

Type 10: Gain Margin and Phase Margin

For Concept, refer to Control System K-Notes, Frequency Response Analysis

Sample Problem 10:

The frequency response of a linear system $G(j\omega)$ is provided in the tabular form below

$ G(j\omega) $	1.3	1.2	1.0	.8	.5	.3
$\angle G(j\omega)$	-130°	-140°	-150°	-160°	-180°	-200°

Gain Margin and phase margin are

- (A) 6 dB and 30° (B) 6 dB and -30°
(C) -6 dB and 30° (D) -6 dB and -30°

Solution: (A) is correct option

Gain margin is simply equal to the gain at phase cross over frequency (ω_p). Phase cross over frequency is the frequency at which phase angle is equal to -180° . From the table we can see that $\angle G(j\omega_p) = -180^\circ$, at which gain is 0.5.

$$\begin{aligned} \text{GM} &= 20 \log_{10} \left(\frac{1}{|G(j\omega_p)|} \right) \\ &= 20 \log_{10} \left(\frac{1}{.5} \right) = 6 \text{ dB} \end{aligned}$$

Phase Margin is equal to 180° plus the phase angle ϕ_g at the gain cross over frequency (ω_g). Gain cross over frequency is the frequency at which gain is unity. From the table it is clear that $|G(j\omega_g)| = 1$, at which phase angle is -150°

$$\begin{aligned} \phi_{\text{PM}} &= 180^\circ + \angle G(j\omega_g) \\ &= 180^\circ - 150^\circ = 30^\circ \end{aligned}$$

Unsolved Problems:

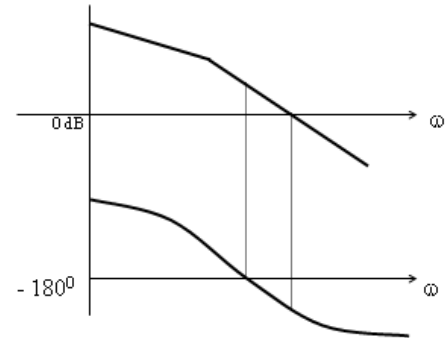
Q.1 The open loop transfer function of a system is $\frac{K}{s(1+0.2s)(1+0.05s)}$. Determine the value

of 'k' such that the phase margin is 60° ?

- (A) 2.3 (B) 3.3 (C) 3.2 (D) 5.2

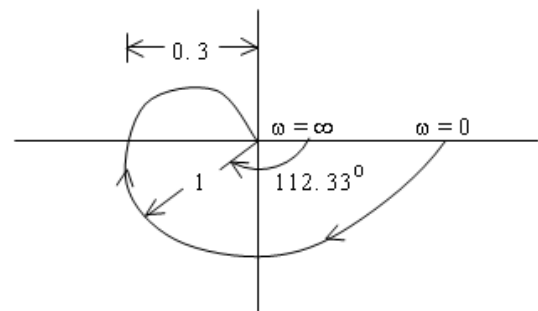
Q.2 The Bode plot of a unity feedback system is shown. The system has

- (A) +ve P.M. and –ve G.M
- (B) +ve P.M. and +ve G.M
- (C) –ve P.M. and –ve G.M
- (D) +ve P.M. and +ve G.M



Q.3 A unity feedback system has OLTF $G(s)$. Polar plot is shown in the figure. The gain margin and phase margin are

- (A) $GM = -0.3$, $PM = 112.33^\circ$
- (B) $GM = 0.3$, $PM = 112.33^\circ$
- (C) $GM = 3.33$, $PM = 67.67^\circ$
- (D) None of the above



Q.4 The open loop transfer function of a unity feedback control system is given as $G(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$. The phase crossover frequency and the gain margin are, respectively.

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|--|---|
| (A) $\frac{1}{\sqrt{T_1 T_2}}$ and $\frac{T_1 + T_2}{T_1 T_2}$ | (B) $\frac{1}{T_1 + T_2}$ and $\frac{T_1 + T_2}{T_1 T_2}$ |
| (C) $\frac{1}{\sqrt{T_1 T_2}}$ and $\frac{T_1 T_2}{T_1 + T_2}$ | (D) $\frac{1}{T_1 + T_2}$ and $\frac{T_1 T_2}{T_1 + T_2}$ |

Q.5 In the $G(s)H(s)$ -plane, the Nyquist plot of the loop transfer function $G(s)H(s) = \frac{\pi e^{-0.25s}}{s}$

passes through the negative real axis at the point

- | | | | |
|-------------------|------------------|-------|---------|
| (A) $(-0.25, j0)$ | (B) $(-0.5, j0)$ | (C) 0 | (D) 0.5 |
|-------------------|------------------|-------|---------|