CBSE Test Paper 05

Chapter 5 Arithmetic Progression

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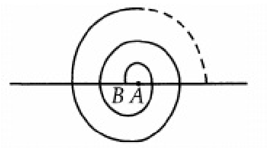
1. The sum of the first 15 multiples of 8 is **(1)**
   a. 900
   b. 960
   c. 1000
   d. 870
2. If the angles of a right angled triangle are in A.P. then the angles of that triangle will be **(1)**
   a. $45^\circ,\ 45^\circ,\ 90^\circ$
   b. $30^\circ,\ 60^\circ,\ 90^\circ$
   c. $40^\circ,\ 50^\circ,\ 90^\circ$
   d. $20^\circ,\ 70^\circ,\ 90^\circ$
3. In an A.P., if $S_n = 3n^2 + 2n$, then the value of 'a_n' is **(1)**
   a. 7n – 2
   b. 9n – 4
   c. 8n – 3
   d. 6n – 1
4. The sum of (a + b), (a – b), (a – 3b), ... to 22nd term is **(1)**
   a. 22a + 440b
   b. 22a – 440b
   c. 20a + 440b
   d. 22a – 400b
5. The first and last terms of an A.P. are 1 and 11. If their sum is 36, then the number of terms will be **(1)**
   a. 7
   b. 5
   c. 8
   d. 6
6. Is series $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, \ldots\ldots$ an A.P.? Give reason. **(1)**
7. The sum of three numbers in AP is 21 and their product is 231. Find the numbers. **(1)**

8. Find a and b such that the numbers a, 9, b, 25 form an AP. **(1)**
9. For an A.P., if a_{25} - a_{20} = 45, then find the value of d. **(1)**
10. Find the common difference of the AP : $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots.$ **(1)**
11. An A.P. consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the past three terms is 429. Find the A.P. **(2)**
12. Write the expression a_n - a_k for the AP: a, a + d, a + 2d, ... and find the common difference of the AP for which 20^{th} term is 10 more than the 18^{th} term. **(2)**
13. The sum of the first three terms of an A.P. is 33. If the product of first and the third term exceeds the second term by 29, find the AP. **(2)**
14. If the m^{th} term of an AP be $\frac{1}{n}$ and its nth term be $\frac{1}{m}$, then show that its (mn)th term is 1. **(3)**
15. Find the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7^{th} term is two more than thrice of its 3rd term. **(3)**
16. The ratio of the sums of first m and first n terms of an A.P. is $m^2 : n^2$. Show that the ratio of its m^{th} and n^{th} terms is (2m - 1):(2n -1). **(3)**
17. A spiral is made up of successive semi-circles with centres alternately at A and B starting with A, of radii 1 cm, 2 cm, 3 cm,.... as shown in the figure. What is the total length of spiral made up of eleven consecutive semi-circles? **(3)**



18. In an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{13}{2}n.$ Find its 25^{th} term. **(4)**
19. Find the sum of all integers between 100 and 550 which are not divisible by 9. **(4)**
20. If the sum of the first n terms of an A.P. is $4n - n^2$, what is the first term? What is the sum of first two terms? What is the second term? Similarly, find the third, the tenth and the nth terms. **(4)**

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Solution

1. b. 960

   Explanation: Multiples of 8 are 8, 16, 24, ...

   Here $a=8, d=16-8=8$ and $n=15$

   Now, $\mathrm{S}_n=\frac{n}{2}[2a+(n-1)d]$

   $\Rightarrow \mathrm{S}_{15}=\frac{15}{2}[2\times 8+(15-1)8]$

   $\Rightarrow \mathrm{S}_{15}=\frac{15}{2}[16+14\times 8]$

   = $\frac{15}{2}\times 128$

   = 15 $\times$ 64

   = 960

2. b. $30^{\circ},\ 60^{\circ},\ 90^{\circ}$

   Explanation: Let the three angles of a triangle be $a-d, a$ and $a+d$.

   $\therefore a-d+a+a+d=180^{\circ}$

   $\Rightarrow 3a=180^{\circ}$

   $\Rightarrow a=60^{\circ}$

   Therefore, one angle is of 60° and other is 90° (given).

   Let third angle be x°, then

   $60^{\circ}+90^{\circ}+x^{\circ}=180^{\circ}$

   $\Rightarrow 150^{\circ}+x^{\circ}=180^{\circ}$

   $\Rightarrow x^{\circ}=180^{\circ}-150^{\circ}=30^{\circ}$

   Therefore, the angles of the right angled triangle are $30^{\circ}, 60^{\circ}, 90^{\circ}$.

3. d. 6n – 1

   Explanation: Given: $S_n=3n^2+2n$

   $\mathrm{S}_1=3(1)^2+2\times 1=3+2=5$

   $\Rightarrow a=5$

   S_2 = $3(2)^2$ + 2 $\times$ 2

   S_2 = 3 $\times$ 4 + 4

   S_2 = 12+4

$S_2 = 16$

$\Rightarrow a_1 + a_2 = 16$

$\Rightarrow a_1 = 5$

$\Rightarrow a_2 = 11$

$\therefore d = a_2 - a_1 = 11 - 5 = 6$

$\therefore a_n = a + (n-1)\,d$

$= 5 + (n-1)\,6 = 5 + 6n - 6 = 6n - 1$

4. b. 22a – 440b

Explanation: Given: $a = a + b, d = a - b - a - b = -2b$

$\therefore S_{22} = \frac{22}{2}[2(a+b) + (22-1)(-2b)]$

$= 11[2a + 2b + (21)(-2b)]$

$\Rightarrow S_{22} = 11[2a + 2b - 42b]$

$= 11[2a - 40b]$

$= 22a - 440b$

5. d. 6

Explanation: Given: $a = 1, l = 11$ and $S_n = 36$

$\therefore S_n = \frac{n}{2}(a + l)$

$\Rightarrow 36 = \frac{n}{2}(1 + 11)$

$\Rightarrow 72 = n \times 12$

$\Rightarrow n = 6$

6. Common difference,

$d_1 = \sqrt{6} - \sqrt{3}$

$= \sqrt{3}(\sqrt{2} - 1)$

$d_2 = \sqrt{9} - \sqrt{6}$

$= \sqrt{3 \times 3} - \sqrt{2 \times 3}$

$= 3 - \sqrt{6}$

$d_3 = \sqrt{12} - \sqrt{9}$

$= \sqrt{4 \times 3} - \sqrt{9}$

$= 2\sqrt{3} - 3$

As common difference does not equal.

Hence, The given series is not in A.P.

7. Let the required numbers be (a-d), a and (a + d).............(1)

   Then, according to question, (a - d) + a + (a + d) = 21

   $\Rightarrow$ 3a = 21

   $\Rightarrow$ a = 7.

   And, (a - d)· a· (a + d) = 231 $\Rightarrow$ a (a^2 - d^2) = 231

   $\Rightarrow$7(49 - d^2) = 231 [∵a = 7]

   $\Rightarrow$7d^2 = 343 - 231 = 112

   $\Rightarrow$$d^2$ = 16

   $\Rightarrow$d = $\pm$ 4.

   Thus, a = 7 and d = $\pm$4. Now substitute these values of a and d in above equation (1).

   Therefore, the required numbers are(3,7,11) or (11,7,3).

8. The numbers $a, 9, b, 25$ form an AP,

   we have

   $9-a=b-9=25-b.$

   Now, b - 9 = 25 - b $\Rightarrow$ 2b = 34 $\Rightarrow$ b = 17.

   And, 9 - a = b - 9 $\Rightarrow$ a + b = 18 $\Rightarrow$ a + 17 = 18 $\Rightarrow$ a = 1.

   Hence, $a=1\ and\ b=17.$

9. Let the first term of an A.P be a and common difference d.

   $a_n=a+(n-1)d$

   ∴a_{25}- a_{20} =$[a+(25-1)d]-[a+(20-1)d]$

   or, 45 = $a+24d-a-19d$

   or, 45 = $5d$

   or, $d=\frac{45}{5}$ = 9

10. Common difference(d) = $n^{th}term-(n-1)^{th}term$

    $\therefore d=a_2-a_1$

    $d=(\frac{1-p}{p})-(\frac{1}{p})=\frac{(1-p)-(1)}{p}=\frac{-p}{p}=-1$

    $d=-1$

11. Let the middle most terms of the A.P. be $(a-d),a,(a+d)$

    Given $a-d+a+a+d=225$

    $3a=225$

or, $a = 75$

and the middle term = $\frac{37+1}{2}$ = 19th term

$\therefore$ A.P. is

(a - 18d),....(a - 2d), {a - d), a, (a + d), (a + 2d),..........$(a + 18d)$

Sum of last three terms

$(a + 18d) + (a + 17d) + (a + 16d) = 429$

or,$3a + 51d = 429$

or, $225 + 51d = 429$

or, d = 4

First term a_1 = a - 18d = 75 - 18 $\times$ 4 = 3

a_2 = 3 + 4 = 7

Hence, A.P. = 3, 7, 11 ,.........., 147

12. a_n = a + (n - 1)d; a_k = a + (k - 1)d

    Now, a_n - a_k = [a + (n - 1)d] - [a + (k - 1)d] = (n - 1)d - (k - 1)d = (n - 1 - k + 1)d

    a_n - a_k = (n - k)d ....(1)

    Let a_{18} = x.

    a_{20} = x + 10

    Taking n = 20 and k = 18, equation (1) becomes

    a_{20} - a_{18} = (20 -18)d $\Rightarrow$ (x + 10) - x = 2d $\Rightarrow$ d = 5

13. Let the first three terms in A.P. be a - d, a, a + d. It is given that the sum of these terms is 33.

    $\therefore$ a - d + a + a + d = 33

    $\Rightarrow$ 3a = 33

    $\Rightarrow$ a = 11

    It is given that

    $a_1 \times a_3 = a_2 + 29$

    (a - d)(a + d) = a + 29

    $a^2 - d^2 = a + 29$

    $121 - d^2 = 11 + 29$

    $d^2 = 121 - 40 = 81$

$d = \pm 9$

If d = 9 then the series is 2,11,20,29

If d = -9 then the series is 20,11,2,-7,-16

14. Let a be the first term and d be the common difference of the given AP. Now, we know that in general mth and nth terms of the given A.P can be written as

    T_m = a + (m-l)d and T_n = a + (n-1)d respectively.

    Now, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$ (given).

    ∴ a + (m-1)d = $\frac{1}{n}$ ....(i)

    and a + (n-1)d = $\frac{1}{m}$ .... (ii)

    On subtracting (ii) from (i), we get

    (m-n)d = $(\frac{1}{n} - \frac{1}{m}) = (\frac{m-n}{mn}) \Rightarrow$ d = $\frac{1}{mn}$

    Putting d = $\frac{1}{mn}$ in (i), we get

    a + $\frac{(m-1)}{mn} \Rightarrow$ a = $\{\frac{1}{n} - \frac{(m-1)}{mn}\} = \frac{1}{mn}$

    Thus, a= $\frac{1}{mn}$ and d= $\frac{1}{mn}$

    ∴Now, in general (mn)th term can be written as T_{mn} = a +(mn-1)d

    = $\{\frac{1}{mn} + \frac{(mn-1)}{mn}\}$ [∵a= $\frac{1}{mn}$]

    = 1.

    Hence, the (mn)th term of the given AP is 1.

15. Here, we have the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7^{th} term is two more than thrice of its 3rd term.

    Let "a" be the first term and" d"be the common difference of the given A.P. Therefore, a_3 = 7 and a_7 = 3a_3 + 2 [Given]

    ⇒a + 2d = 7 and a + 6d = 3(a + 2d) + 2

    ⇒ a + 2d = 7 and a + 6d = 3a + 6d + 2

    ⇒ a + 2d = 7 and a -3a = 6d - 6d + 2

    ⇒a + 2d = 7 and -2a = 2

    ⇒a + 2d = 7 and a = -1

    ⇒-1 + 2d = 7

    ⇒2d = 7 + 1 = 8

    ⇒d = 4

$\Rightarrow$ a = -1 and d = 4

Putting n = 20, a = - 1 and d = 4 in $S_n = \frac{n}{2}\{2a + (n-1)d\}$,we get

$S_{20} = \frac{20}{2}\{2 \times -1 + (20-1) \times 4\} = \frac{20}{2}(-2+76) = 740$

16. Let first term of given A.P. be a and common difference be d also sum of first m and first n terms be S_m and S_n respectively

$\therefore \quad \frac{S_m}{S_n} = \frac{m^2}{n^2}$

or, $\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m^2}{n^2} \times \frac{n}{m}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

or, $m(2a + (n-1)d) = n[2a + (m-1)d]$

Now, $\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$

$= \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$

or, $= \frac{a+2ma-2a}{a+2na-2a}$

or, $= \frac{2ma-a}{2na-a}$

or, $= \frac{a(2m-1)}{a(2n-1)}$

or, $= \frac{(2m-1)}{(2n-1)}$

= $2m-1 : 2n-1$

The ratio of its mth and nth terms is $2m-1 : 2n-1$.

Hence proved

17. Let r_1, r_2,.... be the radii of semicircles and l_1, l_2 .....be the lengths of circumferences of semi-circles, then

l_1 = $\pi r_1 = \pi(1) = \pi\text{cm}$

$l_2 = \pi r_2 = \pi(2) = 2\pi\text{cm}$

$l_3 = \pi r_3 = \pi(3) = 3\pi\text{cm}$

....

$l_{11} = \pi r_{11} = \pi(11) = 11\pi\text{cm}$

$\therefore$ Total length of spiral

= $l_1 + l_2 + \ldots + l_{11}$

= $\pi + 2\pi + 3\pi + \ldots + 11\pi$

= π(1 + 2 + 3 + 4 ....+ 11)
$= \pi \times \frac{11\times12}{2}$
$= 66 \times 3.14$
$= 207.24cm$

18. According to the question,
    Given Sum of n terms $(S_n) = \frac{3n^2}{2} + \frac{13}{2}n$
    Put n = 24, $S_{24} = \frac{3\times24\times24}{2} + \frac{13\times24}{2}$
    = 864 + 156
    = 1020
    Put n = 25, $S_{25} = \frac{3\times25\times25}{2} + \frac{13\times25}{2}$
    $= \frac{1875}{2} + \frac{325}{2}$
    $= \frac{2200}{2} = 1100$
    $\therefore$ 25th term (a_{25}) = S_{25} - S_{24}
    = 1100 - 1020
    = 80

19. All integers between 100 and 550, which are divisible by 9
    = 108, 117, 126,.........., 549
    First term (a) = 108
    Common difference(d) = 117 - 108 = 9
    Last term(a_n) = 549
    $\Rightarrow$a + (n - 1)d = 549
    $\Rightarrow$108 + (n - 1)(9) = 549
    $\Rightarrow$108 + 9n - 9 = 549
    $\Rightarrow$9n = 549 + 9 - 108
    $\Rightarrow$ 9n = 450
    $\Rightarrow \quad n = \frac{450}{9} = 50$
    Sum of 50 terms = $\frac{n}{2}[a + a_n]$
    $= \frac{50}{2}[108 + 549]$
    $= 25 \times 657$
    = 16425
    Now, sum of all integers between 100 and 550 which are not divisible by 9
    = Sum of all integers between 100 and 550 - Sum of all integers between 100 and 550

which are divisible by 9

= [101 + 102 + 130 +......+ 549] - 16425

= $\frac{549 \times 550}{2} - \frac{100 \times 101}{2}$ - 16425

= 150975 - 5050 - 16425

= 129500

20. Given that,

$S_n = 4n - n^2$

First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms = S_2

$= 4(2) - (2)^2 = 8 - 4 = 4$

Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

$a_n = a + (n - 1)d$

$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

Therefore, $a_3 = 5 - 2(3) = 5 - 6 = -1$

$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4. The second term is 1. 3^{rd}, 10^{th} and n^{th} terms are -1, -15, and 5 - 2n respectivey.