5. Playing with Numbers

Exercise 5.1

1. Question

Without performing actual addition and division write the quotient when the sum of 69 and 96 is divided by

(i) 11 (ii) 15

Answer

1) Here, we observe that 69 and 96 are having ten’s and unit place interchanged, ie they are having reverse digits.

Hence, sum of digits is 15.

We know that when ab + ba is divided by 11, quotient is (a + b).

∴ The sum of 69 and 96 is divided by 11, we get 15 (sum of digits) as our quotient.

2) Here, we observe that 69 and 96 are having ten’s and unit place interchanged, ie they are having reverse digits.

Hence, sum of digits is 15.

We know that when ab + ba is divided by (a + b), quotient is 11.

∴ The sum of 69 and 96 is divided by 15 (sum of digits) , we get 11 as our quotient.

2. Question

Without performing actual computations, find the quotient when 94-49 is divided by

(i) 9 (ii) 5

Answer

1) Here, we observe that 94 and 49 are having ten’s and unit place interchanged, ie they are having reverse digits.

Hence, difference of digits is 5.

We know that when ab – ba is divided by 9, the quotient is (a – b).

∴ 94 – 49 is divided by 9, the quotient is 5.

2) Here, we observe that 94 and 49 are having ten’s and unit place interchanged, ie they are having reverse digits.

Hence, difference of digits is 5.

We know that when ab – ba is divided by (a – b), quotient is 9.

∴ the difference of 94 and 49 is divided by 5 (difference of digits) , we get 9 as our quotient.

3. Question

If sum of the number 985 and two other numbers obtained by arranging the digits of 985 in cyclic order is divided by 111,22 and 37 respectively. Find the quotient in each case.

Answer

Here our given numbers are 985, 859 and 598.

Hence, the quotient obtained when the sum of these three numbers is divided by:

1) 111

We know that when sum of three digit numbers, in cyclic order, is done, and then divided by 111, quotient is sum of digits of a number.

Quotient = Sum of digits = 22

2) 22 (Sum of digits)

We know that when sum of three digit numbers, in cyclic order, is done, and then divided by sum of digits , quotient is 111.

Quotient = 111

3) 37

Here, 3 × 37 = 111

∴ Quotient = 3 × (Sum of the digits) = 3 × 22 = 66

4. Question

Find the quotient when the difference of 985 and 958 is divided by 9.

Answer

Here, quotient ,when difference between the numbers 985 and 958 is divided by 9, we know that when ten’s and unit’s place is interchanged, we get quotient as a difference of unit’ s and ten’s place.

∴ Quotient is 8 – 5 = 3

Exercise 5.2

1. Question

Given that the number $\overline{35a64}$ is divisible by 3, where a is a digit, what are the possible values of a?

Answer

Here it is given that $\overline{35a64}$ is divisible by 3.

We know that if a number is divisible by 3, then sum of digits must be a multiple of 3.

∴ 3 + 5+a+6+4 = multiple of 3

∴ a + 18 = 0,3,6,9,12,15,.....

But ‘a’ is a digit, hence, ‘a’ can have value between 0 to 9.

∴ a + 18 = 18 which gives a = 0.

∴ a + 18 = 21 which gives a = 3.

∴ a + 18 = 24 which gives a = 6.

∴ a + 18 = 27 which gives a = 9.

∴ a = 0,3,6,9

2. Question

If x is a digit such that the number $\overline{18x71}$ is divisible by 3, find possible values of x.

Answer

Here it is given that $\overline{18x71}$ is divisible by 3.

We know that if a number is divisible by 3, then sum of digits must be a multiple of 3.

∴ 1 + 8+x+7+1 = multiple of 3

∴ a + 17 = 0,3,6,9,12,15,.....

But ‘x’ is a digit, hence, ‘x’ can have value between 0 to 9.

∴ x + 17 = 18 which gives x = 1.

∴ x+ 17 = 21 which gives x = 4.

∴ x + 17 = 24 which gives x = 7.

∴ x = 1,4,7

3. Question

If x is a digit of the number $\overline{66784x}$ such that it is divisible by 9, find possible values of x.

Answer

Here it is given that $\overline{66784x}$ is divisible by 9.

We know that if a number is divisible by 9, then sum of digits must be a multiple of 9.

∴ 6 + 6 + 7 + 8 + 4 + x = multiple of 9

∴ x + 31 = 0,9,18,27,…..

But ‘x’ is a digit, hence, ‘x’ can have value between 0 to 9.

∴ x + 31 = 36.

∴ x = 5

4. Question

Given that the number $\overline{67y19}$ is divisible by 9, where y is a digit, what are the possible values of y?

Answer

Here it is given that $\overline{67y19}$ is divisible by 9.

We know that if a number is divisible by 9, then sum of digits must be a multiple of 9.

∴ 6 + 7+y+1+9 = multiple of 9

∴ y + 23 = 0,9,18,27,,…..

But ‘y’ is a digit, hence, ‘y’ can have value between 0 to 9.

∴ y + 23 = 27 which gives y = 4.

∴ y = 4

5. Question

If $\overline{3x2}$ is a multiple of 11, where x is a digit, what is the value of x?

Answer

Here, given number is $\overline{3x2}$.

We know that a number is divisible by 11 if and only if the difference between the sum of odd and even place digits is a multiple of 11.

∴ Sum of even placed digits – Sum of odd placed digits = 0, 11, 22,…

∴ x – (3+2) = 0, 11, 22,,…

∴ x – 5 is a multiple of 11

∴ x – 5 = 0

∴ x = 5

6. Question

If $\overline{98215x2}$ is a number with x as its tens digit such that it is divisible by 4. Find all possible values of x.

Answer

Here, given number $\overline{98215x2}$ is divisible by 4

We know that a number is divisible by 4, only when the number formed by its digits in units and tens place is divisible by 4.

∴ x2 is divisible by 4

Expanding x2,

10x + 2 = multiple of 4

Now, x2 can take values 2, 12, 22, 32, 42, 52, 62, 72, 82, 92

Out of these only 12, 32, 52, 72 and 92

∴ x can take values 1,3,5,7 and 9

7. Question

If x denotes the digit at hundreds place of the number $\overline{67x19}$ such that the number is divisible by 11. Find all possible values of x.

Answer

Here, given number is $\overline{67x19}$.

We know that a number is divisible by 11 if and only if the difference between the sum of odd and even place digits is a multiple of 11.

∴ Sum of even placed digits – Sum of odd placed digits = 0, 11, 22,…

∴ (6 + x + 9) – (7+1) = 0, 11, 22,…

∴ x + 7 is a multiple of 11

∴ x + 7 = 11

∴ x = 4

8. Question

Find the remainder when 981547 is divided by 5. Do that without doing actual division.

Answer

We know that if a number is divided by 5, then remainder is remainder obtained by dividing unit place by 5.

∴ Here, 7 ÷ 5 gives 2 as a remainder.

Hence, remainder will be 2 when 981547 is divided by 5.

9. Question

Find the remainder when 51439786 is divided by 3. Do that without performing actual division.

Answer

We know that if a number is divided by 3, then remainder is remainder obtained by dividing sum of digits by 3.

Here, sum of digits is 43.

∴ 43 ÷ 3 gives remainder 1

Hence, remainder will be 1 when 51439786 is divided by 3.

10. Question

Find the remainder, without performing actual division, when 798 is divided by 11.

Answer

We know that if a number is divided by 11, then remainder is difference between sum of even and odd digit places.

∴ Remainder = 7 + 8 – 9 = 6

∴ remainder will be 6 when 798 is divided by 11

11. Question

Without performing actual division, find the remainder when 928174653 is divided by 11.

Answer

We know that if a number is divided by 11, then remainder is difference between sum of even and odd digit places.

∴ Remainder = 9 + 8 + 7 + 6 + 3– 2 – 1 – 4 - 5 = 33 – 12 = 21

∴ 21 ÷ 11 gives 10 as remainder

∴ remainder will be 10 when 928174653 is divided by 11

12. Question

Given an example of a number which is divisible by

1) 2 but not by 4.

2) 3 but not by 6.

3) 4 but not by 8.

4) both 4 and 8 but not by 32

Answer

1) 2 but not by 4.

Any number which follows criteria of 4n + 2 is an example of a number divisible by 2 but not by 4.

For example, 6 , where n= 1.

2) 3 but not by 6.

Any number which follows criteria of 6n + 3 is an example of a number divisible by 3 but not by 6.

For example, 9 , where n= 1.

3) 4 but not by 8.

Any number which follows criteria of 8n + 4 is an example of a number divisible by 4 but not by 8.

For example, 12 , where n= 1.

4) both 4 and 8 but not by 32

Any number which follows criteria of 32n + 8 or 32n + 16 or 32n +24 is an example of a number divisible by both 4 and 8 but not by 32

For example, 40 , where n= 1.

13. Question

Which of the following statements are true?

(i) If a number is divisible by 3, it must be divisible by 9.

(ii) If a number is divisible by 9, it must by divisible by 3.

(iii) If a number is divisible by 4, it must by divisible by 8.

(iv) If a number is divisible by 8, it must be divisible by 4.

(v) A number is divisible by 18, if it is divisible by both 3 and 6.

(vi) If a number is divisible by both 9 and 10, it must be divisible by 90.

(vii) If a number exactly divides the sum of two numbers, it must exactly divide the numbers separately.

(viii) If a number divides three numbers exactly, it must divide their sum exactly.

(ix) If two number are co-prime, at least one of them must be a prime number.

(x) The sum of two consecutive odd numbers is always divisible by 4.

Answer

(i) If a number is divisible by 3, it must be divisible by 9.

False, as any number following criteria of 9n + 3 or 9n + 6 violates the statement.

For example, 6, 12,…

(ii) If a number is divisible by 9, it must by divisible by 3.

True, as 9 is multiple of 3.

Hence, every number which is divisible by 9 must be divisible by 3.

(iii) If a number is divisible by 4, it must by divisible by 8.

False, as any number following criteria of 8n + 4 violates the statement.

For example, 4, 12,20,….

(iv) If a number is divisible by 8, it must be divisible by 4.

True, as 8 is multiple of 4.

Hence, every number which is divisible by 8 must be divisible by 4.

(v) A number is divisible by 18, if it is divisible by both 3 and 6.

False, for example 48, which is divisible to both 3 and 6 but not divisible with 18

(vi) If a number is divisible by both 9 and 10, it must be divisible by 90.

True, as 90 is the GCD of 9 and 10.

Hence, every number which is divisible by both 9 and 10, it must be divisible by 90.

(vii) If a number exactly divides the sum of two numbers, it must exactly divide the numbers separately.

False, for example 6 divides 30, but 6 divides none of 13 and 17 as both are prime numbers.

(viii) If a number divides three numbers exactly, it must divide their sum exactly.

True, if x,y and z are three numbers, where each of x, y and z is divided by a number (say s), then (x+y+z) is divided by s

(ix) If two number are co-prime, at least one of them must be a prime number.

False, as 16 and 21 are co prime but none of them is prime.

(x) The sum of two consecutive odd numbers is always divisible by 4.

True.

Exercise 5.3

1. Question

Solved each of the following Cryptarithms:

$$\begin{array}{cc} & 3 \quad 7 \\ + & A \quad B \\ \hline & 9 \quad A \end{array}$$

Answer

Solving for unit's place,

7 + B = A

Solving for ten’s place,

3 + A = 9

∴ A = 6 and B = -1 which is not possible.

Hence, there is one carry in ten’s place, which means 7 + B >9

∴ now solving for ten’s place with one carry,

3 +A +1 = 9

∴ A = 5

For unit’s place subtracting 10 as one carry is given to ten’s place ,

7 + B - 10= 5

∴ B = 8

Hence, A = 5 and B = 8

2. Question

Solved each of the following Cryptarithms:

$$\begin{array}{rr} A & B \\ +3 & 7 \\ \hline 9 & A \end{array}$$

Answer

Solving for unit’s place,

B + 7 = A

Solving for ten’s place,

A + 3 = 9

∴ A = 6 and B = -1 which is not possible.

Hence, there is one carry in ten’s place, which means B + 7 >9

∴ now solving for ten’s place with one carry,

A +3 +1 = 9

∴ A = 5

For unit’s place subtracting 10 as one carry is given to ten’s place ,

B + 7 - 10= 5

∴ B = 8

Hence, A = 5 and B = 8

3. Question

Solved each of the following Cryptarithms:

$$\begin{array}{rr} A & 1 \\ +1 & B \\ \hline B & 0 \end{array}$$

Answer

Here, in units place,

1 + B = 0

Which means that B = -1 which is not possible.

Hence, one is carry is given to ten’s place.

∴ A + 1 +1 = B …(1)

Now in unit’s place, we need to subtract 10 as one carry is given in ten’s place

∴ 1 + B – 10 = 0

∴ B = 9

Substituting in 1,

A + 1 + 1 = 9

∴ A =7

Hence, A = 7 and B = 9

4. Question

Solved each of the following Cryptarithms:

$$\begin{array}{rrr} 2 & A & B \\ +A & B & 1 \\ \hline B & 1 & 8 \end{array}$$

Answer

Here, in unit’s place,

B + 1 = 8

∴ B = 7

Now in ten’s place,

A + B = 1

∴ A + 7 = 1

∴ A = -6 which is not possible.

Hence, A + B > 9

Now, we carry one in hundred’s place and hence subtract 10 from ten’s place

∴ In ten’s place,

A + B – 10 = 1

∴ A + 7 = 11

∴ A = 4

Now to check whether our values of A and B are correct, we solve for hundred’s place.

2 + A + 1 =B

RHS = 2 + 4 + 1 = 7 = B =LHS

Hence, A = 4 and B = 7

5. Question

Solved each of the following Cryptarithms:

$$\begin{array}{rrr} 1 & 2 & A \\ +6 & A & B \\ \hline A & 0 & 9 \end{array}$$

Answer

Here, in unit’s place,

A + B = 9 …(1)

Now by this condition we know that sum of 2 digits can be greater than 18.

Hence, there is no need to carry one in ten’s place.

Now, for ten’s place,

2 + A = 0

Which means A = -2 which is never possible

Hence, 2 + A > 9

So, we carry one in hundred’s place and hence subtract 10 in ten’s place.

∴ Solving for ten’s place,

2 + A – 10 = 0

∴ A = 8

Now, substituting in 1,

A + B = 9

B = 9 – 8

∴ B = 1

Hence, A = 8 and B = 1

6. Question

Solved each of the following Cryptarithms:

A	B	7
+7	A	B
9	8	A

Answer

For unit’s place,

We have two conditions, when 7 + B ≤ 9 and 7 + B > 9

For 7 + B ≤ 9

7 + B = A

∴ A – B = 7 …(1)

In ten’ place,

B + A = 8 …(2)

Solving 1 and 2 simultaneously,

2A = 15 which means A = 7.5 which is not possible

Hence, our condition 7 + B ≤ 9 is wrong.

∴ 7 + B > 9 is correct condition

Hence, carrying one in ten’s place and subtracting 10 from unit’s place,

7 + B – 10 = A

∴ B – A = 3 … (3)

For ten’s place,

B + A + 1 = 8

∴ B + A =7 …(4)

Solving 3 and 4 simultaneously,

2B = 10

∴ B = 5 and A = 2

7. Question

Show that the Crptarithm $4 \times \overline{AB} = \overline{CAB}$ does not have any solution.

Answer

If B is multiplied by 4 then only 0 satisfies the above condition.

Hence, for unit place to satisfy the above condition, we have, B = 0.

Similarly for ten’s place, only 0 satisfies.

But, AB cannot be 00 as 00 is not a two digit number.

So, A and B cannot be equal to 0

Hence, there is no solution satisfying the condition $4 \times \overline{AB} = \overline{CAB}$.