# Linear Equation in One Variable



A numeric expression can be expressed in many ways. (*Fig.*1)

Similarly, can we express 5x in different ways? (*Fig.*2)



05

Try to write some other expressions for 5x.

Are your solutions different from the solutions of your friends?



Try This

Neha, I have created many new forms of 5x $3x \times 2x$  $4x \times x$ 



What do you think? Who is right? Discuss among your friends.

Therefore, like numbers, algebraic expression can also be represented in many ways.

We will have to take care of the rules of operations for algebraic expressions.



Take some other algebraic expressions and write them in different ways.

# Equation in One Variable

Rinku	:	Jitu there are five coins of same value in my pocket, can you tell, how much money I have?	A 14
Jitu	:	How can I tell? Show me the coins and then I can tell you.	T.D. R
Reshma	a:	If you have one rupee coin then you will have five rupees in your pocket and if you have two rupees coin then you will have ten rupees. That is you have 5 <i>x</i> rupees in your pocket.	-
Rinku	:	There are 25 rupees in my pocket, so which coins do I have?	
Jitu :		Now that's easy. Since you have $5x$ rupees, $5x = 25$	- min
		I will check the value of $5x$ for $x = 1, 2, 5, 10$ and for whatever value both sides of "=" are same, will be the value of your coins.	

Think any two numbers. Like 9 and 21, now take any other two numbers smaller than ten and make relationship using the four operations:-

- (i) If we add 3 to double of nine we get 21
- (ii) If 6 is subtracted from three times of 9 we get 21
- (iii) If we divide 21 by 7 and then add six to it, we get 9
- (iv) If we subtract 3 from 21 and then divide it by 2, we get 9 .....

Similarly write 5 statements of different equations using 9 and 21 with different operations and numbers.

Obviously this can be done with any two numbers.

					1ry 1his	
Create three	relation for eac	ch of these	2-			
(i)	2 and 5	(ii)	11 and 25	(iii)	40 and 123	

See the statements given below:-

- (i) The age of Raju is 9 years and his mother's age is 35 years.
- (ii) There are 25 children in class-4 and 45 children in class-9.
- (iii) Reema has 10 rupees and Meera has 7 rupees.

In each of these we can find some relationship like the sum of Raju's mother's age and his age is 44 years.



- $(9 \times 2) + 3 = 21$ ( × ) - =
- $\frac{21}{7} + 6 = 9$

Raju's age + Mother's age = 44 years.

Children in class-9 – children in class 4 = 20

Many relationships can be developed for all such situations.

Create three relationships for each.

### Let us Now Learn to Solve One Variable Equation

Ramola: I have thought of one number, this number is 5 less than twice of 25, what is the number?

Malini : The number  $= (25 \times 2) - 5$ = 50 - 5 = 45

Therefore the number is 45.

Let us see an other example- 5 is 10 less than any number?

Then its equation will be x - 10 = 5 (Suppose that number is x)

#### Try This

1. 4 is 2 less than any number.	2. Addition of 8 and any number is 12.
---------------------------------	--

## **Making Equations**

An equation shows the equality of both sides. Let us see an example to understand how it is formed.

**EXAMPLE-1.** 4 more than 3 times of a number is equal to 13.

**Solution :** 3x + 4 = 13

**EXAMPLE-2.** The current age of a boy is twice his age four years ago.

**Solution :** Suppose the current age of the boy = b years

His age four years ago = (b-4)

then b = 2(b - 4)

In the previous chapter you saw that ax + b represents a linear algebraic expression with one variable. An equation of one variable is represented as ax + b = c. Where a, b, c are real constant numbers and  $a \neq 0$ . Similarly you can create equation from polynomial  $ax^2 + bx + c$ . Obviously here ax + b = c,  $ax^2 + bx + c = d$  are respectively linear equation with one variable and quadratic equation in one variable where a, b, c, d are real constant numbers and  $a \neq 0$ .



# Make Statement from Given Algebraic Equation

Consider an algebraic equation 5x - 2 = 10. A possible statement for this may be as follows:-

If 2 is subtracted from 5 times of any number then it is equal to 10. What is the number?

**EXAMPLE-3.** Write the mathematical statement for  $\frac{x}{4} + x = 10$ .

**SOLUTION :** Adding  $\frac{1}{4}$  of a number to itself gives 10. What is the number?



# Solution of Equation

To understand what is the solution of an equation see the following example:-

In *x*+2=7

If we place the value of x=3

 $3+2 \neq 7$  the equation is not valid

If we place the value of x=4

 $4+2 \neq 7$  the equation is not valid

If we place the value of x=5

5+2=7 the equation is true because both sides are equal

Therefore, we say that x=5 is a solution of the equation, because equation is valid for this value of x.

We can say that the solution of an equation is that value of variable for which the equation holds true.

# **Properties of Equation**



- 2. The four rules of equality for real numbers a, b, c
  - (i) If a = b then a + c = b + c

that is if two expressions are equal then the sum will remain equal if we add the same number on both the sides of the equation.

- (ii) If a = b then a c = b cthat is if two expressions are equal then the difference will remain equal when we subtract the same number from both side the of the equation.
- (iii) If a = b then ac = bc

that is if two expressions are equal then the product will remain equal if we multiply the same number on both sides of the equation.

(iv) If a = b then  $\frac{a}{c} = \frac{b}{c}$ 

that is if two expressions are equal then the quotient will remain equal when we divide the expressions by the same number on both the sides of the equation.

**EXAMPLE-4.** Solve the equation x - 7 = 3Solution : x - 7 = 3

LINEAR EQUATION IN ONE VARIABLE 105

x - 7 + 7 = 3 + 7 x = 10 (using equality rule (i) we add 7 to both sides and thus cancels (-7))EXAMPLE-5. Solve the equation 3x + 5 = 14SOLUTION : 3x + 5 = 14 3x + 5 - 5 = 14 - 5 (using equality rule (ii) subtract 5 from both sides to cancel 5). 3x = 9  $\frac{3x}{3} = \frac{9}{3}$  (using equality rule (iv) divide both sides by 3 to get rid of 3) x = 3EXAMPLE-6. Solve the equation  $\frac{4x + 5}{3} = 15$ . SOLUTION :  $\frac{4x + 5}{3} = 15$  $\frac{(4x + 5)}{3} \times 3 = 15 \times 3$  (using equality rule (iii))



x = 10EXAMPLE-7. Solve the equation 4x - 2x - 5 = 4 + 6x + 3. Solution : 4x - 2x - 5 = 4 + 6x + 3 2x - 5 = 7 + 6x (Adding of similar terms) 2x - 5 - 6x = 7 + 6x - 6x (using equality rule (ii)) -4x - 5 = 7 -4x - 5 + 5 = 7 + 5 (using equality rule (i)) -4x = 12  $\frac{-4x}{-4} = \frac{12}{-4}$  (using equality rule (iv)) x = -3

4x + 5 - 5 = 45 - 5 (using equality rule (ii))

 $\frac{4x}{4} = \frac{40}{4}$  (using equality rule (iv))

4x + 5 = 45

4x = 40

#### 106 MATHEMATICS - IX

#### Exercise - 5.1



Make equations from the statements given below:-

- (i) The sum of two consecutive numbers is 11.
- (ii) The sum of Tikendra and Tejaram's age is 30 whereas Tikendra's age is twice of Tejaram's age.
- (iii) One side of triangle is twice of the second side and equal to third side. Sum of the sides is 40.
- (iv) The length of the rectangle is 3 units more than its breadth. The perimeter of that rectangle is 15 units.
- (v) Ramesh, Dinesh and Satish have pencil in ratio of 2:3:4. The total number of pencils are 18.
- 2. Solve the equations given below:-

(i)	5x + 2 = 17	(ii)	5p + 1 = 24
(iii)	4x + 8x = 17x - 9 - 1	(iv)	-7 + 3t - 9t = 12t - 5
(v)	3(z-2) + 5z = 2	(vi)	-2 + (x + 4) = 8x

3. Make the statement for the following equations:-

(i) 
$$x+3 = 27$$
 (ii)  $\frac{x}{2} + x = 18$  (iii)  $\frac{x}{x+2} = 30$ 

- 4. Solve the equation:-
  - (i) 6 + (4 m) = 8(3m + 5) (ii) 2(k 5) + 3k = k + 6
  - (iii) 5p + 4(3 2p) = 2 + p 10 (iv)  $\frac{x}{3} + 1 = \frac{7}{75}$
  - (v)  $m \frac{m-1}{2} = 1 \frac{m-2}{3}$  (vi)  $\frac{3t-2}{4} \frac{2t+3}{3} = \frac{2}{3} t$
  - (vii)  $\frac{x}{2} \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$  (viii)  $\frac{9x}{7 6x} = 15$ (ix)  $\frac{3y + 4}{2 - 6y} = \frac{-2}{5}$

# Application of Equation

We use equations to solve many questions in mathematics like- the length of the rectangle is 11 *cm* more than its breadth. If the perimeter of the rectangle is 110 *cm* then find length and breadth of the rectangle. In this we have been given the perimeter but to calculate the length

and breadth of the rectangle we need to establish a relation with the given information. We will learn to find solution to such questions with the help of equations.

**EXAMPLE-8.** The length of a rectangle is 1 *cm* more than twice of its breadth. If perimeter of the rectangle is 110 *cm*. Then find the length and breadth of the rectangle.

**SOLUTION :** Suppose breadth of the rectangle is *w cm*.

Then according to the question, length of the rectangle = (2w + 1) cm.

Given perimeter of the rectangle = 2 (length + breadth)

Putting the values  

$$110 = 2 (2w + 1 + w)$$
  
 $110 = 4w + 2 + 2w$   
 $110 = 6w + 2$   
 $110 - 2 = 6w + 2 - 2$   
 $108 = 6w$   
 $w = \frac{108}{6} = 18 \ cm$ 

Length of rectangle (L) = 2w + 1

$$L = 2 \times 18 + 1 = 37 \ cm$$

- **EXAMPLE-9.** Find four consecutive even integer numbers, if sum of first three number is more than 8 from the fourth number.
- **SOLUTION :** Suppose 2*x* is an even integer number than four consecutive numbers

2x, 2x + 2, 2x + 4, 2x + 6

According to the question:-

Sum of first three numbers = fourth number + 8 2r + (2r + 2) + (2r + 4) = (2r + 6) + 8

$$2x + (2x + 2) + (2x + 4) = (2x + 6) + 8$$
  

$$2x + 2x + 2 + 2x + 4 = 2x + 6 + 8$$
  

$$6x + 6 = 2x + 14$$
  

$$6x + 6 - 2x = 2x + 14 - 2x$$
  

$$4x + 6 = 14$$
  

$$4x + 6 - 6 = 14 - 6$$
  

$$4x = 8$$
  

$$\frac{4x}{4} = \frac{8}{4}$$
  

$$x = 2$$



EXAMPLE-10. What must be added to the double of rational number  $-\frac{7}{3}$  to get  $\frac{3}{7}$ . SOLUTION : Twice of rational numbers is  $-\frac{7}{3}$ ,  $2 \times \left(-\frac{7}{3}\right) = -\frac{14}{3}$ Suppose we add x to it  $-\frac{14}{3}$  to get  $\frac{3}{7}$ . Then  $-\frac{14}{3} + x = \frac{3}{7}$   $7 (-14 + 3x) = 3 \times 3$   $[7 \times (-14)] + (7 \times 3x) = 9$  (-98) + 21x = 9 -98 + 21x = 9 21x = 9 + 98 21x = 107 $x = \frac{107}{21}$ 

Thus, if we add  $\frac{107}{21}$  to double of rational number  $-\frac{7}{3}$  we will get  $\frac{3}{7}$ .

**EXAMPLE-11.** If sum of three continuous multiples of 13 is 390, then what are they? **SOLUTION :** Suppose a multiple of 13 is 13x, then the next multiples will be 13(x+1) and 13(x+2).

$$0 13 13 \times 2 13 \times 3 \dots 13x 13(x+1) 13(x+2)$$

Sum of three continuously multiples of 13 is 390, therefore

-

$$13x + 13(x + 1) + 13(x + 2) = 390$$
  

$$13x + 13x + 13 + 13x + 26 = 390$$
  

$$39x + 39 = 390$$
  

$$39x = 390 - 39$$
  

$$39x = 351$$
  

$$x = \frac{351}{39}$$

= 9 х

so the required multiples of 13 are  $13 \times 9 = 117$ ,  $13 \times (9+1) = 130$  and 13  $\times$  (9+2) = 143.

- **EXAMPLE-12.** In a two digit number the difference between its digits is 3. If we add this number to the number obtained by interchanging the places of its digit, we get 143. A digit at tens place is bigger. Find the number.
- **SOLUTION :** Suppose that the digit at units place is x. The difference between the digit at the unit and tens place is 3, therefore the digit at tens place is = x + 3.

Therefore, the two digit number is:-

$$= 10 (x + 3) + x$$

$$= 10x + 30 + x$$

$$= 11x + 30$$
If the two digit number is 43  
then we can write it as  
$$43 = (10 \times 4) + 3$$

Now, let us interchange the places of the digits of this number, that means write the units digit at the tens place and the tens digit at the unit place, the number thus obtained is:-

$$= 10x + (x + 3)$$
  
= 10x + x + 3  
= 11x + 3

On adding both these numbers we get 143, so:-

$$(11x + 30) + (11x + 3) = 143$$

$$11x + 30 + 11x + 3 = 143$$

$$22x + 33 = 143$$

$$22x = 143 - 33$$

$$x = \frac{110}{22}$$

$$x = 5$$
therefore digit in units place = 5  
the digit in tens place = 5 + 3  
= 8  
So the number is = 85

So

On interchanging the digits of 43 we get the number 34, and we can write it as

$$34 = (10 \times 3) + 4$$

**Verification of the solution:**- On interchanging the digit of 85 we get 58. On adding 85 and 58 we get 143.

- **EXAMPLE-13.** The ratio between Indramani and Sohan's current age is 4 : 5. After 8 years the ratio of their ages would be 5 : 6. Find their present age.
- **SOLUTION :** Suppose the present age of Indramani is 4x years and the present age of Sohan is 5x years.

Age of Indramani after 8 year	= (4x + 8) year
Age of Sohan after 8 year	= (5x + 8) year

According to the question, the ratio of their ages after 8 years will be 5 : 6, therefore:-

4x + 8	_	5	
5x + 8	=	6	
6 (4x + 8)	=	5(5x+8)	
$(6 \times 4x) + (6 \times 8)$	=	$(5\times5x) + (5\times8)$	
24x + 48	=	25x + 40	
24x + 48 - 40	=	25 <i>x</i>	
24x + 8	=	25 <i>x</i>	
8	=	25x - 24x	
8	=	x	
So, present age of Indramani	=	4 <i>x</i>	
	=	$4 \times 8$	
	=	32 year	
And, present age of Sohan	=	5 <i>x</i>	
	=	$5 \times 8$	
	=	40 year	
<b>Checking the solution:</b> Indramani's age after 8 years $= 32 + 8 = 40$ year			
Sohan's age after 8 y	years	= 40 + 8 = 48 year	
The ratio of their age	e	$=\frac{40}{48}=\frac{5}{6}$	

#### Exercise - 5.2



- Area of a triangle is 36 *sq m* and the length of its base is 12 meter then find the height of the triangle.
- The length of the rectangle is 5 *cm* more than its breadth. Its perimeter is five times its breadth. Find the length and breadth of the rectangle.

- 3. If one angle of triangle is 15° more than its second angle. The third angle is 25° more than double the second angle. Find the three angles of the triangle.
- 4. Find three consecutive odd numbers when three times of their sum is 5 more than 8 times the middle number.
- 5. If one side of triangle is  $\frac{1}{4}$  of its perimeter, other side is 7 *cm* and third side is

 $\frac{2}{5}$  of its perimeter, then find the perimeter of the triangle.

- 6. Sum of the digits of a two-digit number is 8. The number obtained by interchanging the digits exceeds the original number by 18. Find the number.
- 7. The ages of Vimla and Sarita are in the ratio of 7 : 5. Four year later, their ages will be in the ratio 4 : 3. Find their ages.
- 8. Alka thinks of a number, she adds 5 to it. To this she adds double of the original number and then subtracts 10 from it to get 40. Find the number.
- 9. Difference between two positive integers is 40. The integers are in the ratio 2:3. Find the integers.
- 10. Sum of three consecutive multiples of 5 is 555. Find these multiples.
- 11. Age of Rohit is 5 more than twice of Pradeep's age. 6 year ago, age of Pradeep $\frac{1}{1}$

was  $\frac{1}{3}$  of Rohit's age. Find their ages.

12. A motor boat goes downstream in a river, covers the distance between two coastal towns in 5 hours. It covers same distance upstream in six hours. The speed of water is  $2 \frac{km}{h}$  find the speed of the motor boat in still water.

#### What Have We Learnt

- 1. Only one variable is used in these equations and these are linear that is the power of the variable is 1.
- 2. Both sides of the equation can be a linear expression.
- 3. Linear equation with one variable is ax + b = c (*a*, *b*, *c* real finite numbers and  $a \neq 0$ )
- 4. A quadratic equation of one variable is-  $ax^2 + bx + c = d$  (*a*, *b*, *c*, *d* are real finite numbers and  $a \neq 0$ ).
- 5. First we simplify the expressions before solving any equation.
- 6. We solve the equations using the rules of equality.

