HOTS (Higher Order Thinking Skills)

Que 1. AB and AC are two chords of a circle of radius r such that AB = 2AC. If P and q are the distances of AB and AC from the centre. Prove that $4q^2 = p^2 + 3r^2$.

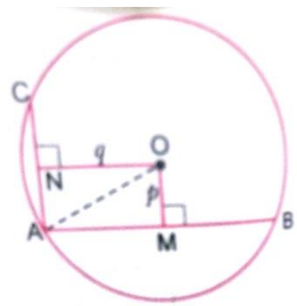

Fig. 10.56

Sol. Draw OM ⊥ AB and ON ⊥ AC

Join OA.

In right ΔOAM,

$$OA^2 = OM^2 + AM^2$$

⇒ $r^2 = p^2 + \left(\frac{1}{2}AB\right)^2$ (∵ OM ⊥ AB, ∴ OM bisects AB)

⇒ $\frac{1}{4}AB^2 = r^2 - p^2$ or $AB^2 = 4r^2 - 4p^2$ …(i)

In right ΔOAN,

$$OA^2 = ON^2 + AN^2$$

⇒ $r^2 = q^2 + \left(\frac{1}{2}AC\right)^2$ (∵ ON ⊥ AC, ∴ ON bisects AC)

⇒ $\frac{1}{4}AC^2 = r^2 - q^2$ or $\frac{1}{4}\left(\frac{1}{2}AB\right)^2 = r^2 - q^2$ (∵ AB = 2AC)

⇒ $\frac{1}{16}AB^2 = r^2 - q^2$ or $AB^2 = 16r^2 - 16p^2$ …(ii)

From (i) and (ii), we have

$$4r^2 - 4p^2 = 16r^2 - 16q^2$$

Or $r^2 - p^2 = 4r^2 - 4q^2$

Or $4q^2 = 3r^2 + p^2$

Que 2. Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect equal chords AD and CE with the circle. Prove that ∠ABC is

equal to half the difference of the angles subtended by the chords AC and DE at the center.

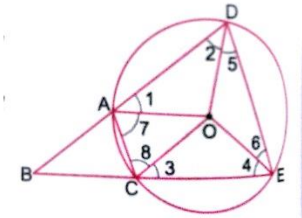

Fig. 10.57

Sol. Given: AD = CE

To prove: $\angle ABC = \frac{1}{2}(\angle DOE - \angle AOC)$

In ΔAOD and ΔCOE

AD = CE (Given)

AO = OC and DO = OE (Radii of same circle)

∴ ΔAOD ≅ ΔCOE (By SSS congruence criterion)

⇒ $\angle 1 = \angle 3$, $\angle 2 = \angle 4$ (CPCT) …(i)

But OA = OD and OC = OE ⇒ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ …(ii)

From (i) and (ii), we have

$\angle 1 = \angle 2 = \angle 3 = \angle 4$ (= x say)

Also, OA = OC and OD = OE

⇒ $\angle 7 = \angle 8$ (= z say) and $\angle 5 = \angle 6$ (= y say)

Now, ADEC is a cyclic quadrilateral

⇒ $\angle DAC + \angle DEC = 180^0$

⇒ $x + z + x + y = 180^0$ ⇒ $y = 180^0 - 2x - z$ …(iii)

In ΔDOE, $\angle DOE = 180^0 - 2y$

And in ΔAOC, $\angle AOC = 180^0 - 2z$

∴ $\angle DOE - \angle AOC = (180^0 - 2y) - (180^0 - 2z) = 2z - 2y$

$= 2z - 2(180^0 - 2x - z)$ (Using (iii))

$= 4z + 4x - 360^0$ …(iv)

Again, $\angle BAC + \angle CAD = 180^0$ ⇒ $\angle BAC = 180^0 - (z + x)$ …(v)

Similarly, ∠BAC = 180^0 – (z + x) …(vi)

In ΔABC, ∠ABC = 180^0 - ∠BAC - ∠BCA

= 180^0 – 2[180^0 – (z + x)] (Using (v) and (vi))

= 2z + 2x - 180^0 = $\frac{1}{2}$ (4z + 4x - 360^0) …(vii)

From (iv) and (vii), we have

∠BAC = $\frac{1}{2}$ (∠DOE - ∠AOC)

Que 3. Bisectors of angles A, B and C of a triangle ABC intersects its circumcircle at D, E and F respectively. Prove that angles of triangle DEF are 90^0 - $\frac{A}{2}$, 90^0 - $\frac{B}{2}$ and 90^0 - $\frac{C}{2}$.

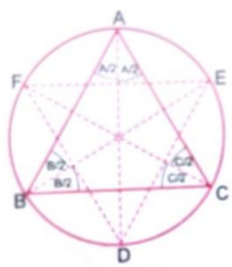

Fig. 10.58

Sol. We have ∠BED = ∠BAD

(Angles in the same segment)

⇒ ∠BED = $\frac{1}{2}$ ∠A …(i)

Also, ∠BEF = ∠BCF (Angles in the same segment)

⇒ ∠BEF = $\frac{1}{2}$ ∠C …(ii)

From (i) and (ii) ∠BED + ∠BEF = $\frac{1}{2}$ ∠A + $\frac{1}{2}$ ∠C

∠DEF = $\frac{1}{2}$ (∠A + ∠C)

⇒ ∠DEF = $\frac{1}{2}$ (180^0 - ∠B) (∵ ∠A + ∠B + ∠C = 180^0)

⇒ ∠DEF = 90^0 - $\frac{1}{2}$ ∠B