

Topics : Simple Harmonic Motion, Friction, Fluid Mechanics, Rigid Body Dynamics, Kinematics, Geometrical Optics

Type of Questions

Single choice Objective ('-1' negative marking) Q.1 to Q.4

(3 marks 3 min.)

M.M., Min.

[12, 12]

Subjective Questions ('-1' negative marking) Q.5 to Q.6

(4 marks 5 min.)

[8, 10]

Comprehension ('-1' negative marking) Q.7 to Q.9

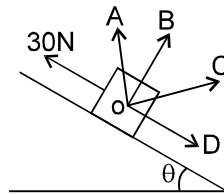
(3 marks 3 min.)

[9, 9]

1. A particle performs S.H.M. on x-axis with amplitude A and time period T. The time taken by the particle to travel a distance A/5 starting from rest is:

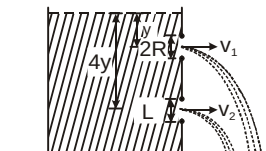
(A) $\frac{T}{20}$ (B) $\frac{T}{2\pi} \cos^{-1} \left(\frac{4}{5} \right)$ (C) $\frac{T}{2\pi} \cos^{-1} \left(\frac{1}{5} \right)$ (D) $\frac{T}{2\pi} \sin^{-1} \left(\frac{1}{5} \right)$

2. A body of mass 10 kg lies on a rough inclined plane of inclination $\theta = \sin^{-1} \frac{3}{5}$ with the horizontal. When a force of 30 N is applied on the block parallel to & upward the plane, the total reaction by the plane on the block is nearly along:



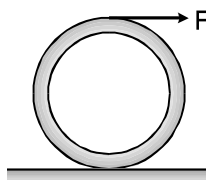
- (A) OA (B) OB (C) OC (D) OD

3. A large open tank has two small holes in its vertical wall as shown in figure. One is a square hole of side 'L' at a depth '4y' from the top and the other is a circular hole of radius 'R' at a depth 'y' from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. Then, 'R' is equal to :



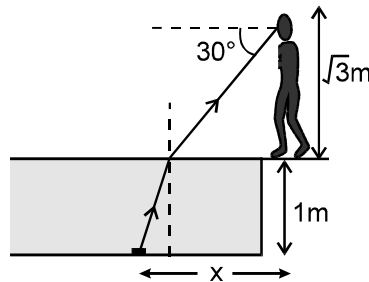
(A) $\frac{L}{\sqrt{2\pi}}$ (B) $2\pi L$ (C) $\sqrt{\frac{2}{\pi}} \cdot L$ (D) $\frac{L}{2\pi}$

4. A ring of mass m and radius R rolls on a horizontal rough surface without slipping due to an applied force 'F'. The friction force acting on ring is : –



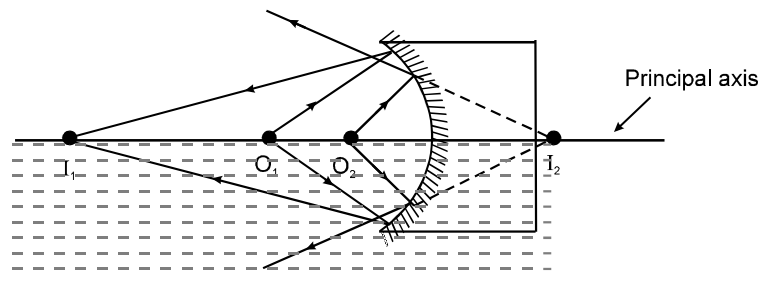
(A) $\frac{F}{3}$ (B) $\frac{2F}{3}$ (C) $\frac{F}{4}$ (D) Zero

5. A particle is projected from the ground level. It just passes through upper ends of vertical poles A, B, C of height 20 m, 30 m and 20 m respectively. The time taken by the particle to travel from B to C is double of the time taken from A to B. Find the maximum height attained by the particle from the ground level.
6. A man is standing at the edge of a 1m deep swimming pool, completely filled with a liquid of refractive index $\sqrt{3/2}$. The eyes of the man are $\sqrt{3}$ m above the ground. A coin located at the bottom of the pool appears to be at an angle of depression of 30° with reference to the eye of man. Then horizontal distance (represented by x in the figure) of the coin from the eye of the man is _____ mm.



COMPREHENSION

A block with a concave mirror of radius of curvature 1m attached to one of its sides floats, with exactly half of its length immersed in water and the other half exposed to air.



Any ray originating from an object O_1 and O_2 (as shown in figure) floating on the surface of water first gets reflected by the mirror. This then gets refracted by the water surface if the image formed by reflection is real (I_1). If the image is virtual (I_2), then the reflected ray never encounters the air-water interface and hence there is no refraction. The image for the next three questions refers to the final image, formed after both the reflection and refraction (if it occurs at all) has taken place.

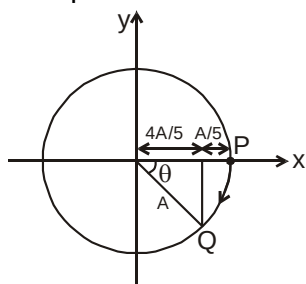
7. The final image formed is unique (i.e. only one image is formed) if the point object floats on the surface of water at a distance x in front of the mirror where
 (A) x is less than 50 cm (B) x is less than 1m
 (C) x is between 50 cm and 1m (D) for any value of x
8. There is no refraction of light rays reflected from the mirror if the point object floats on the surface of water at a distance x in front of the mirror where
 (A) x is less than 50 cm (B) x is less than 1m
 (C) x is between 50 cm and 1m (D) for any value of x
9. The final image is not unique (i.e. more than one image for the same object) if the point object is placed a distance 'y' above the surface of water and a distance x in front of the mirror (rays are not paraxial), where $y = 20$ cm and x is :
 (A) less than 50 cm (B) less than 1m
 (C) between 50 cm and 1m (D) for any value of x

Answers Key

1. (B) 2. (A) 3. (C) 4. (D) 5. $\frac{125}{4}$ m
 6. $d = 4000$ mm 7. (D) 8. (A) 9. (D)

Hints & Solutions

1. Particle is starting from rest, i.e. from one of its extreme position.



As particle moves a distance $\frac{A}{5}$, we can represent it on a circle as shown.

$$\cos \theta = \frac{4A/5}{A} = \frac{4}{5}$$

$$\theta = \cos^{-1} \left(\frac{4}{5} \right)$$

$$\omega t = \cos^{-1} \left(\frac{4}{5} \right)$$

$$t = \frac{1}{\omega} \cos^{-1} \left(\frac{4}{5} \right)$$

$$= \frac{T}{2\pi} \cos^{-1} \left(\frac{4}{5} \right)$$

Method :

As starts from rest i.e. from extreme position $x =$

$$A \sin (\omega t + \phi)$$

$$\text{At } t = 0 ; x = A$$

$$\Rightarrow \phi = \frac{\pi}{2}$$

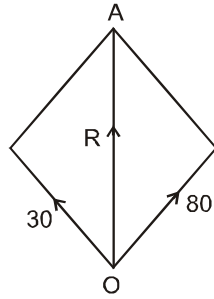
$$\therefore A - \frac{A}{5} = A \cos \omega t$$

$$\frac{4}{5} = \cos \omega t$$

$$\Rightarrow \omega t = \cos^{-1} \frac{4}{5}$$

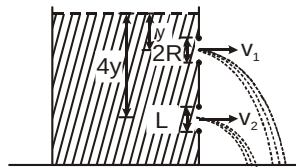
$$t = \frac{T}{2\pi} \cos^{-1} \left(\frac{4}{5} \right)$$

2. Frictional force along the in upward direction
 $= 10 g \sin\theta - 30 = 30 \text{ Nt}$
 $N = 10 g \cos\theta = 80 \text{ Nt}$



Direction of R is along OA.

3. Let v_1 and v_2 be the velocity of efflux from square and circular hole respectively. S_1 and S_2 be cross-section areas of square and circular holes.



$$v_1 = \sqrt{8gy} \quad \text{and} \quad v_2 = \sqrt{2g(y)}$$

The volume of water coming out of square and circular hole per second is

$$Q_1 = v_1 S_1 = \sqrt{8gy} L^2 \quad ; \quad Q_2 = v_2 S_2 = \sqrt{2gy} \pi R^2$$

$$\therefore Q_1 = Q_2$$

$$\therefore R = \sqrt{\frac{2}{\pi}} \cdot L$$

4. (D) $F + f = ma \quad \dots (1)$

$$\text{Also ; } FR - fR = I \frac{a}{R}$$

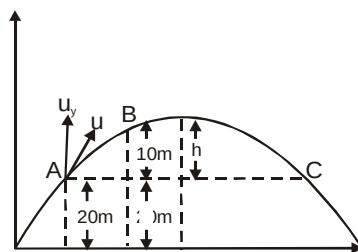
$$F - f = ma \quad \dots (2)$$

$$[I = mR^2]$$

From (1) & (2)

$$f = 0.$$

5. $t_{AB} = t$
 $t_{BC} = 2t$
 So, for ABC part,



Time of flight,

$$t_{AC} = 3t = \frac{2u_y}{g}$$

$$\Rightarrow u_y = \frac{3}{2}gt$$

$$\text{Also, } 10 = u_y t - \frac{1}{2}gt^2 = gt^2$$

$$\Rightarrow t = 1\text{s}$$

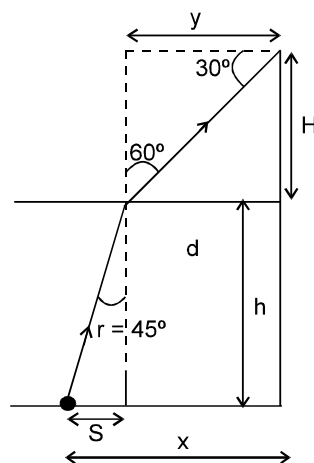
$$\therefore u_y = 15 \text{ m/s}$$

$$\therefore h = \frac{u_y^2}{2g} = \frac{225}{20} = \frac{45}{4} \text{ m.}$$

$$\therefore \text{Maximum height attained} = 20 + \frac{45}{4}$$

$$= \frac{125}{4} \text{ m.}$$

6. Ans. $d = 4000 \text{ mm}$



$$\sin 60^\circ = \sqrt{\frac{3}{2}} \sin r \Rightarrow r = 45^\circ$$

$$\therefore S = h = 1 \text{ m}$$

$$y = H \tan 60^\circ = 3 \text{ m}$$

$$\therefore x = S + y = 4 \text{ m} = 4000 \text{ mm}$$

7. There is only one point image corresponding to a point object, as long as the object lies on the water surface (principal axis of the mirror). Any object lying at some distance from the principal axis results in multiple image points.
8. If light rays diverge outward (forming a virtual image behind the mirror) after reflection, there is no refraction at water surface after reflection. This is the case when the object lies between the focus and the pole.