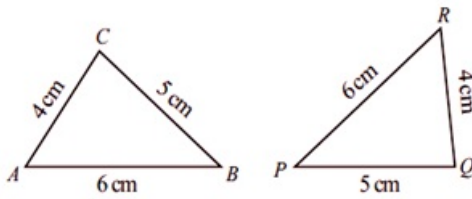


# 1. Equal Triangles

## Questions Pg-11

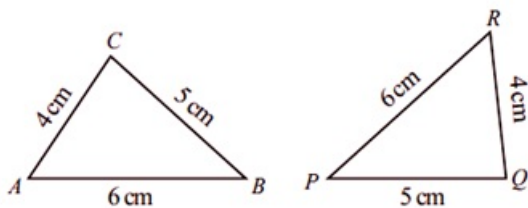
### 1 A. Question

In each pair of triangles below, find all pairs of matching angles and write them down.



### Answer

To find all pairs of matching angles, we always look for the corresponding sides or similar sides in the two triangles and the angles between any two of the corresponding sides in the two figures are matching angles.



In the triangles given above, we first find the corresponding sides or similar sides.

$$AC = RQ = 4 \text{ cm}$$

$$CB = PQ = 5 \text{ cm}$$

$$AB = PR = 6 \text{ cm}$$

To find pairs of matching angles, the angles between any two of the corresponding sides in the two triangles are matching angles.

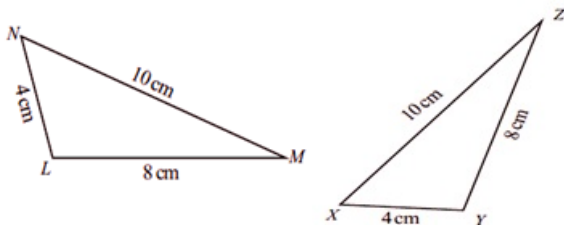
a) Angle between AC(4 cm) and AB(6 cm),  $\angle CAB =$  Angle between RQ(4 cm) and PR(6 cm) ,  $\angle PRQ$ .

b) Angle between AC(4 cm) and CB(5 cm),  $\angle ACB =$  Angle between RQ(4 cm) and PQ(5 cm) ,  $\angle RQP$ .

c) Angle between CB(5 cm) and AB(6 cm),  $\angle CBA =$  Angle between PQ(5 cm) and PR(6 cm) ,  $\angle QPR$ .

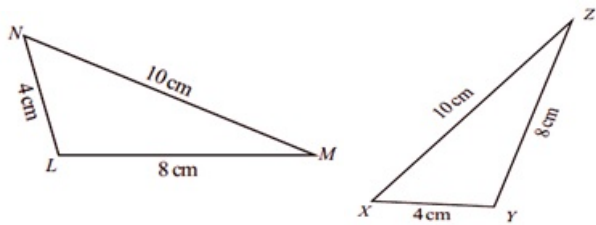
### 1 B. Question

In each pair of triangles below, find all pairs of matching angles and write them down.



### Answer

To find all pairs of matching angles, we always look for the corresponding sides or similar sides in the two triangles and the angles between any two of the corresponding sides in the two figures are matching angles.



In the triangles given above, we first find the corresponding sides or similar sides.

$$NL = XY = 4 \text{ cm}$$

$$LM = YZ = 8 \text{ cm}$$

$$NM = XZ = 10 \text{ cm}$$

To find pairs of matching angles, the angles between any two of the corresponding sides in the two triangles are matching angles.

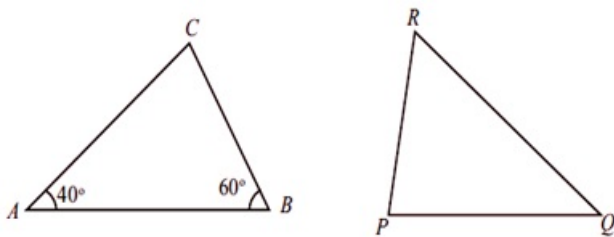
a) Angle between NL (4 cm) and LM (8 cm),  $\angle NLM = \text{Angle between } XY (4 \text{ cm}) \text{ and } YZ (8 \text{ cm}), \angle XYZ$ .

b) Angle between LM (8 cm) and NM (10 cm),  $\angle LMN = \text{Angle between } YZ (8 \text{ cm}) \text{ and } XZ (10 \text{ cm}), \angle YZX$ .

c) Angle between NM (10 cm) and NL (4 cm),  $\angle MNL = \text{Angle between } XZ (10 \text{ cm}) \text{ and } XY (4 \text{ cm}), \angle ZXY$ .

## 2. Question

In the triangles below,  $AB = QR$   $BC = RP$   $CA = PQ$



Compute  $\angle C$  and  $\Delta ABC$  and all angles of  $\Delta PQR$ .

### Answer

We know sum of all interior angles of a triangle is  $180^\circ$ .

In  $\Delta ABC$ ,

$$\angle a + \angle b + \angle c = 180^\circ$$

$$40^\circ + 60^\circ + \angle c = 180^\circ$$

$$100^\circ + \angle c = 180^\circ$$

$$\angle c = 180^\circ - 100^\circ$$

$$\angle c = 80^\circ$$

To find all angles in  $\Delta PQR$ :

Given,  $AB = QR$

$BC = RP$

$CA = PQ$

To find pairs of matching angles, the angles between any two of the corresponding sides in the two figures are matching angles.

Thus,

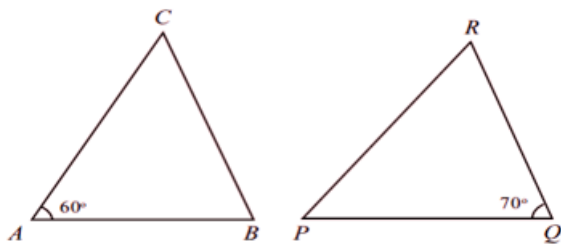
$$\angle CAB = \angle PQR = 40^\circ$$

$$\angle ABC = \angle QRP = 60^\circ$$

$$\angle BCA = \angle RPQ = 80^\circ$$

### 3. Question

In the triangles below,  $AB = QR$   $BC = PQ$   $CA = RP$



Compute the remaining angles of both triangles.

### Answer

Given,  $AB = QR$

$BC = PQ$

$CA = RP$ , which are corresponding sides of both triangles.

Thus, angles between any two of the corresponding sides in the two figures are matching angles.

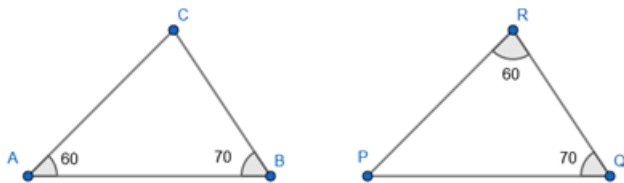
Finding matching angles will help us to compute the remaining angles.

Therefore,

$$\angle ABC = \angle PQR = 70^\circ$$

$$\angle PRQ = \angle CAB = 60^\circ$$

Now, we have,



In  $\triangle ABC$ ,

$$\angle a + \angle b + \angle c = 180^\circ$$

$$60^\circ + 70^\circ + \angle c = 180^\circ$$

$$130^\circ + \angle c = 180^\circ$$

$$\angle c = 180^\circ - 130^\circ$$

$$\angle c = 50^\circ$$

In  $\triangle PQR$ ,

$$\angle p + \angle q + \angle r = 180^\circ$$

$$\angle p + 70^\circ + 60^\circ = 180^\circ$$

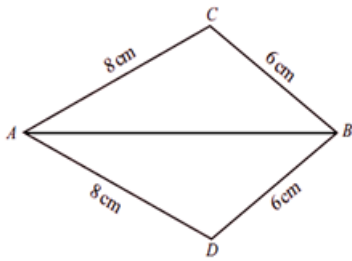
$$\angle p + 130^\circ = 180^\circ$$

$$\angle p = 180^\circ - 130^\circ$$

$$\angle p = 50^\circ$$

### 4. Question

Are the angles of  $\triangle ABC$  and  $\triangle ABD$  equal in the figure above? Why



### Answer

Since  $AC = AD$  and  $CB = DB$ ,  $\angle ACB = \angle ADB$ .

As both the triangles have  $AB$  as a common side, angle between any side and  $AB$  in  $\triangle ABC$  will be a matching angle to angle between

any side and  $AB$  in  $\triangle ABD$ .

So,  $AC = AD$  and  $AB$  which is common,  $\angle CAB = \angle DAB$ .

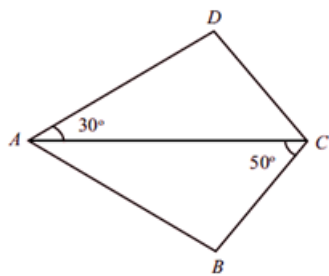
And,  $CB = DB$  and  $AB$  which is common,  $\angle CBA = \angle DBA$ .

Hence, all the angles of  $\triangle ABC$  and  $\triangle ABD$  are equal.

### 5. Question

In the quadrilateral,  $ABCD$  shown below,

$AB = AD$   $BC = CD$



Compute the angles of the quadrilateral.

### Answer

Given,  $AB = AD$

$BC = CD$

$AC$  which is common to both  $\triangle ADC$  and  $\triangle ABC$ .

So,  $\angle DAC = \angle CAB = 30^\circ$  as  $AB = AD$  and  $AC$  which is common.

And,  $\angle DCA = \angle ACB = 50^\circ$  as  $BC = CD$  and  $AC$  which is common.

Now, we know sum of all interior angles of a triangle is  $180^\circ$ .

In  $\triangle ADC$ ,

$$\angle a + \angle d + \angle c = 180^\circ$$

$$30^\circ + \angle d + 50^\circ = 180^\circ$$

$$80^\circ + \angle d = 180^\circ$$

$$\angle d = 180^\circ - 80^\circ$$

$$\angle d = 100^\circ$$

In  $\triangle ABC$ ,

$$\angle a + \angle b + \angle c = 180^\circ$$

$$30^\circ + \angle b + 50^\circ = 180^\circ$$

$$80^\circ + \angle b = 180^\circ$$

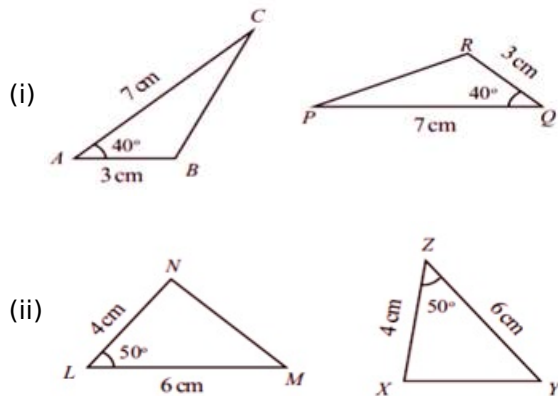
$$\angle b = 180^\circ - 80^\circ$$

$$\angle b = 100^\circ$$

## Questions Pg-15

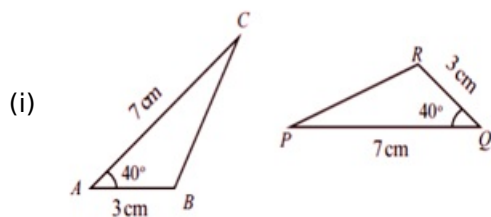
### 1. Question

In each pair of triangles below, find the pairs of matching angles and write then down.



### Answer

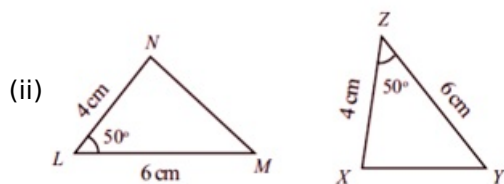
To find all pairs of matching angles, we always look for the corresponding sides or similar sides in the two triangles and the angles between any two of the corresponding sides in the two figures are matching angles.



We can see that  $\angle CAB = \angle PQR = 40^\circ$  as shown in the figure.

Angle between AC (7 cm) and CB,  $\angle ACB =$  Angle between PQ (7 cm) and PR,  $\angle QPR$ .

Angle between AB (3 cm) and CB,  $\angle ABC =$  Angle between RQ (3 cm) and PR,  $\angle PRQ$ .



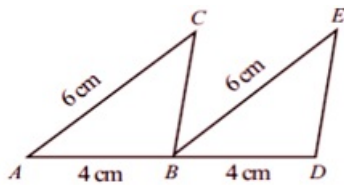
We can see that  $\angle NLM = \angle XZY = 50^\circ$  as shown in the figure.

Angle between LN (4 cm) and NM,  $\angle LNM =$  Angle between XZ (4 cm) and XY,  $\angle ZXY$

Angle between LM (6 cm) and NM,  $\angle LMN =$  Angle between YZ (6 cm) and XY,  $\angle ZYX$

### 2. Question

In the figure below, AC and BE are parallel lines:



i) Are the length of BC and DE equal? Why?

ii) Are BC and DE parallel? Why?

### Answer

$$AC = BE = 6 \text{ cm}$$

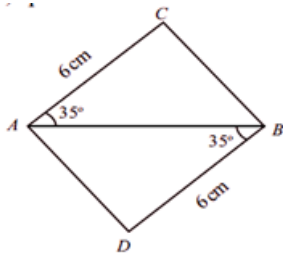
$$AB = BD = 4 \text{ cm}$$

(i) Yes, the length of BC and DE is equal because they are the corresponding sides of two equal triangles.

(ii) Yes, BC and DE are parallel because they are equal and corresponding sides of two equal triangles.

### 3. Question

Is ACBD in the figure, a parallelogram? Why?



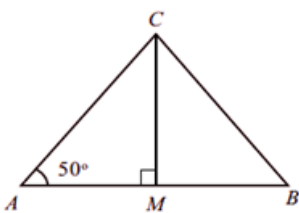
### Answer

We can see that the diagonal AB makes the same angle of  $35^\circ$  with each AC(6 cm) and DB(6 cm). Hence, it acts as a transversal

between AC and DB which makes AC and DB parallel and equal lines. Hence, ABCD is a parallelogram.

### 4. Question

In the figure below, M is the midpoint of the line AB. Compute the other two angles of  $\triangle ABC$ .



### Answer

Given, M is the midpoint of the line AB and  $\angle m = 90^\circ$ .

$$\angle a = 50$$

We know sum of all interior angles of a triangle is  $180^\circ$ .

In  $\triangle AMC$ ,

$$\angle a + \angle b + \angle c = 180^\circ$$

$$50^\circ + 90 + \angle c = 180^\circ$$

$$140^\circ + \angle c = 180^\circ$$

$$\angle c = 180^\circ - 140^\circ$$

$$\angle c = 40^\circ$$

In  $\triangle MBC$ ,

$$\angle c = 40^\circ$$

$$\angle m = 90^\circ$$

$$\angle b = ?$$

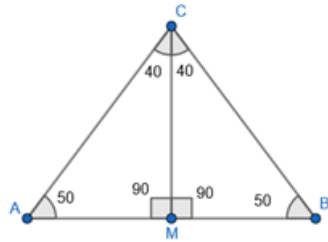
$$\angle m + \angle b + \angle c = 180^\circ$$

$$90^\circ + \angle b + 40^\circ = 180^\circ$$

$$130^\circ + \angle b = 180^\circ$$

$$\angle b = 180^\circ - 130^\circ$$

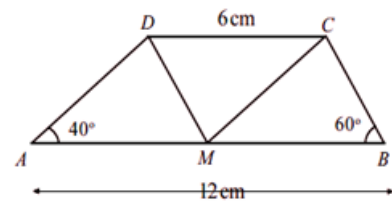
$$\angle b = 50^\circ$$



$$\angle a = 50^\circ, \angle b = 50^\circ, \angle c = 80^\circ$$

### 5. Question

In the figure below, the lines AB and CD are parallel and M is the midpoint of AB.



i) Compute the angles of  $\triangle AMD$ ,  $\triangle MBC$  and  $\triangle DCM$ ?

ii) What is special about the quadrilaterals AMCD and MBCD?

### Answer

Given, lines AB and CD are parallel and M is the midpoint of AB.

As AB and CD are parallel, AD, DM, MC and CB are transversal.

(i)  $\angle AMD = \angle MBC = 60^\circ$  (corresponding angles)

$\angle CMB = \angle DAM = 40^\circ$  (corresponding angles)

$\angle CDM = \angle AMD = 60^\circ$  (alternate interior angles)

$\angle DCM = \angle CMB = 40^\circ$  (alternate interior angles)

On straight line AMB,

$\angle AMD + \angle DMC + \angle CMB = 180^\circ$  (angles in a straight line)

$$60^\circ + \angle DMC + 40^\circ = 180^\circ$$

$$\angle DMC + 100^\circ = 180^\circ$$

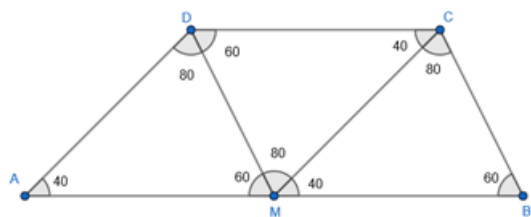
$$\angle DMC = 180^\circ - 100^\circ$$

$$\angle DMC = 80^\circ$$

$\angle ADM = \angle DMC = 80^\circ$  (alternate interior angles)

$\angle MCB = \angle DMC = 80^\circ$  (alternate interior angles)

Therefore, now we have,



(ii) Quadrilaterals AMCD and MBCD both contain two equal triangles. That is what makes special.

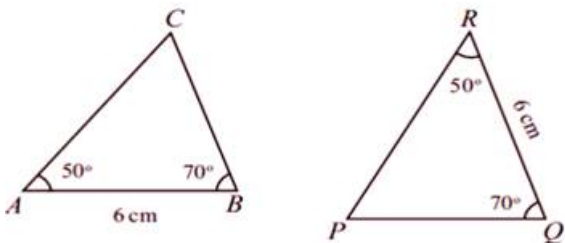
Quad. AMCD consists of  $\triangle AMD$  and  $\triangle DMC$ .

Quad. MBCD consists of  $\triangle MBC$  and  $\triangle DMC$ .

## Questions Pg-21

### 1 A. Question

In each pair of triangles below, find matching pairs of sides and write their names.



### Answer

In  $\triangle ABC$  and  $\triangle PQR$ ,

$(AB) = l(RQ) = 6 \text{ cm}$

$\angle A = \angle R = 50^\circ$

$\angle B = \angle Q = 70^\circ$

According to property,

If one side of a triangle and angle at its ends are equal to one side of another triangle and the angles at its ends, then the third angles are also equal and the sides opposite equal angle are equal.

In  $\triangle ABC$  and  $\triangle PQR$ ,

BC is opposite  $\angle A$  and PQ is opposite  $\angle R$

Also, AC is opposite  $\angle B$  and PR is opposite  $\angle Q$

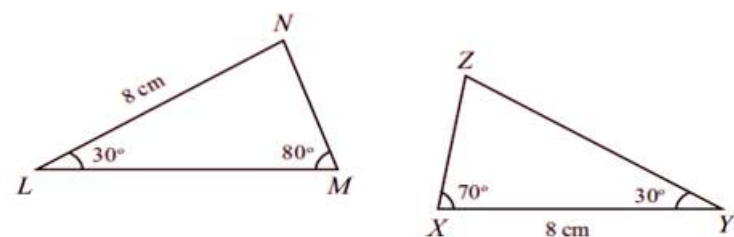
$\therefore BC = PQ$

And  $AC = PR$

Also,  $AB = RQ = 6 \text{ cm}$  (given) opposite  $\angle C$  and  $\angle P$

### 1 B. Question

In each pair of triangles below, find matching pairs of sides and write their names.





## Answer

In  $\triangle LMN$  and  $\triangle XYZ$ ,

$$LN = XY = 8 \text{ cm}$$

$$\angle L = \angle Y = 30^\circ$$

In  $\triangle LMN$ , sum of all angles of triangle is  $180^\circ$

$$\therefore \angle L + \angle M + \angle N = 180^\circ$$

$$30 + 80 + \angle N = 180$$

$$110 + \angle N = 180$$

$$\therefore \angle N = 180 - 110$$

$$= 70^\circ$$

Here,  $\angle N = \angle X = 70^\circ$

According to property,

If one side of a triangle and angle at its ends are equal to one side of another triangle and the angles at its ends, then the third angles are also equal and the sides opposite equal angle are equal.

{Here,  $LN$  and  $XY$  is the required side}

In  $\triangle LMN$  and  $\triangle XYZ$ ,

$NM$  is opposite  $\angle L$  and  $XZ$  is opposite  $\angle Y$

Also,  $LM$  is opposite  $\angle N$  and  $YZ$  is opposite  $\angle X$

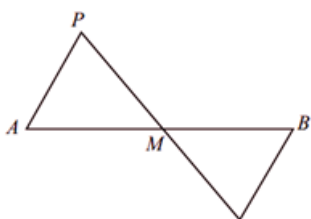
$$\therefore NM = XZ$$

And  $LM = YZ$

Also,  $LN = XY = 8 \text{ cm}$  (given)

## 2. Question

In the figure,  $AP$  and  $BQ$  equal and parallel are lines drawn at the ends of the line  $AB$ . The point of intersection of  $PQ$  and  $AB$  is marked as  $M$ .



- Are the sides of  $\triangle AMP$  equal to the sides of  $\triangle BMQ$ ? Why?
- What is special about the position of  $M$  on  $AB$ ?
- Draw a line 5.5 centimeters long. Using a set square, locate the midpoint of this line.

## Answer

Given-

$$AP = BQ$$

And  $AP \parallel BQ$

According to the property,

If a pair of equal and opposite sides is parallel, then the four points connected form a parallelogram.

$\therefore$  the imaginary quadrilateral  $APBQ$  is a parallelogram.

AB and PQ are diagonals of parallelogram.

Also, the diagonals of parallelogram bisect each other.

$$\therefore PM = MQ$$

$$\text{And } AM = MB$$

Also,  $\angle AMP = \angle BMQ$ , (vertically opposite angles)

According to property,

When two sides of a triangle and angle made by them are equal to the two sides and angle made by them of another triangle, then the third sides and the corresponding two angles are also equal.

$$\therefore AP = BQ$$

Hence, the sides of the two triangles are equal.

ii) Specialty of point M,

It divides both sides in equal ratio i.e. 1:1

(Reason, Discussed above i.e. the imaginary quadrilateral APBQ is a parallelogram and AP, BQ are its diagonals).

iii) Steps for construction,

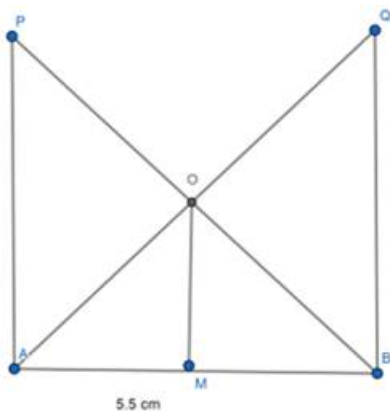
1. Draw a line AB = 5.5 cm

2. Using set-square draw two perpendicular lines on AB at each points A and B of length 5.5 cm as AP( = 5.5 cm) and BQ( = 5.5 cm)

3. Join lines PB and AQ and let them intersect at point O.

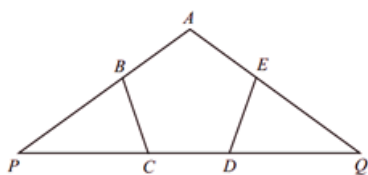
4. Drop a perpendicular from O to AB and let the new point be M.

M is the required midpoint of AB.



### 3. Question

In the figure, ABCDE is pentagon with all sides of the same length and all angles of the same size. The sides AB and AE extended, meet the side CD extended at P and Q.



i) Are the sides of  $\Delta BPC$  equal to the sides of  $\Delta EQD$ ? Why?

ii) Are the side of AP and AQ of  $\Delta APQ$  equal? Why?

### Answer

Here, all angles of the polygon are equal(a regular polygon).

$$\therefore \angle ABC = \angle AED \dots(\text{eq}1)$$

$(\angle PBC, \angle ABC)$  and  $(\angle AED, \angle QED)$  form a linear pair.

$$\angle PBC = 180 - \angle ABC \text{ (Linear angles are supplementary)}$$

$$= 180 - \angle AED \text{ (from eq1)}$$

$$\angle PBC = \angle QED \dots(\text{eq}2) \text{ } (\angle AED \text{ and } \angle QED \text{ form linear pair}).$$

$$\angle BCD = \angle EDC \dots(\text{eq}2)$$

$(\angle BCD, \angle BCP)$  and  $(\angle EDC, \angle EDQ)$  form a linear pair.

$$\angle BCP = 180 - \angle BCD \text{ (Linear angles are supplementary)}$$

$$= 180 - \angle EDC \text{ (from eq2)}$$

$$\angle BCP = \angle EDQ \dots(\text{eq}3)$$

According to property,

If one side of a triangle and angle at its ends are equal to one side of another triangle and the angles at its ends, then the third angles are also equal and the sides opposite equal angle are equal.

In  $\triangle PBC$  and  $\triangle QED$ ,

$$\angle BCD = \angle EDC$$

$$\angle BCP = \angle EDQ$$

Also,  $BC = DE$  (a regular polygon)

Hence,  $PC = DQ$  ( $\angle PBC = \angle DEQ$ )

$$PB = EQ \text{ } (\angle PCB = \angle EDQ)$$

$$\text{And } \angle BPC = \angle EQD \dots(\text{eq}4)$$

Hence, the sides are equal.

ii) From eq4,

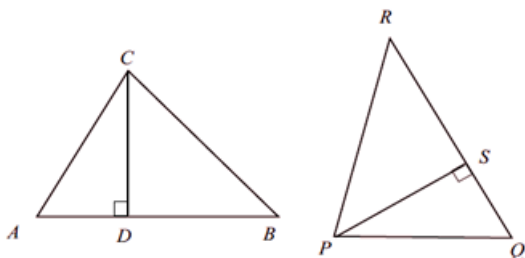
$$\angle BPC = \angle EQD$$

$$\therefore \angle APQ = \angle AQP$$

Hence,  $AP = AQ$  (converse of isosceles angle theorem).

#### 4. Question

In  $\triangle ABC$  and  $\triangle PQR$  shown below.



$$AB = QR \text{ } BC = RP \text{ } CA = PQ$$

i) Are  $CD$  and  $PS$  equal? Why?

ii) What is the relation between the areas of  $\triangle ABC$  and  $\triangle PQR$ ?

#### Answer

In  $\triangle ABC$  and  $\triangle PQR$

$$AB = QR$$

$$BC = RP$$

$$CA = PQ$$

$\therefore \Delta ABC \cong \Delta QRP$  {the corresponding sides on both sides of the equation are equal}

$$\therefore \angle ABC = \angle QRP \dots(\text{eq}1)$$

In  $\Delta CDB$  and  $\Delta PSR$ ,

$$\angle DBC(\angle ABC) = \angle SRP(\angle QRP) \dots(\text{eq}2) \text{ (from eq1)}$$

$$\angle CDB = \angle PSR = 90^\circ$$

Thus, in these triangles 2 angles are equal

Since, sum of all angles of a triangle is  $180^\circ$

$\therefore$  third angle is also equal

$$\therefore \angle DCB = \angle RPS \dots(\text{eq}3)$$

According to property,

If one side of a triangle and angle at its ends are equal to one side of another triangle and the angles at its ends, then the third angles are also equal and the sides opposite equal angle are equal.

{Here, common side is BC and RP

$$\angle DCB = \angle RPS \text{ (from eq3)}$$

$$\angle DBC = \angle SRP \text{ (from eq2)}$$

Applying it in  $\Delta CDB$  and  $\Delta PSR$

$$CD = PS \dots(\text{eq}4) \text{ (CD is opposite } \angle CBD, PS \text{ is opposite } \angle PRS \text{ and } \angle CBD = \angle PRS)$$

ii) For a triangle,  $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

$$\therefore \text{area}(\Delta ABC) = \frac{1}{2} \times AB \times CD$$

$$\text{And area}(\Delta PQR) = \frac{1}{2} \times QR \times PS$$

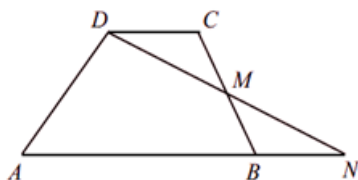
Here,  $AB = QR$  (given)

And,  $CD = PS$  (from eq4)

Hence, areas of both the triangles are equal.

## 5 A. Question

In the quadrilateral ABCD shown below, the sides AB and CD are parallel. M is the midpoint of the side BC.



The lines DM and AB extended, meet at N.

Are the areas of  $\Delta DCM$  and  $\Delta BMN$  equal? Why?

## Answer

Given-

$$AB \parallel CD$$

$$BM = MC$$

In  $\triangle DCM$  and  $\triangle BMN$ ,

$\angle DMC = \angle BMN$  (vertically opposite angles)

$BM = MC$

$\angle DCM = \angle MBN$  (alternate angle test,  $DC \parallel BN$  with  $BC$  as transversal).

Using the property,

If one side of a triangle and angle at its ends are equal to one side of another triangle and the angles at its ends, then the third angles are also equal and the sides opposite equal angle are equal.

$\therefore \angle CDM = \angle MNB$

$DM = MN$

$DC = BN$

As all 3 sides and angles are equal.

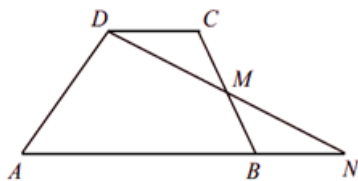
$\triangle DCM \cong \triangle NBM$

Also, congruent triangles have equal areas.

$\therefore \text{area}(\triangle DCM) = \text{area}(\triangle BMN) \dots(\text{eq})1$

### 5 B. Question

In the quadrilateral ABCD shown below, the sides AB and CD are parallel. M is the midpoint of the side BC.



The lines DM and AB extended, meet at N.

What is the relation between the areas of the quadrilateral ABCD and the triangle AND?

### Answer

area of a parallelogram = **base  $\times$  height**

area of a triangle =  $\frac{1}{2} \times$  **base  $\times$  height**

area of parallelogram =  $\text{area}(\text{ABCD})$

=  $\text{area}(\text{ADMB}) + \text{area}(\triangle DCM) \dots(\text{eq})1$

Area of triangle =  $\text{area}(\triangle \text{AND})$

=  $\text{area}(\text{ADMB}) + \text{area}(\triangle BMN)$

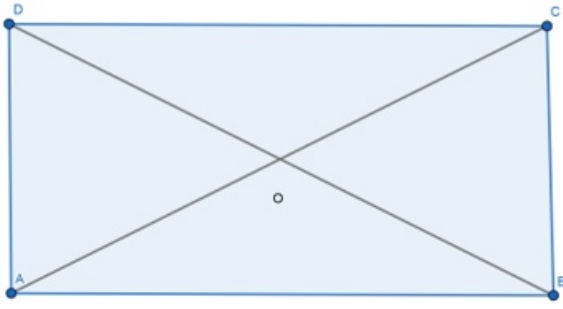
=  $\text{area}(\text{ADMB}) + \text{area}(\triangle DCM)$  (from eq1)  $\dots(\text{eq})2$

$\therefore \text{area of parallelogram} = \text{area of triangle}$

### 6. Question

Are the two diagonals of a rectangle equal? Why?

### Answer



A rectangle is a parallelogram where each angle is a right angle.

Here,  $AD = BC$  ...(eq)1

And  $AB = CD$

According to Pythagoras theorem,

$$(\text{Hypotenuse})^2 = (\text{one side})^2 + (\text{other side})^2$$

In  $\Delta ABD$ ,

Hypotenuse =  $BD$

One side =  $AB$

And other side =  $AD$

$$\therefore (BD)^2 = (AB)^2 + (AD)^2 \text{ ...(eq)2}$$

In  $\Delta ABC$ ,

Hypotenuse =  $AC$

One side =  $AB$

And other side =  $BC$

$$\therefore (AC)^2 = (AB)^2 + (BC)^2$$

$$= (AB)^2 + (AD)^2 \text{ ...(eq)3 ... (from eq1)}$$

As is clear from (eq)2 and (eq)3

$$BD^2 = AC^2$$

Taking square root on both sides,

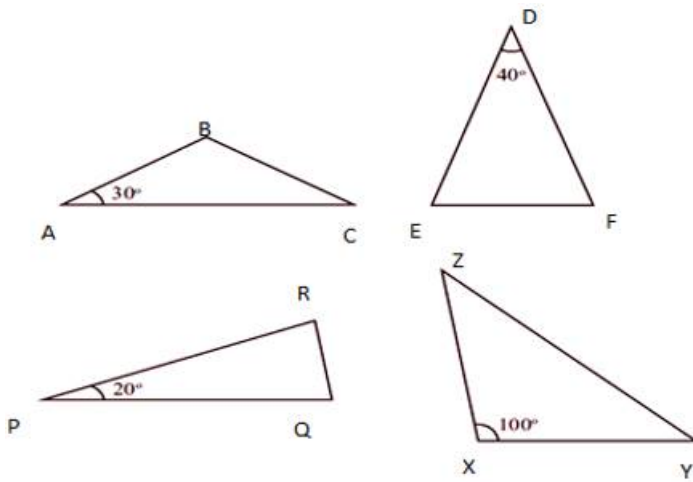
$$BD = AC$$

$\therefore$  Diagonals of a rectangle are equal.

## Questions Pg-26

### 1. Question

Some isosceles triangles are drawn below. In each, one angle is given. Find the other angles.



### Answer

In first figure,

As seen it seems  $AB = BC$  (since it is a isosceles triangle).

Also, if 2 sides of a triangle are equal, the angles opposite equal sides are also equal.

$$\therefore \angle BAC(\angle A) = \angle BCA(\angle C) = 30^\circ$$

In a triangle,

Sum of all angles of a triangle is  $180^\circ$

Hence, in  $\Delta ABC$ ,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\therefore 30 + \angle B + 30 = 180^\circ$$

$$\angle B + 60 = 180^\circ$$

$$\therefore \angle B = 180 - 60$$

$$= 120^\circ$$

In second figure,

As seen it seems  $DE = DF$  (since it is a isosceles triangle).

Also, if 2 sides of a triangle are equal, the angles opposite equal sides are also equal.

$$\therefore \angle DEF(\angle E) = \angle DFE(\angle F) = y^\circ \dots(\text{eq}1)$$

In a triangle,

Sum of all angles of a triangle is  $180^\circ$

Hence, in  $\Delta DEF$ ,

$$\angle D + \angle E + \angle F = 180^\circ$$

$$\therefore 40 + y + y = 180^\circ$$

$$\therefore 40 + 2y = 180$$

$$\therefore 2y = 180 - 40$$

$$= 140^\circ$$

$$\therefore y = \frac{140}{2} = 70^\circ$$

Hence,  $\angle E = \angle F = 70^\circ$  (from eq1)

In 3<sup>rd</sup> figure,

As seen it seems  $PQ = PR$  (since it is a isosceles triangle).

Also, if 2 sides of a triangle are equal, the angles opposite equal sides are also equal.

$$\therefore \angle PQR(\angle Q) = \angle PRQ(\angle R) = y^\circ \dots(\text{eq}1)$$

In a triangle,

Sum of all angles of a triangle is  $180^\circ$

Hence, in  $\Delta PQR$ ,

$$\angle P + \angle Q + \angle R = 180^\circ$$

$$\therefore 20 + y + y = 180^\circ$$

$$\therefore 20 + 2y = 180$$

$$\therefore 2y = 180 - 20$$

$$= 160^\circ$$

$$\therefore y = \frac{160}{2} = 80^\circ$$

Hence,  $\angle Q = \angle R = 80^\circ$  (from eq1)

In 4<sup>th</sup> figure,

As seen it seems  $XY = XZ$  (since it is a isosceles triangle).

Also, if 2 sides of a triangle are equal, the angles opposite equal sides are also equal.

$$\therefore \angle XYZ(\angle Y) = \angle XZY(\angle Z) = m^\circ \dots(\text{eq}1)$$

In a triangle,

Sum of all angles of a triangle is  $180^\circ$

Hence, in  $\Delta XYZ$ ,

$$\angle X + \angle Y + \angle Z = 180^\circ$$

$$\therefore 100 + m + m = 180^\circ$$

$$\therefore 100 + 2m = 180$$

$$\therefore 2m = 180 - 100$$

$$= 80$$

$$\therefore m = \frac{80}{2} = 40^\circ$$

Hence,  $\angle Y = \angle Z = 40^\circ$  (from eq1)

## 2. Question

One angle of an isosceles triangle is  $90^\circ$ . What are the other two angles?

### Answer

Let in  $\Delta ABC$ ,

$$\angle A = 90^\circ$$

It is given that triangle is a isosceles triangle.

Case 1

Two equal angles both equal to  $90^\circ$

If it happens, then sum of two equal angles will be  $= 90 + 90 = 180^\circ$



But, sum of all angles of a triangle is  $180^\circ$

Thus, the third angle =  $180 - 180 = 0$

But, this is not possible.

Case 2

Two equal angles other than right angle.

If  $\angle A = 90^\circ$

Let  $\angle B = \angle C = y^\circ$

Sum of all angles of a triangle is  $180^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$90 + y + y = 180^\circ$$

$$90 + 2y = 180^\circ$$

$$2y = 180 - 90$$

$$= 90$$

$$\therefore y = \frac{90}{2} = 45^\circ$$

Hence the other 2 angles are  $45^\circ$

### 3. Question

One angle of an isosceles triangle is  $60^\circ$ . What are the other two angles?

#### Answer

Let in  $\triangle ABC$ ,

$$\angle A = 60^\circ$$

It is given that triangle is an isosceles triangle.

Case 1

Two equal angles both equal to  $60^\circ$

If it happens, then sum of two equal angles will be  $= 60 + 60 = 120^\circ$

But, sum of all angles of a triangle is  $180^\circ$

Thus, the third angle  $= 180 - 120 = 60$

Hence, other two angles are  $60^\circ$  each.

Case 2

Two equal angles other than given angle ( $\angle A$ )

If  $\angle A = 60^\circ$

Let  $\angle B = \angle C = y^\circ$

Sum of all angles of a triangle is  $180^\circ$

$$\therefore \angle A + \angle B + \angle C = 180^\circ$$

$$60 + y + y = 180^\circ$$

$$60 + 2y = 180^\circ$$

$$2y = 180 - 60$$

$$= 120$$

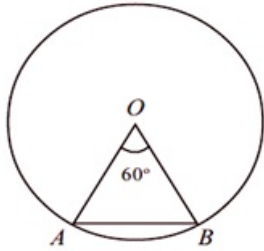
$$\therefore y = \frac{120}{2} = 60^\circ$$

Hence the other 2 angles are also  $60^\circ$

Hence, in both the cases the triangle is equilateral (each angle =  $60^\circ$ )

#### 4. Question

In figure below, O is the center of the circle and A, B are point on the circle.



Compute  $\angle A$  and  $\angle B$ .

#### Answer

In  $\triangle OAB$ ,

$OA = OB = r = \text{Radius of circle}$

In a triangle with 2 equal sides, the angles opposite to equal sides are equal.

$$\therefore \angle OAB(\angle A) = \angle OBA(\angle B) = y^\circ \dots(\text{eq}1)$$

In a triangle,

Sum of all angles of a triangle is  $180^\circ$

$\therefore$  In  $\triangle OAB$ ,

$$\angle O + \angle A + \angle B = 180^\circ$$

$$\therefore 60 + y + y = 180^\circ$$

$$\therefore 60 + 2y = 180^\circ$$

$$\therefore 2y = 180 - 60$$

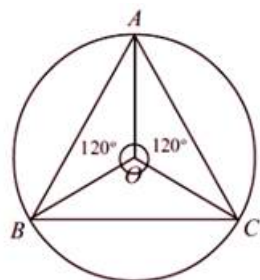
$$= 120$$

$$\therefore y = \frac{120}{2} = 60^\circ$$

$$\therefore \angle A = \angle B = 60^\circ \text{ (from eq1)}$$

#### 5. Question

In the figure below, O is the center of the circle and A, B, C are points on the circle.



What are the angles of  $\triangle ABC$ ?

#### Answer

Here,

$$\angle AOB = \angle AOC = 120^\circ$$

Sum of a complete whole angle is  $360^\circ$

$$\therefore \angle AOB + \angle AOC + \angle BOC = 360^\circ$$

$$120 + 120 + \angle BOC = 360^\circ$$

$$\therefore 240 + \angle BOC = 360^\circ$$

$$\therefore \angle BOC = 360 - 240$$

$$= 120^\circ$$

In  $\triangle OAB$ ,

$OA = OB = \text{radius of circle}$

In a triangle with 2 equal sides, the angles opposite to equal sides are equal.

$$\therefore \angle OAB = \angle OBA = y^\circ$$

In a triangle,

Sum of all angles of a triangle is  $180^\circ$

Hence, in  $\triangle OAB$ ,

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\therefore y + y + 120 = 180^\circ$$

$$\therefore 120 + 2y = 180$$

$$\therefore 2y = 180 - 120$$

$$= 60$$

$$\therefore y = \frac{60}{2} = 30^\circ$$

$$\therefore \angle OAB = \angle OBA = 30^\circ \dots(\text{eq})1$$

In  $\triangle OAC$ ,

$OA = OC = \text{radius of circle}$

In a triangle with 2 equal sides, the angles opposite to equal sides are equal.

$$\therefore \angle OAC = \angle OCA = y^\circ$$

In a triangle,

Sum of all angles of a triangle is  $180^\circ$

Hence, in  $\triangle OAB$ ,

$$\angle OAC + \angle OCA + \angle AOC = 180^\circ$$

$$\therefore y + y + 120 = 180^\circ$$

$$\therefore 120 + 2y = 180$$

$$\therefore 2y = 180 - 120$$

$$= 60$$

$$\therefore y = \frac{60}{2} = 30^\circ$$

$$\therefore \angle OAC = \angle OCA = 30^\circ \dots(\text{eq})2$$

In  $\triangle OBC$ ,

$OB = OC = \text{radius of circle}$

In a triangle with 2 equal sides, the angles opposite to equal sides are equal.

$$\therefore \angle OCB = \angle OBC = y^\circ$$

In a triangle,

Sum of all angles of a triangle is  $180^\circ$

Hence, in  $\Delta OCB$ ,

$$\angle OCB + \angle OBC + \angle COB = 180^\circ$$

$$\therefore y + y + 120 = 180^\circ$$

$$\therefore 120 + 2y = 180$$

$$\therefore 2y = 180 - 120$$

$$= 60$$

$$\therefore y = \frac{60}{2} = 30^\circ$$

$$\therefore \angle OAC = \angle OBC = 30^\circ \dots(\text{eq}3)$$

From equations 1,2 and 3

In  $\Delta ABC$

$$\angle A = \angle BAO + \angle OAC$$

$$= 30 + 30$$

$$= 60^\circ$$

$$\angle B = \angle OBA + \angle OBC$$

$$= 30 + 30$$

$$= 60^\circ$$

$$\angle C = \angle OCB + \angle OCA$$

$$= 30 + 30$$

$$= 60^\circ$$