Sample Question Paper - 37
Mathematics-Standard (041)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *All questions are compulsory.*
3. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
4. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
5. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Calculate the mean for the following frequency distribution :

Class	10-30	30-50	50-70	70-90	90-110
Frequency	15	18	25	10	2

2. A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is recast into a hollow cylinder of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

OR

If the diameter of cross-section of a wire is decreased by 5%, how much percent will the length be increased so that volume remains the same?

3. The difference of mother's age and her daughter's age is 21 years and the twelfth part of the product of their ages is less than the mother's age by 18 years. Find their ages.

4. If the numbers $2n - 1$, $3n + 2$ and $6n - 1$ are in A.P., then find n and hence find the numbers.

OR

Which term of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ...$ is the first negative term?

5. Find the value of k for which the equation $x^2 + k(2x + k - 1) + 2 = 0$ has real and equal roots.

6. Two concentric circles of radii a and b where $a > b$, are given, find the length of a chord of the larger circle which touches the other circle.

SECTION - B

7. At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A.

OR

The angles of depression of the top and bottom of a 12 m tall building, from the top of a multi-storeyed building are 30° and 60° respectively. Find the height of the multi-storeyed building.

8. The following table gives the literacy rate of 40 cities :

Literacy rate (in %)	30-40	40-50	50-60	60-70	70-80	80-90
Number of cities	6	7	10	6	8	3

Find the modal literacy rate.

9. Draw a circle of radius 3.5 cm. Draw two tangents to the circle which are perpendicular to each other.

10. Find the median of the following data :

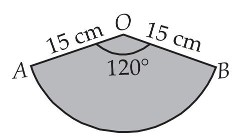
Marks	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Number of students	5	15	25	20	7	8	10

SECTION - C

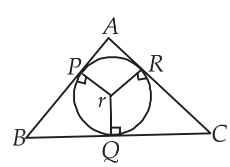
11. The height of a cone is 5 m. Find the height of a cone whose volume is sixteen times its volume and radius equal to its diameter.

OR

A sector of a circle of radius 15 cm has the angle 120°. It is rolled up so that two bounded radii are joined together to form a cone. Find the volume of the cone. $\left(\text{Take } \pi = \frac{22}{7}\right)$



12. In the given fig., ΔABC is circumscribed touching the circle at P, Q, R if $AP = 4$ cm, $BP = 6$ cm, $AC = 12$ cm, then find radius of circle.



Case Study - 1

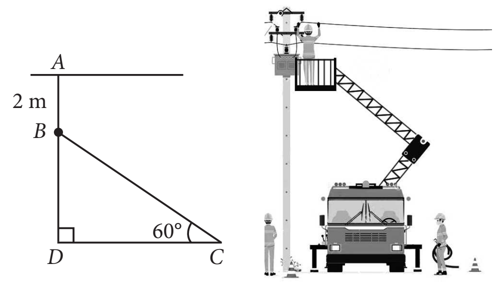
13. Do you know, we can find A.P. in many situations in our day-to-day life. One such example is a tissue paper roll, in which the first term is the diameter of the core of the roll and twice the thickness of the paper is the common difference. If the sum of first n rolls of tissue on a roll is $S_n = 0.1n^2 + 7.9n$, then answer the following questions.

(i) Find S_{n-1}.

(ii) What is the diameter of roll when one tissue sheet is rolled over it?

Case Study - 2

14. An electrician has to repair an electric fault on the pole of height of 8 m. He needs to reach a point 2 m below the top of the pole to undertake the repair work.



Based on the above information, answer the following questions.

(i) What should be the length of ladder, so that it makes an angle of 60° with the ground?

(ii) Find the distance between the foot of ladder and pole.

Solution

MATHEMATICS STANDARD 041

Class 10 - Mathematics

1. The frequency distribution table from the given data can be drawn as :

Class	Class marks (x_i)	Frequency (f_i)	$f_i x_i$
10-30	20	15	300
30-50	40	18	720
50-70	60	25	1500
70-90	80	10	800
90-110	100	2	200
		$\Sigma f_i = 70$	$\Sigma f_i x_i = 3520$

$\therefore$ Mean $= \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{3520}{70} = 50.286$

2. Volume of iron = $(440 \times 260 \times 100)$ cm^3

Internal radius of the pipe = 30 cm

External radius of the pipe = (30 + 5)cm = 35 cm

Let the length of the pipe be h cm.

Volume of iron in the pipe

= (External volume) – (Internal volume)

$= [\pi(35)^2 h - \pi(30)^2 h]\text{cm}^3 = (325\pi h)\text{cm}^3$

$\therefore\ 325\pi h = 440 \times 260 \times 100$

$\Rightarrow\ \text{length}(h) = \left(\dfrac{440 \times 260 \times 100 \times 7}{325 \times 22}\right)\text{cm}$

= 11200 cm = 112 m

OR

Let r be the radius of cross-section of wire and h be its length. Then, volume $= \pi r^2 h$ ...(i)

5% of diameter of cross-section $= \dfrac{5}{100} \times 2r = \dfrac{r}{10}$

$\therefore$ New diameter $= 2r - \dfrac{r}{10} = \dfrac{19r}{10}$

$\Rightarrow$ New radius $= \dfrac{19r}{20}$

Let the new length be h_1. Then,

$$\text{volume} = \pi\left(\frac{19r}{20}\right)^2 h_1 \qquad \text{...(ii)}$$

From (i) and (ii), we have

$$\pi r^2 h = \pi\left(\frac{19r}{20}\right)^2 h_1 \Rightarrow h_1 = \frac{400}{361}h$$

$\therefore$ Increase in length $= h_1 - h = \dfrac{400h}{361} - h = \dfrac{39h}{361}$

Percentage increase in length

$$= \frac{h_1 - h}{h} \times 100 = \frac{39h}{h \times 361} \times 100 = \frac{3900}{361} = 10.8\%$$

Hence, the length of the wire increased by 10.8%.

3. Let the age of mother = x years

and age of daughter = $(x - 21)$ years

Also, $\dfrac{x(x-21)}{12} = x - 18$

$\Rightarrow\ x^2 - 21x = 12x - 216 \Rightarrow x^2 - 33x + 216 = 0$

$\Rightarrow x^2 - 24x - 9x + 216 = 0 \Rightarrow x(x - 24) - 9(x - 24) = 0$

$\Rightarrow\ (x - 24)(x - 9) = 0 \Rightarrow x = 24$ or $x = 9$

$\therefore\ x = 24$ [$\because x = 9$ is not possible]

Hence, mother's age = 24 years

Her daughter's age = (24 – 21) years = 3 years.

4. Let $2n - 1$, $3n + 2$ and $6n - 1$ are in A.P.

$\therefore\ (3n + 2) - (2n - 1) = (6n - 1) - (3n + 2)$

$\Rightarrow\ n + 3 = 3n - 3 \Rightarrow 6 = 2n \Rightarrow n = 3$

$\therefore$ Numbers are : $2(3) - 1 = 5$, $3(3) + 2 = 11$ and $6(3) - 1 = 17$.

OR

Clearly, given sequence is an A.P. in which $a = 20$ and

$d = 19\dfrac{1}{4} - 20 = \dfrac{77}{4} - 20 = \dfrac{-3}{4}$

Let n^{th} term of the given A.P. be the first negative term.

i.e., $a_n < 0$

$\Rightarrow\ a + (n - 1)d < 0 \Rightarrow 20 + (n-1)\left(\dfrac{-3}{4}\right) < 0$

$\Rightarrow\ 80 - 3n + 3 < 0 \Rightarrow 83 - 3n < 0$

$\Rightarrow\ 3n > 83 \Rightarrow n > 27\dfrac{2}{3} \Rightarrow n = 28$

Here, $n = 28$ is smallest natural number $> 27\dfrac{2}{3}$, for which the a_{28} of given A.P. is negative.

5. Given, $x^2 + k(2x + k - 1) + 2 = 0$

or $x^2 + 2xk + k^2 - k + 2 = 0$

Since, equation has real and equal roots.

$\therefore$ Discriminant, $D = 0$

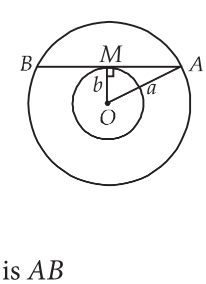
$\Rightarrow\ (2k)^2 - 4(1)(k^2 - k + 2) = 0$

$\Rightarrow\ 4k^2 - 4k^2 + 4k - 8 = 0 \Rightarrow 4k - 8 = 0$

$\Rightarrow\ 4k = 8 \Rightarrow k = 2$

6. Radius of larger circle = a

Radius of smaller circle = b



$\because$ In ΔOAM, we have

$OA^2 = OM^2 + AM^2$

$\Rightarrow\ a^2 = b^2 + AM^2$

$\Rightarrow\ AM^2 = a^2 - b^2$

$\Rightarrow\ AM = \sqrt{a^2 - b^2}$

Now, length of chord of larger circle is AB

$= 2AM = 2\sqrt{a^2 - b^2}$

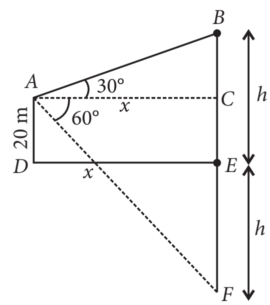
7. Let DE be the level of water and cloud be at position B which is h m above the level of water and reflection of cloud be at F and $AC = DE = x$ m.

$\therefore$ $BC = (h - 20)$m, $CF = (h + 20)$ m

In ΔABC, $\tan 30° = \dfrac{BC}{AC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h-20}{x} \Rightarrow x = \sqrt{3}(h-20)$ ...(i)

In ΔACF,



$\tan 60° = \dfrac{CF}{AC}$

$\Rightarrow \sqrt{3} = \dfrac{h+20}{x}$

$\Rightarrow x = \dfrac{h+20}{\sqrt{3}}$ ...(ii)

From (i) and (ii), we get

$\sqrt{3}(h-20) = \dfrac{h+20}{\sqrt{3}}$

$\Rightarrow 3h - 60 = h + 20 \Rightarrow 2h = 80 \Rightarrow h = 40$

From (i), we have $x = \sqrt{3}(40-20) = 20\sqrt{3}$

Applying Pythagoras theorem in ΔABC,

$AB^2 = BC^2 + AC^2 = (20)^2 + (20\sqrt{3})^2$

$= 400 + 1200 = 1600 \Rightarrow AB = \sqrt{1600} = 40$ m

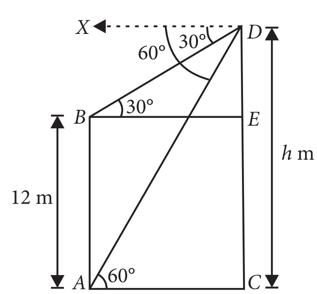
$\therefore$ Distance of the cloud from point A = 40 m

OR

Let AB be the building and CD be the multi-storeyed building of height h m.

Here, $AB = CE = 12$ m and $DE = (h - 12)$ m

In ΔACD, $\tan 60° = \dfrac{CD}{AC}$



$\Rightarrow \sqrt{3} = \dfrac{h}{AC}$

$\Rightarrow AC = \dfrac{h}{\sqrt{3}}$

In ΔBDE, $\tan 30° = \dfrac{DE}{BE}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{(h-12)}{BE}$

$\Rightarrow BE = (h-12)\sqrt{3}$ m

Now, $AC = BE \Rightarrow \dfrac{h}{\sqrt{3}} = (h-12)\sqrt{3}$

$\Rightarrow h = 3(h - 12) \Rightarrow h = 3h - 36$

$\Rightarrow -2h = -36 \Rightarrow h = 18$

Thus, the height of the multi-storeyed building is 18 m.

8. From the given data, we observe that, highest frequency is 10, which lies in the class interval 50-60.

$\therefore$ $l = 50, f_1 = 10, f_0 = 7, h = 10, f_2 = 6$

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 50 + \left(\dfrac{10-7}{2\times 10 - 7 - 6}\right) \times 10$

$= 50 + \dfrac{30}{7} = 50 + 4.29 = 54.29$

9. Steps of construction :

Step-I : Draw a circle of radius 3.5 cm with centre O.

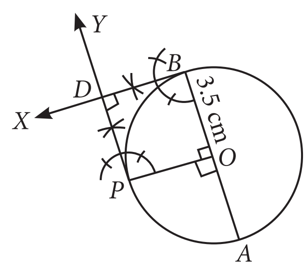
Step-II : Draw a diameter AB.

Step-III : Construct $\angle AOP = 90°$.

Step-IV : At P and B, draw $PY \perp OP$ and $BX \perp OB$.

Step-V : Let PY and BX intersect at D.

Hence, DB and DP are required tangents to the circle perpendicular to each other.



10. The frequency distribution table for the given data can be drawn as :

Marks	Frequency (f_i)	Cumulative frequency ($c.f.$)
20-30	5	5
30-40	15	20
40-50	25	45
50-60	20	65
60-70	7	72
70-80	8	80
80-90	10	90
Total	90	

Here, $N = 90$ and $\dfrac{N}{2} = 45$

Class interval corresponding to 45 is 40-50.

$\therefore$ Median $= 40 + \left[\dfrac{45-20}{25}\right] \times 10$

$= 40 + \left[\dfrac{25}{25}\right] \times 10 = 40 + 10 = 50$

11. Height of the cone (h) = 5 m

Let its radius be r m

$\therefore$ Volume of the cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi r^2 \times 5 \text{m}^3 = \frac{5}{3}\pi r^2 \text{m}^3$

Now, the diameter of the cone = $2r$ m

$\therefore$ Radius of the second cone = $2r$ m

Let the height of the second cone = H m

$\therefore$ Volume of the second cone

$= \frac{1}{3}\pi(2r)^2 H \text{ m}^3 = \frac{4}{3}\pi r^2 H \text{ m}^3$

According to the question,

Volume of the second cone = 16 × Volume of the first cone

$\therefore \quad \frac{4}{3}\pi r^2 H = 16 \times \frac{5}{3}\pi r^2$

$\Rightarrow \; 4H = 80 \quad \Rightarrow \quad H = \frac{80}{4} \Rightarrow H = 20$

$\therefore$ Height of the second cone = 20 m

OR

Radius of the sector of the circle, R = 15 cm

Angle of the sector, $\theta = 120°$

Length of arc of the sector $= \frac{\theta}{360°} \times 2\pi R$

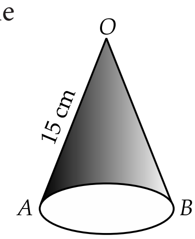
$= \frac{120°}{360°} \times 2\pi \times 15 = 10\,\pi$ cm

Let r be the base radius of the cone.

$\therefore$ Circumference of the base of cone

= Length of arc of sector

$\Rightarrow \; 2\pi r = 10\pi \Rightarrow r = 5$ cm



Slant height of the cone,

l = radius OA of the given sector

$\Rightarrow \; l = 15$ cm

Now, height of the cone,

$h = \sqrt{l^2 - r^2} = \sqrt{(15)^2 - (5)^2}$

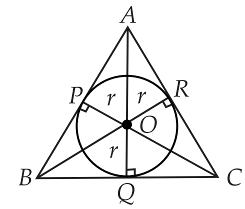
$= \sqrt{225 - 25} = \sqrt{200} = 10\sqrt{2}$ cm

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 5 \times 5 \times 10\sqrt{2}$

$= \frac{5500 \times 1.414}{21} = 370.33 \text{ cm}^3$.

12. We have, AP = 4 cm, BP = 6 cm and AC = 12 cm

Since the length of tangents drawn from an external point to a circle, are equal.



$\therefore \; AP = AR$

$\Rightarrow \; AR = 4$ cm

Also, $BP = BQ$

$\Rightarrow \; BQ = 6$ cm

and $CR = CQ$

$\Rightarrow \; CR = AC - AR = (12 - 4)$ cm = 8 cm

Now, $BC = BQ + CQ = (6 + 8)$ cm = 14 cm

Now, $s = \frac{AB + BC + AC}{2} = 18$

[$\because$ s = Semi-perimeter of ΔABC]

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

[Heron's formula]

$= \sqrt{18(18-14)(18-12)(18-10)} = 24\sqrt{6}$

Now, ar(ΔAOB) + ar(ΔBOC) + ar(ΔCOA) = ar(ΔABC)

$\Rightarrow \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 14 \times r + \frac{1}{2} \times 12 \times r = 24\sqrt{6}$

$\Rightarrow \; r(5 + 7 + 6) = 24\sqrt{6} \Rightarrow r = \frac{24\sqrt{6}}{18} = \frac{4\sqrt{6}}{3}$ cm

13. Here $S_n = 0.1n^2 + 7.9n$

(i) $S_{n-1} = 0.1(n-1)^2 + 7.9(n-1) = 0.1n^2 + 7.7n - 7.8$

(ii) $S_1 = t_1 = a = 0.1(1)^2 + 7.9(1) = 8$ cm

= Diameter of core

So, radius of the core = 4 cm

Now, $S_2 = 0.1(2)^2 + 7.9(2) = 16.2$

$\therefore$ Required diameter $= t_2 = S_2 - S_1 = 16.2 - 8 = 8.2$ cm

14. (i) Total height of pole = 8 m

$\therefore \; BD = AD - AB = (8 - 2)\text{m} = 6$ m

(ii) In ΔBDC, $\frac{BD}{BC} = \sin 60°$

$\Rightarrow \; \frac{6}{BC} = \frac{\sqrt{3}}{2}$

$\Rightarrow \; BC = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 4\sqrt{3}$ m

Now, in ΔBDC,

$\frac{BD}{CD} = \tan 60° \Rightarrow \frac{6}{CD} = \sqrt{3} \Rightarrow CD = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3}$ m

Thus the required distance is $2\sqrt{3}$ m.