\vec{a}}{|\vec{a}|}$

   =$8.\frac{5\hat{i}-\hat{j}+2\hat{k}}{\sqrt{5^2+(-1)^2+2^2}}$

   $= \frac{8(5\hat{i}-\hat{j}+2\hat{k})}{\sqrt{30}}$

   $= \frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$

4. b. $\pm\frac{1}{\sqrt{3}}$

   Explanation: As x$\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector, therefore,

   $\left|x\left(\hat{i} + \hat{j} + \hat{k}\right)\right| = 1 \Rightarrow x\sqrt{1+1+1} = 1 \Rightarrow x = \frac{1}{\sqrt{3}}$

5. a. $\sqrt{3}$

   Explanation: We have : $\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}$, then, $\left|\overrightarrow{a}\right| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

6. [0, 12]

7. $\frac{1}{2}|\vec{a} \times \vec{b}|$

8. $\widehat{a} = \frac{\vec{a}}{|\vec{a}|}$

9. We are given that $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$.

   Now, the angle between $\vec{a}$ and $\vec{b}$ is given by

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\sqrt{6}}{\sqrt{3}\times 2} = \frac{\sqrt{3}\times\sqrt{2}}{\sqrt{3}\times 2} = \frac{\sqrt{2}}{2}$ (simplifying)

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} \quad \left[\because \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$

$\therefore \quad \theta = \frac{\pi}{4}$

10. Here, we are given that $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$
$\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$. Therefore, we have
$3\vec{a} + 2\vec{b} = 3(\hat{i} + \hat{j} - 2\hat{k}) + 2(2\hat{i} - 4\hat{j} + 5\hat{k})$
$= (3\hat{i} + 3\hat{j} - 6\hat{k}) + (4\hat{i} - 8\hat{j} + 10\hat{k})$
$= 7\hat{i} - 5\hat{j} + 4\hat{k}$
Hence, direction ratio of vectors $3\vec{a} + 2\vec{b}$ are 7, - 5 and 4.

11. We have to find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.
Let $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$
Then, $|\vec{a}| = \sqrt{(2)^2 + (-3)^2 + (6)^2}$
$= \sqrt{4 + 9 + 36} = \sqrt{49}$ = 7 units
Now unit vector in the direction of the given vectors $\vec{a}$ is given as
$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{7}(2\hat{i} - 3\hat{j} + 6\hat{k})$
$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$
Now, the vector of magnitude equal to 21 units and in the direction of $\vec{a}$ is given by
$21\hat{a} = 21\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = 6\hat{i} - 9\hat{j} + 18\hat{k}$

12. $\vec{a} + \lambda\vec{b} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$
$= (2 - \lambda)\,\hat{i} + (2 + 2\lambda)\,\hat{j} + (3 + \lambda)\,\hat{k}$
$\left(\vec{a} + \lambda\vec{b}\right).\vec{c} = 0 \quad \left[\because \vec{a} + \lambda\vec{b} \perp \vec{c}\right]$
$\left[(2 - \lambda)\,\hat{i} + (2 + 2\lambda)\,\hat{j} + (3 + \lambda)\,\hat{k}\right].\left(3\hat{i} + \hat{j}\right) = 0$
$3\,(2 - \lambda) + (2 + 2\lambda) = 0$
$-\lambda = -8$
$\lambda = 8$

13. The vector with initial point P(1, 3, 2) and terminal point Q(-1, 0, 8) is given by

$\overrightarrow{PQ}=(-1-1)\,\hat{i}+(0-3)\,\hat{j}+(8-2)\,\hat{k}=-2\hat{i}-3\hat{j}+6\hat{k}$

Thus, $\overrightarrow{QP}=-\overrightarrow{PQ}=2\hat{i}+3\hat{j}-6\hat{k}$

$\Rightarrow\left|\overrightarrow{QP}\right|=\sqrt{2^2+3^2+(-6)^2}=\sqrt{4+9+36}=\sqrt{49}=7$

Therefore, unit vector in the direction of $\overrightarrow{QP}$ is given by

$\widehat{QP}=\frac{\overrightarrow{QP}}{\left|\overrightarrow{QP}\right|}=\frac{2\hat{i}+3\hat{j}-6\hat{k}}{7}$

Hence, the required vector of magnitude 11 in direction of $\overrightarrow{QP}$ is 11

$\overrightarrow{QP}=11\left(\frac{2i+3j-6k}{7}\right)=\frac{22}{7}\hat{i}+\frac{33}{7}\hat{j}-\frac{66}{7}\hat{k}$

14. Given: $\vec{a}=2\hat{i}+2\hat{j}+3\hat{k},\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=3\hat{i}+\hat{j}$

    Now $\vec{a}+\lambda\vec{b}=2\hat{i}+2\hat{j}+3\hat{k}+\lambda\left(-\hat{i}+2\hat{j}+\hat{k}\right)$=

    $=2\hat{i}+2\hat{j}+3\hat{k}-\lambda\hat{i}+2\lambda\hat{j}+\lambda\hat{k}$

    $\Rightarrow\vec{a}+\lambda\vec{b}=(2-\lambda)\,\hat{i}+(2+2\lambda)\,\hat{j}+(3+\lambda)\,\hat{k}$

    Again, $\vec{c}=3\hat{i}+\hat{j}=3\hat{i}+\hat{j}+0\hat{k}$

    Since, $\left(\vec{a}+\lambda\vec{b}\right)$ is perpendicular to $\vec{c}$ therefore, $\left(\vec{a}+\lambda\vec{b}\right).\vec{c}=0$

    $\Rightarrow 6-3\lambda+2+2\lambda=0\Rightarrow-\lambda+8=0$

    $\Rightarrow-\lambda=-8\Rightarrow\lambda=8$

15. According to the question,

    $\vec{b}=\hat{i}-\hat{j}+\hat{k},$

    $\vec{c}=2\hat{i}+\hat{j}-3\hat{k}$ and

    $\vec{d}\;=\hat{i}+\hat{j}+\hat{k}$

    let the required vector is $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

    Given, $\vec{a}\cdot\vec{b}=4,\;\vec{a}\cdot\vec{c}=0$ and $\vec{a}\cdot\vec{d}\;=2$

    Now, $\vec{a}.\vec{b}=4$

    $\Rightarrow a_1-a_2+a_3=4$....(i)

    $\vec{a}.\vec{c}=0$

    $\Rightarrow 2a_1+a_2-3a_3=0$ ...(ii)

    and $\vec{a}.\vec{d}\;=2$

    $\Rightarrow a_1+a_2+a_3=2$ ...(iii)

On subtracting Eq. (iii) from Eq. (i), we get

$-2a_2 = 2$

$\Rightarrow a_2 = -1$

On substituting a_2 = -1 in Eq. (ii) and (iii),

we get

$2a_1 - 3a_3 = 1$ ...(iv)

$a_1 + a_3 = 3$ ...(v)

On multiplying Eq. (v) by 3 and then adding with Eq. (iv), we get

$5a_1 = 1 + 9 = 10$

$\Rightarrow a_1 = 2$

On substituting a_1 = 2 in Eq. (v), we get

$a_3 = 1$

$\therefore$the vector is $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$.

16. $\vec{a} = \sqrt{4+1+9} = \sqrt{14}$
    $\vec{b} = \sqrt{1+9+4} = \sqrt{14}$
    $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 3 & 2 \end{vmatrix}$
    $= -11\hat{i} - \hat{j} + 7\hat{k}$
    $\left|\vec{a} \times \vec{b}\right| = \sqrt{(-11)^2 + (-1)^2 + (7)^2}$
    $= \sqrt{171} = 3\sqrt{19}$
    $\sin\theta = \frac{\left|\vec{a}\times\vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|} = \frac{3\sqrt{19}}{\sqrt{14}.\sqrt{14}} = \frac{3}{14}\sqrt{19}$

17. According to the question,
    To prove $[\vec{a}.\ \vec{b} + \vec{c}.\ \vec{d}\ ]$ = $\left[\vec{a} \quad \vec{b} \quad \vec{d}\right] + \left[\vec{a} \quad \vec{c} \quad \vec{d}\right]$.
    LHS = $[\vec{a}.\vec{b} + \vec{c}.\vec{d}] = \vec{a}.\{(\vec{b} + \vec{c}) \times \vec{d}\}$
    by definition of scalar triple product , we get
    $= \vec{a}.(\vec{b} \times \vec{d} + \vec{c} \times \vec{d})$
    $= \vec{a}.(\vec{b} \times \vec{d}) + \vec{a}.(\vec{c} \times \vec{d}) = \left[\vec{a} \quad \vec{b} \quad \vec{d}\right] + \left[\vec{a} \quad \vec{c} \quad \vec{d}\right]$

18. A (1, 1, 2) B(2, 3, 5) C (1, 5, 5)

$\overrightarrow{OA}=\hat{i}+\hat{j}+2\hat{k}$

$\overrightarrow{OB}=2\hat{i}+3\hat{j}+5\hat{k}$

$\overrightarrow{OC}=\hat{i}+5\hat{j}+5\hat{k}$

$\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k}$

$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=4\hat{j}+3\hat{k}$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 0 & 4 & 3\end{vmatrix}$$

$=-6\hat{i}-3\hat{j}+4\hat{k}$

Area of $\Delta ABC=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$=\frac{1}{2}\sqrt{(-6)^2+(-3)^2+4^2}$

$=\frac{1}{2}\sqrt{61}$ sq. unit