Nuclear Reactions (Part - 1)

Q.249. An alpha-particle with kinetic energy T_α =7 .0 MeV is scattered elastically by an initially stationary Li^6 nucleus. Find the kinetic energy of the recoil nucleus if the angle of divergence of the two particles is Θ = 60°.

Ans. Initial momentum of the a particle is $\sqrt{2mT_\alpha}\,\hat{i}$ (where $\hat{i}$ is a unit vector in the incident direction). Final momenta are respectively $\vec{p}_\alpha$ and $\vec{p}_{Li}$ Conservation of momentum reads

$$\vec{p}_\alpha + \vec{p}_{Li} = \sqrt{2mT_\alpha}\,\hat{i}$$

Squaring $p_\alpha^2 + p_{Li}^2 + 2p_\alpha p_{Li}\cos\Theta = 2mT_\alpha$ (1)

where Θ is the angle between $\vec{p}_\alpha$ and $\vec{p}_{Li}$

Also by energy conservation $\frac{p_\alpha^2}{2m} + \frac{p_{L_i}^2}{2M} = T_\alpha$

(m & M are respectively the masses of a particle and Li^6) So

$$p_\alpha^2 + \frac{m}{M}p_{Li}^2 = 2mT_\alpha \quad (2)$$

Substracting (2) from (1) we see that

$$p_{Li}\left[\left(1-\frac{m}{M}\right)p_{Li} + 2p_\alpha\cos\Theta\right] = 0$$

Thus if $p_{Li} \neq 0$

$$p_\alpha = -\frac{1}{2}\left(1-\frac{m}{M}\right)p_{Li}\sec\Theta.$$

Since pα, p_{Li} are both positive number (being magnitudes of vectors) we must have

$$-1 \le \cos\Theta < 0 \quad \text{if} \quad m < M.$$

This being understood, we write

$$\frac{p_{Li}^2}{2M}\left[1+\frac{M}{4m}\left(1-\frac{m}{M}\right)^2 \sec^2\Theta\right] = T_\alpha$$

Hence the recoil energy of the L_i, nucleus is

$$\frac{p_{Li}^2}{2M} = \frac{T_\alpha}{1+\frac{(M-m)^2}{4mM}\sec^2\Theta}$$

As we pointed out above $\Theta \neq 60°$. If we take $\Theta = 120°$, we get recoil energy of Li = 6 MeV

Q.250. A neutron collides elastically with an initially stationary deuteron. Find the fraction of the kinetic energy lost by the neutron (a) in a head-on collision; (b) in scattering at right angles.

Ans. (a) In a head on collision

$$\sqrt{2mT} = p_d + p_n$$

$$T = \frac{p_d^2}{2M} + \frac{p_n^2}{2m}$$

Where p_d and p_n are the momenta of deuteron and neutron after the collision. Squaring

$$p_d^2 + p_n^2 + 2p_d p_n = 2mT$$

$$p_n^2 + \frac{m}{M}p_d^2 = 2mT$$

or since $p_d = 0$ in a head on collisions

$$p_n = -\frac{1}{2}\left(1-\frac{m}{M}\right)p_d.$$

Going back to energy conservation

$$\frac{p_d^2}{2M}\left[1+\frac{M}{4m}\left(1-\frac{m}{M}\right)^2\right] = T$$

So $\frac{p_d^2}{2M} = \frac{4mM}{(m+M)^2} T$

This is the energy lost by neutron. So the fraction of energy lost is

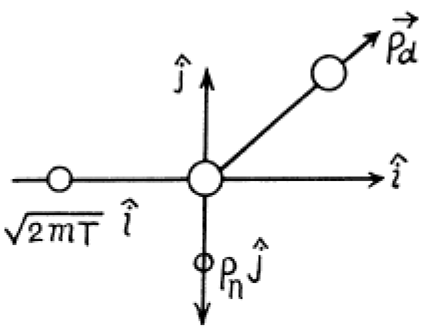
$$\eta = \frac{4mM}{(m+M)^2} = \frac{8}{9}$$

(b) In this case neutron is scattered by 90°. Then we have from the diagram

$$\vec{p_d} = p_n \hat{j} + \sqrt{2mT}\,\hat{i}$$

Then by eneigy conservation

$$\frac{p_n^2 + 2mT}{2M} + \frac{p_n^2}{2m} = T$$



$$\frac{p_n^2 + 2mT}{2M} + \frac{p_n^2}{2m} = T$$

or $$\frac{p_n^2}{2m}\left(1 + \frac{m}{M}\right) = T\left(1 - \frac{m}{M}\right)$$

or $$\frac{p_n^2}{2m} = \frac{M-m}{M+m} \cdot T$$

The energy lost by neutron in then

$$T - \frac{p_n^2}{2m} = \frac{2m}{M+m} T$$

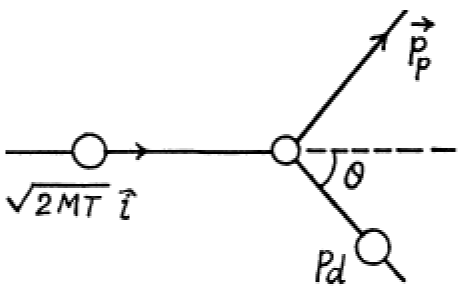
or fraction of energy lost is $\eta = \frac{2m}{M+m} = \frac{2}{3}$

Q.251. Find the greatest possible angle through which a deuteron is scattered as a result of elastic collision with an initially stationary proton.

Ans. From conservation of momentum

$$\sqrt{2MT}\,\hat{i} = \vec{p_d} + \vec{p_p}$$

$$\text{or } p_p^2 = 2MT + p_d^2 - 2\sqrt{2MT}\,p_d \cos\theta$$



From energy conservation

$$T = \frac{p_d^2}{2M} + \frac{p_p^2}{2m}$$

(M = mass of denteron, m = mass of proton)

So $p_p^2 = 2mT - \frac{m}{M} p_d^2$.

Hence $p_d^2\left(1 + \frac{m}{M}\right) - 2\sqrt{2MT}\; p_d \cos\theta + 2(M - m)T = 0$

For real roots $4(2MT)\cos^2\theta - 4 \times 2(M - m)T\left(1 + \frac{m}{M}\right) \geq 0$

$$\cos^2\theta \geq \left(1 - \frac{m^2}{M^2}\right)$$

Hence $\sin^2\theta \leq \frac{m^2}{M^2}$

i.e. $\theta \leq \sin^{-1}\frac{m}{M}$

For deuteron-proton scaltering $\theta_{max} = 30°$.

Q.252. Assuming the radius of a nucleus to be equal to $= 0.13 \sqrt[3]{A}$ pm, where A is its mass number, evaluate the density of nuclei and the number of nucleons per unit volume of the nucleus.

Ans. This problem has a misprint Actually the radius R of a nucleus is given by

$$R = 1{\cdot}3 \sqrt[3]{A}\ fm$$

where $fm = 10^{-15}$ m.

Then the number of nucleous per unit volume

is $$\frac{A}{\frac{4\pi}{3}R^3} = \frac{3}{4\pi} \times (1{\cdot}3)^{-3} \times 10^{+39}\ \text{cm}^{-3} = 1{\cdot}09 \times 10^{38}\ \text{per cc}$$

The corresponding mass density is (1.09 x 10^{-38} x mass of a nucleon) per cc = 1.82 x 10^{11}kg/cc

Q.253. Write missing symbols, denoted by x, in the following nuclear reactions: (a) B^{10} (x, α) Be^{8}
(b) O^{17} (d, n) x; (c) Na^{23} (p, x) Ne^{20}; (d) x (p, n) Ar^{37}.

Ans. (a) The particle x must carry two nucleons and a unit of positive charge.
The reaction is

$$B^{10}(d, \alpha) B_e^8$$

(b) The particle x must contain a proton in addition to the constituents of O^{17}. Thus the reaction is

$$O^{17}(d, n) F^{18}$$

(c) The particle x must carry nucleon number 4 and two units of +ve charge. Thus the particle must be x = α and the reaction is

$$Na^{23}(p, \alpha) Ne^{20}$$

(d) The particle x must carry mass number 37 and have one unit less of positive charge. Thus x = Cl ^{37} and the reaction is

$$Cl^{37}\,(p\,,n)\,Ar^{37}$$

Q.254. Demonstrate that the binding energy of a nucleus with mass number A and charge Z can be found from Eq. (6.6b).

Ans. From the basic formula

$$E_b = Z\,\Delta_H + (A - Z)\,\Delta_n - \Delta$$

We define
AH = mH - 1 amu
An = - 1 amu
A = M - A amu
Then clearly Eb - Z A# + (A - Z) An - A

Q.255. Find the binding energy of a nucleus consisting of equal numbers of protons and neutrons and having the radius one and a half times smaller than that of A1^{27} nucleus.

Ans. The mass number of the given nucleus must be

$$27 \Big/ \left(\frac{3}{2}\right)^3 = 8$$

Thus the nucleus is Be^{8}. Then The binding eneigy is
E b - 4 x 0-00867 + 4 + x 0-00783 - 0-00531 amu
= 0-06069 amu = 56-5 MeV
On using 1 amu = 931 MeV.

Q.256. Making use of the tables of atomic masses, find: (a) the mean binding energy per one nucleon in O^{16} nucleus; (b) the binding energy of a neutron and an alpha-particle in a B^{11} nucleus; (c) the energy required for separation of an O^{16} nucleus into four identical particles.

Ans. (a) Total binding eneigy of Ihe O^{16} nucleus is
E_b = 8 x .00867 + 8 x .00783 + 0.00509 amu
= 0.13709 amu = 127.6 MeV
So B.E. per nucleon is 7.98 Mev/nucleon

(b) B.E. of neutron in B^{11} nucleus

$$= \text{B.E. of } B^{11} - \text{B.E. of } B^{10}$$

(since on removing a neutron from B^{11} we get B^{10})

$$= \Delta_n - \Delta_{B_{11}} + \Delta_{B_{10}} = \cdot 00867 - \cdot 00930 + \cdot 01294$$

= 0.01231 amu = 11.46 MeV
B.E. of (an α-particle in B^{11})
= B.E. of B^{1} - B.E. of Li^{7} - B.E. of α
(since on removing an a from B^{11} we get Li^{7})

$$= -\Delta_{B_{11}} + \Delta_{Li_7} + \Delta_{\alpha}$$

= - 0.00930 + 0.01601 + 0.00260
= 0.00931 amu = 8.67 MeV

(c) This eneigy is

[B.E. of O^{16} + 4 (B.E. of a particles)]

$$= -\Delta_{O^{16}} + 4\,\Delta_{\alpha}$$

= 4 x 0-00260 + 0.00509
= 0.01549 amu - 14.42 MeV

Q.257. Find the difference in binding energies of a neutron and a proton in a B^{11} nucleus. Explain why there is the difference.

Ans. B.E. o f a neutron in B^{11} - B.E. of a proton in B^{11}

$$= (\Delta_n - \Delta_{B^{11}} + \Delta_{B^{10}}) - (\Delta_p - \Delta_{B^{11}} + \Delta_{B_e^{10}})$$

$= \Delta_n - \Delta_p + \Delta_{B^{10}} - \Delta_{B_e^{10}}$ = 0.00867 - 0.00783

+ 0.01294 - 0.01354 = 0.00024 amu = 0.223 MeV
The difference in binding energy is essentially due to the coulomb repulsion between the proton and the residual nucleus Be^{10} which together constitute B^{11}.

Q.258. Find the energy required for separation of a Ne^{20} nucleus into two alpha-particles and a C^{12} nucleus if it is known that the binding energies per one nucleon in Ne^{20}, He^4, and C^{12} nuclei are equal to 8.03, 7.07, and 7.68 MeV respectively.

Ans. Required energy is simply the difference in total binding energies

= B.E. of Ne^{20} - 2 (BE. of He^4) - B.E. of C^{12}

$$= 20\,\varepsilon_{Ne} - 8\,\varepsilon_{\alpha} - 12\,\varepsilon_{C}$$

(ε is binding energy per unit nucleon.) Substitution gives 11.88MeV .

Q.259. Calculate in atomic mass units the mass of (a) a Li^8 atom whose nucleus has the binding energy 41.3 MeV; (b) a C^{10} nucleus whose binding energy per nucleon is equal to 6.04 MeV.

Ans. We have for Li^8

41.3 MeV = 0.044361 amu = $3\Delta_H + 5\Delta_n - \Delta$
Hence Δ = 3 x 0.00783 + 5 x 0.00867 - 0.09436 - 0.02248 amu

(b) For C^{10} 10 x 6.04 = 60.4 MeV
- 0-06488 amu

$$= 6\,\Delta_H + 4\,\Delta_n - \Delta$$

Hence Δ = 6 x 0.00783 + 4 x 0-00867 - 0.06488 = 0.01678 amu
Hence the mass of C^{10} is 10.01678 amu

Q.260. The nuclei involved in the nuclear reaction $A_1 + A_2 \rightarrow \rightarrow A_{3} + A_4$ have the binding energies E_1, E_2, E_3, and E_4. Find the energy of this reaction.

Ans. Suppose M_1 , M_2 , M_3 , M_4 are the rest masses of the nuclei A_1 lf A_2 , A_3 and

A_4participating in the reaction

$$A_1 + A_2 \rightarrow A_3 + A_4 + Q$$

Here Q is the energy released. Then by conservation of energy.

$$Q = c^2 (M_1 + M_2 - M_3 - M_4)$$

Now $M_1 c^2 = c^2 (Z_1 m_H + (A_1 - Z) m_n) - E_1$ etc. and

$Z_1 + Z_2 = Z_3 + Z_4$ (conservation of change)
$A_1 + A_2 = A_3 + A_4$ (conservation of heavy particles)
Hence $Q = (Es + E4) - (Ex + E2)$

Q.261. Assuming that the splitting of a U^{236} nucleus liberates the energy of 200 MeV, find: (a) the energy liberated in the fission of one kilogram of U236 isotope, and the mass of coal with calorific value of 30 kJ/g which is equivalent to that for one kg of U^{235}; (b) the mass of U^{235} i sotope split during the explosion of the atomic bomb with 30 kt trotyl equivalent if the calorific value of trotyl is 4.1 kJ/g.

Ans. (a) the energy liberated in the fission of 1 kg of U^{235} is

$$\frac{1000}{235} \times 6.023 \times 10^{23} \times 200\text{MeV} = 8.21 \times 10^{10}\text{kJ}$$

The mass of coal with equivalent calorific value is

$$\frac{8{\cdot}21 \times 10^{10}}{30000} \text{ kg} = 2.74 \times 10^{6} \text{ kg}$$

(b) The required mass' is

$$\frac{30 \times 10^{9} \times 4{\cdot}1 \times 10^{3}}{200 \times 1{\cdot}602 \times 10^{-13} \times 6{\cdot}023 \times 10^{23}} \times \frac{235}{1000} \text{ kg} = 1{\cdot}49 \text{ kg}$$

Q.262. What amount of heat is liberated during the formation of one gram of He^4 from deuterium H^2? What mass of coal with calorific value of 30 kJ/g is thermally equivalent to the magnitude obtained?

Ans. The reaction is (in effect).

$$H^2 + H^2 \rightarrow He^4 + Q$$

Then

$$Q = 2\Delta_{H^2} - \Delta_{He^4} + Q$$

= 0.02820-0.00260
= 0.02560 amu = 23.8 MeV

Hence the energy released in 1 gm of He^4 is

$$\frac{6 \cdot 023 \times 10^{23}}{4} \times 23.8 \times 16.02 \times 10^{-13} \text{ Joule} = 5.75 \times 10^8 \text{ kJ}$$

This eneigy can be derived from

$$\frac{5 \cdot 75 \times 10^8}{30000} \text{ kg} = 1.9 \times 10^4 \text{ kg of Coal.}$$

Q.263. Taking the values of atomic masses from the tables, calculate the energy per nucleon which is liberated in the nuclear reaction $Li^6 + H^2 \rightarrow 2He^4$. Compare the obtained magnitude with the energy per nucleon liberated in the fission of U^{235} nucleus.

Ans. The energy released in the reaction

$$Li^6 + H^2 \rightarrow 2\,He^4$$

is $\Delta_{Li^6} + \Delta_{H^2} - 2\,\Delta_{He^4}$

= 0.01513 + 0.01410 - 2 x 0.00 260 amu
= 0.02403 amu = 22.37 MeV

$$\frac{22 \cdot 37}{8} = 2.796 \text{ MeV/nucleon.}$$

This should be compared with the value $\frac{200}{235}$ = 0.85 MeV/nucleon

Q.264. Find the energy of the reaction $Li^7 + p \rightarrow 2He^4$ if the binding energies per nucleon in Li^7 and He^4 nuclei are known to be equal to 5.60 and 7.06 MeV respectively.

Ans. The eneigy of reaction

$$Li^7 + p \rightarrow 2\,He^4$$

is, 2 x B.E. of He^4 - B.E. of Li^7
$= 8\varepsilon\alpha - 7\varepsilon_{Li} = 8 \times 7.06 - 7 \times 5.60 = 17.3$ MeV

Q.265. Find the energy of the reaction N^{14} (α, p) O^{17} if the kinetic energy of the incoming alpha-particle is $T_\alpha = 4.0$ MeV and the proton outgoing at an angle $\theta = 60°$ to the motion direction of the alpha-particle has a kinetic energy $T_p = 2.09$ MeV.

Ans. The reaction is $N^{14}(a, p)O^{17}$
It is given that (in the Lab frame where N^{14} is at rest) $T_\alpha = 4.0$MeV. The momentum of incident α particle is
The momentum of outgoing proton is

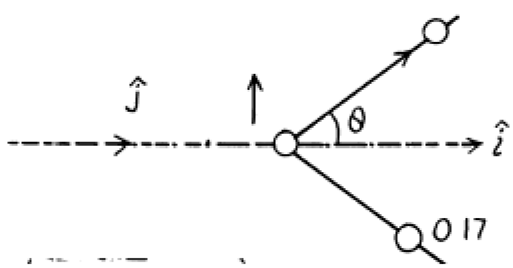
$$\sqrt{2 m_\alpha T_\alpha}\, \hat{i} = \sqrt{2 \eta_\alpha m_0 T_\alpha}\, \hat{i}$$

$$\sqrt{2 m_p T_p}\, (\cos\theta\, \hat{i} + \sin\theta\, \hat{j})$$

$$= \sqrt{2 \eta_p m_0 T_p}\, (\cos\theta\, \hat{i} + \sin\theta\, \hat{j})$$

Where $\eta_p = \frac{m_p}{m_0},\ \eta_\alpha = \frac{m_\alpha}{m_0},$

and m_o is the mass of O^{17}
The momentum of O^{17} is



$$\left(\sqrt{2 \eta_\alpha m_0 T_\alpha} - \sqrt{2 \eta_p m_0 T_p}\cos\theta\right) \hat{i}$$

$$- \sqrt{2 m_0 \eta_p T_p}\sin\theta\, \hat{j}$$

By energy conservation (conservation of energy including rest mass energy and kinetic energy)

$$M_{14} c^2 + M_\alpha c^2 + T_\alpha$$
$$= M_p c^2 + T_p + M_{17} c^2$$

$$+ \left[\left(\sqrt{\eta_\alpha T_\alpha} - \sqrt{\eta_p T_p}\cos\theta\right)^2 + \eta_p T_p \sin^2\theta + \eta_p T_p \sin^2\theta \right]$$

Hence by definition of the Q of reaction

$$Q = M_{14} c^2 + M_\alpha c^2 - M_p c^2 - M_{17} c^2$$

$$= T_p + \eta_\alpha T_\alpha + \eta_p T_p - 2\sqrt{\eta_p \eta_\alpha T_\alpha T_p} \times \cos\theta - T_\alpha$$

$$= (1 + \eta_p) T_p + T_\alpha (1 - \eta_\alpha)$$

$$- 2\sqrt{\eta_p \eta_\alpha T_\alpha T_p} \cos\theta = -1{\cdot}19 \text{ MeV}$$

Q.266. Making use of the tables of atomic masses, determine the energies of the following reactions: (a) Li^7 (p, n) Be^7; (b) Be^9 (n,γ) Be^{10}; (c) Li^7 (α, n) B^{10}); (d) O^{16} (d, α) N^{14}.

Ans. (a) The reaction is $Li^7 (p, n) Be^7$ and the energy of reaction is

$$Q = (M_{Be^7} + M_{Li^7}) c^2 + (M_p - M_n) c^2$$

$$= (\Delta_{Li_7} - \Delta_{Be^7}) c^2 + \Delta_p - \Delta_n$$

$$= [0{\cdot}01601 + 0{\cdot}00783 - 0{\cdot}01693 - 0{\cdot}00867] \text{ amu} \times c^2$$

=-1.64 MeV

(b) The reaction is $Be^9 (n, \gamma) Be^{10}$.

Mass of γ is taken zero. Then

$$Q = (M_{Be^9} + M_n - M_{Be^{10}}) c^2$$

$$= (\Delta_{Be^9} + \Delta_n - \Delta_{Be^{10}}) c^2$$

= (0.01219 + 0.00867 - 0.01354) amu × c^2

= 6.81 MeV

(c) The reaction is Li^2 (α,n) B^{10}. The energy is

$$Q = (\Delta_{Li^7} + \Delta_\alpha - \Delta_n - \Delta_{B^{10}}) c^2$$

= (0.01601 + 0.00260 - 0.00867 - .01294) amu x c^2
= - 2.79 MeV

(d) The reaction is O^{16} (d , α) N^{14} The energy of reaction is

$$Q = (\Delta_{O^{16}} + \Delta_d - \Delta_\alpha - \Delta_{N}^{14})\, c^2$$

= (- 0.00509 + 0.01410 - 0.00260 - 0.00307) amu x c^2
= 3.11 MeV

Q.267. Making use of the tables of atomic masses, find the velocity with which the products of the reaction B^{10} (n, α) Li^7 come apart; the reaction proceeds via interaction of very slow neutrons with stationary boron nuclei.

Ans. The reaction is B^{10} (n, ct) Li^1. The energy of the reaction is

$$Q = (\Delta_{B^{10}} + \Delta_n - \Delta_\alpha - \Delta_{Li^7})\, c^2$$

= (0.01294 + 0-00867 - 0.00260 - 0.01601) amu x c^2
= 2-79 MeV

Since the incident neutron is very slow and B^{10} is stationary, the final total momentum must also be zero. So the reaction products must emerge in opposite directions. If their speeds are, repectively, v_a and v_{Li}

then $4v_a = 7v_{Li}$

and $\frac{1}{2}(4\,v_\alpha^2 + 7\,v_{Li}^2) \times 1{\cdot}672 \times 10^{-24}$ = 2.79 x 1.602 x 10^{-6}

So $\frac{1}{2} \times 4\,v_\alpha^2 \left(1 + \frac{4}{7}\right) =$ 2.70 x 10^{18} cm^2/s^2

or $v_a = 9.27 \times 10^6$ m/s

Then $v_{Li} = 5.3 \times 10^6$ m/s

Q.268. Protons striking a stationary lithium target activate a reaction Li^7 (p, n) Be^7. At what value of the proton's kinetic energy can the resulting neutron be stationary?

Ans. Q of this reaction (Li^7(p, n)Be^7) was calculated in problem 266 (a). If is - 1.64 MeV.
We have by conservation of momentum and energy $P_p = P_{Be}$ (since initial Li and final neutron are both at rest)

$$\frac{p_p^2}{2\,m_p} = \frac{p_{Be}^2}{2\,m_{Li}} + 1{\cdot}64$$

Then $\frac{P_p^2}{2\,m_p}\left(1-\frac{m_p}{m_{Be}}\right) = 1{\cdot}64$

Hence $T_p = \frac{p_p^2}{2\,m_p} = \frac{7}{6} \times 1{\cdot}64\,\text{MeV}$ = 1.91MeV

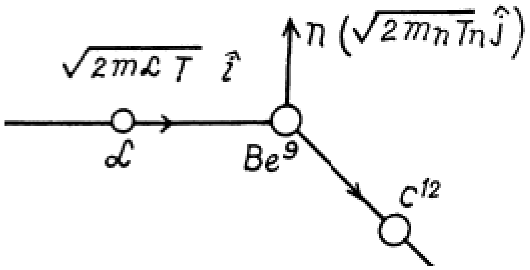
Q.269. An alpha particle with kinetic energy T = 5.3 MeV initiates a nuclear reaction Be^9 (α, n) C^{12} with energy yield Q = + 5.7 MeV. Find the kinetic energy of the neutron outgoing at right angles to the motion direction of the alpha-particle.

Ans. It is understood that Be^9 is initially at rest. The moment of the outgoing neutron is

$\sqrt{2\,m_n T_n}\,\hat{j}$. The momentum of C^{12} is

$\sqrt{2\,m_\alpha T}\,\hat{i} - \sqrt{2\,m_n T_n}\,\hat{j}$

Then by energy conservation



$$T + Q = T_n + \frac{2\,m_\alpha T + 2\,m_n T_n}{2\,m_{ci}}$$

(m_c is the mass of C^{12})

Thus $T_n = \frac{m_c\,(T+Q) - m_\alpha T}{m_c + m_n}$

$$= \frac{(m_c - m_\alpha)\,T + m_c Q}{m_c + m_n} \quad \frac{Q + \left(1 - \frac{m_\alpha}{m_c}\right)T}{1 + \frac{m_n}{m_c}} = 8{\cdot}52\,\text{MeV}$$

Q.270. Protons with kinetic energy T =1.0 MeV striking a lithium target induce a nuclear reaction p + Li^7 → 2He^4. Find the kinetic energy of each alpha-particle and the angle of their divergence provided their motion directions are symmetrical with respect to that of incoming protons.

Ans. The Q value of the reaction $Li^7(p, \alpha) He^4$ is

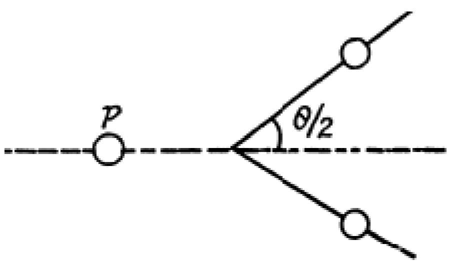
$$Q = (\Delta_{Li^7} + \Delta_H - 2\,\Delta_{He^4})\,c^2$$

= (0.01601 + 0.00783 - 0.00520) amu x c^2
= 0.01864 amu x c^2 - 17.35 MeV
Since the direction of He^4 nuclei is symmetrical, their momenta must also be equal. Let T be the K.E. of each He^4. Then

$$p_p = 2\sqrt{2\,m_{He}\,T}\,\cos\frac{\theta}{2}$$

(p_p is the momentum of proton). Also

$$\frac{p_p^2}{2\,m_p} + Q = 2\,T = T_p + Q$$



Hence $$T_p + Q = 2\,\frac{p_p^2 \sec^2\frac{\theta}{2}}{8\,m_{He}}$$

$$= T_p\,\frac{m_p}{2\,m_{He}}\sec^2\frac{\theta}{2}$$

Hence $$\cos\frac{\theta}{2} = \sqrt{\frac{m_p}{2\,m_{He}}\,\frac{T_p}{T_p + Q}}$$

Substitution gives $\theta = 170.53°$

Also $$T = \frac{1}{2}(T_p + Q) = 9{\cdot}18\ \text{MeV}.$$

Nuclear Reactions (Part - 2)

Q.271. A particle of mass m strikes a stationary nucleus of mass M and activates an endoergic reaction. Demonstrate that the threshold (minimal) kinetic energy required to initiate this reaction is defined by Eq. (6.6d).

Ans. Energy required is minimum when the reaction products all move in the direction of the incident particle with the same velocity (so that the combination is at rest in the centre of mass frame). We then have

$$\sqrt{2mT_{th}} = (m+M)\,v$$

(Total mass is constant in the nonrelativistic limit).

$$T_{th} - |Q| = \frac{1}{2}(m+M)\,v^2 = \frac{m\,T_{th}}{m+M}$$

or $$T_{th}\frac{M}{m+M} = |Q|$$

Hence $$T_{th} = \left(1+\frac{m}{M}\right)|Q|$$

Q.272. What kinetic energy must a proton possess to split a deuteron H^2 whose binding energy is E_b = 2.2 MeV?

Ans. The result of the previous problem applies and we End that energy required to split a deuteron is

$$T \geq \left(1+\frac{M_p}{M_d}\right)E_b = 3{\cdot}3\ \text{MeV}$$

Q.273. The irradiation of lithium and beryllium targets by a monoergic stream of protons reveals that the reaction Li^7(p, n)Be^7 -- 1.65 MeV is initiated whereas the reaction Be^9 (p, n) B^9 — 1.85 MeV does not take place. Find the possible values of kinetic energy of the protons.

Ans. Since the reaction $Li^7\,(p,n)\,Be^7\,(Q = -1{\cdot}65\ \text{MeV})$ is initiated, the incident proton energy must be

$$\geq \left(1 + \frac{M_p}{M_{Li}}\right) \times 1{\cdot}65 = 1{\cdot}89 \text{ MeV}$$

since the reaction $Be^9 (p, n) B^9 (Q = -1{\cdot}85 \text{ MeV})$ is not initiated,

$$T \leq \left(1 + \frac{M_p}{M_{Ve}}\right) \times 1{\cdot}85$$ = 2.06 MeV Thus 1.89 MeV $\leq T_p \leq$ 2.06 MeV

Q.274. To activate the reaction (n, a) with stationary B^{11} nuclei, neutrons must have the threshold kinetic energy T_{th} = 4.0 MeV. Find the energy of this reaction.

Ans. We have $= \left(1 + \frac{m_n}{M_{B^{11}}}\right) |Q|$

or $Q = -\frac{11}{12} \times 4 \text{ MeV} =$ -3.67 MeV

Q.275. Calculate the threshold kinetic energies of protons required to activate the reactions (p, n) and (p, d) with Li^7 nuclei.

Ans. The Q o f the reaction Li^7 (p, n) Be^7 was calculated in problem 266 (a). It is - 1.64 MeV Hence, the threshold K.E. of protons for initiating this reaction is

$$T_{th} = \left(1 + \frac{m_p}{m_{Li}}\right) |Q| = \frac{8}{7} \times 1{\cdot}64$$

For the reaction Li^7 (p, d) Li^6

we find $Q = (\Delta_{Li^7} + \Delta_M - \Delta_d - \Delta_{Li^6}) c^2$

= (0.01601 + 0.00783 - 0.01410 - 0.01513) amu x c^2
= - 5 02 MeV

The threshold proton energy for initiating this reaction is

$$T_{th} = \left(1 + \frac{m_p}{m_{Li^7}}\right) \times |Q| =$$ 5.73 MeV

Q.276. Using the tabular values of atomic masses, find the threshold kinetic energy of an alpha particle required to activate the nuclear reaction Li^7 (α, n) Be^9. What is the velocity of the B^{10} nucleus in this case?

Ans. The Q of Li^7 (a, n) B^{10} was calculated in problem 266 (c). It is Q = 2.79 MeV Then the threshold energy of a-particle is

$$T_{th} = \left(1 + \frac{m_\alpha}{m_{Li}}\right)|Q| = \left(1 + \frac{4}{7}\right)2{\cdot}79 = 4.38\text{MeV}$$

The velocity of B^{10} in this case is simply the volocity of centre of mass

$$v = \frac{\sqrt{2 m_\alpha T_{th}}}{m_\alpha + m_{Li}} = \frac{1}{1 + \frac{m_{Li}}{m_\alpha}}\sqrt{\frac{2 T_{th}}{m_\alpha}}$$

This is because both B^{10} and n are at rest in the CM frame at theshold. Substituting the values of various quantities
we get v = 5.27 x 10^6 m/s

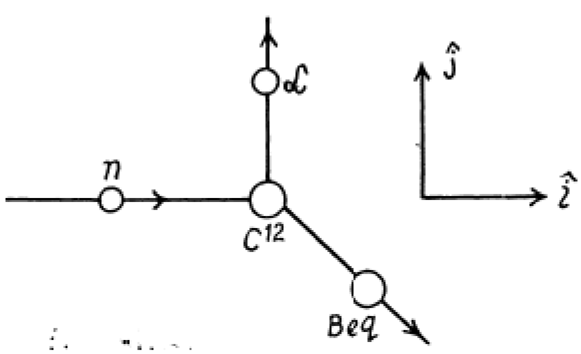
Q.277. A neutron with kinetic energy T = 10 MeV activates a nuclear reaction C^{12} (n, α) Be^9 whose threshold is T_{th} = 6.17 MeV. Find the kinetic energy of the alpha-particles outgoing at right angles to the incoming neutrons' direction.

Ans. The momentum of incident neutron is $\sqrt{2 m_n T}\ \hat{i}$, that of α particle is $\sqrt{2 m_\alpha T_\alpha}\ \hat{j}$ and of

Be^9 is
$$-\sqrt{2 m_\alpha T_\alpha}\ \hat{j} + \sqrt{2 m_n T}\ \hat{i}$$

By conservation of energy

$$T = T_\alpha + \frac{m_\alpha T_\alpha + m_n T}{M} + |Q|$$



(M is the mass of Be^9). Thus

$$T_\alpha = \left[T\left(1-\frac{m_n}{M}\right)-|Q|\right]\frac{M}{M+m_\alpha}.$$

Using $T_{th} = \left(1+\frac{m_n}{M}\right)|Q|$

we get

$$T_\alpha = \frac{M}{M+m_\alpha}\left[\left(1-\frac{m_n}{M}\right)T-\frac{T_{th}}{1+\frac{m_n}{M}}\right]$$

M' is the mass of C^{12} nucleus.

or $T_\alpha = \frac{1}{M+m_\alpha}\left[(M-m_n)\,T-\frac{M M'}{M'+m_n}T_{th}\right]$ = 2.21MeV

Q.278. How much, in per cent, does the threshold energy of gamma quantum exceed the binding energy of a deuteron (E_b = 2.2 MeV) in the reaction Y + H^2 → n + p?

Ans. The formula of problem 6.271 does not apply here because the photon is always

reletivistic. At threshold, the energy of the photon E_y implies a momentum $\frac{E_\gamma}{c}$. The velocity of centre of mass with respect to the rest frame of initial H^2 is

$$\frac{E_\gamma}{(m_n+m_p)\,c}$$

Since both n & p are at rest in CM frame at threshold, we write

$$E_\gamma = \frac{E_\gamma^2}{2\,(m_n+m_p)\,c^2}+E_b$$

by conservation of energy. Since the first term is a small correction, we have

$$E_\gamma = E_b + \frac{E_b^2}{2\,(m_n+m_p)\,c^2}$$

Thus $\frac{\delta E}{E_b} = \frac{E_b}{2(m_n + m_p)c^2} = \frac{2 \cdot 2}{2 \times 2 \times 938} = 5{\cdot}9 \times 10^{-4}$

or nearly 0.06%

Q.279. A proton with kinetic energy T = 1.5 MeV is captured by a deuteron H^2. Find the excitation energy of the formed nucleus.

Ans. The reaction is

$$p + d \rightarrow He^3$$

Excitation energy of He^3 is just the energy available in centre of mass. The velocity of the centre of mass is

$$\frac{\sqrt{2 m_p T_p}}{m_p + m_d} = \frac{1}{3}\sqrt{\frac{2 T_p}{m_p}}$$

In the CM frame, the kinetic energy available is ($m_d \approx 2 m_p$)

$$\frac{1}{2} m_p \left(\frac{2}{3}\sqrt{\frac{2 T_p}{m_p}}\right)^2$$

$$+ \frac{1}{2} 2 m_p \left(\frac{1}{3}\sqrt{\frac{2 T}{m_p}}\right)^2 = \frac{2T}{3}$$

The total energy available is then $Q + \frac{2T}{3}$

where $Q = c^2 (\Delta_n + \Delta_d - \Delta^3_{He})$

= c2 x (0.00783 + 0.01410 - 0.01603) amu
= 5.49 McV
Finally E = 6.49 MeV.

Q.280. The yield of the nuclear reaction C^{13} (d, n)N^{14} has maximum magnitudes at the following values of kinetic energy T_1 of bombarding deuterons: 0.60, 0.90, 1.55, and 1.80 MeV. Making use of the table of atomic masses, find the corresponding energy levels of the transitional nucleus through which this reaction proceeds.

Ans. The reaction is

$$d + C^{13} \rightarrow N^{15*} \rightarrow n + N^{14}$$

Maxima of yields deteimine the eneigy levels of N^{15}*. As in the previous problem the excitation eneigy is

$$E_{exc} = Q + E_K$$

where E_K = available kinetic eneigy. This is found is as in the previous problem. The velocity of the centre of mass is

$$\frac{\sqrt{2\, m_d T_i}}{m_d + m_c} = \frac{m_d}{m_d + m_c}\sqrt{\frac{2\, T_i}{m_d}}$$

so $$E_K = \frac{1}{2} m_d \left(1 - \frac{m_d}{m_d + m_c}\right)^2 \frac{2\, T_i}{m_d} + \frac{1}{2} m_c \left(\frac{m_d}{m_d + m_c}\right)^2 \frac{2\, T_i}{m_d} = \frac{m_c}{m_d + m_c} T_i$$

Q is the Q value for the ground state of N^{15} : We have

$$Q = c^2 \times (\Delta_d + \Delta_{C^{13}} - \Delta_{N^{15}})$$

= c^2 x (0.01410 + 0.00335 - 0.00011) amu
= 16.14 MeV
The excitation energies then are
16.66 MeV, 16.92 MeV
17.49 MeV and 17.70 MeV.

6.281. A narrow beam of thermal neutrons is attenuated η = 360 times after passing through a cadmium plate of thickness d = 0.50 mm. Determine the effective cross-section of interaction of these neutrons with cadmium nuclei.
Ans. We have the relation

$$\frac{1}{\eta} = e^{-n\sigma d}$$

Here $\frac{1}{\eta}$ attenuation factor

n = no. of Cd nuclei per unit volume
σ = effective cross section

d = thickness of the plate

Now $n = \frac{\rho N_A}{M}$

(p = density, M = Molar weight of Cd, N_A = Avogadro number.)

Thus $\sigma = \frac{M}{\rho N_A d} \ln \eta = 2{\cdot}53 \text{ kb}$

Q.282. Determine how many times the intensity of a narrow beam of thermal neutrons will decrease after passing through the heavy water layer of thickness d = 5.0 cm. The effective cross-sections of interaction of deuterium and oxygen nuclei with thermal neutrons are equal to σ_1 = 7.0 b and σ_2 --- 4.2 b respectively.

Ans. Here

$$\frac{1}{\eta} = e^{-(n_2 \sigma_2 + n_1 \sigma_1) d}$$

where 1 refers to O^1 and 2 to D nuclei
Using $n_2 = 2n$, $n_1 = n$ = concentration of O nuclei in heavy water we get

$$\frac{1}{\eta} = e^{-(2\sigma_2 + \sigma_1) n d}$$

Now using the data for heavy water

$$n = \frac{1{\cdot}1 \times 6{\cdot}023 \times 10^{23}}{20} = 3.313 \times 10^{22} \text{ per cc}$$

Thus substituting the values

$$\eta = 20{\cdot}4 = \frac{I_0}{I}.$$

Q.283. A narrow beam of thermal neutrons passes through a plate of iron whose absorption and scattering effective cross-sections are equal to σ_a = 2.5b and σ_8 = 11b respectively. Find the fraction of neutrons quitting the beam due to scattering if the thickness of the plate is d = 0.50 cm.

Ans. In traversing a distance d the fraction which is either scattered or absorbed is clearly

$$1 - e^{-n(\sigma_s + \sigma_a)d}$$

by the usual definition of the attenuation factor. Of this, the fraction scattered is (by definition of scattering and absorption cross section)

$$w = \left\{1 - e^{-n(\sigma_s + \sigma_a)d}\right\} \frac{\sigma_s}{\sigma_s + \sigma_a}$$

In iron $n = \frac{\rho \times N_A}{M} = 8{\cdot}39 \times 10^{22}$ per cc

Substitution gives w = 0.352

Q.284. The yield of a nuclear reaction producing radionuclides may be described in two ways: either by the ratio w of the number of nuclear reactions to the number of bombarding particles, or by the quantity k, the ratio of the activity of the formed radionuclide to the number of bombarding particles, Find: (a) the half-life of the formed radionuclide, assuming w and k to be known; (b) the yield w of the reaction $Li^7(p, n)Be^7$ if after irradiation of a lithium target by a beam of protons (over t = 2.0 hours and with beam current I = 10μA) the activity of Be^7 became equal to A = = $1.35.10^8$ dis/s and its half-life to T = 53 days.

Ans. (a) Assuming of course, that each reaction produces a radio nuclide of the same type, the decay constant α of the radionuclide is k/w . Hence

$$T = \frac{\ln 2}{\lambda} = \frac{w}{k} \ln 2$$

(b) number of bombarding particles is : $\frac{It}{e}$

(e = charge on proton). Then the number of Be^7 produced is : $\frac{It}{e} w$

If λ = decay constant of $Be^7 = \frac{\ln 2}{T}$, , then the activity is $A = \frac{It}{e} w \cdot \frac{\ln 2}{T}$

Hence $w = \frac{eAT}{It \ln 2} = 1{\cdot}98 \times 10^{-3}$

Q.285. Thermal neutrons fall normally on the surface of a thin gold foil consisting of stable Au^{197} nuclide. The neutron flux density is J = $1.0.10^{10}$ part./(s- cm^2). The mass of the foil is m = 10 mg. The neutron capture produces beta-active Au^{188} nuclei with half-life T = 2.7 days. The effective capture cross-section is σ = 98 b. Find: (a) the irradiation time after which the number of Au^{187} nuclei decreases

by = 1.0%; (b) the maximum number of Au^{198}nuclei that can be formed during protracted irradiation.

Ans. (a) Suppose - N_o. of Au^{197} nuclei in the foil. Then the number of Au^{197} nuclei transformed in time t is

$$N_0 \cdot J \cdot \sigma \cdot t$$

For this to equal ηN_0, we must have

$$t = \eta / (J \cdot \sigma)$$ = 323 years

(b) Rate of formation of the Au^{198} nuclei is N_0 • J • a per sec
and rate of decay is λn, where n is the number of Au^{198} at any in stant
Thus

$$\frac{dn}{dt} = n_0 \cdot J \cdot \sigma - \lambda n$$

The maximum number of Au^{198} is clearly

$$n_{max} = \frac{N_0 J \sigma}{\lambda} = \frac{N_0 J \sigma T}{\ln 2}$$

because if n is smaller, $\frac{dn}{dt} > 0$ and n increase further and if n is larger

$\frac{dn}{dt} < 0$ and n will decrease. (Actually n_{max} is approached steadily as $t \to \infty$)

Substitution give susing $N_0 = 3{\cdot}057 \times 10^{19}$, $n_{max} = 1{\cdot}01 \times 10^{13}$

Q.286. A thin foil of certain stable isotope is irradiated by thermal neutrons falling normally on its surface. Due to the capture of neutrons a radionuclide with decay constant λ appears. Find the law describing accumulation of that radionuclide N (t) per unit area of the foil's surface. The neutron flux density is J, the number of nuclei per unit area of the foil's surface is n, and the effective crosssection of formation of active nuclei is α.

Ans. Rate of formation of the radionuclide is n.J.σ per unit area per-sec. Rate of decay is λN.
Thus

$\frac{dN}{dt} = n \cdot J \cdot \sigma - \lambda N$ per unit area per second

Then $\left(\frac{dN}{dt} + \lambda N\right) e^{\lambda t} = n \cdot J \cdot \sigma \, e^{\lambda t}$ or $\frac{d}{dt}(N e^{\lambda t}) = n \cdot J \cdot \sigma \cdot e^{\lambda t}$

Hence $N e^{\lambda t} = \text{Const} + \frac{n \cdot J \cdot \sigma}{\lambda} e^{\lambda t}$

The number of radionuclide at t = 0 when the process starts is zero. So constant $= -\frac{n \cdot J \cdot \sigma}{\lambda}$

Then $N = \frac{n \cdot J \cdot \sigma}{\lambda}(1 - e^{-\lambda t})$

Q.287. A gold foil of mass m = 0.20g was irradiated during t = 6.0 hours by a thermal neutron flux falling normally on its surface. Following ζ = 12 hours after the completion of irradiation the activity of the foil became equal to A = 1.9.10[7] dis/s. Find the neutron flux density if the effective cross-section of formation of a radioactive nucleus is σ = 96b, and the half-life is equal to T = 2.7 days.

Ans. We apply the formula of the previous problem except that have N = no. of radio nuclide and no. of host nuclei originally.

Here $n = \frac{0 \cdot 2}{197} \times 6 \cdot 023 \times 10^{23} =$ 6.115 $\times 10^{20}$

Then after time t $N = \frac{n \cdot J \cdot \sigma \cdot T}{\ln 2}\left(1 - e^{-\frac{t \ln 2}{T}}\right)$

T = half life of the radionuclide.

After the source of neutrons is cut off the activity after time T will be

$$A = \frac{n \cdot J \cdot \sigma \cdot T}{\ln 2}(1 - e^{-t \ln 2/T}) \; e^{-\tau \ln 2/T \times \frac{\ln 2}{T}} = n \cdot J \cdot \sigma \, (1 - e^{-t \ln 2/T}) \, e^{-\tau \ln 2/T}$$

Thus $J = A e^{\tau \ln 2/T} \big/ n \sigma (1 - e^{-t \ln 2/T}) = 5 \cdot 92 \times 10^9 \text{ part/cm}^2 \cdot \text{s}$

Q.288. How many neutrons are there in the hundredth generation if the fission process starts with N_o = 1000 neutrons and takes place in a medium with multiplication constant k = 1.05?

Ans. No. of nuclei in the first generation = No .of nuclei initially = N_0
N_0 in the second generation = N_0 x multiplication factor = $N_0.k$

N_0 in the the 3rd generation $= N_0 \cdot k \cdot k = N_0 k^2$

N_0 in the nth generation $= N_0 k^{n-1}$

Substitution gives 1.25 x 10^5 neutrons

Q.289. Find the number of neutrons generated per unit time in a uranium reactor whose thermal power is P = 100 MW if the average number of neutrons liberated in each nuclear splitting is v = 2.5. Each splitting is assumed to release an energy E = 200 MeV.

Ans. N_o- of fissions per unit time is clearly P/E. Hence no. of neutrons produced per unit time to

$\frac{\nu P}{E}$. Substitution gives 7.80 x 10^{18} neutrons/sec

Q.290. In a thermal reactor the mean lifetime of one generation of thermal neutrons is ζ = 0.10 s. Assuming the multiplication constant to be equal to k = 1.010, find: (a) how many times the number of neutrons in the reactor, and consequently its power, will increase over t = 1.0 min;
(b) the period T of the reactor, i.e. the time period over which its power increases e-fold.

Ans. (a) This number is k^{n-1} where n = no. of generations in time t = t/T
Substitution gives 388.

(b) We write $k^{n-1} = e^{\left(\frac{T}{\tau}-1\right)\ln k} = e$

or $\frac{T}{\tau} - 1 = \frac{1}{\ln k}$ and $T = \tau\left(1 + \frac{1}{\ln k}\right) = 10{\cdot}15 \text{ sec}$