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State Space Analysis

General Representation of State Model

- State equation

$$\dot{x} = Ax + Bu$$

- Output equation

$$y = Cx + Du$$

where,
$\dot{x}$ = Velocity vector $(n \times 1)$
x = State vector $(n \times 1)$
u = Input vector $(m \times 1)$
y = Output vector $(p \times 1)$
A = State matrix $(n \times n)$
B = Input matrix $(n \times m)$
C = Output matrix $(p \times n)$
D = Transmission matrix $(p \times m)$
n = Number of state variables
p = Number of outputs
m = Number of inputs

Controllability

$$Q_c = \left[B : AB : A^2B : \ldots\ldots : A^{n-1}B\right]$$

where, Q_c = Controllability test matrix $(n \times nm)$

Condition for State Controllability

$$|Q_c| \neq 0$$ .... (Matrix be non singular)

Observability

$$Q_o = \left[C^T : A^TC^T : (A^T)^2C^T : \ldots\ldots : (A^T)^{n-1}C^T\right]$$

where, Q_o = Observability test matrix $(n \times np)$

Condition for State Observability

$$|Q_o| \neq 0$$ .... (Matrix be non singular)

Note:

- If AB is controllable, A^TB^T is observable.
- If AC is observable, A^TC^T is controllable.
- If input-output transfer function of a linear time invariant system does not have pole-zero cancellation, the system can always be represented by completely controllable and observable state model.

- Transfer function (T.F.)

$$\text{T.F.} = C[sI - A]^{-1}B + D$$

State transition matrix (STM)

$$e^{At} = \phi(t) = L^{-1}\left[(sI - A)^{-1}\right]$$

Properties of STM

(i) $\phi(0) = I$(identity matrix)
(ii) $\phi^{-1}(t) = \phi(-t)$
(iii) $[\phi(t)]^K = \phi(Kt)$
(iv) $\phi(t_1 + t_2) = \phi(t_1)\,\phi(t_2)$
(v) $\phi(t_2 - t_1)\,\phi(t_1 - t_0) = \phi(t_2 - t_0)$
(vi) $\dot{\phi}(t) = A\phi(t)$

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