Maharashtra State Board
Class X Mathematics - Algebra
Board Paper – 2015

Time: 2 hours **Total Marks: 40**

Note:- (1) All questions are compulsory.
(2) Use of calculator is not allowed.

1. Attempt any five question from the following: **5**

i. State whether the following sequence is an A.P. or not?
1, 4, 7, 10, ……..

ii. A card is drawn from the pack of 25 cards labeled with numbers 1 to 25. Write the sample space for this random experiment.

iii. Find the value of x + y, if 12x + 13y = 29 and 13x + 12y = 21

iv. For a sequence if $S_n = \frac{n}{n+1}$ then find the value of S_{10}.

v. Verify whether 1 is the root of the quadratic equation :
$x^2 + 3x - 4 = 0$

vi. If x + y = 5 and x = 3, then find the value of y.

2. Attempt any four sub-questions from the following: **8**

i. Solve the following quadratic equation by factorization method :
$x^2 - 5x + 6 = 0$.

ii. Find the term t_{11} of an A.P. :
4, 9, 14, ……..

iii. If the point A(3, 2) lies on the graph of the equation 5x + ay = 19, then find a.

iv. A die is thrown. If A is the event that the number on upper face is less than 5, then write sample space and event A in set notation.

v. For a certain frequency distribution, the value of Mean is 101 and Median is 100. Find the value of Mode.

vi. If one root of the quadratic equation
$kx^2 - 7x + 12 = 0$ is 3, then find the value of k.

3. Attempt any three of the following sub questions: **9**

i. Area under different crops in a certain village is given below. Represent it by a pie diagram :

Corps	Area (in Hectares)
Jowar	80
Wheat	20
Sugarcane	60
Vegetables	20

ii. If two coins are tossed, then find the probability of the event that at least one head turns up.

iii. Solve the following simultaneous equations by using graphical method :

$x + y = 6$;

$x - y = 4$.

iv. There is an auditorium with 35 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row and so on. Find the number of seats in the twenty-first row.

v. Solve the following quadratic equation by completing square method :

$x^2 + 10x + 21 = 0$.

4. Attempt any two sub-questions from the following: **8**

i. Two digit numbers are formed using the digits 0, 1, 2, 3, 4, 5 where digits are not repeated.

P is the event that the number so formed is even.

Q is the event that the number so formed is greater than 50.

R is the event that the number so formed is divisible by 3.

Then write the sample space S and events P, Q, R using set notation.

ii. The following table shows ages of 3000 patients getting medical treatment in a hospital on a particular day :

Age (in years)	No. of Patients
10-20	60
20-30	42
30-40	55
40-50	70
50-60	53
60-70	20

Find the median age of the patients.

iii. If $\alpha + \beta = 5$ and $\alpha^3 + \beta^3 = 35$, find the quadratic equation whose roots are α and β.

5. Attempt any two of the following subquestions: 10

i. Babubhai borrows Rs. 4,000 and agrees to repay with a total interest of Rs. 500 in 10 instlments, each instalment being less than the preceding instalment by Rs. 10. What should be the first and the last instalments?

ii. On the first day of the sale of tickets of a drama, in all 35 tickets were sold. If the rates tickets were Rs. 20 and Rs. 40 per ticket and the total collection was Rs, 900, find the number of tickets sold of each rate.

iii. Given below is the frequency distribution of driving speeds (in km/hour) of the vehicles of 400 college students:

Speed (in km/hr)	No. of Students
20-30	6
30-40	80
40-50	156
50-60	98
60-70	60

Draw Histogram and hence the frequency polygon for the above data.

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1.

i. 1, 4, 7, 10, ……
Let 'd' be the common difference, 'a' be the common ratio, and t_2 , t_3, t_4 be the 2nd, 3rd and 4th term respectively.
Here $t_2 = 4$, $t_3 = 7$, $t_4 = 10$
$d = 4 - 1 = 3$
$a = 1$
We know $t_2 - a = t_3 - t_2 = 3$
Since the common difference is same throughout the series, therefore the series is an A.P.

ii. Sample Space for this experiment
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25}

iii. $12x + 13y = 29$…….(a)
$13x + 12y = 21$……..(b)
Adding (a) and (b), we have,
$25x + 25y = 50$
$\Rightarrow x + y = \frac{50}{25} = 2$
$\therefore x + y = 2$

iv. $S_n = \frac{n}{n+1}$
For n = 10,
$S_n = \frac{n}{n+1}$
$= \frac{10}{10+1} = \frac{10}{11}$

v. $x^2 + 3x - 4 = 0$
If $x = 1$ satisfies the quadratic equation, then 1 is a root of the quadratic equation.
$1^2 + 3 \times 1 - 4 = 0$
Hence Proved.

vi. $x + y = 5$.....(a)

If $x = 3$ is substituted in the equation (a), then

$\Rightarrow 3 + y = 5$

$\Rightarrow y = 5 - 3$

$\therefore y = 2$

2.

i. The given quadratic equation is $x^2 - 5x + 6 = 0$

Rewriting the above equation, we have,

$x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x - 3) - 2(x - 3) = 0$

$\Rightarrow (x - 2)(x - 3) = 0$

$\Rightarrow x - 2 = 0$ or $x - 3 = 0$

$\Rightarrow x = 2$ or $x = 3$

ii. We need to find the 11^{th} term of the A.P., 4, 9, 14

Here, the initial term is, $a = 4$.

Common difference = 14 - 9 = 9 - 4 = 5

The general term of an A.P is given by the formula,

$t_n = a + (n - 1)d$

$\Rightarrow t_{11} = 4 + (11 - 1) \times 5$

$\Rightarrow t_{11} = 4 + 10 \times 5$

$\Rightarrow t_{11} = 4 + 50$

$\Rightarrow t_{11} = 54$

iii. Given that the point A(3, 2) lies on the graph of the equation $5x + ay = 19$.

Thus, the point satisfies the equation of the graph.

Substituting the values $x = 3$ and $y = 2$, we have,

$5 \times 3 + a \times 2 = 19$

$\Rightarrow 15 + 2a = 19$

$\Rightarrow 2a = 19 - 15$

$\Rightarrow 2a = 4$

$\Rightarrow a = \frac{4}{2}$

$\Rightarrow a = 2$

Therefore the value of a is 2.

iv. The sample space (S) is

S = {1, 2, 3, 4, 5, 6}

No. of sample points = n(S) = 6

Let A be the event of getting numbers less than 5.

$\therefore$ A = {1, 2, 3, 4}

$\Rightarrow$ n(A) = 4

v. The interrelation between the measures of central tendency is given by

Mean – Mode = 3(Mean - Median)

Given that Mean = 101 and Median = 100

Thus from the above relation, we have,

$\Rightarrow$ 101 – Mode = 3 (101 - 100)

$\Rightarrow$ 101 - Mode = 3

$\Rightarrow$ 101 - 3 = Mode

$\Rightarrow$ Mode = 98

vi. The given quadratic equation is $kx^2 - 7x + 12 = 0$.

Let α and β be the roots of the given equation.

Comparing the given equation with the standard equation,

$ax^2 + bx + c = 0$, we have,

$a = k, b = -7$ and $c = 12$.

Thus, $\alpha + \beta = \frac{-b}{a} = \frac{-(-7)}{k}$

and $\alpha\beta = \frac{c}{a} = \frac{12}{k}$

Since one of the roots is 3, we have,

$3 + \beta = \frac{7}{k}$ and $3\beta = \frac{12}{k}$

$\Rightarrow 3 + \beta = \frac{7}{k}$ and $\beta = \frac{4}{k}$

Substituting the value of $\beta = \frac{4}{k}$ in $3 + \beta = \frac{7}{k}$, we have,

$3 + \frac{4}{k} = \frac{7}{k}$

$\Rightarrow \frac{3k + 4}{k} = \frac{7}{k}$

$\Rightarrow 3k + 4 = 7$

$\Rightarrow 3k = 7 - 4$

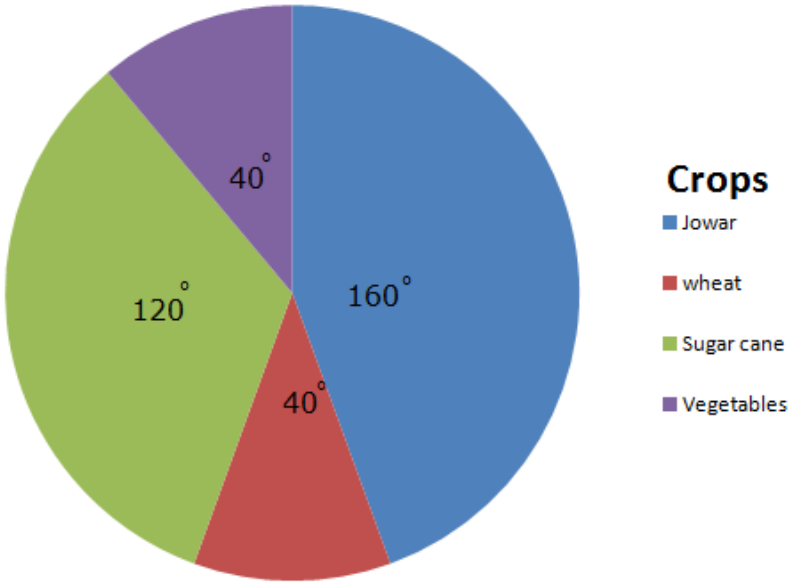
$\Rightarrow 3k = 3$

$\Rightarrow k = 1$

3.

i. We compute the central angle for each crop as shown in the following table.

Crops	Area (in Hectares)	Measure of central angle
Jowar	80	$\frac{80}{180}\times 360° = 160°$
wheat	20	$\frac{20}{180}\times 360° = 40°$
Sugar cane	60	$\frac{60}{180}\times 360° = 120°$
Vegetables	20	$\frac{20}{180}\times 360° = 40°$
Total	180	360°



ii. Let S be the sample space.

Then S = {HH, HT, TH, TT}

∴ n(S) = 4

Let A be the event where at least one tail turns up.

At least one head means 1 head or more than 1 heads.

If S = {HH, HT, TH, TT}

∴ A = {HT, TH, HH}

∴ n(A) = 3

$$\text{Now, } P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$$

iii. First we find three solutions for each equation and make tables.

$x + y = 6 \Rightarrow y = 6 - x$

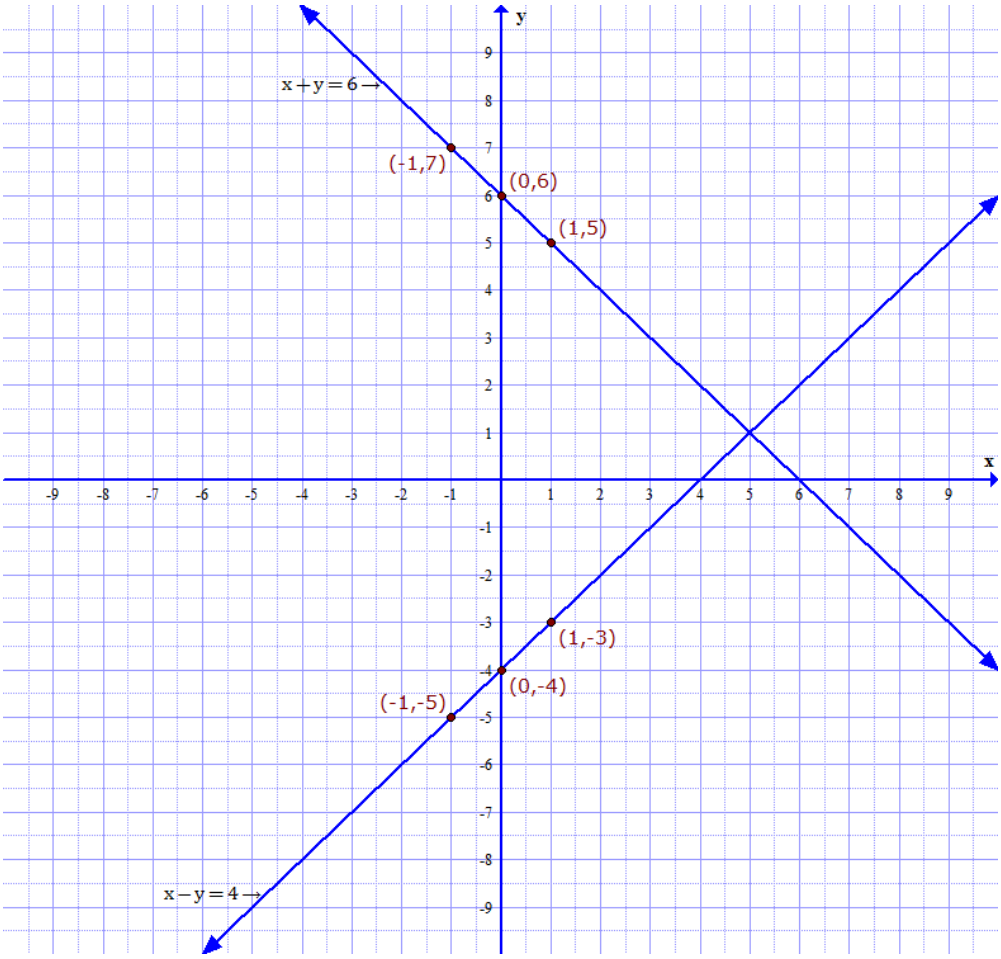
$x - y = 4 \Rightarrow y = x - 4$

$x + y = 6$			
x	1	−1	0
y	5	7	6

Now, plot the points (1, 5), (−1, 7) and (0, 6) and draw the line passing through them.

$x - y = 4$			
x	1	−1	0
y	− 3	−5	−4

Now, plot the points (1, −3), (−1, −5) and (0, −4) and draw the line passing through them.



Thus the point of intersection of two lines is (5, 1).

Hence x = 5 and y = 1 is the solution of the given equations.

iv. There is an auditorium with 35 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. Find the number of seats in the twenty-fifth row.

Number of seats in the first row = 20

$\therefore$ a = 20

Increase in the number of seats in consecutive rows = 2

$\therefore$ d = 2

To find the number of seats in the 25^{th} row, find t_{25}

$t_n = a + (n - 1)d$ ... (Formula)

$\therefore t_{25} = 20 + (25 - 1) \times 2$ ... (Substituting the values)

$= 20 + (24 \times 2)$

$= 20+48$

$\therefore t_{25} = 68$

Thus, the number of seats in the twenty-fifth row is 68.

v. $x^2 + 10x + 21 = 0$

$\therefore x^2 + 10x = -21$

To complete the square, find the third term.

$$\text{Third term} = \left(\frac{1}{2}\text{coefficient of } x\right)^2$$

Coefficient of $x = 10$

$$\therefore \text{Third term} = \left(\frac{1}{2} \times 10\right)^2 = (5)^2 = 25$$

Add 25 on both the sides,

$x^2 + 10x + 25 = -21 + 25$

$\therefore (x + 5)^2 = 4$

$x + 5 = \pm 2$ ...(Taking square root on both sides)

$\therefore x + 5 = 2$ or $x + 5 = -2$

$\therefore x = -3$ or $x = -7$

4.

i. As we have to form two-digit numbers, 0 cannot be at the tens place.
The sample space is
S = {10, 12, 13, 14, 15, 20, 21, 23, 24, 25, 30, 31, 32, 34, 35, 40, 41,
42, 43, 45, 50, 51, 52, 53, 54}
Let P be the event that the number formed is an even number.
∴ P = {10, 12, 14, 20, 24, 30, 32, 34, 40, 42, 50, 52, 54}
Let Q be the event that the number formed is greater than 50.
∴ Q = {51, 52, 53, 54}
Let R be the event that the number formed is divisible by 3.
∴ R = {12, 15, 21, 24, 30, 42, 45, 51, 54}

ii.

Age (in years)	Number of patients (f)	c.f. (less than type)
10 – 20	60	60
20 – 30	42	$102 \rightarrow c.f.$
30 – 40 Median class	55 → f	157
40 – 50	70	227
50 – 60	53	280
60 – 70	20	300

Here, N = 300

$\therefore \frac{N}{2} = 150.$

Cumulative frequency just greater than 150 is 157.
∴ Corresponding class (30 – 40) is the median class.
L = 30, f = 55, c.f. = 102, h = 10

$$\text{Median} = L + \left(\frac{N}{2} - \text{c.f.}\right)\frac{h}{f}$$

$$= 30 + (150 - 102) \times \frac{10}{55}$$

$$= 30 + 48 \times \frac{10}{55}$$

$$= 30 + 8.73$$

$$= 38.73$$

Thus, the median age of patients is 38.73 years.

iii. $\alpha+\beta=5 \quad \ldots.(1)$

$\alpha^3+\beta^3=35 \ldots. (2)$

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

$=(5)^3-3\alpha\beta(5)$

$=125-15\alpha\beta$

$\therefore 125-15\alpha\beta=35 \ [\text{from}(2)]$

$\therefore 15\alpha\beta=125-35$

$\therefore 15\alpha\beta=90$

$\therefore \alpha\beta=6$

The required quadratic equation is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow x^2-5x+6=0$

5.

i. Each instalment is Rs. 10 less than the preceding one.

$\therefore$ The instalments are in A.P. with common difference -10

Babubhai repays Rs. 4000 with interest of Rs. 500 in 12 instalments

i.e. $S_{10}=4000+500=4500$

Here $n=12$, $d=-10$ and $S_{10}=4500$.

we have to find the 1st instalment i.e. a,

$S_n=\frac{n}{2}[2a+(n-1)d]$

$\therefore S_{10}=4500=\frac{10}{2}[2a+(10-1)(-10)]$

$\therefore 4500=5[2a+9\times(-10)]$

$\therefore \frac{4500}{5}=2a-90$

$\therefore 900=2a-90$

$\therefore 2a=900+90=990$

$\therefore a = \frac{990}{2} = 495$

$\therefore a = 495$

t_n = last instalment.

$t_n = a + (n-1)d$

$t_{10} = 495 + (10-1)\times(-10)$

$t_{10} = 495 + 9\times(-10)$

$t_{10} = 495 - 90$

$\therefore t_{10} = 405$

$\therefore$ The first instalment is Rs. 495 and the last instalment is Rs. 405.

ii. Let x tickets be sold at the rate of Rs.20 and y tickets at the rate of Rs.40.

Therefore, by first condition,

$x + y = 35$ ....(1)

By second condition,

$20x + 40y = 900$

$\therefore x + 2y = 45$ .....(2)

Subtracting equation (1) from the equation (2),

$$\begin{array}{r} x + 2y = 45 \\ x + y = 35 \\ - \quad - \quad - \\ \hline y = 10 \end{array}$$

Substituting $y = 10$ in equation (1),

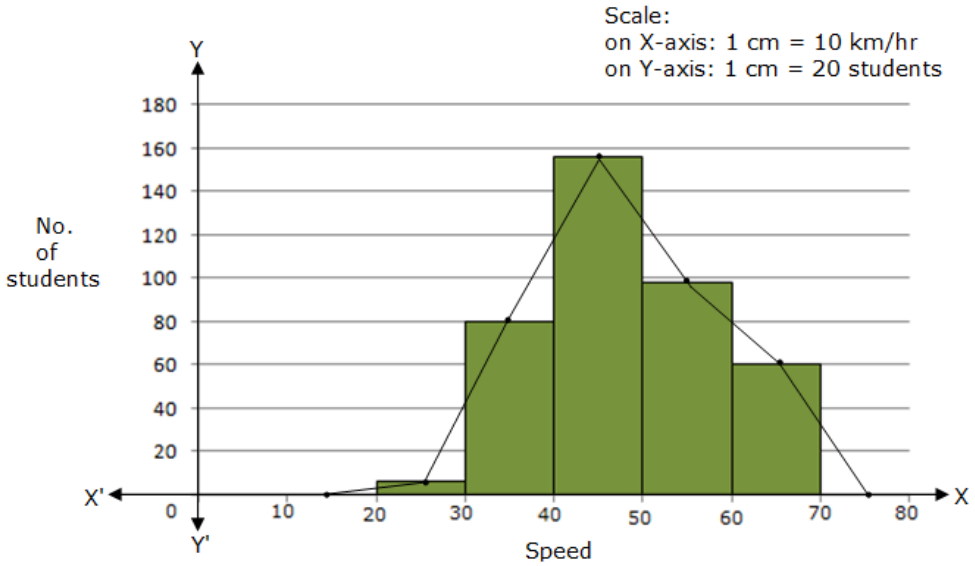
$x + 10 = 35$

$x = 35 - 10$

$x = 25$

Therefore, 25 tickets at the rate of Rs. 20 each and 10 tickets at the rate of Rs. 40 were sold.

iii.


Scale:
on X-axis: 1 cm = 10 km/hr
on Y-axis: 1 cm = 20 students
Y
Y'
X'
X
No. of students
180
160
140
120
100
80
60
40
20
0
10
20
30
40
50
60
70
80
Speed