

TRIGONOMETRY

CONTENTS

- Right Angle Triangle
- Trigonometric Ratio (T.R.) of some Specific Angles
- Trigonometric Ratios of Complementary Angles
- Trigonometric Identities

Trigonometry is the branch of mathematics in which we study of relationships between the sides & angles of a triangle.

Fact : In Greek words :

Tri = three

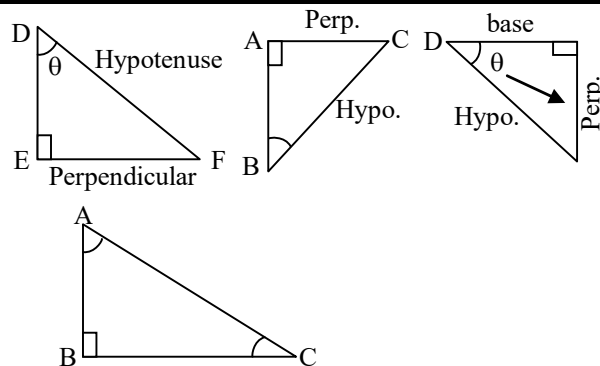
gon = sides

metron = measure

The ratio of sides of a right angle triangle with respect to acute angles are called "Trigonometric ratios of the angle".

➤ RIGHT ANGLE TRIANGLE

1. A Δ having one angle equal to 90° is called right angle Δ .
2. The sum of other two acute (Less than 90°) angles is 90° . (or both acute angles are complementary)
3. The side opposite to 90° , is called hypotenuse, it is longest side in Δ .
4. The side opposite to given one acute angle is perpendicular.
5. The rest (IIIrd) side is base.

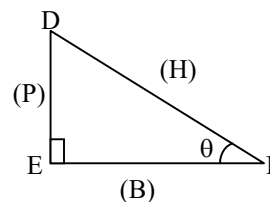


	Hypotenuse	Perpendicular	Base
for $\angle A$	AC	BC	AB
for $\angle C$	AC	AB	BC

The trigonometry ratio are

sine of $\angle\theta$, cosine of $\angle\theta$, tangent of $\angle\theta$, cotangent of $\angle\theta$, secant of $\angle\theta$, cosecant of $\angle\theta$.

These ratios are abbreviated as $\sin \theta$, $\cos \theta$, $\tan \theta$, $\cot \theta$, $\sec \theta$, $\csc \theta$ and the relation with sides are



$\sin \theta$	$= P/H = DE/DF$
$\cos \theta$	$= B/H = EF/DF$
$\tan \theta$	$= P/B = DE/EF$
$\cot \theta$	$= B/P = EF/DE$
$\sec \theta$	$= H/B = DF/EF$
$\csc \theta$	$= H/P = DF/DE$

By above table $\sin \theta = \frac{1}{\csc \theta}$, $\cos \theta = \frac{1}{\sec \theta}$,

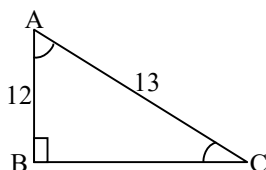
$\tan \theta = \frac{1}{\cot \theta}$ also $\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{P/H}{B/H} = \frac{P}{B}$

$\therefore$ we can say "Trigonometric Ratio" represents ratio between acute angles & sides of triangle.

❖ EXAMPLES ❖

Ex.1 If ABC is right angle triangle, $\angle B = 90^\circ$, $AB = 12$ cm, $AC = 13$ cm then find $\sin A$ and $\cos C$.

Sol. Using Pythagoras theorem



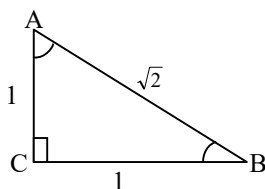
$$BC = \sqrt{AC^2 - AB^2} = \sqrt{169 - 144} = 5 \text{ cm}$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{5}{13}$$

$$\cos C = \frac{AB}{AC} = \frac{12}{13} \quad \text{Ans.}$$

Ex.2 If $\sin A = \frac{1}{\sqrt{2}}$ in right triangle ABC, then find value of $\tan A$, $\operatorname{cosec} A$, $\tan B$, $\operatorname{cosec} B$.

Sol.



$$\sin A = \frac{1}{\sqrt{2}} = \frac{BC}{AB}$$

$$\therefore AC = \sqrt{AB^2 - BC^2} = \sqrt{(\sqrt{2}k)^2 - (k)^2}$$

$$= \sqrt{2k^2 - k^2} = \sqrt{k^2} = k$$

$$\therefore \tan A = \frac{BC}{AC} = \frac{k}{k} = 1$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sqrt{2}k}{k} = \sqrt{2}$$

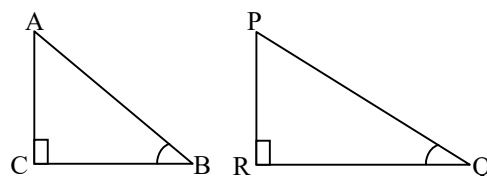
$$\tan B = \frac{AC}{BC} = \frac{k}{k} = 1$$

$$\operatorname{cosec} B = \frac{AB}{AC} = \frac{\sqrt{2}k}{k} = \sqrt{2}$$

Ex.3 If $\angle B$ and $\angle Q$ are acute angles such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$.

[NCERT]

Sol. Let us consider two right triangles ABC and PQR where $\sin B = \sin Q$.



$$\text{We have } \sin B = \frac{AC}{AB}$$

$$\text{and } \sin Q = \frac{PR}{PQ}$$

$$\text{Then } \frac{AC}{AB} = \frac{PR}{PQ}$$

$$\text{Therefore, } \frac{AC}{PR} = \frac{AB}{PQ} = k, \text{ say } \dots(1)$$

Now, using Pythagoras theorem,

$$BC = \sqrt{AB^2 - AC^2}$$

$$\text{and } QR = \sqrt{PQ^2 - PR^2}$$

$$\begin{aligned} \text{So, } \frac{BC}{QR} &= \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} \\ &= \frac{\sqrt{k^2 PQ^2 - k^2 PR^2}}{\sqrt{PQ^2 - PR^2}} \\ &= \frac{k \sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k \dots(2) \end{aligned}$$

From (1) and (2), we have

$$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$$

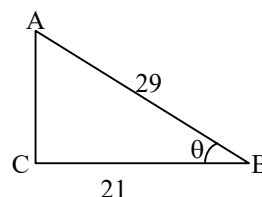
Then, by using Theorem, $\triangle ACB \sim \triangle PRQ$ and therefore, $\angle B = \angle Q$.

Ex.4 Consider $\triangle ACB$, right-angled at C, in which $AB = 29$ units, $BC = 21$ units and $\angle ABC = \theta$ (see figure). Determine the value of

(i) $\cos^2 \theta + \sin^2 \theta$,

(ii) $\cos^2 \theta - \sin^2 \theta$

[NCERT]



Sol. In $\triangle ACB$, we have

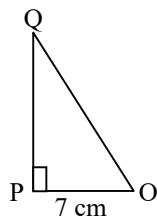
$$\begin{aligned} AC &= \sqrt{AB^2 - BC^2} = \sqrt{(29)^2 - (21)^2} \\ &= \sqrt{(29-21)(29+21)} = \sqrt{(8)(50)} \\ &= \sqrt{400} = 20 \text{ units} \end{aligned}$$

$$\text{So, } \sin \theta = \frac{AC}{AB} = \frac{20}{29}, \cos \theta = \frac{BC}{AB} = \frac{21}{29}$$

$$\begin{aligned} \text{Now, (i) } \cos^2 \theta + \sin^2 \theta &= \left(\frac{20}{29}\right)^2 + \left(\frac{21}{29}\right)^2 \\ &= \frac{20^2 + 21^2}{29^2} = \frac{400 + 441}{841} = 1, \end{aligned}$$

$$\begin{aligned} \text{and (ii) } \cos^2 \theta - \sin^2 \theta &= \left(\frac{21}{29}\right)^2 - \left(\frac{20}{29}\right)^2 \\ &= \frac{(21+20)(21-20)}{29^2} = \frac{41}{841}. \end{aligned}$$

Ex.5 In $\triangle OPQ$, right-angled at P, $OP = 7$ cm and $OQ - PQ = 1$ cm (see figure). Determine the values of $\sin Q$ and $\cos Q$. [NCERT]



Sol. In $\triangle OPQ$, we have

$$\begin{aligned} OQ^2 &= OP^2 + PQ^2 \\ \text{i.e. } (1 + PQ)^2 &= OP^2 + PQ^2 \\ \text{i.e. } 1 + PQ^2 + 2PQ &= OP^2 + PQ^2 \\ \text{i.e. } 1 + 2PQ &= 7^2 \\ \text{i.e. } PQ &= 24 \text{ cm and } OQ = 1 + PQ = 25 \text{ cm} \end{aligned}$$

$$\text{So, } \sin Q = \frac{7}{25} \text{ and } \cos Q = \frac{24}{25}$$

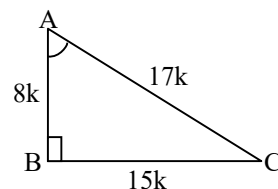
Note :

1. The values of $\sin \theta$ & $\cos \theta$ are always less than or equal to 1 & greater than or equal to -1.
2. Value of $\tan \theta$ & $\cot \theta$ lie between $-\infty$ to $+\infty$.
3. $\sin A$, $\cos A$, etc. are not product of $\sin$ and A .
4. $(\sin A)^2 \neq \sin A^2$ etc.

Ex.6 Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

[NCERT]

Sol. $\cot A = \frac{8}{15} = \frac{\text{base}}{\text{perpendicular}}$



$$\begin{aligned} AC &= \sqrt{AB^2 + BC^2} = \sqrt{64k^2 + 225k^2} \\ &= \sqrt{289k^2} = 17k \end{aligned}$$

$$\sin A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{AC}{AB} = \frac{17k}{8k} = \frac{17}{8} \quad \text{Ans.}$$

Ex.7 Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios. [NCERT]

Sol. $\sec \theta = \frac{13}{12} = \frac{\text{Hypotenuse}}{\text{Base}}$

$$\begin{aligned} \therefore \text{perpendicular} &= \sqrt{(13k)^2 - (12k)^2} \\ &= \sqrt{(169 - 144)k^2} = 5k \end{aligned}$$

$$\therefore \sin \theta = \frac{P}{H} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos \theta = \frac{1}{\sec \theta} = \frac{12}{13}$$

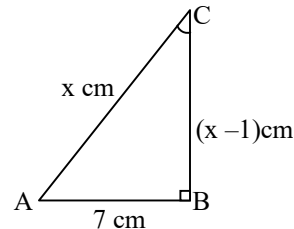
$$\tan \theta = \frac{P}{B} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot \theta = \frac{B}{P} = \frac{12k}{5k} = \frac{12}{5}$$

$$\operatorname{cosec} \theta = \frac{H}{P} = \frac{13k}{5k} = \frac{13}{5}$$

Ex.8 In $\triangle ABC$, right-angled at B, $AB = 7$ cm and $(AC - BC) = 1$ cm. Find the values of $\sin C$ and $\cos C$.

Sol. Consider $\triangle ABC$ in which $\angle B = 90^\circ$, $AB = 7$ cm and $(AC - BC) = 1$ cm.



Let $AC = x$ cm.

Then, $BC = (x - 1)$ cm

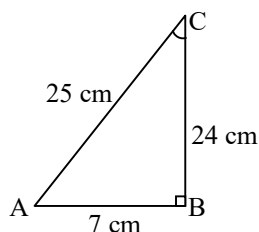
By Pythagoras theorem, we have :

$$AB^2 + BC^2 = AC^2 \Rightarrow (7)^2 + (x-1)^2 = x^2$$

$$\Rightarrow 49 + x^2 - 2x + 1 = x^2$$

$$\Rightarrow 2x = 50$$

$$\Rightarrow x = 25$$



$\therefore AC = 25$ cm, $BC = (25 - 1)$ cm = 24 cm
and $AB = 7$ cm.

For T-ratios of $\angle C$, we have

base = $BC = 24$ cm,

perpendicular = $AB = 7$ cm and

hypotenuse = $AC = 25$ cm.

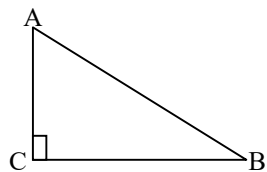
$$\therefore \sin C = \frac{AB}{AC} = \frac{7}{25} \text{ and } \cos C = \frac{BC}{AC} = \frac{24}{25}.$$

Ex.9 If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

[NCERT]

Sol. $\Theta \cos A = \cos B$

$$\frac{AC}{AB} = \frac{BC}{AB}$$



$$\therefore AC = BC$$

$\therefore \Delta$ is an isosceles Δ

$\therefore \angle A = \angle B$ Proved.

Ex.10 If $\cot \theta = \frac{7}{8}$, evaluate : [NCERT]

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}, (ii) \cot^2 \theta$$

Sol. $\Theta \cot \theta = \frac{7}{8} = \frac{\text{Base}}{\text{Perpendicular}} = \frac{B}{P}$

$$\therefore H = \sqrt{(8k)^2 + (7k)^2} = \sqrt{(64 + 49)k} \\ = \sqrt{113} k$$

$$\therefore \sin \theta = \frac{P}{H} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos \theta = \frac{B}{H} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

$$(i) \frac{(1 + \sin \theta)(1 - \sin \theta)}{(1 + \cos \theta)(1 - \cos \theta)}$$

$$= \frac{\left(1 + \frac{8}{\sqrt{113}}\right)\left(1 - \frac{8}{\sqrt{113}}\right)}{\left(1 + \frac{7}{\sqrt{113}}\right)\left(1 - \frac{7}{\sqrt{113}}\right)}$$

$$= \frac{(\sqrt{113} + 8)(\sqrt{113} - 8)}{(\sqrt{113} + 7)(\sqrt{113} - 7)}$$

$$= \frac{113 - 64}{113 - 49} = \frac{49}{64}$$

Ans.

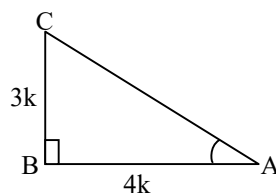
$$(ii) \cot^2 \theta = \left(\frac{B}{P}\right)^2 = \left(\frac{7k}{8k}\right)^2 = \frac{49}{64} \text{ Ans.}$$

Ex.11 If $3 \cot A = 4$, check whether

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A \text{ or not.}$$

[NCERT]

Sol. $\Theta \cot A = \frac{4}{3} \therefore \tan A = \frac{3}{4}$



$$\therefore AC = \sqrt{AB^2 + BC^2} = \sqrt{16k^2 + 9k^2} \\ = \sqrt{25k^2} = 5k$$

$$\therefore \sin A = \frac{3k}{5k} = \frac{3}{5}$$

$$\cos A = \frac{4k}{5k} = \frac{4}{5}$$

$$\text{LHS} = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{1 - \frac{9}{16}}{1 + \frac{9}{16}}$$

$$= \frac{(16-9)/16}{(16+9)/16} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A$$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

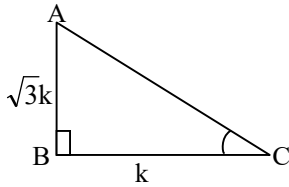
$$\text{LHS} = \text{RHS}$$

Ex.12 In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$, find the value of: [NCERT]

$$(i) \sin A \cos C + \cos A \sin C$$

$$(ii) \cos A \cos C - \sin A \sin C$$

Sol. $\tan A = \frac{1}{\sqrt{3}} = \frac{P}{B}$



$$\therefore AC = \sqrt{(\sqrt{3}k)^2 + (k)^2} = \sqrt{3k^2 + k^2} = 2k$$

$$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2};$$

$$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2};$$

$$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2};$$

$$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$(i) \sin A \cos C + \cos A \sin C$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$

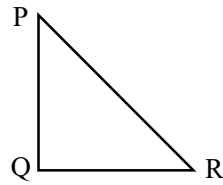
$$(ii) \cos A \cos C - \sin A \sin C$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Ex.13 In $\triangle PQR$, right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$. [NCERT]

Sol.



$$\Theta \text{ } PR + QR = 25 \text{ cm}$$

$$PQ = 5 \text{ cm}$$

$$\text{Let } PR = x \text{ cm}$$

$$\therefore QR = (25 - x) \text{ cm}$$

Using Pythagoras theorem

$$PR^2 = PQ^2 + QR^2$$

$$x^2 = 5^2 + (25 - x)^2$$

$$x^2 = 25 + 625 + x^2 - 50x$$

$$\Rightarrow 50x = 650$$

$$\Rightarrow x = 13 \text{ cm} = PR$$

$$\therefore QR = 25 - 13 = 12 \text{ cm.}$$

$$\sin P = \frac{QR}{PR} = \frac{12k}{13k} = \frac{12}{13}$$

$$\cos P = \frac{PQ}{PR} = \frac{5k}{13k} = \frac{5}{13}$$

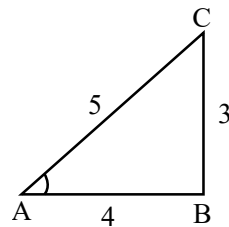
$$\tan P = \frac{QR}{PQ} = \frac{12k}{5k} = \frac{12}{5}$$

Ans.

Ex.14 If $\sin A = \frac{3}{5}$, find $\cos A$ and $\tan A$.

Sol. Since $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$, so

We draw a triangle ABC, right angled at B such that



Perpendicular = $BC = 3$ units,

and, Hypotenuse = $AC = 5$ units.

By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow 5^2 = AB^2 + 3^2$$

$$\Rightarrow AB^2 = 5^2 - 3^2$$

$$\Rightarrow AB^2 = 16 \Rightarrow AB = 4$$

When we consider the t-ratio of $\angle A$, we have
Base = $AB = 4$, Perpendicular = $BC = 3$,
Hypotenuse = $AC = 5$.

$$\therefore \cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$$

$$\text{and, } \tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{3}{4}$$

Ex.15 If $\operatorname{cosec} A = \sqrt{10}$, find other five trigonometric ratios.

Sol. Since $\operatorname{cosec} A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\sqrt{10}}{1}$,

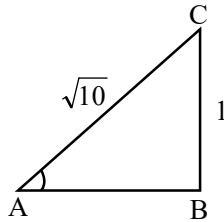
so we draw a right triangle ABC, right angled at B such that

Perpendicular = BC = 1 unit. and,

Hypotenuse = AC = $\sqrt{10}$ units.

By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$



$$\Rightarrow (\sqrt{10})^2 = AB^2 + 1^2$$

$$\Rightarrow AB^2 = 10 - 1 = 9$$

$$\Rightarrow AB = \sqrt{9} = 3$$

When we consider the trigonometric ratios of $\angle A$, we have

Base = AB = 3, Perpendicular = BC = 1, and

Hypotenuse = AC = $\sqrt{10}$.

$$\therefore \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{\sqrt{10}};$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{3}{\sqrt{10}};$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{3};$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\sqrt{10}}{3};$$

$$\text{and } \cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{3}{1} = 3$$

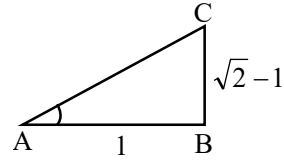
Ex.16 If $\tan A = \sqrt{2} - 1$, show that $\sin A \cos A = \frac{\sqrt{2}}{4}$.

Sol. Since $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\sqrt{2}-1}{1}$, so we draw a right triangle ABC, right angled at

B such that Base = AB = 1 and Perpendicular = BC = $\sqrt{2} - 1$.

By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$



$$\Rightarrow AC^2 = 1^2 + (\sqrt{2} - 1)^2$$

$$\Rightarrow AC^2 = 1 + 2 + 2 - 2\sqrt{2}$$

$$\Rightarrow AC^2 = 4 - 2\sqrt{2} \Rightarrow AC = \sqrt{4 - 2\sqrt{2}}$$

$$\text{Now, } \sin A = \frac{BC}{AC} = \frac{\sqrt{2}-1}{\sqrt{4-2\sqrt{2}}}, \text{ and}$$

$$\cos A = \frac{AB}{AC} = \frac{1}{\sqrt{4-2\sqrt{2}}}$$

$$\therefore \sin A \cos A = \frac{\sqrt{2}-1}{\sqrt{4-2\sqrt{2}}} \times \frac{1}{\sqrt{4-2\sqrt{2}}}$$

$$= \frac{\sqrt{2}-1}{4-2\sqrt{2}} = \frac{\sqrt{2}-1}{2\sqrt{2}(\sqrt{2}-1)} = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4}.$$

$\diamond \sin^2 \theta = (\sin \theta)^2$ $\cos^2 \theta = (\cos \theta)^2$ $\tan^2 \theta = (\tan \theta)^2$ $\operatorname{cosec}^2 \theta = (\operatorname{cosec} \theta)^2$ $\sec^2 \theta = (\sec \theta)^2$ $\cot^2 \theta = (\cot \theta)^2$

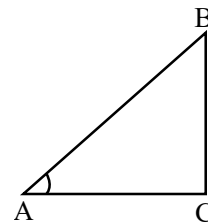
❖ EXAMPLES ❖

Ex.17 In a $\triangle ABC$ right angled at C, if $\tan A = \frac{1}{\sqrt{3}}$

and $\tan B = \sqrt{3}$. Show that

$$\sin A \cos B + \cos A \sin B = 1.$$

Sol. Let us draw a $\triangle ABC$, right angled at C in which $\tan B = \sqrt{3}$ and $\tan A = \frac{1}{\sqrt{3}}$.



Now, $\tan A = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{BC}{AC} = \frac{1}{\sqrt{3}} \quad \left[\because \tan A = \frac{BC}{AC} \right]$$

$$\Rightarrow BC = x \text{ and } AC = \sqrt{3}x \quad \dots(i)$$

And, $\tan B = \sqrt{3}$

$$\Rightarrow \frac{AC}{BC} = \frac{\sqrt{3}}{1} \quad \left[\because \tan B = \frac{AC}{BC} \right]$$

$$\Rightarrow AC = \sqrt{3}x \text{ and } BC = x \quad \dots(ii)$$

From (i) and (ii), we have

$$BC = x, AC = \sqrt{3}x$$

By Pythagoras theorem, we have

$$AB^2 = AC^2 + BC^2$$

$$\Rightarrow AB^2 = (\sqrt{3}x)^2 + x^2$$

$$\Rightarrow AB^2 = 3x^2 + x^2$$

$$\Rightarrow AB^2 = 4x^2$$

$$\Rightarrow AB = 2x$$

When we find the t-ratios of $\angle A$, we have

Base = $AC = \sqrt{3}x$, Perpendicular = $BC = x$,
and Hypotenuse = $AB = 2x$.

$$\therefore \sin A = \frac{BC}{AB} = \frac{x}{2x} = \frac{1}{2} \text{ and}$$

$$\cos A = \frac{AC}{AB} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$

When we consider the t-ratios of $\angle B$, we have

Base = $BC = x$, Perpendicular = $AC = \sqrt{3}x$,
and Hypotenuse = $AB = 2x$.

$$\therefore \cos B = \frac{BC}{AB} = \frac{x}{2x} = \frac{1}{2} \text{ and}$$

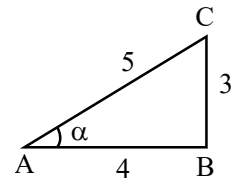
$$\sin B = \frac{AC}{AB} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$

Now,

$$\begin{aligned} \sin A \cos B + \cos A \sin B &= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \\ &= \frac{1}{4} + \frac{3}{4} = 1. \end{aligned}$$

Ex.18 If $\sec \alpha = \frac{5}{4}$, evaluate $\frac{1 - \tan \alpha}{1 + \tan \alpha}$.

Sol. Since $\sec \alpha = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{5}{4}$, so we draw a right triangle ABC, right angled at B such that Hypotenuse = $AC = 5$ units,
Base = $AB = 4$ units, and $\angle BAC = \alpha$.



By Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow 5^2 = 4^2 + BC^2$$

$$\Rightarrow BC^2 = 5^2 - 4^2 = 9$$

$$\Rightarrow BC = \sqrt{9} = 3$$

$$\therefore \tan \alpha = \frac{BC}{AB} = \frac{3}{4}$$

$$\text{Now, } \frac{1 - \tan \alpha}{1 + \tan \alpha} = \frac{1 - \frac{3}{4}}{1 + \frac{3}{4}} = \frac{\frac{4}{4}}{\frac{7}{4}} = \frac{1}{7}.$$

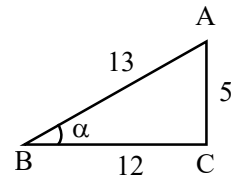
Ex.19 If $\cot B = \frac{12}{5}$, prove that

$$\tan^2 B - \sin^2 B = \sin^4 B \cdot \sec^2 B.$$

Sol. Since $\cot B = \frac{\text{Base}}{\text{Perpendicular}} = \frac{12}{5}$, so we

draw a right triangle ABC, right angled at C such that Base = $BC = 12$ units.

Perpendicular = $AC = 5$ units.



By Pythagoras theorem, we have

$$AB^2 = BC^2 + AC^2$$

$$\Rightarrow AB^2 = 12^2 + 5^2 = 169$$

$$\Rightarrow AB = \sqrt{169} = 13$$

$$\therefore \sin B = \frac{AC}{AB} = \frac{5}{13}, \tan B = \frac{AC}{BC} = \frac{5}{12}$$

$$\text{and } \sec B = \frac{AB}{BC} = \frac{13}{12}$$

$$\text{Now, LHS} = \tan^2 B - \sin^2 B = (\tan B)^2 - (\sin B)^2$$

$$= \left(\frac{5}{12} \right)^2 - \left(\frac{5}{13} \right)^2 = \frac{25}{144} - \frac{25}{169}$$

$$= 25 \left(\frac{1}{144} - \frac{1}{169} \right) = 25 \left(\frac{169 - 144}{144 \times 169} \right)$$

$$= 25 \times \frac{25}{144 \times 169} = \frac{25 \times 25}{144 \times 169}$$

$$= \frac{5^2 \times 5^2}{12^2 \times 13^2} \quad \dots(i)$$

$$\begin{aligned}
 \text{and, RHS} &= \sin^4 B \sec^2 B \\
 &= (\sin B)^4 (\sec B)^2 \\
 &= \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2 \\
 &= \frac{5^4}{13^2 \times 12^2} \\
 &= \frac{5^2 \times 5^2}{13^2 \times 12^2} \quad \dots(ii)
 \end{aligned}$$

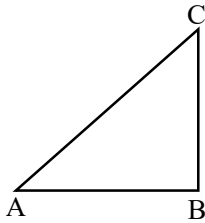
From (i) and (ii), we have
 $\tan^2 B - \sin^2 B = \sin^4 B \sec^2 B$.

Ex.20 In a right triangle ABC, right angled at B, the ratio of AB to AC is $1 : \sqrt{2}$. Find the values of

$$(i) \frac{2 \tan A}{1 + \tan^2 A} \quad \text{and} \quad (ii) \frac{2 \tan A}{1 - \tan^2 A}$$

Sol. We have, $AB : AC = 1 : \sqrt{2}$ i.e. $\frac{AB}{AC} = \frac{1}{\sqrt{2}}$

$$\therefore AB = x \text{ and } AC = \sqrt{2} x.$$



By Pythagoras theorem, we have

$$\begin{aligned}
 AC^2 &= AB^2 + BC^2 \\
 \Rightarrow (\sqrt{2} x)^2 &= x^2 + BC^2 \\
 \Rightarrow BC^2 &= 2x^2 - x^2 = x^2 \\
 \Rightarrow BC &= x
 \end{aligned}$$

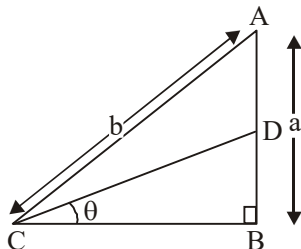
$$\therefore \tan A = \frac{BC}{AB} = \frac{x}{x} = 1$$

$$\text{Now, } \frac{2 \tan A}{1 + \tan^2 A} = \frac{2 \times 1}{1 + 1^2} = \frac{2}{2} = 1$$

$$\text{Now, } \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \times 1}{1 - 1} = \frac{2}{0}, \text{ which is undefined.}$$

Ex.21 In fig. $AD = DB$ and $\angle B$ is a right angle. Determine

- (i) $\sin \theta$ (ii) $\cos \theta$
 (iii) $\tan \theta$ (iv) $\sin^2 \theta + \cos^2 \theta$



Sol. We have,
 $AB = a$
 $\Rightarrow AD + DB = a \quad [\ominus AD = DB]$
 $\Rightarrow AD + AD = a$
 $\Rightarrow 2AD = a \Rightarrow AD = \frac{a}{2}$

$$\text{Thus, } AD = DB = \frac{a}{2}$$

By Pythagoras theorem, we have

$$\begin{aligned}
 AC^2 &= AB^2 + BC^2 \\
 \Rightarrow b^2 &= a^2 + BC^2 \\
 \Rightarrow BC^2 &= b^2 - a^2 \Rightarrow BC = \sqrt{b^2 - a^2}
 \end{aligned}$$

Thus, in $\triangle BCD$, we have

$$\begin{aligned}
 \text{Base} &= BC = \sqrt{b^2 - a^2} \\
 \text{and Perpendicular} &= BD = \frac{a}{2}
 \end{aligned}$$

Applying Pythagoras theorem in $\triangle BCD$, we have

$$\begin{aligned}
 BC^2 + BD^2 &= CD^2 \\
 \Rightarrow (\sqrt{b^2 - a^2})^2 + \left(\frac{a}{2}\right)^2 &= CD^2 \\
 \Rightarrow CD^2 &= b^2 - a^2 + \frac{a^2}{4} \\
 \Rightarrow CD^2 &= \frac{4b^2 - 4a^2 + a^2}{4} \\
 \Rightarrow CD &= \frac{\sqrt{4b^2 - 3a^2}}{2}
 \end{aligned}$$

Now,

$$\begin{aligned}
 (i) \sin \theta &= \frac{BD}{CD} \\
 \Rightarrow \sin \theta &= \frac{a/2}{\frac{\sqrt{4b^2 - 3a^2}}{2}} = \frac{a}{\sqrt{4b^2 - 3a^2}}
 \end{aligned}$$

$$\begin{aligned}
 (ii) \cos \theta &= \frac{BC}{CD} \\
 \Rightarrow \cos \theta &= \frac{\sqrt{b^2 - a^2}}{\frac{\sqrt{4b^2 - 3a^2}}{2}} = \frac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}
 \end{aligned}$$

$$\begin{aligned}
 (iii) \tan \theta &= \frac{BD}{CD} \\
 \Rightarrow \tan \theta &= \frac{a/2}{\frac{\sqrt{b^2 - a^2}}{2}} = \frac{a}{2\sqrt{b^2 - a^2}}, \text{ and}
 \end{aligned}$$

$$(iv) \sin^2 \theta + \cos^2 \theta$$

$$\begin{aligned}
&= \left(\frac{a}{\sqrt{4b^2 - 3a^2}} \right)^2 + \left(\frac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}} \right)^2 \\
&= \frac{a^2}{4b^2 - 3a^2} + \frac{4(b^2 - a^2)}{4b^2 - 3a^2} \\
&= \frac{4b^2 - 3a^2}{4b^2 - 3a^2} = 1
\end{aligned}$$

➤ TRIGONOMETRIC RATIO (T.R.) OF SOME SPECIFIC ANGLES

The angles 0° , 30° , 45° , 60° , 90° are angles for which we have values of T.R.

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} A$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

- $\sin \theta \uparrow$ when $\theta \uparrow$, $0 \leq \theta \leq 90^\circ$
- $\cos \theta \downarrow$ when $\theta \uparrow$, $0 \leq \theta \leq 90^\circ$
- $\tan \theta$, $\cot \theta$ are not defined for $\theta = 90^\circ$ & 0 respectively.
- $\operatorname{cosec} \theta$, $\sec \theta$ are not defined when $\theta = 0$ & 90° respectively.
- $\sin \theta = \cos \theta$ for only $\theta = 45^\circ$
- $\ominus \quad 180^\circ = \pi^c$
- $\therefore 30^\circ = \left(\frac{\pi}{6}\right)^c; 45^\circ = \left(\frac{\pi}{4}\right)^c$

$$60^\circ = \left(\frac{\pi}{3}\right)^c; 90^\circ = \left(\frac{\pi}{2}\right)^c$$

❖ EXAMPLES ❖

Ex.22 Evaluate each of the following in the simplest form :

- $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$
- $\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

Sol. (i) $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} + \frac{1}{4} = 1$$

(ii) $\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$$\begin{aligned}
&= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}} \\
&= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}
\end{aligned}$$

Ex.23 Evaluate the following expression :

(i) $\tan 60^\circ \operatorname{cosec}^2 45^\circ + \sec^2 60^\circ \tan 45^\circ$

(ii) $4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + \cos^2 90^\circ$

Sol. (i) $\tan 60^\circ \operatorname{cosec}^2 45^\circ + \sec^2 60^\circ \tan 45^\circ$

$$\tan 60^\circ (\operatorname{cosec} 45^\circ)^2 + (\sec 60^\circ)^2 \tan 45^\circ$$

$$= \sqrt{3} \times (\sqrt{2})^2 + (2)^2 \times 1$$

$$= \sqrt{3} \times 2 + 4 = 4 + 2\sqrt{3}$$

(ii) $4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + \cos^2 90^\circ$

$$= 4(\cot 45^\circ)^2 - (\sec 60^\circ)^2 + (\sin 60^\circ)^2 + (\cos 90^\circ)^2$$

$$= 4 \times (1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + 0$$

$$= 4 - 4 + \frac{3}{4} + 0 = \frac{3}{4}$$

Ex.24 Show that :

(i) $2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ) = 6$

(ii) $2(\cos^4 60^\circ + \sin^4 30^\circ) - (\tan^2 60^\circ + \cot^2 45^\circ)$

$$+ 3 \sec^2 30^\circ = \frac{1}{4}$$

Sol. (i) $2(\cos^2 45^\circ + \tan^2 60^\circ) - 6(\sin^2 45^\circ - \tan^2 30^\circ)$

$$= 2 \left(\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2 \right) - 6 \left(\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 \right)$$

$$= 2 \left(\frac{1}{2} + 3 \right) - 6 \left(\frac{1}{2} - \frac{1}{3} \right) = 2 \left(\frac{1+6}{2} \right) - 6 \left(\frac{3-2}{6} \right)$$

$$= 2 \times \frac{7}{2} - 6 \times \frac{1}{6} = 7 - 1 = 6$$

(ii) $2(\cos^4 60^\circ + \sin^4 30^\circ) - (\tan^2 60^\circ + \cot^2 45^\circ)$

$$+ 3 \sec^2 30^\circ$$

$$= 2 \left(\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 \right) - ((\sqrt{3})^2 + (1)^2) + 3 \left(\frac{2}{\sqrt{3}} \right)^2$$

$$= 2 \left(\frac{1}{16} + \frac{1}{16} \right) - (3 + 1) + 3 \times \frac{4}{3}$$

$$= 2 \times \frac{1}{8} - 4 + 4 = \frac{1}{4}$$

Ex.25 Find the value of x in each of the following :

(i) $\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

(ii) $\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

Sol.(i) $\tan 3x = \sin 45^\circ \cos 45^\circ + \sin 30^\circ$

$$\Rightarrow \tan 3x = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{2}$$

$$\Rightarrow \tan 3x = \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \tan 3x = 1$$

$$\Rightarrow \tan 3x = \tan 45^\circ$$

$$\Rightarrow 3x = 45^\circ \Rightarrow x = 15^\circ$$

(ii) $\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

$$\Rightarrow \cos x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\Rightarrow \cos x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$\Rightarrow \cos x = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos x = \cos 30^\circ$$

$$\Rightarrow x = 30^\circ$$

Ex.26 If $x = 30^\circ$, verify that

(i) $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$

(ii) $\sin x = \sqrt{\frac{1 - \cos 2x}{2}}$

Sol.(i) When $x = 30^\circ$, we have $2x = 60^\circ$.

$$\therefore \tan 2x = \tan 60^\circ = \sqrt{3}$$

$$\text{And, } \frac{2 \tan x}{1 - \tan^2 x} = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2/\sqrt{3}}{1 - \frac{1}{3}} = \frac{2/\sqrt{3}}{2/3} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3}$$

$$\therefore \tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

(ii) When $x = 30^\circ$, we have $2x = 60^\circ$.

$$\therefore \sqrt{\frac{1 - \cos 2x}{2}} = \sqrt{\frac{1 - \cos 60^\circ}{2}}$$

$$= \sqrt{\frac{1 - \frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\text{And, } \sin x = \sin 30^\circ = \frac{1}{2}$$

$$\therefore \sin x = \frac{\sqrt{1 - \cos 2x}}{2}$$

Ex.27 Find the value of θ in each of the following :

(i) $2 \sin 2\theta = \sqrt{3}$

(ii) $2 \cos 3\theta = 1$

Sol.(i) $2 \sin 2\theta = \sqrt{3}$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin 2\theta = \sin 60^\circ$$

$$\Rightarrow 2\theta = 60^\circ \Rightarrow \theta = 30^\circ$$

(ii) $2 \cos 3\theta = 1$

$$\Rightarrow \cos 3\theta = \frac{1}{2}$$

$$\Rightarrow \cos 3\theta = \cos 60^\circ$$

$$\Rightarrow 3\theta = 60^\circ \Rightarrow \theta = 20^\circ$$

Ex.28 If θ is an acute angle and $\sin \theta = \cos \theta$, find the value of $2 \tan^2 \theta + \sin^2 \theta - 1$.

Sol. $\sin \theta = \cos \theta$

$$\Rightarrow \frac{\sin \theta}{\cos \theta} = \frac{\cos \theta}{\cos \theta}$$

[Dividing both sides by $\cos \theta$]

$$\Rightarrow \tan \theta = 1$$

$$\Rightarrow \tan \theta = \tan 45^\circ \Rightarrow \theta = 45^\circ$$

$$\therefore 2 \tan^2 \theta + \sin^2 \theta - 1$$

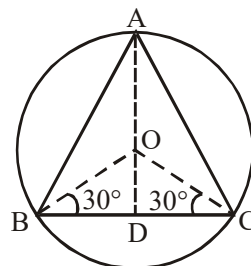
$$= 2 \tan^2 45^\circ + \sin^2 45^\circ - 1$$

$$= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$$

$$= 2 + \frac{1}{2} - 1 = \frac{5}{2} - 1 = \frac{3}{2}$$

Ex.29 An equilateral triangle is inscribed in a circle of radius 6 cm. Find its side.

Sol. Let ABC be an equilateral triangle inscribed in a circle of radius 6 cm. Let O be the centre of the circle.



Then, $OA = OB = OC = 6$ cm.

Let OD be perpendicular from O on side BC. Then, D is mid-point of BC and OB and OC are bisectors of $\angle B$ and $\angle C$ respectively.

$$\therefore \angle OBD = 30^\circ$$

In $\triangle OBD$, right angled at D, we have

$$\angle OBD = 30^\circ \text{ and } OB = 6 \text{ cm.}$$

$$\therefore \cos \angle OBD = \frac{BD}{OB} \Rightarrow \cos 60^\circ = \frac{BD}{6}$$

$$\Rightarrow BD = 6 \cos 60^\circ = 6 \times \frac{\sqrt{3}}{2} = 3\sqrt{3} \text{ cm.}$$

$$\Rightarrow BC = 2 BD = 2(3\sqrt{3}) \text{ cm} = 6\sqrt{3} \text{ cm.}$$

Hence, the side of the equilateral triangle is $6\sqrt{3}$ cm.

Ex.30 Using the formula,

$\sin(A - B) = \sin A \cos B - \cos A \sin B$, find the value of $\sin 15^\circ$.

Sol. Let $A = 45^\circ$ and $B = 30^\circ$. Then $A - B = 15^\circ$. Putting $A = 45^\circ$ and $B = 30^\circ$ in the given formula, we get

$$\sin(45^\circ - 30^\circ) = \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$$

$$\text{or, } \sin(45^\circ - 30^\circ) = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \Rightarrow \sin 15^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

Ex.31 If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$;

$0^\circ < A + B \leq 90^\circ$; $A > B$, find A and B.

Sol. $\tan(A + B) = \sqrt{3} = \tan 60^\circ$

$$\& \tan(A - B) = 1/\sqrt{3} = \tan 30^\circ$$

$$A + B = 60^\circ \dots\dots(1)$$

$$A - B = 30^\circ \dots\dots(2)$$

$$2A = 90^\circ \Rightarrow A = 45^\circ \text{ Ans.}$$

adding (1) & (2)

$$A + B = 60$$

$$A - B = 30$$

Sub fact equation (2) from (1)

$$A + B = 60$$

$$A - B = 30$$

$$- \quad + \quad -$$

$$2B = 30^\circ$$

$$\Rightarrow B = 15^\circ. \text{ Ans.}$$

Note : $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$$\sin(A + B) \neq \sin A + \sin B.$$

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES

Θ We know complementary angles are pair of angles whose sum is 90°

Like $40^\circ, 50^\circ$; $60^\circ, 30^\circ$; $20^\circ, 70^\circ$; $15^\circ, 75^\circ$; etc,

Formulae :

$$\sin(90^\circ - \theta) = \cos \theta, \cot(90^\circ - \theta) = \tan \theta$$

$$\cos(90^\circ - \theta) = \sin \theta, \sec(90^\circ - \theta) = \operatorname{cosec} \theta$$

$$\tan(90^\circ - \theta) = \cot \theta, \operatorname{cosec}(90^\circ - \theta) = \sec \theta$$

Ex.32 Evaluate $\frac{\tan 65^\circ}{\cot 25^\circ}$. [NCERT]

Sol. Θ $65^\circ + 25^\circ = 90^\circ$

$$\therefore \frac{\tan 65^\circ}{\cot 25^\circ} = \frac{\tan(90^\circ - 25^\circ)}{\cot 25^\circ} = \frac{\cot 25^\circ}{\cot 25^\circ} = 1$$

Ans.

Ex.33 Without using trigonometric tables, evaluate the following :

$$(i) \frac{\cos 37^\circ}{\sin 53^\circ} \quad (ii) \frac{\sin 41^\circ}{\cos 49^\circ} \quad (iii) \frac{\sin 30^\circ 17'}{\cos 59^\circ 43'}$$

Sol.(i) We have

$$\frac{\cos 37^\circ}{\sin 53^\circ} = \frac{\cos(90^\circ - 53^\circ)}{\sin 53^\circ} = \frac{\sin 53^\circ}{\sin 53^\circ} = 1$$

$$[\Theta \cos(90^\circ - \theta) = \sin \theta]$$

(ii) We have,

$$\frac{\sin 41^\circ}{\cos 49^\circ} = \frac{\sin(90^\circ - 49^\circ)}{\cos 49^\circ} = \frac{\cos 49^\circ}{\cos 49^\circ} = 1$$

$$[\Theta \sin(90^\circ - \theta) = \cos \theta]$$

(iii) We have,

$$\frac{\sin 30^\circ 17'}{\cos 59^\circ 43'} = \frac{\sin(90^\circ - 59^\circ 43')}{\cos 59^\circ 43'} = \frac{\cos 59^\circ 43'}{\cos 59^\circ 43'} = 1.$$

Ex.34 Without using trigonometric tables evaluate the following :

$$(i) \sin^2 25^\circ + \sin^2 65^\circ \quad (ii) \cos^2 13^\circ - \sin^2 77^\circ$$

Sol.(i) We have,

$$\sin^2 25^\circ + \sin^2 65^\circ = \sin^2(90^\circ - 65^\circ) + \sin^2 65^\circ$$

$$= \cos^2 65^\circ + \sin^2 65^\circ = 1$$

$$[\Theta \sin(90^\circ - \theta) = \cos \theta]$$

(ii) We have,

$$\cos^2 13^\circ - \sin^2 77^\circ = \cos^2(90^\circ - 77^\circ) - \sin^2 77^\circ$$

$$= \sin^2 77^\circ - \sin^2 77^\circ = 0$$

$$[\Theta \cos(90^\circ - \theta) = \sin \theta]$$

Ex.35 Without using trigonometric tables, evaluate the following :

(i) $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$

(ii) $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Sol.(i) We have,

$$\begin{aligned} & \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2 \\ &= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot(90^\circ - 20^\circ)} - 2 \\ &= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\tan 20^\circ} - 2 = 1 + 1 - 2 = 0 \end{aligned}$$

(ii) We have,

$$\begin{aligned} & \sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ \\ &= \sec(90^\circ - 40^\circ) \sin 40^\circ \\ & \quad + \cos 40^\circ \operatorname{cosec}(90^\circ - 40^\circ) \\ &= \operatorname{cosec} 40^\circ \sin 40^\circ + \cos 40^\circ \sec 40^\circ \\ &= \frac{\sin 40^\circ}{\sin 40^\circ} + \frac{\cos 40^\circ}{\cos 40^\circ} = 1 + 1 = 2 \end{aligned}$$

Ex.36 Express each of the following in terms of trigonometric ratios of angles between 0° and 45° ;

(i) $\operatorname{cosec} 69^\circ + \cot 69^\circ$

(ii) $\sin 81^\circ + \tan 81^\circ$

(iii) $\sin 72^\circ + \cot 72^\circ$

Sol.(i) We have,

$$\begin{aligned} & \operatorname{cosec} 69^\circ + \cot 69^\circ \\ &= \operatorname{cosec}(90^\circ - 21^\circ) + \cot(90^\circ - 21^\circ) \\ &= \sec 21^\circ + \tan 21^\circ \\ & [\Theta \operatorname{cosec}(90^\circ - \theta) = \sec \theta \text{ and} \\ & \quad \cot(90^\circ - \theta) = \tan \theta] \end{aligned}$$

(ii) We have,

$$\begin{aligned} & \sin 81^\circ + \tan 81^\circ \\ &= \sin(90^\circ - 9^\circ) + \tan(90^\circ - 9^\circ) \\ &= \cos 9^\circ + \cot 9^\circ \\ & [\Theta \sin(90^\circ - \theta) = \cos \theta \text{ and} \\ & \quad \tan(90^\circ - \theta) = \cot \theta] \end{aligned}$$

(iii) We have,

$$\begin{aligned} & \sin 72^\circ + \cot 72^\circ \\ &= \sin(90^\circ - 18^\circ) + \cot(90^\circ - 18^\circ) \\ &= \cos 18^\circ + \tan 18^\circ \\ & [\Theta \sin(90^\circ - 18^\circ) = \cos 18^\circ \text{ and} \\ & \quad \tan(90^\circ - 18^\circ) = \cot 18^\circ] \end{aligned}$$

Ex.37 Without using trigonometric tables, evaluate the following :

$$\frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \frac{\sin(90^\circ - \theta) \sin \theta}{\tan \theta} + \frac{\cos(90^\circ - \theta) \cos \theta}{\cot \theta}$$

Sol.

$$\begin{aligned} & \frac{\sin^2 20^\circ + \sin^2 70^\circ}{\cos^2 20^\circ + \cos^2 70^\circ} + \frac{\sin(90^\circ - \theta) \sin \theta}{\tan \theta} + \frac{\cos(90^\circ - \theta) \cos \theta}{\cot \theta} \\ &= \frac{\sin^2 20^\circ + \sin^2(90^\circ - 20^\circ)}{\cos^2 20^\circ + \cos^2(90^\circ - 20^\circ)} + \frac{\sin(90^\circ - \theta) \sin \theta}{\tan \theta} + \frac{\cos(90^\circ - \theta) \cos \theta}{\cot \theta} \\ &= \frac{\sin^2 20^\circ + \cos^2 20^\circ}{\cos^2 20^\circ + \sin^2 20^\circ} + \frac{\cos \theta \sin \theta}{\frac{\sin \theta}{\cos \theta}} + \frac{\sin \theta \cos \theta}{\frac{\cos \theta}{\sin \theta}} \\ & \quad \left[\begin{array}{l} \sin(90^\circ - \theta) = \cos \theta \text{ and} \\ \cos(90^\circ - \theta) = \sin \theta \end{array} \right] \\ &= \frac{1}{1} + \cos^2 \theta + \sin^2 \theta = 1 + 1 = 2 \end{aligned}$$

Ex.38 If $\tan 2\theta = \cot(\theta + 6^\circ)$, where 2θ and $\theta + 6^\circ$ are acute angles, find the value of θ .

Sol.

$$\begin{aligned} & \text{We have,} \\ & \tan 2\theta = \cot(\theta + 6^\circ) \\ & \Rightarrow \cot(90^\circ - 2\theta) = \cot(\theta + 6^\circ) \\ & \Rightarrow 90^\circ - 2\theta = \theta + 6^\circ \Rightarrow 3\theta = 84^\circ \\ & \Rightarrow \theta = 28^\circ \end{aligned}$$

Ex.39 If A, B, C are the interior angles of a triangle

ABC, prove that $\tan \frac{B+C}{2} = \cot \frac{A}{2}$

Sol.

$$\begin{aligned} & \text{In } \triangle ABC, \text{ we have} \\ & A + B + C = 180^\circ \\ & \Rightarrow B + C = 180^\circ - A \\ & \Rightarrow \frac{B+C}{2} = 90^\circ - \frac{A}{2} \\ & \Rightarrow \tan \left(\frac{B+C}{2} \right) = \tan \left(90^\circ - \frac{A}{2} \right) \\ & \Rightarrow \tan \left(\frac{B+C}{2} \right) = \cot \frac{A}{2} \end{aligned}$$

Ex.40 If $\tan 2A = \cot(A - 18^\circ)$, where $2A$ is an acute angle, find the value of A. [NCERT]

Sol.

$$\begin{aligned} & \tan 2A = \cot(A - 18^\circ) \\ & \cot(90^\circ - 2A) = \cot(A - 18^\circ) \\ & (\Theta \cot(90^\circ - \theta) = \tan \theta) \\ & 90^\circ - 2A = A - 18^\circ \\ & 3A = 108^\circ \\ & A = 36^\circ \text{ Ans.} \end{aligned}$$

Ex.41 If $\tan A = \cot B$, prove that $A + B = 90^\circ$.

Sol. $\Theta \tan A = \cot B$

$$\tan A = \tan (90^\circ - B)$$

$$A = 90^\circ - B$$

$$A + B = 90^\circ. \text{ Proved}$$

Ex.42 If A, B and C are interior angles of a triangle ABC , then show that

$$\sin \left(\frac{B+C}{2} \right) = \cos \frac{A}{2} \quad \text{[NCERT]}$$

Sol. $\Theta A + B + C = 180^\circ$ (a.s.p. of Δ)

$$B + C = 180^\circ - A$$

$$\left(\frac{B+C}{2} \right) = 90^\circ - \frac{A}{2}$$

$$\sin \left(\frac{B+C}{2} \right) = \sin \left(90^\circ - \frac{A}{2} \right)$$

$$\sin \left(\frac{B+C}{2} \right) = \cos \frac{A}{2} \quad \text{Proved.}$$

Ex.43 Express $\sin 67^\circ + \cos 75^\circ$ in terms of trigonometric ratios of angles between 0° and 45° .

Sol. $\Theta 23 = 90 - 67 \quad \& \quad 15 = 90 - 75$

$$\therefore \sin 67^\circ + \cos 75^\circ$$

$$= \sin (90 - 23)^\circ + \cos (90 - 15)^\circ$$

$$= \cos 23^\circ + \sin 15^\circ. \quad \text{Ans.}$$



TRIGONOMETRIC IDENTITIES

$$(1) \tan \theta = \frac{\sin \theta}{\cos \theta} \quad (\text{linear})$$

$$\left. \begin{array}{l} (2) \sin^2 \theta + \cos^2 \theta = 1 \\ (3) 1 + \tan^2 \theta = \sec^2 \theta \\ (4) 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta \end{array} \right\} \text{square identities}$$

❖ EXAMPLES ❖

Ex.44 Prove the following trigonometric identities :

$$(i) (1 - \sin^2 \theta) \sec^2 \theta = 1$$

$$(ii) \cos^2 \theta (1 + \tan^2 \theta) = 1$$

Sol.(i) We have,

$$\text{LHS} = (1 - \sin^2 \theta) \sec^2 \theta = \cos^2 \theta \sec^2 \theta$$

$$[\Theta 1 - \sin^2 \theta = \cos^2 \theta]$$

$$= \cos^2 \theta \cdot \left(\frac{1}{\cos^2 \theta} \right) \quad \left[\Theta \sec \theta = \frac{1}{\cos \theta} \right]$$

$$= 1 = \text{RHS}$$

(ii) We have,

$$\text{LHS} = \cos^2 \theta (1 + \tan^2 \theta)$$

$$= \cos^2 \theta \cdot \sec^2 \theta$$

$$[\Theta 1 + \tan^2 \theta = \sec^2 \theta]$$

$$= \cos^2 \theta \cdot \left(\frac{1}{\cos^2 \theta} \right) \quad \left[\Theta \sec \theta = \frac{1}{\cos \theta} \right]$$

Ex.45 Prove the following trigonometric identities :

$$(i) \frac{\sin \theta}{1 - \cos \theta} = \operatorname{cosec} \theta + \cot \theta$$

$$(ii) \frac{\tan \theta + \sin \theta}{\tan \theta - \sin \theta} = \frac{\sec \theta + 1}{\sec \theta - 1}$$

Sol.(i) We have,

$$\text{LHS} = \frac{\sin \theta}{(1 - \cos \theta)} \times \frac{(1 + \cos \theta)}{(1 + \cos \theta)}$$

[Multiplying numerator and denominator by $(1 + \cos \theta)$]

$$= \frac{\sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta} = \frac{\sin \theta (1 + \cos \theta)}{\sin^2 \theta}$$

$$[\Theta 1 - \cos^2 \theta = \sin^2 \theta]$$

$$= \frac{1 + \cos \theta}{\sin \theta} = \frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}$$

$$= \operatorname{cosec} \theta + \cot \theta = \text{RHS}$$

$$\left[\Theta \frac{1}{\sin \theta} = \operatorname{cosec} \theta \text{ and } \frac{\cos \theta}{\sin \theta} = \cot \theta \right]$$

(ii) We have,

$$\text{LHS} = \frac{\tan \theta + \sin \theta}{\tan \theta - \sin \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta} + \sin \theta}{\frac{\sin \theta}{\cos \theta} - \sin \theta} = \frac{\sin \theta \left(\frac{1}{\cos \theta} + 1 \right)}{\sin \theta \left(\frac{1}{\cos \theta} - 1 \right)}$$

$$\frac{\frac{1}{\cos \theta} + 1}{\frac{1}{\cos \theta} - 1} = \frac{\sec \theta + 1}{\sec \theta - 1} = \text{RHS}$$

Ex.46 Prove the following identities :

$$(i) (\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$$

$$(ii) (\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2 = (1 + \sec \theta \operatorname{cosec} \theta)^2$$

$$(iii) \sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$$

Sol.(i) We have,

$$\text{LHS} = (\sin \theta + \operatorname{cosec} \theta)^2 + (\cos \theta + \sec \theta)^2$$

$$= (\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \operatorname{cosec} \theta)$$

$$(\cos^2 \theta + \sec^2 \theta + 2 \cos \theta \sec \theta)$$

$$\begin{aligned}
&= \left(\sin^2 \theta + \operatorname{cosec}^2 \theta + 2 \sin \theta \cdot \frac{1}{\sin \theta} \right) \\
&\quad + \left(\cos^2 \theta + \sec^2 \theta + 2 \cos \theta \cdot \frac{1}{\cos \theta} \right) \\
&= (\sin^2 \theta + \operatorname{cosec}^2 \theta + 2) + (\cos^2 \theta + \sec^2 \theta + 2) \\
&= \sin^2 \theta + \cos^2 \theta + \operatorname{cosec}^2 \theta + \sec^2 \theta + 4 \\
&= 1 + (1 + \cot^2 \theta) + (1 + \tan^2 \theta) + 4 \\
&\quad [\ominus \operatorname{cosec}^2 \theta = 1 + \cot^2 \theta, \sec^2 \theta = 1 + \tan^2 \theta] \\
&= 7 + \tan^2 \theta + \cot^2 \theta = \text{RHS}.
\end{aligned}$$

(ii) We have,

$$\begin{aligned}
\text{LHS} &= (\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2 \\
&= \left(\sin \theta + \frac{1}{\cos \theta} \right)^2 + \left(\cos \theta + \frac{1}{\sin \theta} \right)^2 \\
&= \sin^2 \theta + \frac{1}{\cos^2 \theta} + \frac{2 \sin \theta}{\cos \theta} + \cos^2 \theta \\
&\quad + \frac{1}{\sin^2 \theta} + \frac{2 \cos \theta}{\sin \theta} \\
&= (\sin^2 \theta + \cos^2 \theta) + \left(\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta} \right) + \\
&\quad 2 \left(\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} \right) \\
&= (\sin^2 \theta + \cos^2 \theta) + \left(\frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta \cos^2 \theta} \right) \\
&\quad + \frac{2(\sin^2 \theta + \cos^2 \theta)}{\sin \theta \cos \theta} \\
&= 1 + \frac{1}{\sin^2 \theta \cos^2 \theta} + \frac{2}{\sin \theta \cos \theta} \\
&= \left(1 + \frac{1}{\sin \theta \cos \theta} \right)^2 = (1 + \sec \theta \operatorname{cosec} \theta)^2 = \text{RHS}
\end{aligned}$$

(iii) We have, LHS = $\sec^4 \theta - \sec^2 \theta$

$$\begin{aligned}
&= \sec^2 \theta (\sec^2 \theta - 1) = (1 + \tan^2 \theta) (1 + \tan^2 \theta - 1) \\
&\quad [\ominus \sec^2 \theta = 1 + \tan^2 \theta] \\
&= (1 + \tan^2 \theta) \tan^2 \theta = \tan^2 \theta + \tan^4 \theta = \text{RHS}.
\end{aligned}$$

Ex.47 Prove the following identities :

- (i) $\cos^4 A - \cos^2 A = \sin^4 A - \sin^2 A$
- (ii) $\cot^4 A - 1 = \operatorname{cosec}^4 A - 2 \operatorname{cosec}^2 A$
- (iii) $\sin^6 A + \cos^6 A = 1 - 3 \sin^2 A \cos^2 A$.

Sol.(i) We have,

$$\begin{aligned}
\text{LHS} &= \cos^4 A - \cos^2 A = \cos^2 A (\cos^2 A - 1) \\
&= -\cos^2 A (1 - \cos^2 A) = -\cos^2 A \sin^2 A \\
&= -(1 - \sin^2 A) \sin^2 A = -\sin^2 A + \sin^4 A \\
&= \sin^4 A - \sin^2 A = \text{RHS}
\end{aligned}$$

(ii) We have,

$$\begin{aligned}
\text{LHS} &= \cot^4 A - 1 = (\operatorname{cosec}^2 A - 1)^2 - 1 \\
&[\ominus \cot^2 A = \operatorname{cosec}^2 A - 1] \\
&\therefore \cot^4 A = (\operatorname{cosec}^2 A - 1)^2] \\
&= \operatorname{cosec}^4 A - 2 \operatorname{cosec}^2 A + 1 - 1 \\
&= \operatorname{cosec}^4 A - 2 \operatorname{cosec}^2 A = \text{RHS}
\end{aligned}$$

(iii) We have,

$$\begin{aligned}
\text{LHS} &= \sin^6 A + \cos^6 A = (\sin^2 A)^3 + (\cos^2 A)^3 \\
&= (\sin^2 A + \cos^2 A) \{ (\sin^2 A)^2 + (\cos^2 A)^2 \\
&\quad - \sin^2 A \cos^2 A \} \\
&\quad [\ominus a^3 + b^3 = (a + b)(a^2 - ab + b^2)] \\
&= \{ (\sin^2 A)^2 + (\cos^2 A)^2 + 2 \sin^2 A \cos^2 A \\
&\quad - \sin^2 A \cos^2 A \} \\
&= [(\sin^2 A + \cos^2 A)^2 - 3 \sin^2 A \cos^2 A] \\
&= 1 - 3 \sin^2 A \cos^2 A = \text{RHS}
\end{aligned}$$

Ex.48 Prove the following identities :

- (i) $\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A} = \frac{1}{\sin^2 A \cos^2 A} - 2$
- (ii) $\frac{\cos A}{1 - \tan A} + \frac{\sin^2 A}{\sin A - \cos A} = \sin A + \cos A$
- (iii) $\frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{\cos^2 \theta} = 2 \left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \right)$

Sol.(i) We have,

$$\begin{aligned}
\text{LHS} &= \frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A} = \frac{\sin^4 A + \cos^2 A}{\sin^2 A \cos^2 A} \\
&\quad [\text{on taking LCM}] \\
&= \frac{(\sin^2 A)^2 + (\cos^2 A)^2 + 2 \sin^2 A \cos^2 A - 2 \sin^2 A \cos^2 A}{\sin^2 A \cos^2 A} \\
&= \frac{(\sin^2 A + \cos^2 A)^2 - 2 \sin^2 A \cos^2 A}{\sin^2 A \cos^2 A} \\
&= \frac{1 - 2 \sin^2 A \cos^2 A}{\sin^2 A \cos^2 A} \\
&= \frac{1}{\sin^2 A \cos^2 A} - 2 = \text{RHS}
\end{aligned}$$

(ii) We have,

$$\begin{aligned}
\text{LHS} &= \frac{\cos A}{1 - \tan A} + \frac{\sin^2 A}{\sin A - \cos A} \\
&= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin^2 A}{\sin A - \cos A}
\end{aligned}$$

$$\begin{aligned}
&= \frac{\cos A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} \\
&= \frac{\cos A}{\cos A} + \frac{\sin^2 A}{\sin A - \cos A} \\
&= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} \\
&= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A} \\
&= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} \\
&= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A} \\
&= \cos A + \sin A = \text{RHS}
\end{aligned}$$

(iii) We have,

$$\begin{aligned}
\text{LHS} &= \frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{\cos^2 \theta} \\
&= \frac{(1 + 2 \sin \theta + \sin^2 \theta) + (1 - 2 \sin \theta + \sin^2 \theta)}{\cos^2 \theta} \\
&= \frac{2 + 2 \sin^2 \theta}{\cos^2 \theta} = \frac{2(1 + \sin^2 \theta)}{1 - \sin^2 \theta} = 2 \left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \right) \\
&= \text{RHS}.
\end{aligned}$$

Ex.49 Prove the following identities :

- (i) $2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 = 0$
(ii) $(\sin^8 \theta - \cos^8 \theta) = (\sin^2 \theta - \cos^2 \theta)(1 - 2 \sin^2 \theta \cos^2 \theta)$

Sol.(i) We have,

$$\begin{aligned}
\text{LHS} &= 2(\sin^6 \theta + \cos^6 \theta) - 3(\sin^4 \theta + \cos^4 \theta) + 1 \\
&= 2[(\sin^2 \theta)^3 + (\cos^2 \theta)^3] - 3[(\sin^2 \theta)^2 + (\cos^2 \theta)^2] + 1 \\
&= 2[(\sin^2 \theta + \cos^2 \theta) \{(\sin^2 \theta)^2 + (\cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta\}] \\
&\quad - 3[(\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta] + 1 \\
&= 2[(\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 3 \sin^2 \theta \cos^2 \theta] \\
&\quad - 3[(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta] + 1 \\
&= 2[(\sin^2 \theta + \cos^2 \theta)^2 - 3 \sin^2 \theta \cos^2 \theta] - 3[1 - 2 \sin^2 \theta \cos^2 \theta] + 1 \\
&= 2(1 - 3 \sin^2 \theta \cos^2 \theta) - 3(1 - 2 \sin^2 \theta \cos^2 \theta) + 1 \\
&= 2 - 6 \sin^2 \theta \cos^2 \theta - 3 + 6 \sin^2 \theta \cos^2 \theta + 1 \\
&= 0 = \text{RHS}
\end{aligned}$$

(ii) We have,

$$\begin{aligned}
\text{LHS} &= \sin^8 \theta - \cos^8 \theta = (\sin^4 \theta)^2 - (\cos^4 \theta)^2 \\
&= (\sin^4 \theta - \cos^4 \theta)(\sin^4 \theta + \cos^4 \theta) \\
&= (\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta) \\
&\quad (\sin^4 \theta + \cos^4 \theta)
\end{aligned}$$

$$\begin{aligned}
&= (\sin^2 \theta - \cos^2 \theta) \{(\sin^2 \theta)^2 + (\cos^2 \theta)^2 + 2 \sin^2 \theta \cos^2 \theta - 2 \sin^2 \theta \cos^2 \theta\} \\
&= (\sin^2 \theta - \cos^2 \theta) \{(\sin^2 \theta + \cos^2 \theta)^2 - 2 \sin^2 \theta \cos^2 \theta\} \\
&= (\sin^2 \theta - \cos^2 \theta)(1 - 2 \sin^2 \theta \cos^2 \theta) = \text{RHS}
\end{aligned}$$

Ex.50 If $(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)$ prove that each of the side is equal to ± 1 .

Sol.

We have,

$$\begin{aligned}
&(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \\
&= (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) \\
&\text{Multiplying both sides by} \\
&(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) \text{ we get} \\
&(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) \\
&(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) \\
&= (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2 \\
&\Rightarrow (\sec^2 A - \tan^2 A)(\sec^2 B - \tan^2 B)(\sec^2 C - \tan^2 C) \\
&= (\sec A - \tan A)^2 (\sec B - \tan B)^2 (\sec C - \tan C)^2 \\
&\Rightarrow 1 = [(\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C)]^2 \\
&\Rightarrow (\sec A - \tan A)(\sec B - \tan B)(\sec C - \tan C) = \pm 1 \\
&\text{Similarly, multiplying both sides by} \\
&(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C), \\
&\text{we get} \\
&(\sec A + \tan A)(\sec B + \tan B)(\sec C + \tan C) = \pm 1
\end{aligned}$$

Ex.51 If $\tan \theta + \sin \theta = m$ and $\tan \theta - \sin \theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Sol.

We have,

$$\begin{aligned}
\text{LHS} &= m^2 - n^2 = (\tan \theta + \sin \theta)^2 - (\tan \theta - \sin \theta)^2 \\
&= 4 \tan \theta \sin \theta \quad [\Theta(a + b)^2 - (a - b)^2 = 4ab]
\end{aligned}$$

And, $\text{RHS} = 4\sqrt{mn}$

$$= 4\sqrt{(\tan \theta + \sin \theta)(\tan \theta - \sin \theta)}$$

$$= 4\sqrt{\tan^2 \theta - \sin^2 \theta}$$

$$= 4\sqrt{\frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta}$$

$$= 4\sqrt{\frac{\sin^2 \theta - \sin^2 \theta \cos^2 \theta}{\cos^2 \theta}}$$

$$= 4\sqrt{\frac{\sin^2 \theta(1 - \cos^2 \theta)}{\cos^2 \theta}} = 4\sqrt{\frac{\sin^4 \theta}{\cos^2 \theta}}$$

$$= 4 \frac{\sin^2 \theta}{\cos \theta} = 4 \sin \theta \frac{\sin \theta}{\cos \theta} = 4 \sin \theta \tan \theta$$

Thus we have

$$\text{LHS} = \text{RHS, i.e. } m^2 - n^2 = 4\sqrt{mn}$$

Ex.52 If $\cos\theta + \sin\theta = \sqrt{2} \cos\theta$, show that

$$\cos\theta - \sin\theta = \sqrt{2} \sin\theta.$$

Sol. We have,

$$\begin{aligned}\cos\theta + \sin\theta &= \sqrt{2} \cos\theta \\ \Rightarrow (\cos\theta + \sin\theta)^2 &= 2 \cos^2\theta \\ \Rightarrow \cos^2\theta + \sin^2\theta + 2 \cos\theta \sin\theta &= 2 \cos^2\theta \\ \Rightarrow \cos^2\theta - 2 \cos\theta \sin\theta &= \sin^2\theta \\ \Rightarrow \cos^2\theta - 2 \cos\theta \sin\theta + \sin^2\theta &= 2 \sin^2\theta \\ \Rightarrow (\cos\theta - \sin\theta)^2 &= 2 \sin^2\theta \\ \Rightarrow \cos\theta - \sin\theta &= \sqrt{2} \sin\theta\end{aligned}$$

Ex.53 If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, show that $q(p^2 - 1) = 2p$

Sol. We have,

$$\begin{aligned}\text{LHS} &= q(p^2 - 1) \\ &= (\sec\theta + \operatorname{cosec}\theta) [(\sin\theta + \cos\theta)^2 - 1] \\ &= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta} \right) \{ \sin^2\theta + \cos^2\theta + 2 \sin\theta \cos\theta - 1 \} \\ &= \left(\frac{\sin\theta + \cos\theta}{\cos\theta \sin\theta} \right) [1 + 2 \sin\theta \cos\theta - 1] \\ &= \left(\frac{\sin\theta + \cos\theta}{\cos\theta \sin\theta} \right) (2 \sin\theta \cos\theta) \\ &= 2(\sin\theta + \cos\theta) = 2p = \text{RHS}\end{aligned}$$

Ex.54 If $\sec\theta + \tan\theta = p$, show that $\frac{p^2 - 1}{p^2 + 1} = \sin\theta$.

Sol. We have,

$$\begin{aligned}\text{LHS} &= \frac{p^2 - 1}{p^2 + 1} = \frac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1} \\ &= \frac{\sec^2\theta + \tan^2\theta + 2 \sec\theta \tan\theta - 1}{\sec^2\theta + \tan^2\theta + 2 \sec\theta \tan\theta + 1} \\ &= \frac{(\sec^2\theta - 1) + \tan^2\theta + 2 \sec\theta \tan\theta}{\sec^2\theta + 2 \sec\theta \tan\theta + (1 + \tan^2\theta)} \\ &= \frac{\tan^2\theta + \tan^2\theta + 2 \sec\theta \tan\theta}{\sec^2\theta + 2 \sec\theta \tan\theta + \sec^2\theta} \\ &= \frac{2 \tan^2\theta + 2 \tan\theta \sec\theta}{2 \sec^2\theta + 2 \sec\theta \tan\theta} \\ &= \frac{2 \tan\theta (\tan\theta + \sec\theta)}{2 \sec\theta (\sec\theta + \tan\theta)} \\ &= \frac{\tan\theta}{\sec\theta} = \frac{\sin\theta}{\cos\theta \sec\theta} = \sin\theta = \text{RHS}\end{aligned}$$

Ex.55 If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$ show that $(m^2 + n^2)\cos^2\beta = n^2$.

$$\begin{aligned}\text{Sol.} \quad \text{LHS} &= (m^2 + n^2)\cos^2\beta \\ &= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta} \right) \cos^2\beta \\ &\quad \left[\Theta \quad m = \frac{\cos\alpha}{\cos\beta} \text{ and } n = \frac{\cos\alpha}{\sin\beta} \right] \\ &= \left(\frac{\cos^2\alpha \sin^2\beta + \cos^2\alpha \cos^2\beta}{\cos^2\beta \sin^2\beta} \right) \cos^2\beta \\ &= \cos^2\alpha \left(\frac{1}{\cos^2\beta \sin^2\beta} \right) \cos^2\beta \\ &= \frac{\cos^2\alpha}{\sin^2\beta} = \left(\frac{\cos\alpha}{\sin\beta} \right)^2 = n^2 = \text{RHS}\end{aligned}$$

Ex.56 If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, prove that $a^2 + b^2 = m^2 + n^2$.

Sol. We have,

$$\begin{aligned}\text{RHS} &= m^2 + n^2 \\ &= (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2 \\ &= (a^2\cos^2\theta + b^2\sin^2\theta + 2ab \cos\theta \sin\theta) \\ &\quad + (a^2\sin^2\theta + b^2\cos^2\theta - 2ab \sin\theta \cos\theta) \\ &= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) \\ &= a^2 + b^2 = \text{LHS}.\end{aligned}$$

Ex.57 If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm \sqrt{a^2 + b^2 - c^2}$

Sol. We have,

$$\begin{aligned}& (a\cos\theta - b\sin\theta)^2 + (a\sin\theta + b\cos\theta)^2 \\ &= (a^2\cos^2\theta + b^2\sin^2\theta - 2ab \sin\theta \cos\theta) \\ &\quad + (a^2\sin^2\theta + b^2\cos^2\theta + 2ab \sin\theta \cos\theta) \\ &= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) \\ &= a^2 + b^2 \\ \Rightarrow c^2 + (a\sin\theta + b\cos\theta)^2 &= a^2 + b^2 \\ &\quad [\Theta \quad a\cos\theta - b\sin\theta = c] \\ \Rightarrow (a\sin\theta + b\cos\theta)^2 &= a^2 + b^2 - c^2 \\ \Rightarrow a\sin\theta + b\cos\theta &= \pm \sqrt{a^2 + b^2 - c^2}.\end{aligned}$$

Ex.58 Prove that : $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$

$$\begin{aligned}\text{Sol.} \quad (1 - \sin\theta + \cos\theta)^2 &= 1 + \sin^2\theta + \cos^2\theta - 2\sin\theta + 2\cos\theta - 2\sin\theta \cos\theta \\ &= 2 - 2\sin\theta + 2\cos\theta - 2\sin\theta \cos\theta \\ &= 2(1 - \sin\theta) + 2\cos\theta(1 - \sin\theta) \\ &= 2(1 - \sin\theta)(1 + \cos\theta) = \text{RHS}\end{aligned}$$

Ex.59 If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$.

Sol. We have,

$$\begin{aligned}\sin\theta + \sin^2\theta &= 1 \\ \Rightarrow \sin\theta &= 1 - \sin^2\theta \\ \Rightarrow \sin\theta &= \cos^2\theta \\ \text{Now, } \cos^2\theta + \cos^4\theta &= \cos^2\theta + (\cos^2\theta)^2 \\ &= \cos^2\theta + \sin^2\theta = 1\end{aligned}$$

Ex.60 Prove that :

$$\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} + \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} = \frac{2}{2 \sin^2 \theta - 1}$$

Sol. We have,

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} + \frac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta} \\ &= \frac{(\sin \theta - \cos \theta)^2 + (\sin \theta + \cos \theta)^2}{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)} \\ &= \frac{2(\sin^2 \theta + \cos^2 \theta)}{\sin^2 \theta - \cos^2 \theta} \\ &\quad [\Theta (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)] \\ &= \frac{2}{\sin^2 \theta - (1 - \sin^2 \theta)} \\ &= \frac{2}{(2 \sin^2 \theta - 1)} = \text{RHS.} \end{aligned}$$

Ex.61 Express the ratios $\cos A$, $\tan A$ and $\sec A$ in terms of $\sin A$.

Sol. Since $\cos^2 A + \sin^2 A = 1$, therefore,

$$\cos^2 A = 1 - \sin^2 A, \text{ i.e., } \cos A = \pm \sqrt{1 - \sin^2 A}$$

This gives

$$\cos A = \sqrt{1 - \sin^2 A} \quad (\text{Why ?})$$

Hence,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}} \text{ and}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}}.$$

Ex.62 Prove that $\frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{1}{\sec \theta - \tan \theta}$,
 using the identity $\sec^2 \theta = 1 + \tan^2 \theta$.

Sol.

$$\begin{aligned} \text{LHS} &= \frac{\sin \theta - \cos \theta + 1}{\sin \theta + \cos \theta - 1} = \frac{\tan \theta - 1 + \sec \theta}{\tan \theta + 1 - \sec \theta} \\ &= \frac{(\tan \theta + \sec \theta) - 1}{(\tan \theta - \sec \theta) + 1} \\ &= \frac{\{(\tan \theta + \sec \theta) - 1\}(\tan \theta - \sec \theta)}{\{(\tan \theta - \sec \theta) + 1\}(\tan \theta - \sec \theta)} \\ &= \frac{(\tan^2 \theta - \sec^2 \theta) - (\tan \theta - \sec \theta)}{\{\tan \theta - \sec \theta + 1\}(\tan \theta - \sec \theta)} \\ &= \frac{-1 - \tan \theta + \sec \theta}{(\tan \theta - \sec \theta + 1)(\tan \theta - \sec \theta)} \\ &= \frac{-1}{\tan \theta - \sec \theta} = \frac{1}{\sec \theta - \tan \theta}, \end{aligned}$$

which is the RHS of the identity, we are required to prove.

EXERCISE # 1

Q.1 If $\sin \alpha = \frac{1}{\sqrt{2}}$ and $\tan \beta = 1$, find the value of $\sin(\alpha + \beta)$, where α and β both are acute.

Q.2 If $\cos \alpha = \frac{1}{2}$ and $\tan \beta = \frac{1}{\sqrt{3}}$, find the value of $\sin(\alpha + \beta)$, where α and β both are acute.

Q.3 Without using trigonometric tables evaluate the following :

$$\begin{array}{lll} \text{(i)} \frac{\sin 20^\circ}{\cos 70^\circ} & \text{(ii)} \frac{\cos 19^\circ}{\sin 71^\circ} & \text{(iii)} \frac{\sin 21^\circ}{\cos 69^\circ} \\ \text{(iv)} \frac{\tan 10^\circ}{\cot 80^\circ} & \text{(v)} \frac{\sec 11^\circ}{\operatorname{cosec} 79^\circ} & \text{(vi)} \frac{\sin 20^\circ 30'}{\cos 69^\circ 30'} \end{array}$$

Q.4 Without using trigonometric tables evaluate the following :

$$\begin{array}{ll} \text{(i)} \left(\frac{\sin 49^\circ}{\cos 41^\circ} \right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ} \right)^2 & \\ \text{(ii)} \frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cot 35^\circ}{\sin 55^\circ} \right) & \end{array}$$

Q.5 Without using trigonometric tables evaluate the following :

$$\begin{array}{ll} \text{(i)} \frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1 & \\ \text{(ii)} \operatorname{cosec}^2 67^\circ - \tan^2 23^\circ & \end{array}$$

Q.6 Without using trigonometric tables evaluate the following :

$$\begin{array}{ll} \text{(i)} \sin^2 20^\circ + \sin^2 70^\circ - \tan^2 45^\circ & \\ \text{(ii)} \sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ & \end{array}$$

Q.7 Without using trigonometric tables prove the following :

$$\begin{array}{ll} \text{(i)} \tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1 & \\ \text{(ii)} \sin 48^\circ \sec 42^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ = 2 & \\ \text{(iii)} \sin 63^\circ \cos 27^\circ + \cos 63^\circ \sin 27^\circ = 1 & \\ \text{(iv)} \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - \cos 70^\circ \operatorname{cosec} 20^\circ = 1 & \\ \text{(v)} \frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2 & \end{array}$$

Q.8 Prove the following :

$$\begin{array}{ll} \text{(i)} \sin \theta \sin(90^\circ - \theta) - \cos \theta \cos(90^\circ - \theta) = 0 & \\ \text{(ii)} \frac{\sin \theta \cos(90^\circ - \theta) \cos \theta}{\sin(90^\circ - \theta)} + \frac{\cos \theta \sin(90^\circ - \theta) \sin \theta}{\sin(90^\circ - \theta)} = 1 & \\ \text{(iii)} \frac{\sin \theta}{\sin(90^\circ - \theta)} + \frac{\cos \theta}{\cos(90^\circ - \theta)} = \sec \theta \operatorname{cosec} \theta & \\ \text{(iv)} \sin(90^\circ - \theta) \cos(90^\circ - \theta) = \frac{\tan \theta}{1 + \cot^2(90^\circ - \theta)} & \\ \text{(v)} \frac{\cos(90^\circ - \theta)}{1 + \sin(90^\circ - \theta)} + \frac{1 + \sin(90^\circ - \theta)}{\cos(90^\circ - \theta)} = 2 \operatorname{cosec} \theta & \\ \text{(vi)} \frac{1}{1 + \cos(90^\circ - \theta)} + \frac{1}{1 - \cos(90^\circ - \theta)} = 2 \operatorname{cosec}^2(90^\circ - \theta) & \\ \text{(vii)} \sin^2(90^\circ - \theta)(1 + \cot^2(90^\circ - \theta)) = 1 & \\ \text{(viii)} \frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta} = 2 & \\ \text{(ix)} \frac{\tan(90^\circ - A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0 & \\ \text{(x)} \frac{\cos(90^\circ - A) \sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A & \end{array}$$

Q.9 Without using trigonometric tables, evaluate each of the following :

$$\begin{array}{ll} \text{(i)} \sec^2 10^\circ - \cot^2 80^\circ + \frac{\sin 15^\circ \cos 75^\circ + \cos 15^\circ \sin 75^\circ}{\cos \theta \sin(90^\circ - \theta) + \sin \theta \cos(90^\circ - \theta)} & \\ \text{(ii)} \sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ & \\ \text{(iii)} \cot \theta \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \operatorname{cosec} \theta + \sin^2 25^\circ + \sin^2 65^\circ + \sqrt{3} (\tan 5^\circ \tan 45^\circ \tan 85^\circ) & \\ \text{(iv)} \cot \theta \tan(90^\circ - \theta) - \sec(90^\circ - \theta) \operatorname{cosec} \theta + \sqrt{3} (\tan 5^\circ \tan 30^\circ \tan 85^\circ) + \sin^2 25^\circ + \sin^2 65^\circ & \\ \text{(v)} \frac{-\tan \theta \cot(90^\circ - \theta) + \sec \theta \operatorname{cosec}(90^\circ - \theta)}{\tan 10^\circ \tan 20^\circ \tan 45^\circ \tan 70^\circ \tan 80^\circ} + \frac{\sin^2 35^\circ + \sin^2 55^\circ}{\tan 10^\circ \tan 20^\circ \tan 45^\circ \tan 70^\circ \tan 80^\circ} & \end{array}$$

Q.10 The round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β . Prove that the height of the centre of the balloon is $r \sin \beta \operatorname{cosec} \alpha/2$.

Q.11 If $\tan \theta = 8/15$ and $0^\circ < \theta < 90^\circ$, find $\sin \theta$.

Q.12 If $\sin \theta = 8/17$ and $0^\circ < \theta < 90^\circ$, find $\tan \theta$.

Q.13 If $\sin A = \frac{24}{25}$, find the value of $\tan A + \sec A$, where $0^\circ < A < 90^\circ$.

Q.14 If $5 \tan \theta = 12$, find the value of $\frac{2 \cos \theta + \sin \theta}{\sin \theta - \cos \theta}$.

Q.15 If $\tan \theta = \frac{3}{4}$, find the value of $\frac{1 - \cos \theta}{1 + \cos \theta}$.

Q.16 If $\tan \theta = \frac{12}{5}$, find the value of $\frac{1 + \sin \theta}{1 - \sin \theta}$.

Q.17 If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, by using

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}, \text{ prove that}$$

$$A + B = 45^\circ$$

Q.18 If $4 \tan \theta = 3$, find the value of

$$\frac{4 \sin \theta - 2 \cos \theta}{4 \sin \theta + 3 \cos \theta}.$$

Q.19 If $\operatorname{cosec} \theta = \frac{13}{12}$, find the value of

$$\frac{2 \sin \theta - 3 \cos \theta}{4 \sin \theta - 9 \cos \theta}.$$

ANSWER KEY

1. 1

4. (i) 2, (ii) $\frac{1}{2}$

9. (i) 2, (ii) 1, (iii) $\sqrt{3}$, (iv) 1, (v) 2

13. 7

16. 25

2. 1

5. (i) 1, (ii) 1

11. $\frac{8}{17}$

14. $\frac{22}{7}$

18. $\frac{1}{6}$

3. (i) 1, (ii) 1, (iii) 1, (iv) 1, (v) 1, (vi) 1

6. (i) 0, (ii) 2

12. $\frac{8}{15}$

15. $\frac{1}{9}$

19. 3

EXERCISE # 2

- Q.1** Find the value of $\left(\frac{3\pi}{5}\right)$ radians in degrees.
- Q.2** Find the value of 150° in radians.
- Q.3** If $\sin\theta = \frac{5}{13}$, then find the values of $\tan\theta$ and $\sec\theta$.
- Q.4** If $\tan\theta = \frac{x}{y}$, then find the value of $\left(\frac{x \sin\theta + y \cos\theta}{x \sin\theta - y \cos\theta}\right)$.
- Q.5** If $5\tan\theta = 4$, find the value of $\left(\frac{5 \sin\theta - 3 \cos\theta}{5 \sin\theta + 2 \cos\theta}\right)$.
- Q.6** If $16\cot x = 12$, then find the value of $\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)$.
- Q.7** If $\tan\theta = (3/4)$ and $0 < \theta < 90^\circ$, then find the value of $(\sin\theta \cos\theta)$.
- Q.8** If $8 \tan x = 15$, then find the value of $(\sin x - \cos x)$.
- Q.9** If $\tan\theta = \frac{1}{\sqrt{7}}$, then find the value of $\left(\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}\right)$.
- Q.10** If $\cot\theta = \frac{1}{\sqrt{3}}$, then find the value of $\left(\frac{1 - \cos^2\theta}{2 - \sin^2\theta}\right)$.
- Q.11** If $\tan\theta = \frac{4}{3}$, then find the value of $\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}$.
- Q.12** If $3\cos\theta = 5\sin\theta$, then find the value of $\left(\frac{5 \sin\theta - 2 \sec^3\theta + 2 \cos\theta}{5 \sin\theta + 2 \sec^3\theta - 2 \cos\theta}\right)$.
- Q.13** If $\tan\theta = (3/4)$, then find the value of $(\cos^2\theta - \sin^2\theta)$.
- Q.14** Find the value of $\tan 75^\circ$.
- Q.15** If $\tan\theta = \frac{a}{x}$, then find the value of $\frac{x}{\sqrt{a^2 + x^2}}$.
- Q.16** If $3\sin x + 5\cos x = 5$, then the value of $(3\cos x - 5\sin x)^2$.
- Q.17** Find the value of $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$.
- Q.18** Find the value of $\sqrt{\frac{1 + \sin A}{1 - \sin A}}$.
- Q.19** Find the value of $\sqrt{\frac{1 - \sin A}{1 + \sin A}}$.
- Q.20** Find the value of $\sqrt{\frac{1 - \cos x}{1 + \cos x}}$.
- Q.21** Find the value of $\sqrt{\frac{1 + \cos x}{1 - \cos x}}$.
- Q.22** Find the value of $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$.
- Q.23** Find the value of $\left(\frac{\cot\theta}{\cot\theta - \cot 3\theta} + \frac{\tan\theta}{\tan\theta - \tan 3\theta}\right)$.
- Q.24** Find the value of $\left(\frac{\sin A + \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A - \sin B}\right)$.

Q.25 Find the value of $\sin 15^\circ$.

Q.26 Find the value of $(\sin 40^\circ - \cos 50^\circ)$.

Q.27 If $x = r \sin A \cos B$, $y = r \sin A \sin B$ and $z = r \cos A$, then which is correct ?

(A) $x^2 + y^2 + z^2 = r^2$ (B) $x^2 - y^2 + z^2 = r^2$

(C) $x^2 + y^2 - z^2 = r^2$ (D) $-x^2 + y^2 + z^2 = r^2$

Q.28 Find the value of

$(\cot 15^\circ \cot 16^\circ \cot 17^\circ \dots \cot 73^\circ \cot 74^\circ \cot 75^\circ)$.

ANSWER KEY

1. 108°

4. $\frac{x^2 + y^2}{x^2 - y^2}$

7. $\frac{12}{25}$

10. $\frac{3}{5}$

13. $\frac{7}{25}$

16. 9

18. $\sec A + \tan A$

21. $\operatorname{cosec} x + \cot x$

24. 0

27. (A)

2. $\left(\frac{5\pi}{6}\right)^c$

5. $\frac{1}{6}$

8. $\frac{7}{17}$

11. $\frac{1}{3}$

14. $2 - \sqrt{3}$

17. 2

19. $\sec A - \tan A$

22. $\sec x - \tan x$

25. $\frac{\sqrt{3} - 1}{2\sqrt{2}}$

28. 1

3. $\frac{5}{12}$ and $\frac{13}{12}$

6. $\frac{1}{7}$

9. $\frac{3}{4}$

12. $\frac{271}{979}$

15. $\cos \theta$

20. $\operatorname{cosec} x - \cot x$

23. 1

26. 0