CBSE Board
Class XII Mathematics
Sample Paper 4

Time: 3 hours **Total Marks: 100**

General Instructions:

1. **All** the questions are **compulsory**.
2. The question paper consists of **37** questions divided into **three parts** A, B, and C.
3. **Part A** comprises of **20** questions of **1 mark** each. **Part B** comprises of **11** questions of **4 marks** each. **Part C** comprises of **6** questions of **6 marks** each.
4. There is no overall choice. However, an internal choice has been provided in **three questions of 4 marks** each, **four questions of 6 marks** each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculator is **not** permitted.

Part A

Q1 – Q20 are multiple choice type questions. Select the correct option.

1. The value of the determinant $\begin{vmatrix} a^2 & a & 1 \\ \cos nx & \cos(n+1)x & \cos(n+2)x \\ \sin nx & \sin(n+1)x & \sin(n+2)x \end{vmatrix}$ is independent of

A. n
B. a
C. x
D. none of these

2. If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$, then find (A + 2B)'

A. $\begin{bmatrix} -4 & 3 \\ 3 & 6 \end{bmatrix}$

B. $\begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

C. $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

D. $\begin{bmatrix} -4 & -3 \\ -3 & 6 \end{bmatrix}$

3. The value of $\hat{i}\cdot\left(\hat{j}\times\hat{k}\right)+\hat{j}\cdot\left(\hat{i}\times\hat{k}\right)+\hat{k}\cdot\left(\hat{i}\times\hat{j}\right)$, is

A. 0
B. –1
C. 1
D. 3

4. A and B are two events such that P (A) = 0.25 and P (B) = 0.50. The probability of both happening together is 0.14. The probability of both A and B not happening is

A. 0.16
B. 0.25
C. 0.11
D. 0.39

5. The solution of the differential equation $\frac{dy}{dx}+\frac{2y}{x}=0$ with y(1) = 1 is given by

A. $y=\frac{1}{x^2}$

B. $x=\frac{1}{y^2}$

C. $x=\frac{1}{y}$

D. $y=\frac{1}{x}$

6. If 4 $\cos^{-1}x+\sin^{-1}x=\pi$, then the value of x is

A. $\frac{3}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{3}}$

7. The area bounded by the parabola $x = 4 - y^2$ and y-axis, in square units, is

A. $\frac{3}{32}$

B. $\frac{32}{3}$

C. $\frac{33}{2}$

D. $\frac{16}{3}$

8. The interval of increase of the function f(x) = x − e^x + tan (2π/7) is

A. (0, ∞)

B. (−∞, 0)

C. (1, ∞)

D. (−∞, 1)

9. The intercept cut off by the plane 2x + y − z = 5 on x-axis is

A. $\frac{5}{2}$

B. $\frac{2}{5}$

C. 2

D. 5

10. The distance between the point (3, 4, 5) and the point where the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ meets the plane $x + y + z = 17$, is

A. 1

B. 2

C. 3

D. 4

11. Consider the binary operation * defined on Q − {1} by the rule a * b = a + b − ab for all a, b ∈ Q − {1}. The identity element in Q − {1} is

A. 1

B. 0

C. a

D. −1

12. The value of k, for which the function $f(x) = \begin{cases} kx^2, & x \le 2 \\ 3, & x > 2 \end{cases}$ is continuous at x = 2 is

A. $\frac{3}{4}$

B. $\frac{3}{2}$

C. $\frac{4}{3}$

D. 12

13. Find the slope of the tangent to the curve $y = x^3 - x + 1$ at the point where the curve cuts the y-axis.

A. 1
B. −1
C. 2
D. 0

14. What is the range of the function $f(x) = \frac{|x - 1|}{(x - 1)}$?

A. R − {1, −1}
B. R
C. R − {1}
D. {−1, 1}

15. If $[2a+4b \quad c \quad d] = \lambda[a \quad c \quad d] + \mu[b \quad c \quad d]$, then $\lambda + \mu$ is equal to

A. 2
B. −2
C. 6
D. 8

16. If $\sin^{-1} x = y$, then

A. $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

B. $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

C. $0 \le y \le \frac{\pi}{2}$

D. $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

17. The differential equation which represents the family of curves $y = e^{Cx}$ is

A. $\ln y = \frac{dy}{dx}$

B. $y \ln y = x\frac{dy}{dx}$

C. $x \ln y = y\frac{dy}{dx}$

D. $y \ln y = \frac{dy}{dx}$

18. The value of $\int \frac{1}{5+3\cos x} dx$ is

A. $\tan^{-1}\left(\frac{x}{2}\right) + c$

B. $\frac{1}{2}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{2}\right) + c$

C. $\tan^{-1}\left(\frac{\tan\frac{x}{2}}{2}\right) + c$

D. $\frac{x}{4} + c$

19. Evaluate: $\int \frac{x^2}{1+x^3} dx$

A. $\frac{1}{3}\log\left|1+x^3\right| + c$

B. $\log\left|1+x^3\right| + c$

C. $\log|x| + c$

D. $\log\left|x^3\right| + c$

20. The differential equation obtained on eliminating A and B from y = A cos ωt + B sin ωt, is

A. $y'' = -\omega^2 y'$

B. $y' = \omega^2 y$

C. $y'' = -\omega^2 y$

D. $y' = \omega^2 y''$

Part B

21. Prove that $\tan^{-1}\left(\frac{\sqrt{1+\cos x}+\sqrt{1-\cos x}}{\sqrt{1+\cos x}-\sqrt{1-\cos x}}\right)=\frac{\pi}{4}-\frac{x}{2}$ if $\pi < x < \frac{3\pi}{2}$

22. Show that the differential equation $2ye^{x/y}\,dx + (y - 2x\,e^{x/y})\,dy = 0$ is homogeneous.

OR

Obtain the differential equation of all the circles touching the x-axis at the origin.

23. Given that $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$, such that the scalar product of $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and unit vector along the sum of the given two vectors $\vec{b}$ and $\vec{c}$ is unity. Find λ.

24. Let A = Q × Q, where Q is the set of all rational numbers and * is a binary operation on A defined by (a, b) * (c, d) = (ac, b + ad) for (a, b), (c, d) ∈ A. Then find:

(i) The identity element of * in A.

(ii) Invertible element of A, and hence write the inverse of elements (5, 3) and $\left(\frac{1}{2}, 4\right)$.

25. Evaluate the integral: $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$

26. Prove that: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a - b)(b - c)(c - a)(a + b + c)$

27. Differentiate the following function w.r.t. x: $y = (\sin x)^x + \sin^{-1}\sqrt{x}$

OR

Find $\frac{dy}{dx}$ if $(x^2 + y^2)^2 = xy$.

28. Prove that $\int_0^{\pi} \frac{x}{1 + \sin x}\,dx = \pi$

OR

Evaluate: $\int_0^{\frac{\pi}{2}} 2\sin x\cos x\tan^{-1}(\sin x)\,dx$.

29. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

30. Find the Cartesian equation of the plane passing through the points A(0, 0, 0) and B(3, -1, 2) and parallel to the line $\frac{x-4}{1} = \frac{y+3}{-4} = \frac{z+1}{7}$

31. Find the value of 'a' for which the function f defined as

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$$

is continuous at x = 0.

Part C

32. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftsman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftsman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. If the profit on a racket and on a bat is Rs20 and Rs10 respectively, find the number of tennis rackets and crickets bats that the factory must manufacture to earn the maximum profit. Make it as an L.P.P and solve graphically.

33. Suppose 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

OR

A girl throws a cie. If she get a 5 OR 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 OR 4, she tosses a coin two times and notes the number of heads obtained. If she obtained exactly two heads, what is the probability that she threw 1, 2, 3 OR 4 with the die?

34. Using the method of method of integration, find the area of the region bounded by the following lines:

$3x - y - 3 = 0$,

$2x + y - 12 = 0$,

$x - 2y - 1 = 0$

OR

Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

35. Show that a closed right circular cylinder of a given total surface area and maximum volume is such that its height is equal to the diameter of the base.

OR

Show that the semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

36. Find the equation of plane passing through points (1, 2, 3), (0, –1, 0) and parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$

OR

Find the equation of the plane passing through the line of intersection of the planes 2x + y – z = 3, 5x - 3y + 4z + 9 = 0 and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$.

37. Two schools A and B want to award their selected students on the values of sincerity, truthfulness and helpfulness. The school A wants to award x each, y each and z each for the three respective values to 3, 2 and 1 students respectively with a total award money of 1,600. School B wants to spend 2,300 to award its 4, 1 and 3 students on the respective values (by giving the same award money to the three values as before). If the total amount for one prize on each value is 900, using matrices, find the award money for each value. Apart from these three values, suggest one more value which should be considered for award.