EXERCISE 5.4 (SQUARE-ROOT OF COMPLEX NUMBER)

Find the Square-root of the following:

QNo 1 $-15-8i$

Sol Let $\sqrt{-15-8i} = x-iy$

$\Rightarrow -15-8i = x^2+i^2y^2-2xyi$ [squaring both sides]

or $-15-8i = (x^2-y^2)-2xyi$

Equating Real and Imaginary parts

$x^2-y^2 = -15$ ----(1)

$2xy = 8$

Now $(x^2+y^2)^2 = (x^2-y^2)^2+4x^2y^2$

$= (-15)^2+(8)^2 = 225+64 = 289 = (17)^2$

$\Rightarrow x^2+y^2 = 17$ ---(2) [Rejecting $x^2+y^2=-17$ as sum of squares of two real numbers can not be negative]

Adding (1) and (2)

$2x^2 = 2 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

When $x = 1$

We have $2xy = 8$

$\Rightarrow xy = 4$

$\Rightarrow y = \frac{4}{x} = \frac{4}{1} = 4$

$\therefore x-iy = 1-4i$

When $x=-1$

$2xy = 8$

$\Rightarrow xy = 4$

$\Rightarrow y = \frac{4}{x} = \frac{4}{-1} = -4$

$\therefore x-iy = -1+4i$

Combining the two results we get

$x-iy = \pm(1-4i)$

$\Rightarrow \sqrt{-15-8i} = \pm(1-4i)$

Q.No.2 $-8-6i$

Sol. Let $\sqrt{-8-6i} = x-iy$.

$\Rightarrow \quad -8-6i = x^2+i^2y^2-2xyi$

$\Rightarrow \quad -8-6i = (x^2-y^2)-2xyi$

Comparing real and imaginary parts we get

$x^2-y^2 = -8 \quad \text{.....(1)}$

$2xy = 6$

Now $(x^2+y^2)^2 = (x^2-y^2)^2+4x^2y^2$

$= (-8)^2+(6)^2$

$= 64+36 = 100 = (10)^2$

$\Rightarrow \quad x^2+y^2 = 10 \quad \text{....(2)}$ [$\because$ x^2+y^2 can not be -ve so rejecting $x^2+y^2=-10$]

Adding (1) and (2)

$2x^2 = 2$

$\Rightarrow \quad x^2 = 1 \Rightarrow x = \pm 1$

When $x = 1$

$2xy = 6$

$\Rightarrow y = \frac{6}{2x} = \frac{6}{2} = 3$

$\therefore \quad x-iy = 1-3i$

When $x=-1$

$2xy = 6$

$\Rightarrow \quad y = \frac{6}{2x} = \frac{6}{-2} = -3$

$\therefore \quad x-iy = -1+3i$

Combining the two results we get

$x-iy = \pm(1-3i)$

$\therefore \sqrt{-8-6i} = \pm(1-3i)$

Q No 3 $\quad 1-i$

Sol : Let $\sqrt{1-i} = x - iy$.

$\Rightarrow \quad 1 - i = x^2 + i^2y^2 - 2xyi$

or $\quad 1 - i = (x^2 - y^2) - 2xyi$

Comparing Real and Imaginary parts

$x^2 - y^2 = 1 \quad \dots (1)$

$2xy = 1$.

Now $(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$= (1)^2 + (1)^2$

$= 2 = (\sqrt{2})^2$

$\therefore \quad x^2+y^2 = \sqrt{2} \quad \dots (2)$ [Rejecting $x^2+y^2 = -\sqrt{2}$ as sum of squares of real nos. can not be $-$ve]

Adding (1) and (2) we get

$2x^2 = 1 + \sqrt{2}$

$\Rightarrow \quad x^2 = \frac{1+\sqrt{2}}{2} \quad \Rightarrow \quad x = \pm\sqrt{\frac{\sqrt{2}+1}{2}}$

Subtracting (1) from (2)

$2y^2 = \sqrt{2} - 1 \quad \Rightarrow \quad y^2 = \frac{\sqrt{2}-1}{2}$

$\Rightarrow \quad y = \pm\sqrt{\frac{\sqrt{2}-1}{2}}$

Now As (xy) is +ve $\therefore$ x and y will be of same sign.

$\therefore$ When $x = +\sqrt{\frac{\sqrt{2}+1}{2}} \qquad y = +\sqrt{\frac{\sqrt{2}-1}{2}}$

When $x = -\sqrt{\frac{\sqrt{2}+1}{2}} \qquad y = -\sqrt{\frac{\sqrt{2}-1}{2}}$

$\therefore \sqrt{1-i} = x - iy = +\sqrt{\frac{\sqrt{2}+1}{2}} - i\sqrt{\frac{\sqrt{2}-1}{2}}, \quad \sqrt{\frac{\sqrt{2}+1}{2}} + i\sqrt{\frac{\sqrt{2}-1}{2}}$

QNo 4 $-i$ or $0-i$

Sol. Let $\sqrt{0-i} = x-iy$

$\Rightarrow \quad 0-i = (x^2-y^2) - 2xyi$

Comparing real and imaginary parts

$x^2-y^2 = 0$ ---- (1)

$2xy = 1$

Now $(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$= (0)^2 + (1)^2$

$= 1 = (1)^2$

$\Rightarrow x^2+y^2 = 1$ --- (2) [Rejecting $x^2+y^2=-1$ as sum of +ve number can not be -ve]

Adding (1) and (2)

$2x^2 = 1$

$\Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$

Subtracting (1) from (2) we get

$2y^2 = 1 \Rightarrow y^2 = \frac{1}{2} \Rightarrow y = \pm\frac{1}{\sqrt{2}}$

Now as xy is +ve $\therefore$ x and y will have same sign

$\therefore \quad x-iy = \frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$

$\therefore \quad \sqrt{-i} = \pm\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

Q No. 5 i or $0+i$

Sol. Let $\sqrt{i} = x+iy$.

$\Rightarrow \quad i = (x^2-y^2) + 2xyi$

Equaling real and imaginary parts

$$x^2-y^2 = 0 \quad \text{---(1)}$$

$$2xy = 1$$

Now $(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$$= (0)^2 + (1)^2$$

$$= 1^2$$

$\Rightarrow \quad x^2+y^2 = 1$ ---(2) $\left[\begin{array}{l}\text{Rejecting } x^2+y^2 = -1 \\ \text{as } x^2+y^2 > 0\end{array}\right]$

Adding (1) and (2) we get

$$2x^2 = 1$$

$$\Rightarrow x^2 = \frac{1}{2} \Rightarrow x = \pm\frac{1}{\sqrt{2}}$$

Subtracting (1) from (2)

$$2y^2 = 1$$

$$\Rightarrow y^2 = \frac{1}{2} \Rightarrow y = \pm\frac{1}{\sqrt{2}}$$

Since xy is +ve

$\therefore$ x and y will have same sign

$\therefore \quad x+iy = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$ or $-\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}$

$\therefore \quad \sqrt{i} = x+iy = \pm\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$

Q No 6 $1+i$

Sol: Let $\sqrt{1+i} = x+iy$

$\Rightarrow$ $1+i = (x^2-y^2)+2xyi$

Equating real and imaginary parts

$$x^2-y^2 = 1 \quad ----(1)$$

$$2xy = 1$$

Now $(x^2+y^2)^2 = (x^2-y^2)^2 + 4x^2y^2$

$$= (1)^2+(1)^2$$

$$= 2$$

$\Rightarrow$ $x^2+y^2 = \sqrt{2}$ ----(2) [Rejecting $x^2+y^2 = -\sqrt{2}$ as sum of squares of real nos. can not be -ve

Adding (1) and (2)

$$2x^2 = \sqrt{2}+1$$

$\Rightarrow$ $x^2 = \frac{\sqrt{2}+1}{2}$ $\Rightarrow$ $x = \pm\sqrt{\frac{\sqrt{2}+1}{2}}$

Subtracting (1) from (2)

$$2y^2 = \sqrt{2}-1$$

$\Rightarrow$ $y^2 = \frac{\sqrt{2}-1}{2}$ $\Rightarrow$ $y = \pm\sqrt{\frac{\sqrt{2}-1}{2}}$

Since xy is +ve

$\therefore$ x and y are of same sign

$\therefore$ $x+iy = \pm\left(\sqrt{\frac{\sqrt{2}+1}{2}} + \sqrt{\frac{\sqrt{2}-1}{2}}\, i\right)$

$\therefore$ $\sqrt{1+i} = \pm\left(\sqrt{\frac{\sqrt{2}+1}{2}} + \sqrt{\frac{\sqrt{2}-1}{2}}\, i\right)$

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