History of Mathematics

UNIT - 1

It is difficult to tell exactly where and in what form mathematics came into existence. Even before the existence of oldest written documents, we come across a few pictures or symbols which indicate towards basic knowledge of math. For example, Palaeontologists have discovered Ochre rocks in the caves of South Africa where some geometrical patterns can be seen which have been made by etching. It is believed that Ishango bone near Nile River in Congo is the oldest representation of a series of prime numbers. This might be 20, 000 years old.

Around 5000 B.C., some geometrical spatial designs were depicted by the people of Egypt. In ancient India, the oldest known knowledge of Mathematics goes back to 3000-2600 B.C. Indus Valley civilization of North India developed a system of same weights and measures. We also get the proof of a surprisingly advanced brick technique in which ratio and proportion was used. Roads which cut each other at right angles, cuboidal, conical and cylindrical, circular and triangular shapes indicate that math was highly developed at that time.

Mohan-jo-daro

The oldest example of Chinese Mathematics goes back to Chang Dynasty (1600-1046 B.C) where etched on the shells of a tortoise were found. We also get the proof of written math from Sumerian civilization that developed the oldest civilization of Mesopotamia. They had developed a very complicated method of measurement science around 3000-2500 B.C.

Ancient civilizations of Egypt, Greek Babylon and Arabia have contributed a great deal in the field of Math. After Christ, math has been developed continuously in different parts of the world. This knowledge has become richer by sharing with each other.

The story of development of math can be gathered from different sources. In class 9th, we have included a part of the development of Indian mathematics. Hope it will motivate you to see and understand how mathematics was developed in the world.

History of Mathematics

01

History of Mathematics

Mathematics is the backbone of Science and Technology. Hence, in Vedang Jyotish, Rishi Lagadh has written:

यथा शिखा मयूराणाम् नागानाम् मणयो यथा।
तद्वद् वेदांगशास्त्राणाम् गणितम् मूर्धनिस्थितम् ।।

Meaning, mathematics adorns the head of all Vedang Shastras(Sciences) like the plume on top of a peacock's head or the bead on top of the cobra's head.

When we see the history of mathematics, the contribution of India has been very distinctive and famous. Work on the various branches of mathematics have been done since the ancient times in India. We will discuss the same briefly.

Arithmetic: Arithmetic is the main branch of mathematics. Its usage can be seen a lot in the day to day affairs. The basis of Arithmetic is the Number system in which zero has a special position.

Discovery of zero: The concept of zero is very much a part of the Vedas. In the Richa of Yajurveda, chapter 40, Verse 17 'Aum kham brahma' has used the word 'kham'. The word 'kham' indicates the sky and also Shunya (zero). So in books like the Jyotish, the word 'kham' has been used to indicate zero.

Bhaskaracharya, in his book Lilawati has used 'kham' as a zero in Shunya Parikarmaashtak (शून्य परिकर्माष्टक).

योगेखंक्षेपसमं, वर्गादौखं, खभाजितो राशिः।
खहरः स्यात्, खगुणः खं, खगुणश्चिन्त्यश्च शेष विधौ।।

On adding any number to zero, the sum is the number itself. The square,cube etc. of zero is zero. If any number is divided by zero, the denominator of that numeric is zero. On multiplying a number by zero, the product is zero.

Note: Shlokas have just been given in context of the Vedic hymn and so would not be desirable to ask to be quoted in the examination.

The credit of conceptualizing zero, has been given to the great Sanskrit Grammarian, Panini (500 B.C.) and to Pingal (200 B.C.) We also find a reference to the discovery of zero being made by Vedic Rishi Grutsamad.

The first proof of the symbol assigned to zero has been found in the Bakshaali Pandulipi (300- 400 AD). The existence of zero and its symbol in the number system of ancient India have been the most important contribution.

Prof. G.B.Halstead has said:-

"None of the concepts in Mathematics have proved to be as important as zero for the development of the brain and brawn."

Number System: Since ancient times, various countries have been using different methods of representing numbers. Devnagri, Roman and Hindu-Arabic systems are some which we have studied before. We will now see the historical background of these.

Prof.Guinsberg says: Around 770 C.E. a Hindu Scholar by the name of Kank was invited to the famous court of Baghdad by Abbasayyed Khaleefa Alamansur. In this way the Hindu numeric system came to Arabia. Kank taught Astrological Science and Mathematics to the Arab Scholars. With Kank's help, they also translated Brahmagupta's "Brahmasphut Siddhanta" into Arabic.

The discovery of French Scholar, M.F.Nau, proves that in the mid 7th century, Hindu numerics were well known in Syria and they were considered praiseworthy.

B.B.Dutt says: "The Indian number system went slowly from Arabia, Egypt and Northern Arabia towards Europe and by eleventh century it had completely reached Europe. Europeans referred to them as Arabic numbers because they had got them from Arabia, but the Arabs were unanimous in acknowledging them as Hindu Numbers (Al- Arkaan Al-Hindu). These ten numbers were referred to by the Arabs as "Hindsa"."

Place Value: To express any number using ten digits, including zero and to give each digit a face value and place value has made this number system scientific. The place value system is the specialty of the modern number system(the Hindu-Arabic system).

The great French Mathematician, Pierre Laplace has written: It was India that gave us an excellent system of expressing every number using the ten digits(where each digit has a face and a place value). The base of the decimal system is ten. That is the reason this system is called the Metric or the Decimal number system.

History of Hindu Numerals and big numbers: The Hindu numerals developed as follows:

- Kharoshthi System (fourth century b.c.)
- Brahmi system (third century b.c.)
- Gwalior system(ninth century)
- Devnagri system(eleventh century)
- Modern system

From fourth century B.C. to second century A.D., one can find the use of the Kharoshthi system. In the Brahmi system, besides ten, multipliers of ten till hundred and multipliers of hundred upto nine hundred have been found.

In Yajurveda Samhita, Ramayan and religious books thereafter have given numbers from 1 to 10^{53} different names:

- Niyutam 10^{11}
- Utsang 10^{21}
- Hetuheelam 10^{31}
- Nitravaadyam 10^{41}
- Tallakshana 10^{53}

Introduction to Coded Numbers (कूटांक): When an alphabet is used to represent a number, it is called a "coded" number. Ancient Mathematicians had used this concept to express numbers. The use of which can be seen in the Astrology and other vedic books.

- Alphanumeric (वर्णांक) system
- Shabdank (शब्दांक) system
- Vyanjanank (व्यंजनांक) system

Algebra: There are quite a few similarities in the formation and principles of Algebra and Arithmetic. The major difference in these two is Arithmetic deals with expressed (known) quantities and Algebra deals with unexpressed (unknown) quantities. By unexpressed quantities we mean quantities that are not known in the beginning. This is known as an algebraic quantity, and so the branch is Unexpressed Mathematics or Algebra.

The use of Algebra can be seen in era of Shulvsutras when there came up several problems while constructing of Yajna altars(vedis), requiring the use of finding solutions to linear and infinite equations. The contribution of Aryabhatt is creditable in both the fields of Arithmetic as well as Algebra. Algebra developed as an independent branch right during the time of Brahmagupta. It is also known as Coded Mathematics or Implicit Mathematics. Mathematician called Pruthudak swami (860 CE) named it Beej Ganit.

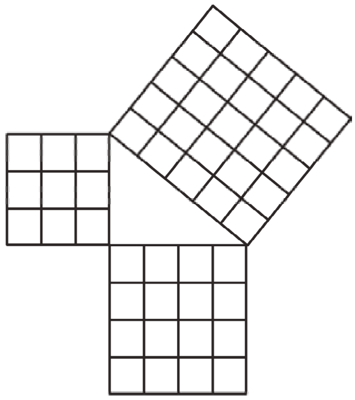
Geometry: When we see the history of Indian Mathematics, we realize that the base of Geometry had been already laid during the Vedic Period. We get to see the mathematics in a Vedang named "Kalp" in the form of Shulvasutras. The rope used in measuring the altars was known as shulva. Sutra means to express the information in the precise form. The shulva sutras were named after their creators - Baudhayan, Aapstambha, Kaatyaayan, Maanav, Maitraayan etc. The shulvasutras contain the information of how to make vedis (altars) of different shapes: • Garun Vedi • Koorma Vedi • Shri Yantra

The **examples** of the shulvasutras geometry are as follows:-

The formation of triangles, squares, rectangles and other complicated geometrical shapes, forming such geometrical shapes whose area are equal to the sum or difference of the areas of some given shapes.

The contributions made by Aryabhatt (476-550 CE), Brahmagupta (600 CE), Bhaskar first(629 CE), Mahavir (850 CE) in the field of Geometry have been commendable.

Baudhayan Theorem



दीर्घ चतुरस्त्रस्य अक्षण्या रज्जुः पार्श्वमानी तिर्यक् मानी च।
यत पृथग्भूते कुरूतः तत् उभयं करोति (इति क्षेत्र ज्ञानम्) ।।

।। 48 (1) बौधायन शुल्व सूत्र।।

Meaning: The area of the square drawn on the diagonal of a rectangle is equal to the sum of the areas of the squares drawn on the two sides of the rectangle. We know that the diagonal of the rectangle divides it into two right angled triangles and in such a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the remaining two sides.

This relation between the sides of the right angled triangle has been known as the Pythagoras Theorem. However, in the book by Dr. Brijmohan - History of Mathematics (page 243), there is a reference that now a lot of historians agree that this Pythagoras Theorem was known to the shulva sutra writers some hundreds of years prior to the birth of Pythagoras. The life of the Greek philosopher Pythagoras is believed to have been from 572 B.C. to 501 B.C. Whereas Indian Mathematician Baudhayan has described this theorem in great detail several years before that. Hence this theorem is in fact the Baudhayan Theorem. Or also known as Shulva Theorem.

Indian history of Pi (π) :

(1) **Aryabhatt (476 - 550 AD)** was the first Mathematician to give a reasonably close estimate of the value of Pi (π) which is the ratio of the circumference and diameter.

चतुरधिकम् शतमष्टगुणं द्वाषष्टिस्तथा सहस्त्राणाम्।
अयुतद्वय विष्कम्भस्य आसन्नो वृत्त परिपाहः ।।

Add 4 to 100, multiply the sum by 8 and add it to 62000. This sum will be approximately the circumference of a circle whose diameter is 20000 that is a circle with 20000 diameter will have a circumference of 62832.

$$\text{Pi}\ (\pi) = \frac{\text{Circumference}}{\text{Diameter}} = \frac{62832}{20000}$$

Thus, according to him, Pi = 3.1416, which is correct upto four decimals even today.

(2) **Bhaskaracharya (1114-1193 AD)** has given the value of Pi in his Granth Leelawati.

व्यासे भनन्दाग्नि हते विभक्ते खबाण सूर्यैः परिधिः स सूक्ष्मः।
द्वाविंशतिघ्ने विहृतेऽथशैलैः स्थूलोऽथवास्याद् व्यवहार् योग्यः।।

If you multiply the diameter by 3927 and divide this by 1250, we get the precise circumference. Or if we multiply the diameter by 22 and divide that by 7 we get a useable approximate value of the circumference.

(3) **Swaamibharti Krishnateertha(1884-1960 A.D.)** has given the value of pi/10 in the well known Anushtup verses, using the alphabets as codes:

गोपी भाग्यमधुव्रात–श्रृंगिशोदधिसंधिग ।
खलजीवित खाताव गलहाला रसंधर ।।

According to swamiji, there can be three useful interpretations of these verses. In the first interpretation, it is a praise of the Lord Krishna. In the second interpretation, it can be considered the praise of Lord Shiva and the third interpretation is that it is the value of pi/10 correct upto 32 decimal places.

$\pi/10 = 0.3\,1\,4\,1\,5\,9\,2\,6\,5\,3\,5\,8\,9\,7\,9\,3\,2\,3\,8\,4\,6\,2\,6\,4\,3\,3\,8\,3\,2\,7\,9\,2$

(4) **Shrinivas Ramanujan (1887-1920 AD)** The first research paper published in Europe by Ramanujan was titled "Modular equation and approximation to π". He found several formulae to get the approximate value of π.

Trigonometry: Trigonometry is that branch of Mathematics in which the relation between the sides and angles of a triangle are studied. This is a very old and important branch of Mathematics. The use of this knowledge is made in calculating the positions of the planets in Indian astrology and astronomy.

The ancient Indian Mathematicians like Aryabhatt, Varahmihira and Brahmagupta made significant contributions to this field.

You can see the description of the trigonometric concepts, formulae and statements in "Soorya Siddhanta"(400 AD), in the Panch Siddhanta by Varahmihira and in Brahmasphoot Siddhanta(630 AD) by Brahmagupta.

There is a clear reference in the book by Dr. Brijmohan, "History of Mathematics" (Page 314), that there is no doubt that three of the trigonometric functions have been defined first by the Hindus.

Aaryabhatt was the first to have used the word 'jya' (around 510 AD). He was also the first to give the tables related to jya and utkram jya (ujjya).

So the word 'jya' went from India to Arab countries, where it became 'Jeeba'. After a while this became 'Jaib'. In Arabic 'jaib' means 'breast' or 'bust'. When the translations were made into Latin in 1150 AD, 'jaib' was replaced by 'Sinus', which has the same meaning in Latin is also breast.

Brahmagupta used the word 'kramjya' to imply jya. It was so named to differentiate it from 'utkram jya'. In Arabic this was then converted to 'karaj'. Alkhwarijamee also used 'karaj'. Indian jya and kotijyaa became sine and cosine in European languages.

The use of trigonometry is seen in Astrology, Astronomy, Engineering and Navigation as also in the study of heights and distances.

Excercise - 1.1

Q.1. Match the following:

Bharti Krishnateertha	Brahmasphoot Siddhanta
Varahamihira	Siddhanta Shiromani
Brahmagupta	Aaryabhatiya
Bhaskaracharya	Panch Siddhanta
Aaryabhatt	Vedic Mathematics

Q.2. Fill in the blanks:

1. The word used for the sky and zero was ____________.
2. Tallakshana was used to denote the number _____________.
3. The alphanumeric system was used by the Mathematician ___________ in his book _______________________.
4. ________________ was also known as Implicit Mathematics.
5. Vedic Mathematics have the __________Sutras.

Q.3 Write the importance of the Modern Number system.

Q.4 Write a few points on the discovery of zero.

Q.5 Give a brief description of the Alphanumeric system.

Q.6 What is the Baudhaayan Theorem?

Q.7 Write the contribution of Aaryabhatt in the estimation of π.

Q.8 Write the name of the creator of Vedic Mathematics. Also give a brief description of its contents.

Activity:

(1) Form a Mathematics Society in your school.

(2) Make a collection of Mathematical books in your school.

(3) Develop a Mathematical Laboratory in your school.

Simple techniques for multiplication: We are going to study some simple multiplying techniques, before which we shall obtain a brief introduction of the book and its researcher mathematician.

The unparalleled mathematician, Swami Bharti Krishna Teertha Shankaraachrya Govardhanmath Jagganaathpuri (1884-1960 AD) created a book named "Vedic Mathematics" and thus made an innovative contribution.

In this book, he has described 16 exceptional sutras and 13 upsutras with their properties and experiments. This book has 40 chapters. It has been presented with a very unique viewpoint.

Digit sum (बीजांक): When you obtain a single digit after adding the different digits in a number, that is known as its Digit sum.

EXAMPLE-1. Find the Digit sum of 10, 11, 321and 78

SOLUTION : Digit sum of 10 → 1 + 0 → 1

Digit sum of 11 → 1 + 1 → 2

Digit sum of 321→ 3 + 2 + 1 → 6

Digit sum of 78 → 7 + 8 → 15, but 15 is not a Digit sum, so we have to add these digits further 1 + 5 → 6, hence the Digit sum of 78 is 6

EXAMPLE-2. Find the Digit sum of 8756904

SOLUTION :

Method 1: Add all the digits of the given number.

8 + 7 + 5 + 6 + 9 + 0 + 4 = 39, again adding these we get 3 + 9 = 12, further adding, we get, 1+2 = 3, which is the Digit sum.

Method 2: Consecutively keep adding the digits until you get a single digit number.

8 + 7 → 15 → 1 + 5 → 6 + 5 → 11 → 1 + 1 → 2 + 6 → 8 + 9 → 17 → 1 + 7 → 8 + 0 → 8 + 4 → 12 → 1 + 2 → 3 is the Digit sum.

Method 3: Inspect the digits of the number 8756904, Add the digits besides 0,9 and other pair which adds to give 9 i.e 4 + 5, so 8 + 7 + 6 → 21 →2 + 1 →3 is the Digit sum.

Points to be noted when finding a Digit sum:

(1) As soon as you add the digits, if you get a two digit number, add the digits further to obtain the Digit sum.

(2) Adding 0 and 9 or leaving them out, will not affect the Digit sum.

(3) The Digit sum of a number is actually the remainder left when you divide the number by 9. Thus finding a Digit sum is the same as dividing the number by 9 and finding the remainder.

(4) A number whose Digit sum is 9, is completely divisible by 9. In that case the Digit sum of the number is 9 not 0.

(5) We can also test the divisibility of a number by 3. So a number which has a Digit sum of 3, 6 or 9 will be divisible by 3.

(6) You can also test the solution you have obtained using Digit sum, hence one needs to have an adequate knowledge of finding Digit sums orally.

Excercise - 1.2

Q.1 What do you mean by Digit sum?

Q.2 Find the Digit sum of the following numbers:

15, 38, 88, 99, 412, 867, 4852, 9875, 24601, 48956701

Q.3 What should be added to the following numbers to make them divisible by 9?

241, 861, 4441, 83504

Q.4 Write the usefulness of Digit sums.

Use of Digit sums in checking the solutions: In Vedic Mathematics there are several ways of checking the correctness of a solution. We will see here how Digit sum helps in checking the correctness of a solution.

Adding:

Check for additions: The digit sum of the numbers to be added should be = to the sum of the digits in the solution.

EXAMPLE-3. Find the sum of 3469, 2220 and 1239 and check if the solution is right using Digit sum.

SOLUTION : Check

		Check
	3469	4
	2220	6
+	1239	6
	6928	7

(1) The Digit sums of the numbers are respectively 4, 6 and 6.

(2) The Digit sum of the Digit sums of the numbers is

$4 + 6 + 6 = 16 \rightarrow \quad 1 + 6 \rightarrow \quad 7$

(3) The Digit sum of the sum of the three numbers is

$6 + 9 + 2 + 8 = 25 \rightarrow \quad 2 + 5 \rightarrow 7$

(4) Since the Digit sum of both (2) and (3) is 7, hence the solution is correct.

Subtraction:

Check for subtraction : In this the Digit sum of subtracting (below) number + the Digit sum of the answer = Digit sum of the upper number.

EXAMPLE-4. 7816 - 3054. Solve this and check the answer.

SOLUTION : Check

		Check
	7816	4
–	3054	3
	4762	1

(1) The digit sum of the quantity being reduced(lower) is 3

And the Digit sum of the answer 4762 is 1.

(2) The sum of the two digit sums is 3 + 1 = 4

(3) The digit sum of the upper quantity 7816 is = 4.

(4) Since both (2) and (3) have 4 as their digit sum, the answer is correct.

Multiplication:

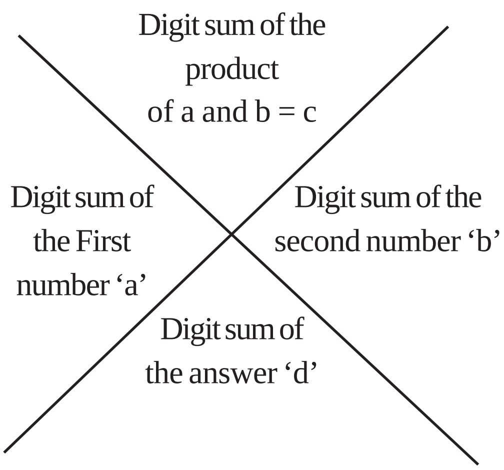
Check for multiplication: Digit sum of first number × Digit sum of second number = Digit sum of the product of the two numbers.

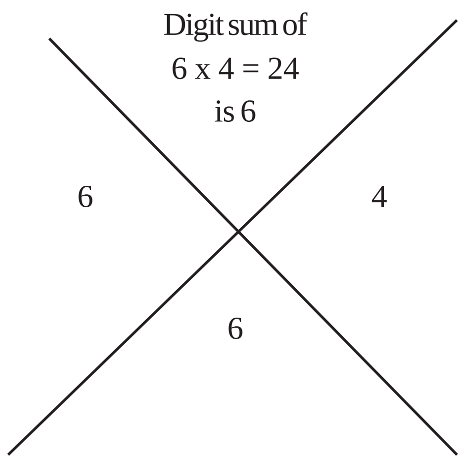
EXAMPLE-5. 456 × 814. Find the solution and check the answer.

SOLUTION :

456 × 814	(1) $6 \times 4 = 24$, the digit sum of which is 6
1824	(2) The product of numbers is 371184, the Digit sum of which
4560	is 6
364800	(3) Since (1) and (2) are the same digit sums that is 6, the
371184	answer is correct.

Another way of putting this is





If ‘c’ and ‘d’ are different then the answer is definitely wrong.

Division:

Check for division : Digit sum of the dividend = (Digit sum of divisor x digit sum of quotient) + Digit sum of remainder

EXAMPLE-6. 7481 ÷ 31. Find the solution and check your answer.

SOLUTION :

Divisor dividend quotient

$$\begin{array}{l} 31)\ 7481\ (241 \\ \underline{-62} \\ \quad 128 \\ \underline{-124} \\ \quad 0041 \\ \quad\underline{-31} \\ \quad\ \ 10 \end{array}$$

Digit sum of dividend → 7 + 4 + 8 + 1 → 20 → 2

Digit sum of divisor x digit sum of quotient + digit sum of remainder

(7 × 4) + 1 → 28 + 1 → 29 →11 →2

Since above two are equal, the answer is correct.

Vedic Methods to multiply

(1) **Urdhva Tiryak Vidhi,** using the sutra Urdhvatiryagbhyam.

The meaning of the sutra is Urdhva = Vertical (↑)

And tiryak = diagonal ↖↗ = ⤧

EXAMPLE-7.

$$\begin{array}{r} 41 \\ \times\ 38 \\ \hline \end{array}$$

SOLUTION :

$$\begin{array}{r} 41 \\ \times\ 38 \\ \hline 15|5|8 \\ 3\ \ \ \ \ \ \end{array}$$

$\boxed{1558 \quad \text{(answer)}}$

Formula: Urdhwatiryagbhyaam

(1) First column (multiplication of units place with units place)

$$\begin{array}{r} 1 \\ \times 8 \\ \hline 8 \end{array}$$

↑ Urdhwaguna

(Vertical multiplication)

and write the product in units position in the solution.

(2) First and second column (units and tens place)

$$\begin{array}{r} 41 \\ \times 38 \\ \hline \end{array}$$

Multiply diagonally and add

(4×8) + (1×3)

32+3=35 (5 of the 35 is written in tens place and 3 is carried to hundreds place)

(3) Second column (tens place)

$$\begin{array}{r} 4 \\ \times 3 \\ \hline 12 \end{array} \uparrow \text{ Vertical multiplication}$$

+ 3 (carried over) = 15 (write this as it is to the left)

EXAMPLE-8.

$$\begin{array}{r} 56 \\ \times\ 82 \\ \hline \end{array}$$

SOLUTION :

$$\begin{array}{r} 56 \\ \times\ 82 \\ \hline \end{array}$$

Formula: Urdhwatiryagbhyaam (observe and understand)

45	9	2
5	1	

Vertical multiplication

$$\begin{array}{cc} 5 & 5 \\ \uparrow & \\ 8 & 8 \end{array} \quad \begin{array}{cc} 5 & 6 \\ \times & \\ 8 & 2 \end{array} \quad \begin{array}{cc} 6 & \\ \uparrow & \\ 2 & \end{array}$$

Vertical multiplication

$\downarrow$

Add after doing Diagonal multiplication

EXAMPLE-9.

$$\begin{array}{r} 231 \\ \times\ 425 \\ \hline \end{array}$$

Solve using formula Urdhwatiryagbhyaam.

SOLUTION :

$$\begin{array}{r} 231 \\ \times\ 425 \\ \hline \end{array}$$

9	8	1	7	5
1	2	1		

$$\begin{array}{c} 2 \\ \uparrow \\ 4 \end{array} \quad \begin{array}{cc} 2 & 3 \\ \times & \\ 4 & 2 \end{array} \qquad \begin{array}{ccc} 2 & 3 & 1 \\ & \times & \\ 4 & 2 & 5 \end{array} \qquad \begin{array}{cc} 3 & 1 \\ \times & \\ 2 & 5 \end{array} \quad \begin{array}{c} 1 \\ \uparrow \\ 5 \end{array}$$

(1) frist column (units place with units place product)

$$\begin{array}{r} 1 \\ \times\ 5 \\ \hline 5 \end{array} \uparrow \text{ Vertical multiplication}$$

(2) First and second column (units and tens place product)

$$\begin{array}{c} 31 \\ \times\ 25 \\ \hline (3 \times 5) + (1 \times 2) \end{array}$$

Diagonal multiplication

15 + 2 = 17 (write the 7 in tens place and carry over 1)

(3) First, second and third column (units, tens and hundreds place)

$$\begin{array}{c} 231 \\ \times 425 \\ \hline (2\times5)+(1\times4)+(3\times2) \end{array}$$

As shown by arrows, product of first by third digit of these two numbers and add it to the product of second by second digits of both.

$10 + 4 + 6 = 20$

20 + 1 (carried over) = 21 (keep 1 in hundreds place and carry over 2)

(4) Second and third column (drop the units place)

$$\begin{array}{r} 23 \\ \times\ 42 \\ \hline (2\times2) + (3\times4) \end{array}$$

$4 + 12 = 16$

16 + 2 (carried over) = 18 (write 8 in the thousands position and keep 1 as a carry over)

(5) Third column (drop the units and tens place)

$$\begin{array}{r} 2 \\ \times\ 4 \\ \hline 8 \end{array}$$

Vertical multiplication

8 + 1 (carried over) = 9 (write 9 in the ten thousands place)

Exercise - 1.3

Find the following products using formula Urdhwatirygbhyaam:

(1) $\begin{array}{r} 23 \\ \times 32 \\ \hline \end{array}$ (2) $\begin{array}{r} 44 \\ \times\ 52 \\ \hline \end{array}$ (3) $\begin{array}{r} 92 \\ \times\ 37 \\ \hline \end{array}$ (4) $\begin{array}{r} 55 \\ \times\ 55 \\ \hline \end{array}$

(5) $\begin{array}{r} 123 \\ \times\ 321 \\ \hline \end{array}$ (6) $\begin{array}{r} 414 \\ \times\ 232 \\ \hline \end{array}$ (7) $\begin{array}{r} 504 \\ \times\ 618 \\ \hline \end{array}$ (8) $\begin{array}{r} 812 \\ \times\ 453 \\ \hline \end{array}$

(2) Ekanyunena Poorvena Vidhi (method) (meaning one less than before) : This formula is used when one of the numbers is made up of 9s. There are three conditions that occur between the multiplier and multiplicand:

1. The number of digits is same.
2. The number of digits is more in the multiplier than the multiplicand that is there are more 9s.
3. The number of digits is less in the multiplier than the multiplicand that is there are less 9s.

Condition 1:

EXAMPLE-10. Solve 63 x 99 using Ekanyunenapoorven method.

SOLUTION :

$$\begin{array}{c|c} \multicolumn{2}{c}{63 \times 99} \\ \hline 62 & 37 \end{array}$$

(1) There are two digits in both.

(2) The left side of the answer is 1 less than the multiplicand. i.e. 1 less than 63 is 62.

(3) The right side of the answer is 99 - 62 = 37

EXAMPLE-11. 3452 × 9999

SOLUTION :

$$\begin{array}{c|c} \multicolumn{2}{c}{3452 \times 9999} \\ \hline 3451 & 6548 \end{array}$$

(1) Left side of the answer 3451 (1 less than 3452)

(2) Right side of the answer is 9999 - 3451 = 6548

Condition 2.

EXAMPLE-12. Find the product 43 × 999 using formula Ekanyunena poorvena.

SOLUTION :

$$\begin{array}{c|c} \multicolumn{2}{c}{043 \times 999} \\ \hline 042 & 957 \end{array}$$

$\boxed{42957 \text{ Answer}}$

(1) Add a zero to the left of 43, to make the digits equal

(2) The left side of the answer is 1 less than 043 ie 042

(3) The right side of the answer is 999-042 = 957

EXAMPLE-13. Solve 347 × 99999 using the formula Ekanyunenpoorvena.

SOLUTION :

$$\begin{array}{c|c} \multicolumn{2}{c}{00347 \times 99999} \\ \hline 00346 & 99653 \end{array}$$

$\boxed{34699653 \text{ Answer}}$

(1) Add two zeros to the left of 347 and make the digits equal

(2) The left side of the answer is 1 less tha 00347 is 00346

(3) The right side of the answer is

$$\begin{array}{r} 99999 \\ -\ 00346 \\ \hline 99653 \end{array}$$

Condition 3:

EXAMPLE-14. Solve 438 x 99 using Ekanyunenapoorvena.

SOLUTION :

$$\begin{array}{r} 438 \times 99 \\ 43799 \\ -\ 437 \\ \hline 43362 \end{array}$$

(1) Reduce 438 by 1 and keep 99 as it is following this. Then subtract 437 from this number to get the solution. Thus subtract 437 from 43799 to get 43362

Exercise - 1.4

Solve using formula Eknyunenapoorvena (also check your answers):

(1) 57×99 (2) 4378×9999 (3) 87×999

(4) 345×99999 (5) 48×9 (6) 9457×999

(3) Ekaadhikena poorvena Vidhi (method) : Here the use of the sutra Ekaadhikenapoovena and Antyayordeshakepi is made.

This method is used when the sum of the units place digits of the two numbers is 10 and the remaining digits are the same.

Example-15. Find the product 12 x 18.

Solution :

$$\begin{array}{r|l} \multicolumn{2}{c}{12 \times 18} \\ \hline 2 & 16 \end{array}$$

Formula Ekaadhikenapoorvena and antyayordeshakepi

(1) Left side of the answer (one more than the tens place x tens digit) = $2 \times 1 = 2$

(2) Right side of the answer = product of units digits = $2 \times 8 = 16$

$\therefore\ 12 \times 18 = 216$

Example-16. 21×29

Solution :

$$\begin{array}{r|l} \multicolumn{2}{c}{21 \times 29} \\ \hline 6 & 09 \end{array}$$

Formula Ekaadhikenapoorvena and Antyardeshkepi.

(1) Left side of the answer (one more than tens digit x tens digit) = $3 \times 2 = 6$

(2) Right side of the answer = product of the units digits = $1 \times 9 = 9$

$\therefore\ 21 \times 29 = 609$

Note: The sum of the units digits is 10. So the product of these units digits should have two digits, since there is only one, we add a 0 before 9.

Example-17. Solve 102×108

Solution :

$$\begin{array}{r|l} \multicolumn{2}{c}{102 \ \times 108} \\ \hline 110 & 16 \end{array}$$

Formula Ekaadhikenapoorvena and Antyardeshkepi.

(1) Left side of the answer (1 more than 10 x ten) = $11 \times 10 = 110$

(2) Right side of the answer = product of units

digits = $2 \times 8 = 16$

$\therefore 102 \times 108 = 11016$

EXAMPLE-18. Solve 194 × 196.

SOLUTION :

194 × 196 Formula Ekaadhikenapoorvena and Antyardeshkepi.

380 | 24

(1) Left side of the answer = (1 more than19×19)

= 20 × 19 = 380

(2) Right side of the answer = product of units digits. = 4 × 6 = 24

∴ 194 × 380 = 38024

Exercise-1.5

Use the formula Ekaadhikenapoorvena and Antyardeshkepi to find the answers and check using Digit sums:

(1) 13 × 17	(2) 22 × 28	(3) 34 × 36	(4) 91 × 99
(5) 35 × 35	(6) 42 × 48	(7) 72 × 78	(8) 93 × 97
(9) 104 × 106	(10) 105 × 105	(11) 203 × 207	(12) 405 × 405
(13) 502 × 508	(14) 603 × 607	(15) 704 × 706	(16) 905 × 905
(17) 193 × 197	(18) 292 × 298	(19) 392 × 398	(20) 495 × 495

Nikhilam Vidhi : This method is used to find products, when the numbers are close in value to the base or the sub base.

Base : 10,100,1000............etc. are called bases

Sub base: 20,30,......200,300, etc. are called sub bases.

Deviation from the base:

(1) Firstly find the base which is a power of 10 and is closest to the number.

(2) If the given number is greater than this base, subtract the base from the number and write the deviation with a positive sign.

(3) If the given number is less than the base, then subtract it from the base and write the deviation with a negative sign..

Number	Base	Deviation
12	10	+2
9	10	-1
104	100	+4
98	100	-2
1002	1000	+002
992	1000	-008

.... And so on.

Gunaa Nikhilam : Base

EXAMPLE-19. Solve 12×14

SOLUTION :

Number	Deviation
12	+2
×14	+4
16	8

(1) Both numbers have a base 10

(2) The deviation of 12 from 10 is +2

(3) The deviation of 14 from 10 is +4

(4) The right side of the answer is product of the deviations, $2 \times 4 = 8$

(5) The left side of the answer = (First number + deviation of the second number) or (Second number + deviation of the first number)

$= 12 + (+4)$ or $14 + (+2)$

$= 16$

168 Answer.

Note: In this method we keep the same number of digits in the right side of the answer as the number of zeros in the base

EXAMPLE-20. Solve 16×15 using Nikhilam Formula.

SOLUTION :

Number	Deviation
16	+6
× 15	+5
24	0

(1) Both the numbers have the same base 10.

(2) The deviation of 16 from 10 is +6

The deviation of 15 from 10 is +5

(3) The right side of the answer

= Product of the deviations

$= 6 \times 5 = 30$

(4) The left side of the answer

= One number + deviation of other

$= 16 + 5 + 3$ (carried over) $= 24$

Or $= 15 + 6 + 3$ (carried over) $= 24$

Thus answer is 240

EXAMPLE-21. Solve 8×13 using the formula Nikhilam.

SOLUTION :

Number	Deviation
8	–2
× 13	+3
11	$\bar{6}$

(1) Base is 10

(2) Right side of answer = Product of the deviations

$= -2 \times (+3) = -6$

(3) Left side of the answer = One number + deviation of other

= 8 + (+3) = 11

Or = 13 + (-2) = 11

Hence answer is 110 - 6 = 104

EXAMPLE-22. Use Nikhilam Sutra to find 104 × 108

SOLUTION :

Number	Deviation
104	+04
×108	+08
112	32

(1) The base is 100. This has two zeros hence the right side of the answer will also have two digits.

(2) The right side of the answer = product of deviations

= 04 × 08 = 32

Answer is 11232

(3) The left side of the answer = One number + deviation of other

= 104 + (+8) = 112

or = 108 + (+4) = 112

EXAMPLE-23. Solve 103 × 101 using Nikhilam Sutra.

SOLUTION :

Number	Deviation
103	+03
× 101	+01
104	03

(1) The right side of the answer = Product of deviations

= 3 × 1 = 3

(2) This right side will have two digits as the base has two zeros, hence we take it as 03

(3) The left side of the answer = One number + deviation of other

Hence answer is 10403

= 103 + (+1) = 104

Or = 101 + (+3) = 104

EXAMPLE-24. Solve using the Nikhilam sutra : 92 × 107

SOLUTION :

Number	Deviation
92	–08
×107	+07
99	$\overline{56}$

(1) The base is 100 and the deviations are respectively, -08 and +07.

(2) The right side of the answer = Product of deviations

= -8 × (+7) = -56 or $\overline{56}$

(3) The left side of the answer = One number + deviation of other

= 92 + (+7) = 99

Or = 107 + (-8) = 99

Hence answer is 9900 - 56 = 9844

EXAMPLE-25. Solve 1014 × 994 using Nikhilam Sutra.

SOLUTION :

$$\begin{array}{c|c} \text{Number} & \text{Deviation} \\ 1014 & +014 \\ \times 994 & -006 \\ \hline 1008 & \overline{084} \end{array}$$

(1) Base is 1000. The deviations are respectively, +014, -006 .

(2) The right side of the answer = Product of deviations

= 14 × (-006) = -084 = $\overline{084}$

(3) The left side of the answer = One number + deviation of other

= 1014 + (–6) = 1008

Or = 994 + (+ 014)= 1008

(4) There are 3 zeros in the base so there has to be three digits in the right side of the answer, so we take it as 1008000.

Answer is 1008000 - 084 = 1007916

Exercise 1.6

Find the products using Nikhilam Sutra and verify the answers using Digit sums:

(1) 13 × 13

(2) 104 × 102

(3) 105 × 106

(4) 98 × 94

(5) 122 × 102

(6) 96 × 107

(7) 1012 × 1004

(8) 998 × 974

(9) 1016 × 998

Square

We have seen four methods of finding a product:

(1) Urdhwatiryak Method (2) Ekanyunena poorvena Method (3) Ekaadhikena poorvena Method (4) Nikhilam Method. We could use these easily to find the square of a number.

We will now find the square of a number using Ekaadhikena poorvena method and some other special methods.

(1) Ekaadhikena poorvena and Antyardeshkepi : We can find the square of numbers that have 5 in the units place orally.

EXAMPLE-26. Find 65^2.

SOLUTION : $65^2 = 65 \times 65$ Using the formula Ekaadhikena poorvena

42 | 25

(1) Left side of the answer = tens digit × one more than tens digit $= 6 \times 7 = 42$

(2) Right side of the answer $= 5 \times 5 = 25$

Hence answer is 4225

Exercise - 1.7

Find the following squares orally, using Ekaadhikena Poorvena method:

15^2, 25^2, 35^2, 45^2, 55^2, 75^2, 85^2, 95^2, 105^2, 115^2

(2) Anurupyena Vidhi : This method is used normally to find the square of a two digit number. We know that $(a + b)^2 = a^2 + 2ab + b^2$, which we will write as follows:

$(a \mid b)^2 = a^2 \mid 2ab \mid b^2$ and using this for the two digits of the number to be squared, we start with the digit in the units place and keep one digit in each of the right and middlemost columns and carry the other digits to the leftmost column.

EXAMPLE-27. Find 64^2

SOLUTION : 64^2 will be solved using $(a \mid b)^2 = a^2 \mid 2ab \mid b^2$

(1) $b^2 = 4^2 = 16$

(2) $2ab = 2 \times 6 \times 4 = 48$

$48 + 1$ (carried from 16) $= 49$

(3) $a^2 = 6^2 = 36$

$36 + 4$ (carried from 49) $= 40$

$64^2 = 36 \mid 48 \mid 16$

$= 36 \quad 49 \quad 6$

$40 \quad 9 \quad 6$ Answer

EXAMPLE-28. Find 48^2.

SOLUTION : We will use $(a \mid b)^2 = a^2 \mid 2ab \mid b^2$

$48^2 = 4^2 \mid 2 \times 4 \times 8 \mid 8^2$

$= 16 \mid 64 \mid 64$

$= 16 \quad 70 \quad 4$

$= 23 \quad 0 \quad 4$ Answer

Exercise - 1.8

Use $(a + b)^2 = a^2 + 2ab + b^2$ ($(a \mid b)^2 = a^2 \mid 2ab \mid b^2$) to find the following squares:

(1) 34^2 (2) 19^2 (3) 54^2 (4) 64^2 (5) 92^2

(3) Yaavat oonam taavat oonee krutya vargam ch yojayet Sutra- In this method, we find the deviation of the number (whose square one has to find) from its base. If the deviation is less, we reduce that from the number and if it is more, we add it in the number to get the left side of the answer. The right side is obtained by squaring the deviation.

We can take help of the following formula to get the square:

$\text{Number}^2 = \text{number} \pm \text{deviation} \mid (\pm\text{deviation})^2$

Example: $13^2 = 13 + 3 \mid 3^2$ (13 is 3 more than base 10)

$= 169$ Answer.

Example: $7^2 = 7 - 3 \mid 3^2$ (7 is 3 less than base 10)

$= 49$ Answer

Example: $98^2 = 98 - 2 \mid (02)^2$

$= 96\ 04$ Answer (04 because the base has two zeros)

Example: $106^2 = 106 + 6 \mid 6^2$

$= 112\ \ 36$ Answer

Exercise - 1.9

Solve using the Formula: यावत ऊनं तावत् ऊनी कृत्य वर्गं च योजयेत ...... :

12^2, 14^2, 102^2, 105^2, 108^2, 94^2, 996^2,

Square Root

We already know the methods of finding square root of a number using prime factorization and division method. We shall now see how we can find the square root using Vilokanam Vidhi.

Vidhi -Vilokanam: We can find the square root of a four or five digits perfect square by inspection. Observe the following table:

Number	1	2	3	4	5	6	7	8	9	10
Number^2	1	4	9	16	25	36	49	64	81	100
Digit sum	1	4	9	7	7	9	4	1	9	1

Memorize the following:

Unit place of square number	-	1	4	5	6	9	0
Unit place of square root	-	1 or 9	2 or 8	5	4 or 6	3 or 7	0

Note : (1) If the units place of a square number is 2,3,7 or 8, they are not perfect squares.

(2) The number of pairs that you can make in the square number, those many digits will be there in the square root of the number.

(3) Numbers whose digit sum is 2,3,5,6 or 8 are not perfect squares.

EXAMPLE-29. Find the square root of the perfect square 6889 using Vilokanam vidhi.

SOLUTION : $\sqrt{6889}$ sutra vilokanam

$\sqrt{6889}$ = 83 or 87

(1) You can make two pairs in the square number, hence square root will have two digits.

(2) The right pair (89) decides the units place and the left pair(68) decides the tens digit.

(3) Since units place of square is 9, hence the units place of square root will be 3 or 7.

(4) The left pair(68)helps decide the tens place. The closest square root to that is 8 as $8^2 = 64$ and $9^2 = 81$, which is definitely more than 68. So we take tens place digit as 8.

(5) The answer could be 83 or 87.

(6) As 6889 is smaller than 7225 which is the square of 85, hence the square root is 83. $\sqrt{6889}$ = 83

Exercise - 1.10

Find the square root of the following using Vilokanam Vidhi:

(1) 9409 (2) 7569 (3) 8281 (4) 3249

Algebra

Multiplication (by Urdhwatiryak Vidhi): This formula is used in Arithmetic but works equally well with multiplication of algebraic expressions.

Example-30. Multiply $3x + 1$ with $2x + 4$ and check the product.

Solution : Urdhwatiryak Vidhi

$$\begin{array}{r} 3x + 1 \\ \times 2x + 4 \\ \hline 6x^2 + 14x + 4 \end{array}$$

(1) First Column

$$\begin{array}{r} +1 \\ \times +4 \\ \hline +4 \end{array}$$

Verticle multiplication

(2) First and second column

$$\begin{array}{r} 3x + 1 \\ \times\ 2x + 4 \\ \hline \end{array}$$

Diagonal multiplication

adding $(3x \times 4) + (2x \times 1)$

$= 12x + 2x = 14x$

(3) Third column

$$\begin{array}{r} 3x \\ \times 2x \\ \hline 6x^2 \end{array}$$

Verticle multiplication

Answer is $6x^2 + 14x + 4$

Check: Digit sum of (digit sum of numerical coefficients of the first expression x digit sum of numerical coefficients of the second expressions) = The digit sum of the numerical coefficients of the answer.

(1) $4 \times 6 = 24$, whose digit sum is 6

(2) $6 + 14 + 4 = 24$, whose digit sum is 6

As (1) and (2) give the same digit sum, our answer is correct.

Example-31. Solve using Urdhwa tiryak Vidhi $(2x + y) \times (3x + 5y)$

Solution :

$$\begin{array}{r} 2x + y \\ \times\ 3x - 5y \\ \hline + 6x^2 - 7xy - 5y^2 \end{array}$$

$$\begin{array}{cccc} 2x & 2x & +y & +y \\ \uparrow & \times & & \uparrow \\ 3x & 3x & -5y & -5y \end{array}$$

Example-32. Find the product of polynomials $x^2 + 3x + 2$ and $5x^2 + x + 1$ using Urdhwa tiryak Vidhi.

SOLUTION :

$$\begin{array}{r} x^2 + 3x + 2 \\ \times\ 5x^2 + x + 1 \\ \hline 5x^4 + 16x^3 + 14x^2 + 5x + 2 \end{array}$$

formula Urdhwa tiryagbhyaam

(1) First column

$$\begin{array}{r} +2 \\ +1 \\ \hline +2 \end{array}$$

$\uparrow$ Vertical multiplication

(2) First and second column

$$\begin{array}{r} +3x + 2 \\ \times\ \ +x + 1 \\ \hline \end{array}$$

Multiply diagonally and add

$(3x \times 1) + (x \times 2)$

$3x + 2x = 5x$

(3) First, second and third column

$$\begin{array}{r} x^2 + 3x + 2 \\ 5x^2 + x + 1 \\ \hline \end{array}$$

as per signs, multiply first with third terms diagonally and second by second vertically and add.

$(x^2 \text{ x} 1) + (5x^2 \text{ x } 2) + (3x \text{ x } x)$

$= x^2 + 10x^2 + 3x^2 = 14x^2$

(4) Third and second column

$$\begin{array}{r} x^2 + 3x \\ \times\ 5x^2 + x \\ \hline \end{array}$$

multiply diagonally and add

$(x^2 \times x) + (5x^2 \times 3x)$

$x^3 + 15x^3 = 16x^3$

(5) Third column

$$\begin{array}{r} x^2 \\ \times\ 5x^2 \\ \hline 5x^4 \end{array}$$

$\uparrow$ Vertical multiplication

Exercise - 1.11

Solve using Urdhwatiryak Vidhi and check the answers:

(1) $\begin{array}{r} 4x+1 \\ \times\ x+5 \\ \hline \end{array}$ (2) $\begin{array}{r} 4x+2y \\ \times\ 3x+3y \\ \hline \end{array}$ (3) $\begin{array}{r} x-3y \\ \times\ x+3y \\ \hline \end{array}$ (4) $\begin{array}{r} x+4 \\ \times\ x+4 \\ \hline \end{array}$

(5) $\begin{array}{r} x^2+2x+1 \\ \times\ x^2+3x+4 \\ \hline \end{array}$ (6) $\begin{array}{r} 2x^2+3y-4 \\ \times\ 3x^2+4y+5 \\ \hline \end{array}$

Division

Paraavartya Vidhi: Division in Arithmetic and Algebra is done using Paraavartya Vidhi.

EXAMPLE-33. Divide $7x^2 - 5x + 3$ by $x + 1$

SOLUTION : Divisor $x + 1$

Modified divisor is –1

Divisor $x+1$	$7x^2$	$-5x$	$+3$
Modified divisor is –1	$+7$	-5	$+3$
		-7	$+12$
	$+7$	-12	$+15$

(1) Write the dividend and divisor in their respective places. The deviation of the divisor x + 1 is +1 and the inverse (revised value) of this is -1, so the modified divisor is -1

(2) Write the coefficients of the terms of the dividend with its signs.

(3) Since modified divisor has one digit, hence we leave one digit in the units place of the dividend and make the line for division.

(4) The first digit of the dividend that is 7 is the first digit of the answer. Write it down as it is.

(5) (First digit of answer x modified divisor), the product of these two is written below –5.

$+7 \times (-1) = -7$

(6) $(-5) + (-7) = -12$ is the second digit of the answer.

(7) The second digit of the answer x modified divisor, the product of these two is written below +3 of the dividend.

$-12 \times (-1) = +12$

(8) Now we have been crossed the division line. Hence +3 +12 = +15

(9) The quotient is 7x - 12 and the remainder is +15

EXAMPLE-34. Divide $x^3 + 2x + 12$ by $x + 2$.

SOLUTION : Divisor $x + 2$

Modified divisor –2

Divisor $x+2$	$x^3 + 0x^2$	$+2x$	$+12$
Modified divisor –2	$+1 + 0$	$+ 2$	$+12$
	-2		
		$+4$	-12
	$+1 -2$	$+6$	0

(1) The divisor is x + 2 and the deviation is +2, hence the modified divisor is the inverse of this, that is -2.

(2) Write the dividend in the reducing power of x. it does not have the term x^2, so we write the coefficient of this as 0.

(3) Modified divisor has one digit, so we draw the division line leaving a digit from the units place in the dividend.

(4) The first coefficient of the dividend, +1, is the first digit of the answer.

(5) First digit of the answer × modified divisor = +1 × (–2) = –2 is written below 0.

(6) +0 + (–2) = -2, is the second digit of the answer.

(7) Second digit of the answer x modified divisor = –2 × (–2) = + 4, is written below +2.

(8) +2 + (+4) = +6 is the third digit of the answer.

(9) Third digit of the answer x modified divisor = +6 × (-2) = –12, is written after the line for division below +12. –12 + 12 = 0 is the remainder after division line.

(10)Hence, the quotient +1 –2 +6 is written in the increasing order of the powers of x from units place.

Quotient $= x^2 - 2x + 6$

Remainder = 0

EXAMPLE-35. Divide $4x^3 - 5x - 9$ by $2x + 1$.

SOLUTION :

divisor	dividend	dividend in the reducing power of x
$2x + 1$	$4x^3 - 5x - 9$	$4x^3 + 0\,x^2 - 5x - 9$

(1) Divide the divisor by 2, so we get 1 as the coefficient of x, as the maximum power of x in divisor needs to have a coefficient 1 for this method.

Hence $\frac{2x+1}{2} = x + \frac{1}{2}$

Dividend written with the signs of the coefficients in the reducing power of x

New divisor $x+\frac{1}{2}$	+4	+0	–5	–9
Modified divisor $-\frac{1}{2}$		–2	+1	+2
	+4	–2	–4	–7

(1) First digit of answer is +4

(2) First digit of the answer × modified divisor = –½ × 4 = –2, is written below 0.

(3) +0 – 2 = –2, is the second digit of the answer.

(4) Second digit of the answer x modified divisor = –½ × –2 = 1, is written below -5.

(5) –5 + 1 = -4 is the third digit of the answer.

(6) Third digit of the answer × modified divisor = –4 × (–½) = +2, is written after the division line, below -9

(7) –9 + 2 = –7 is the remainder.

(8) In the quotient, +4 – 2 – 4, we have to divide by 2 because we have divided the divisor by 2, hence the quotient is +2 – 1 – 2 which when written with x is, $2x^2 - x - 2$

Remainder is –7.

EXAMPLE-36. Divide p(x) by g(x) when p(x) = $x^4 + 1$ and g(x) = $x + 1$

SOLUTION : Divisor dividend

$x + 1$ $x^4 + 1$

(1) We write the dividend in the reducing power of x, and write the coefficient of those terms that are not there as 0.

Divisor Dividend

$x + 1$ $x^4 + 0x^3 + 0x^2 + 0x + 1$

Write the coefficients of the dividend with the signs

Divisor $x + 1$	+1	+0	+0	+0	+1
Modified divisor –1		–1	+1	–1	
					+1
	+1	–1	+1	–1	+2

Quotient $x^3 - x^2 + x - 1$ (we write the quotient along with x as x^0 in units, x^1 in the tens and so on in the increasing power)

Remainder = 2

Check of the answer:

Digit sum of the dividend should be = Digit sum of (product of the digit sum of the divisor x digit sum of the quotient) + digit sum of remainder

LHS = 2

RHS = $(2 \times 0) + 2 = 2$ Since LHS = RHS, the answer is correct.