CBSE Test Paper 03

Chapter 1 Relations and Functions

1. The range of the function f(x) = [sin x] is
   a. [1, 1]
   b. {–1, 1}
   c. { – 1, 0, 1}
   d. (–1, 1)
2. Let A = {a, b, c}, then the range of the relation R= {(a, b), (a, c), (b, c)} defined on A is
   a. {b, c}
   b. {c}
   c. {a, b}
   d. {a, b, c}
3. R is a relation from {11, 12, 13} to {8, 10, 12} defined by y = x – 3. The relation R^{-1} =.
   a. {(8, 11), (9, 12), (10, 13)}
   b. {(11, 8), (13, 10)}
   c. {(8,11),(10,13),(12,15)}
   d. {(8, 11), (10, 13)}
4. A function $f : X \rightarrow Y$ is said to be one – one and onto if
   a. f is one – one
   b. f is onto
   c. f is both one – one and onto
   d. f is either one – one or onto
5. Let A = {1, 2, 3}, then the domain of the relation R = {(1, 1), (2, 3), (2, 1)} defined on A is
   a. {1, 3}
   b. {1, 2}
   c. None of these.
   d. {1, 2, 3}
6. A relation R defined on a set A is said to be ________, if (x, y) R (y, x) R, where x, y A.
7. If n(A) = p and n(B) = q, then n(A$\times$B) = ________.
8. A relation R defined on a set A is said to be ________, if (x, x) $\in$ R, where x $\in$ A.
9. If A = { 1, 2, 3}, B = { 4, 5, 6, 7} and f = {(1, 4), (2, 5), (3, 6)} is a function from A to B. State

whether f is one-one or not.

10. Let * : R $\times$ R $\rightarrow$ R is defined as a * b = 2a + b. Find (2 * 3) * 4

11. A Relation R:A $\rightarrow$ A is said to be Transitive if .... $\forall$ a, b, c $\in$ A.

12. Find gof and fog, if:

    i. f(x) = $|x|$ and g(x) = $|5x - 2|$
    ii. f(x) = $8x^3$ and g(x) = $x^{\frac{1}{3}}$

13. Are the following set of ordered pairs functions? If so, examine whether the mapping is injective or surjective.

    i. {(x, y) : x is a person, y is the mother of x}.
    ii. {(a, b) : a is a person, b is an ancestor of a}.

14. Show that the relation in the set A = {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

15. Show that the relation R defined by (a, b) R (c, d) $\Rightarrow$ a + d = b + c on the set N$\times$N is an equivalence relation.

16. If f: R $\rightarrow$ R is defined as f(x) = 10x + 7. Find the function g : R $\rightarrow$ R, such that gof = fog = I_R.

17. Consider the binary operation ^ on the set {1, 2, 3, 4, 5} defined by a ^ b = min {a, b}. Write the operation table of the operation ^.

18. Given a non-empty set X, consider the binary operation * : P (X) x P (X) $\rightarrow$ P (X) given by A * B = A $\cap$ B $\forall$ A, B in P (X), where P (X) is the power set of X. Show that X is the identity element for this operation and X is the only invertible element in P (X) with respect to the operation *.

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Solution

1. c. { – 1, 0, 1}

   Explanation: The only possible integral values of sin x are { -1 ,0, 1 }. As

2. a. {b, c}

   Explanation: Since the range is represented by the y- coordinate of the ordered pair (x , y). Therefore, range of the given relation is { b , c }.

3. d. {(8, 11), (10, 13)}

   Explanation: R is a relation from {11, 12, 13} to {8, 10, 12} defined by y = x – 3. The relation R^{-1} is given by x = y + 3, from {8, 10, 12} to { 11, 12, 13} $\Rightarrow$ relation = {(8,11),(10,13)}.

4. c. f is both one – one and onto

   Explanation: A function $f : X \rightarrow Y$ is defined to be one – one (or injective),if f$x_1 \neq x_2$ in X $\Rightarrow$ $f(x_1) \neq f(x_2)$ in Y. and $R_f = Y$.

5. b. {1, 2}

   Explanation: Since the domain is represented by the x- coordinate of the ordered pair (x, y). Therefore, domain of the given relation is {1, 2}.

6. symmetric
7. pq
8. reflexive

9. Given, A = {1, 2, 3}, B = {4, 5, 6, 7} and
   f : A $\rightarrow$ B is defined as f = {(1, 4), (2, 5), (3, 6)} i.e. f(1) = 4, f(2) = 5 and f(3) = 6.
   It can be seen that the images of distinct elements of A under f are distinct. So, f is one-one.In other words, no two elements of set A are associated with set B which implies that there is one to one correspondence between X and Y.

10. We are given that * : R $\times$ R $\rightarrow$ R such that a * b = 2a+ b.
    Now, on putting a = 2 and b = 3, we get
    (2 * 3) = 2(2) + 3= 4 + 3 = 7

$\therefore$(2 * 3) * 4

= 7 * 4 = 2 (7) + 4 {now considering a= 7 and b =4}

= 14 + 4

= 18.

11. (a, b)$\in$R, and (b, c)$\in$R $\Rightarrow$ (a, c) $\in$R.

12. To find: gof and fog

    i. f(x) = |x| and g(x) = |5x - 2|
       gof = g[f(x)] = g[|x|] = |5|x|-2| and fog=f(g(x))=f(|5x-2|) =||5x-2|| = |5x-2|
    ii. f(x) = $8x^3$ and g(x) = $x^{\frac{1}{3}}$
       gof = g[f(x)] = g[$8x^3$] $\left(8x^3\right)^{\frac{1}{3}}$ = 2x
       and fog = f[g(x)] $= f\left[\left(x^{\frac{1}{3}}\right)\right] = 8\left(x^{\frac{1}{3}}\right)^3$ = 8x

13. i. Given set of ordered pair is {(x, y) : x is a person, y is the mother of x}.
       It represents a function. Here, the image of distinct elements of x under **f** are not distinct, so it is not injective but it is surjective.
    ii. Set of ordered pairs = {(a, b) : a is a person, b is an ancestor of a}.
       Here, each element of domain does not have a unique image. So, it does not represent function.

14. R = {(1, 2), (2, 1)}, so for (a, a), (1, 1) $\notin$ R. $\therefore$R is not reflexive.
    Also if (a, b) $\in$R then (b, a) $\in$ R $\therefore$ R is symmetric.
    Now (a, b) $\in$ R and (b, c) $\in$ R ,then does not imply (a, c) $\notin$ R as (1,2) $\in$ R and (2,1) $\in$ R but (1,1) $\notin$ R $\therefore$ R is not transitive.
    Therefore, R is symmetric but neither reflexive nor transitive.

15. (a, b) R (c, d) $\Rightarrow$a + d = b + c where a, b, c, d $\in$N
    (a, b) R (a, b) $\Rightarrow$a + b = a + b for all (a, b) $\in$N $\times$N
    R is reflexive
    Now
    (a, b) R (c, d) $\Rightarrow$ a + d = b + c for (a, b) (c, d) $\in$N $\times$N
    $\Rightarrow$d + a = c + b
    $\Rightarrow$c + b = d + a

$\Rightarrow$(c, d) R (a, b) for (a, b), (c, d) $\in N \times N$

Hence R is symmetric.

(a, b) R (c, d) $\Rightarrow$ a + d = b + c ....(1)

for (a, b), (c, d) $\in N \times N$

And (c, d) R (e, f) $\Rightarrow$ c + f = d + e ....(2)

for(c, d), (e, f) $\in N \times N$

Adding (1) and (2)

(a + d) + (c+f) = (b + c) + (d + e)

a + f = b+ e

(a, b) R (e, f)

Hence, R is transitive

So, R is equivalence.

16. f: R $\rightarrow$ R is defined as f(x) = 10x + 7.We have to find the function g : R $\rightarrow$ R, such that gof = fog = l_R.

Given, f(x) = 10x + 7

Also, gof = fog = I_R

Now, gof = $I_R \Rightarrow$ gof(x) = I_R(x)

$\Rightarrow$ g[f(x)]= x, $\forall$ x $\in$ R [$\because$ I_R(x) = x, $\forall$ x $\in$ R]

$\Rightarrow$ g(10x + 7) = x, $\forall$ x $\in$ R

Let 10x + 7 = y $\Rightarrow$ 10x = y- 7

$\Rightarrow x = \frac{y-7}{10} \Rightarrow g(y) = \frac{y-7}{10} \cdot \forall y \in R$

or, g(x) = $\frac{x-7}{10}, \forall x \in R$

17. Let A = {1, 2, 3, 4, 5} defined by a ^ b = min{a, b} i.e., minimum of a and b.

$\wedge$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

18. i. Let E ∈ P(X) be the identity element .Then,

A*E = E*A = A ∀ A ∈ P(X)

$\Rightarrow A \cap E = E \cap A = A \;\forall\; A \in P(X)$

$\Rightarrow X \cap E = X [\because\; X \in P(X)] \Rightarrow X \subset E$

Also, $E \subset X \;[\because E \in P(X)]$

Thus, E = X.

Hence,X is the identity element.

ii. Let A ∈ P(X) be invertible.

Then,there exists B ∈ P(X) such that A*B = B*A = X,

where X is the identity element.

$A \cap B = B \cap A = X \Rightarrow X \subset A, X \subset B$

Also, $A, B \subset X \;[\because A, B \in P(X)]$

∴ A = B = X

Hence, X is the only invertible element and X^{-1} = B = X.