UNIT – II

Algebra

CHAPTER

2 POLYNOMIALS

Syllabus

- *Zeroes of a polynomial. Relationship between zeroes and coefficients of quadratic polynomials. Statement and simple problems on division algorithm of polynomials with real coefficients.*

Chapter Analysis

List of Topics	2016			2017			2018
	Delhi	**Outside Delhi**	**Foreign**	**Delhi**	**Outside Delhi**	**Delhi**	**Delhi & Outside Delhi**
Zeroes of Polynomial	**Summative Assessment-I**						**1 Q (3 M)**
Finding the value of variable by putting the values of zeroes in a polynomial							

TOPIC-1

Zeroes of a Polynomial and Relationship between Zeroes and Coefficients of Quadratic Polynomials

Revision Notes

- **Polynomial :** An algebraic expression in the form of $a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_2x^2 + a_1x + a_0$, (where n is a whole number and $a_0, a_1, a_2, \ldots\ldots, a_n$ are real numbers) is called a polynomial in one variable x of degree n.
- **Value of a Polynomial at a given point :** If $p(x)$ is a polynomial in x and 'α' is any real number, then the value obtained by putting $x = \alpha$ in $p(x)$, is called the value of $p(x)$ at $x = \alpha$.
- **Zero of a Polynomial :** A real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$.

  Geometrically, the zeroes of a polynomial $p(x)$ are precisely the x-co-ordinates of the points, where the graph of $y = p(x)$ intersects the x-axis.

  (i) A linear polynomial has one and only one zero.

  (ii) A quadratic polynomial has at most two zeroes.

(iii) A cubic polynomial has at most three zeroes.

(iv) In general, a polynomial of degree n has at most n zeroes.

- **Graphs of Different types of Polynomials :**
- **Linear Polynomial :** The graph of a linear polynomial $p(x) = ax + b$ is a straight line that intersects x-axis at one point only.
- **Quadratic Polynomial : (i)** Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens upwards, if a > 0 and intersects x-axis at a maximum of two distinct points.

  (ii) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola which opens downwards, if $a < 0$ and intersects x-axis at a maximum of two distinct points.

  Graph of a cubic polynomial : Graph of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ intersects x-axis at a maximum of three distinct points.

- **Relationship between the Zeroes and the Coefficients of a Polynomial :**

(i) Zero of a linear polynomial $= \dfrac{(-1)^1 \text{Constant term}}{\text{Coefficient of } x}$

If $ax + b$ is a given linear polynomial, then zero of linear polynomial is $\dfrac{-b}{a}$

(ii) In a quadratic polynomial,

$$\text{Sum of zeroes of a quadratic polynomial} = \frac{(-1)^1 \text{ Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes of a quadratic polynomial} = \frac{(-1)^2 \text{ Constant term}}{\text{Coefficient of } x^2}$$

$\therefore$ If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then

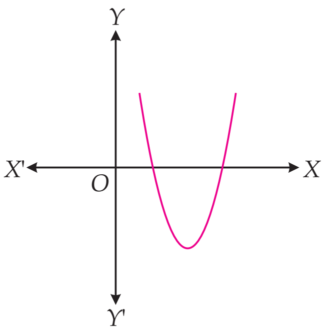
$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

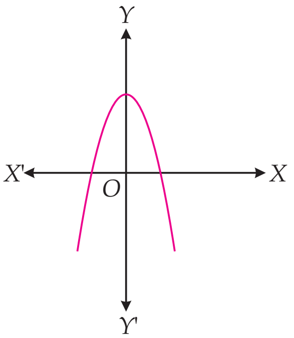
(iii) If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then

$$\alpha + \beta + \gamma = (-1)^1 \frac{b}{a} = -\frac{b}{a},\ \alpha\beta + \beta\gamma + \gamma\alpha = (-1)^2 \frac{c}{a} = \frac{c}{a} \text{ and } \alpha\beta\gamma = (-1)^3 \frac{d}{a} = -\frac{d}{a}$$

- **Discriminant of a Quadratic Polynomial :** For $f(x) = ax^2 + bx + c$, where $a \neq 0$, $b^2 - 4ac$ is called its discriminant D. The discriminant D determines the nature of roots/zeroes of a quadratic polynomial.

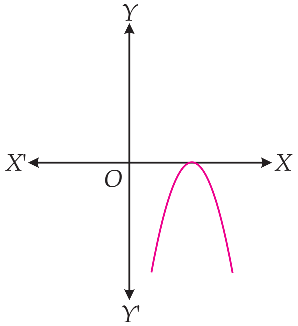
  Case I : If $D > 0$, graph of $f(x) = ax^2 + bx + c$ will intersect the x-axis at two distinct points, x-co-ordinates of points of intersection with x-axis is known as 'zeroes' of $f(x)$.

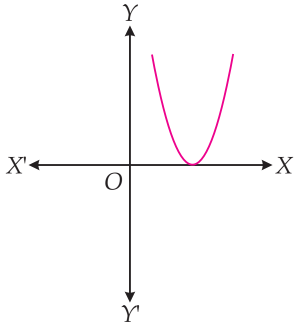




$\therefore$ $f(x)$ will have two zeroes and we can say that roots/zeroes of the to given polynomials are real and unequal.

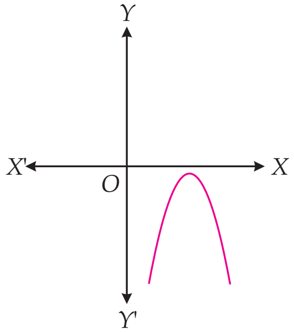
Case II : If $D = 0$, graph of $f(x) = ax^2 + bx + c$ will touch the x-axis at one point only.

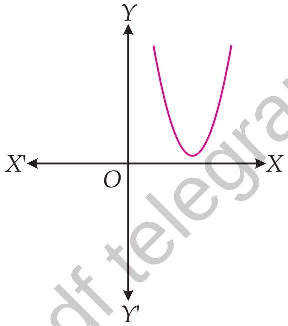




$\therefore$ $f(x)$ will have only one 'zero' and we can say that roots/zeroes of the given polynomial are real and equal.

Case III : If $D < 0$, graph of $f(x) = ax^2 + bx + c$ will neither touch nor intersect the x-axis.





$\therefore$ $f(x)$ will not have any real zero.

Know the Formulae

Relationship between the zeroes and the coefficients of a Polynomial :

S. No.	Type of polynomial	General form	Maximum Number of zeroes	Relationship between zeroes and coefficients
1.	Linear	$ax + b$, *where* $a \neq 0$	1	$k = -\dfrac{b}{a}$, *i.e.*, $k = \dfrac{-\text{Constant term}}{\text{Coefficient of } x}$
2.	Quadratic	$ax^2 + bx + c$, *where* $a \neq 0$	2	Sum of zeroes $(\alpha + \beta)$ $= \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2} \Rightarrow -\dfrac{b}{a}$ Product of zeroes $(\alpha\beta)$ $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} \Rightarrow \dfrac{c}{a}$
3.	Cubic	$ax^3 + bx^2 + cx + d$, *where* $a \neq 0$	3	Sum of zeroes $(\alpha + \beta + \gamma)$ $= \dfrac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3} \Rightarrow -\dfrac{b}{a}$ Product of sum of zeroes taken two at a time $(\alpha\beta + \beta\gamma + \gamma\alpha)$ $= \dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^3} \Rightarrow \dfrac{c}{a}$ Product of zeroes $(\alpha\beta\gamma)$ $= \dfrac{-\text{Constant term}}{\text{Coefficient of } x^3} \Rightarrow -\dfrac{d}{a}$

How it is done on GREENBOARD?

Q. If α and β are the roots of equation $x^2 - 5x + 6 = 0$, then find the value of α – β.

Sol. : **Step I :** Compare $x^2 - 5x + 6 = 0$ with $ax^2 + bx + c = 0$ to get

$a = 1$, $b = -5$ and $c = 6$.

Step II : Sum of roots $\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1}$

or, $\alpha + \beta = 5$ ...(i)

Step III : Product of roots

$$\alpha\beta = \frac{c}{a} = \frac{6}{1}$$

or, $\alpha\beta = 6$ ...(ii)

Step IV : $\alpha - \beta = \pm\sqrt{(\alpha+\beta)^2 - 4\alpha\beta}$

or, $\alpha - \beta = \pm\sqrt{(5)^2 - 4\times 6}$

$= \pm\sqrt{25-24}$

$= \pm\sqrt{1} = \pm 1$

$= 1$ or -1

Objective Type Questions

(1 mark each)

[A] Multiple Choice Questions :

Q. 1. The number of polynomials having zeroes as –2 and 5 is :

(a) 1 **(b) 2**

(c) 3 **(d) more than 3**

U [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : We know that if we divide or multiply a polynomial by any constant (real number), then the zeroes of polynomial remains same.

Here, $\alpha = -2$ and $\beta = +5$

$\therefore\ \alpha + \beta = -2 + 5 = 3$ and $\alpha\beta = -2 \times 5 = -10$

So, required polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 3x - 10$

If we multiply this polynomial by any real number, let 5 and 2, we get $5x^2 - 15x - 50$ and $2x^2 - 6x - 20$ which are different polynomials but having same zeroes –2 and 5. So, we can obtain so many (infinite polynomials) from two given zeroes.

Q. 2. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is :

(a) $-\frac{c}{a}$ **(b)** $\frac{c}{a}$

(c) 0 **(d)** $-\frac{b}{a}$

A [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Let $f(x) = ax^3 + bx^2 + cx + d$

If α, β, γ are the zeroes of $f(x)$, then

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

One root is zero (given) so, $\alpha = 0$. $\beta\gamma = \frac{c}{a}$

Q. 3. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is :

(a) $b - a + 1$ **(b)** $b - a - 1$

(c) $a - b + 1$ **(d)** $a - b - 1$

A [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Let $f(x) = x^3 + ax^2 + bx + c$

$\because$ One of the zeroes of $f(x)$ is –1 so

$$f(-1) = 0$$

$$(-1)^3 + a(-1)^2 + b(-1) + c = 0$$

$$-1 + a - b + c = 0$$

$$a - b + c = 1$$

$$c = 1 + b$$

Now, $\alpha\beta\gamma = \frac{-d}{a}$ [$\because a = 1, d = c$]

$$-1\beta\gamma = \frac{-c}{1}$$

$$\beta\gamma = c$$

$$\beta\gamma = 1 + b - a$$

Q. 4. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are :

(a) both positive

(b) both negative

(c) one positive and one negative

(d) both equal U [NCERT Exemp.]

Sol. Correct option : (b)

Explanation : Let given quadratic polynomial be

$$p(x) = x^2 + 99x + 127$$

On comparing $p(x)$ with $ax^2 + bx + c$. we get

$a = 1$, $b = 99$ and $c = 127$

We know that,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{[By quadratic formula]}$$

$$= \frac{-99 \pm \sqrt{(99)^2 - 4 \times 1 \times 127}}{2 \times 1}$$

$$= \frac{-99 \pm \sqrt{9801 - 508}}{2}$$

$$= \frac{-99 \pm \sqrt{9293}}{2} = \frac{-99 \pm 96.4}{2}$$

$$= \frac{-99 + 96.4}{2}, \frac{-99 - 96.4}{2}$$

$$= \frac{-2.6}{2}, \frac{-195.4}{2}$$

$$= -1.3, \ -97.7$$

Hence, both zeroes of the given quadratic polynomial $p(x)$ are negative.

Q. 5. The zeroes of the quadratic polynomial $x^2 + kx + k$, $k \neq 0$,

(a) cannot both be positive

(b) cannot both be negative

(c) are always unequal

(d) are always equal U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : ***P*** Let $f(x) = x^2 + kx + k$, $k \neq 0$.

On comparing the given polynomial with $ax^2 + bx + c$, we get $a = 1$, $b = k$, $c = k$

If α and β be the zeroes of the polynomial (x). We know that,

Sum of zeroes, $\alpha + \beta = -\dfrac{b}{a}$

$$\alpha + \beta = -\frac{k}{1} = -k \quad \text{...(i)}$$

And product of zeroes, $\alpha\beta = \dfrac{c}{a}$

$$\alpha\beta = \frac{k}{1} = k \quad \text{...(ii)}$$

Case I : If k is negative, $\alpha\beta$ [from equation (ii)] is negative. It means α and β are of opposite sign.

Case II : If k is positive, then $\alpha\beta$ [from equation (ii)] is positive but $\alpha + \beta$ is negative. If, the product of two numbers is positive, then either both are negative or both are positive. But the sum of these numbers is negative, so numbers must be negative. Hence, in any case zeroes of the given quadratic polynomial cannot both be positive.

Q. 6. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ are equal, then :

(a) c and a have opposite signs

(b) c and b have opposite signs

(c) c and a have the same sign

(d) c and b have the same sign U [NCERT Exemp.]

Sol. Correct option : (c)

Explanation : For equal roots $b^2 - 4ac = 0$ or $b^2 = 4ac$

b^2 is always positive so $4ac$ must be positive, *i.e.*, product of a and c must be positive, *i.e.*, a and c must have same sign either positive or negative.

Q. 7. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive. U [NCERT Exemp.]

Sol. Correct option : (a)

Explanation : Let $f(x) = x^2 + ax + b$ and α, β are the roots of it.

Then, $\beta = -\alpha$ (Given)

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

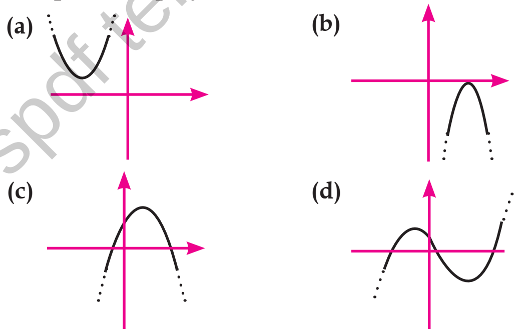
$$\alpha - \alpha = -\frac{a}{1} \text{ and } \alpha(-\alpha) = \frac{b}{1}$$

$$-a = 0 \quad -\alpha^2 = b$$

$$a = 0 \Rightarrow b < 0 \text{ or } b \text{ is negative.}$$

So, $f(x) = x^2 + b$ shows that it has no linear term.

Q. 8. Which of the following is not the graph of a quadratic polynomial?



U [NCERT Exemp.]

Sol. Correct option : (d)

Explanation : Graph (d) intersect at three points on x-axis so the roots of polynomial of graph is three, so it is cubic polynomial. Other graphs are of quadratic polynomial. Graph a have no real zeroes and Graph b has coincident zeroes.

[B] Very Short Answer Type Questions :

Q. 1. If zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is –3, then find the value of k.

R [NCERT Exemp.]

Sol. Let $p(x) = (k - 1)x^2 + kx + 1$

As –3 is a zero of $p(x)$, then

$$p(-3) = 0$$

$$\Rightarrow (k-1)(-3)^2 + k(-3) + 1 = 0$$

$$\Rightarrow 9k - 9 - 3k + 1 = 0$$

$$\Rightarrow 9k - 3k = +9 - 1$$

$$\Rightarrow 6k = 8$$

$$\Rightarrow k = \frac{4}{3}$$

Q. 2. Find a quadratic polynomial, whose zeroes are –3 and 4. R [NCERT Exemp.]

Sol. $x^2 - (\alpha + \beta) + \alpha\beta$

Here, roots of the quadratic are given as -3 and 4.

$\therefore$ Sum of the roots $= -3 + 4 = 1$

and Product of the roots = $-3 \times 4 = -12$

$\therefore$ The quadratic polynomial is

$= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - 1x - 12$

Hence the required polynomial is $\frac{x^2}{2} - \frac{x}{2} - 6$.

Q. 3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then find the value of a and b.

R [NCERT Exemp.]

Sol. Let $f(x) = x^2 + (a + 1)x + b$

As 2, and (-3) are zeroes of polynomial $f(x) = x^2 + (a + 1)x + b$, then

$$f(2) = 0$$
$$(2)^2 + (a+1)(2) + b = 0$$
$$4 + 2a + 2 + b = 0$$
$$2a + b = -6 \qquad ...(i)$$

And $f(-3) = 0$

$$(3)^2 + (a+1)(-3) + b = 0$$
$$9 - 3a - 3 + b = 0$$
$$-3a + b = -6$$
$$3a - b = 6 \qquad ...(ii)$$
$$5a = 0 \qquad \text{[Adding (i) and (ii)]}$$
$$a = 0$$

But, $2a + b = -6$ [From (i)]

$$2(0) + b = -6$$
$$\Rightarrow \quad b = -6$$

Hence, the value of $a = 0$ and $b = -6$.

Q. 4. If α and β are the roots of $ax^2 - bx + c = 0$, where $(a \neq 0)$, then calculate $\alpha + \beta$.

C + R [Board Term – 1, 2014]

Sol. Sum of the roots $= \dfrac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

or, $\alpha + \beta = -\left(-\dfrac{b}{a}\right)$

$= \dfrac{b}{a}$ **1**

Q. 5. If one root of $5x^2 + 13x + k = 0$ is the reciprocal of the other root, then find value of k.

C + U [CBSE Comp Set I/II/III 2018]

Sol. Let α and $\frac{1}{\alpha}$ be the roots of the given quadratic polynomial.

$\therefore \quad \alpha.\frac{1}{\alpha} = \frac{k}{5} = 1$ ½

$\Rightarrow \quad k = 5$ ½

[CBSE Marking Scheme, 2018]

[C] True / False :

Are the following statements 'True' or 'False'? Justify your answers.

(i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

(ii) If the graph of a polynomial intersects the x-axis at only one point, it cannot be a quadratic polynomial.

(iii) If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.

(iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.

(vi) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b and c is non-negative.

(vii) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeroes is $\frac{1}{2}$.

[NCERT Exemp.]

Sol. (i) False, let α and β be the roots of the quadratic polynomial. If α and β are positive then

$\alpha + \beta = \frac{-b}{a}$ it shows that $\frac{-b}{a}$ is negative but sum of two positive numbers (α, β) must be +ve *i.e.*, either b or a must be negative. So, a, b and c will have different signs.

(ii) False, because when two zeroes of a quadratic polynomial are equal, then two intersecting points coincide to become one point.

(iii) True, if a polynomial of degree more than two has two real zeroes and other zeroes are not real or are imaginary, and then graph of the polynomial will intersect at two points on x-axis.

(iv) True, let $\beta = 0, \gamma = 0$

$$f(x) = (x - \alpha)(x - \beta)(x - \gamma)$$
$$= (x - \alpha)\, x \cdot x$$
$$\Rightarrow f(x) = x^3 - \alpha x^2$$

which has no linear (coefficient of x) and constant terms.

(v) True, if $f(x) = ax^3 + bx^2 + cx + d = 0$. Then, for all negative roots, a, b, c and d must have sign.

(vi) False, all the zeroes of cubic polynomial are positive only when all the constants a, b, and c are negative.

(vii) False, $f(x) = kx^2 + x + k$ $(a = k, b = 1, c = k)$

For equal roots $b^2 - 4ac = 0$

$$\Rightarrow (1)^2 - 4(k)(k) = 0$$
$$\Rightarrow \quad 4k^2 = 1$$
$$\Rightarrow k^2 = \frac{1}{4}$$
$$\Rightarrow \quad k = \pm\frac{1}{2}$$

So, there are $+\frac{1}{2}$ and $-\frac{1}{2}$ values of k so that the given equation has equals roots.

Short Answer Type Questions-I (2 marks each)

Q. 1. If zeroes of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$, then find the value of a.

C + U [Board Term-1, 2016 Set-O4YP6G7]

Sol. Product of (zeroes) roots $= \frac{c}{a}$

$$= \frac{2a}{1} = \alpha \cdot \frac{2}{\alpha} \qquad 1$$

or, $2a = 2$

$\therefore$ $a = 1$ 1

[CBSE Marking Scheme, 2016]

Q. 2. Find all the zeroes of $f(x) = x^2 - 2x$.

U [Board Term-1, 2013, LK–59]

Sol. Given, $f(x) = x^2 - 2x$ 1

$= x(x - 2)$

$f(x) = 0 \Rightarrow x = 0$ or $x = 2$

Hence, zeroes are 0 and 2. 1

Q. 3. Find the zeroes of the quadratic polynomial $\sqrt{3}\,x^2 - 8x + 4\sqrt{3}$. U [Board Term-1, 2013, LK–59]

Sol. Let $p(x) = \sqrt{3}\,x^2 - 8x + 4\sqrt{3}$

$\because$ $p(x) = 0$

$\sqrt{3}\,x^2 - 6x - 2x + 4\sqrt{3} = 0$ 1

$\sqrt{3}\,x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3}) = 0$

$(\sqrt{3}\,x - 2)(x - 2\sqrt{3}) = 0$

$x = \frac{2}{\sqrt{3}}$ or $2\sqrt{3}$ 1

$\therefore$ Zeroes are $\frac{2}{\sqrt{3}}$ and $2\sqrt{3}$.

Q. 4. Find a quadratic polynomial, the sum and product of whose zeroes are 6 and 9 respectively. Hence find the zeroes.

A [Board Term-1, 2016 Set– LGRKEGO]

Sol. Sum of zeroes = 6, Product of zeroes = 9

Quadratic polynomial $= x^2 -$ (Sum of zeroes)x + Product of zeroes

$\therefore$ Quadratic polynomial is $x^2 - 6x + 9$ 1

Also, $x^2 - 6x + 9 = 0$

or, $(x - 3)(x - 3) = 0$

$\therefore$ $x = 3$ or 3

Hence, zeroes are 3 and 3. 1

AI **Q. 5. Find the quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively.**

A [Board Term-1, 2012, Set–35]
[Board Term-1, 2012, Set–64]

Sol. Given,

Sum of zeroes $= \frac{21}{8}$

and Product of zeroes $= \frac{5}{16}$ 1

So, quadratic polynomial

$= x^2 -$ (Sum of zeroes) x + Product of zeroes

$= x^2 - \left(\frac{21}{8}\right)x + \frac{5}{16}$

$= \frac{1}{16}(16x^2 - 42x + 5)$ 1

AI **Q. 6. What should be added to the polynomial $x^3 - 3x^2 + 6x - 15$ so that it is completely divisible by $x - 3$.**

A [Board Term-1, 2016 Set-ORDAWEZ]
[Board Term-I, Set-2015]

Sol.

$$\begin{array}{r|l} x-3 & x^3 - 3x^2 + 6x - 15 \;(\; x^2 + 6 \\ & \underline{\;x^3 - 3x^2\;} \\ & \quad\;\; - \quad + \\ & \qquad\qquad 6x - 15 \\ & \qquad\qquad \underline{+6x - 18} \\ & \qquad\qquad\; - \quad + \\ & \qquad\qquad\quad +3 \end{array} \qquad 1$$

Remainder = 3

Hence, – 3 must be added. 1

[CBSE Marking Scheme, 2016]

Q. 7. If m and n are the zeroes of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

A [Board Term-1, 2012, Set– 40]

Sol. Let $p(x) = 3x^2 + 11x - 4 = 0$

$3x^2 + 12x - x - 4 = 0$

$3x(x + 4) - 1(x + 4) = 0$ ½

$(3x - 1)(x + 4) = 0$

So, zeroes are, $x = \frac{1}{3} \Rightarrow m = \frac{1}{3}$ and $x = -4$

$\Rightarrow n = -4$ ½

Now, $\frac{m}{n} + \frac{n}{m} = \frac{\left(\frac{1}{3}\right)}{-4} + \frac{-4}{\left(\frac{1}{3}\right)} = \frac{1}{-12} - 12$ ½

$= \frac{-145}{12}$ ½

Alternative Solution :

Let $p(x) = 3x^2 + 11x - 4 = 0$

Given, m and n are zeroes of $p(x)$

Then, $m + n =$ sum of zeroes

$= -\frac{11}{3}$

$m \times n =$ product of zeroes

$= -\frac{4}{3}$

Now, $\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn}$

$$= \frac{(m+n)^2 - 2mn}{mn}$$

$$= \frac{\left(-\frac{11}{3}\right)^2 - 2\left(-\frac{4}{3}\right)}{-\frac{4}{3}}$$

$$= \frac{\frac{121}{9} + \frac{8}{3}}{-\frac{4}{3}}$$

$$= \frac{121+24}{-12} = \frac{-145}{12}$$

Q. 8. If p and q are the zeroes of polynomial $f(x) = 2x^2 - 7x + 3$, find the value of $p^2 + q^2$.

A [Board Term-1, 2012, Set–21]

Sol. Given $f(x) = 2x^2 - 7x + 3$

Sum of roots $= p + q = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$$= -\left(\frac{-7}{2}\right) = \frac{7}{2} \qquad \frac{1}{2}$$

and Product of roots $= pq = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$$= \frac{3}{2} \qquad \frac{1}{2}$$

Since, $(p+q)^2 = p^2 + q^2 + 2pq$

so, $p^2 + q^2 = (p+q)^2 - 2pq$ ½

$$= \left(\frac{7}{2}\right)^2 - 3 = \frac{49}{4} - \frac{3}{1} = \frac{37}{4} \qquad \frac{1}{2}$$

Hence, the value of $p^2 + q^2 = \frac{37}{4}$

[CBSE Marking Scheme, 2012]

Q. 9. Find the condition that zeroes of polynomial $p(x) = ax^2 + bx + c$ are reciprocal of each other.

A [Board Term-1, 2012, Set–50]

Sol. Given $p(x) = ax^2 + bx + c$

Let α and $\frac{1}{\alpha}$ be the zeroes of $p(x)$, then

Product of zeroes $= \alpha \times \frac{1}{\alpha} = \frac{c}{a}$ or $\frac{c}{a} = 1$ **1**

So, required condition is, $c = a$ **1**

Q. 10. Find the value of k, if –1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$.

C + U [Board Term-1, 2012, Set–62]

Sol. Since, – 1 is a zero of the polynomial

and $p(x) = kx^2 - 4x + k,$

then $p(-1) = 0$ **1**

$\therefore$ $k(-1)^2 - 4(-1) + k = 0$

$\Rightarrow$ $k + 4 + k = 0$

$\Rightarrow$ $2k + 4 = 0$

$\Rightarrow$ $2k = -4$

Hence, $k = -2$ **1**

Q. 11. If α and β are the zeroes of a polynomial $x^2 - 4\sqrt{3}\,x + 3$, then find the value of $\alpha + \beta - \alpha\beta$.

C + A [Board Term-1, 2015, Set-DDE–M]

Sol. Let, $x^2 - 4\sqrt{3}x + 3 = 0$

If α and β are the zeroes of $x^2 - 4\sqrt{3}\,x + 3$.

then $\alpha + \beta = -\frac{b}{a}$

$\Rightarrow$ $\alpha + \beta = -\frac{(-4\sqrt{3})}{1}$

$\Rightarrow$ $\alpha + \beta = 4\sqrt{3}$ **1**

and $\alpha\beta = \frac{c}{a}$

$\alpha\beta = \frac{3}{1}$

$\Rightarrow$ $\alpha\beta = 3$

$\therefore$ $\alpha + \beta - \alpha\beta = 4\sqrt{3} - 3.$ **1**

Q. 12. Find the values of a and b, if they are the zeroes of polynomial $x^2 + ax + b$.

C + A [Board Term-1, 2013, FFC]

Sol. Sum of zeroes $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$\therefore$ $a + b = -a$

$\Rightarrow$ $2a + b = 0$ **1**

and product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ $ab = b$

or, $a = 1$

Substituting $a = 1$ in $2a + b = 0$, we get $b = -2$ **1**

Q. 13. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, find the value of k, such that $\alpha^2 + \beta^2 = 40$.

C + A [Board Term-1, 2015 ,Set WJQZQBN]

Sol. $\alpha + \beta = -\frac{b}{a}$

$$= \frac{-(-6)}{1} = 6$$

and $\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$ **1**

Given, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 40$

$\Rightarrow$ $(6)^2 - 2k = 40$

$\Rightarrow$ $36 - 2k = 40$

$\Rightarrow$ $-2k = 4$

$\therefore$ $k = -2$ **1**

[CBSE Marking Scheme, 2015]

Q. 14. If one of the zeroes of the quadratic polynomial $f(x) = 14x^2 - 42k^2x - 9$ is negative of the other, find the value of 'k'. C + U [Board Term-1, 2012, Set– 48]

Sol. Given, $f(x) = 14x^2 - 42k^2x - 9$

Let one zero be α

Then, the other zero $= -\alpha,$

$\therefore$ Sum of zeroes $= \alpha + (-\alpha) = 0$

Since, sum of zeroes $= -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ **1**

Hence, sum of zeroes $= \dfrac{42k^2}{14} = 3k^2$

$\therefore$ $3k^2 = 0$ or $k = 0$. **1**

AI **Q. 15. Find the value of p, for which one root of the quadratic equation $px^2 - 14x + 8 = 0$ is 6 times the other.**

C + A [O.D. Set III 2017]

Sol. 9. Let α and β be the roots of given quadratic equation.

$\beta = 6\alpha$

Here, $a = p$ $b = -14$, $c = 8$.

$\alpha + \beta = \dfrac{-(-14)}{p} = \dfrac{-b}{a}$

$7\alpha = \dfrac{+14}{p}$

$\alpha = \dfrac{2}{p}$ —(1)

Also, $\alpha\beta = \dfrac{8}{p} = \dfrac{c}{a}$

$\alpha \times 6\alpha = \dfrac{8}{p}$

$6\alpha^2 = \dfrac{8}{p}$

from (1),

$6\left(\dfrac{2}{p}\right)^2 = \dfrac{8}{p}$

$6 \times \dfrac{\cancel{4}}{p^2} = \dfrac{\cancel{8}\,2}{p}$

$\dfrac{6}{p^2} = \dfrac{2}{p}$

$\dfrac{\cancel{6}\,3}{\cancel{2}} = \dfrac{p^2}{\cancel{p}}$

$\therefore p = 3$

[Toppers' Answer, 2017]

Q. 16. If one zero of the polynomial $2x^2 + 3x + \lambda$ is $\frac{1}{2}$, find the value of λ and the other zero.

C + A [Board Term-1, 2012, Set–71]

Sol. Let the zero of $2x^2 + 3x + \lambda$ be $\frac{1}{2}$ and β.

$\therefore$ $\dfrac{1}{2}\beta = \dfrac{\lambda}{2}$

$\Rightarrow$ $\beta = \lambda$

and $\alpha + \beta = -\dfrac{3}{2}$ **1**

$\beta = -\dfrac{3}{2} - \dfrac{1}{2} \Rightarrow -2$

Hence, $\lambda = \beta = -2$

Thus, other zero $= -2$ **1**

Q. 17. If α and β are zeroes of the polynomial $f(x) = x^2 - x - k$, such that $\alpha - \beta = 9$, find k.

C + A [Board Term-1, 2013, Set FFC]

Sol. Since, α and β are the zeroes of the polynomial, then

$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\alpha + \beta = -\left(\frac{-1}{1}\right) = 1 \quad \text{...(i)}$$

Given, $\alpha - \beta = 9$ ...(ii) **1**

Solving (i) and (ii), $\alpha = 5$ and $\beta = -4$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2},$$

$\alpha\beta = -k$

$\Rightarrow$ $(5)(-4) = -k$

$\Rightarrow$ $k = 20$ **1**

Q. 18. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

C + A **[Board Term-1, 2012, Set–39]**

Sol. Let $f(x) = 2x^2 - 5x - 3$

and the zeroes of polynomial be α and β, then

$$\text{Sum of zeroes} = \alpha + \beta = \frac{5}{2}$$

$$\text{Product of zeroes} = \alpha\beta = -\frac{3}{2}$$

According to the question, zeroes of $x^2 + px + q$ are 2α and 2β

$$\text{Sum of zeroes} = -\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2} \Rightarrow \frac{-p}{1}$$

$$-p = 2\alpha + 2\beta = 2(\alpha + \beta)$$

or, $$-p = 2 \times \frac{5}{2} = 5 \text{ or } p = -5 \quad \textbf{1}$$

$$\text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coeff. of } x^2} \Rightarrow \frac{q}{1}$$

or, $$q = 2\alpha \times 2\beta = 4\alpha\beta$$

or, $$q = 4\left(-\frac{3}{2}\right) = -6$$

∴ $$p = -5 \text{ and } q = -6. \quad \textbf{1}$$

[CBSE Marking Scheme, 2012]

Short Answer Type Questions-II

(3 marks each)

Q. 1. Verify whether 2, 3 and $\frac{1}{2}$ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$.

U **[Board Term-1, 2013, LK–59]**

Sol. (i) Given, $p(x) = 2x^3 - 11x^2 + 17x - 6$

Then,
$$\begin{aligned} p(2) &= 2(2)^3 - 11(2)^2 + 17(2) - 6 \\ &= 16 - 44 + 34 - 6 \\ &= 50 - 50 \\ &= 0 \end{aligned} \quad \textbf{1}$$

(ii) Again,
$$\begin{aligned} p(3) &= 2(3)^3 - 11(3)^2 + 17(3) - 6 \\ &= 54 - 99 + 51 - 6 \\ &= 105 - 105 = 0 \end{aligned} \quad \textbf{1}$$

(iii) Again,
$$\begin{aligned} p\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 - 11\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6 \\ &= \frac{1}{4} - \frac{11}{4} + \frac{17}{2} - 6 \\ &= \frac{(1 - 11 + 34 - 24)}{4} \\ &= 0 \end{aligned}$$

Hence, 2, 3 and $\frac{1}{2}$ are the zeroes of $p(x)$. **1**

Q. 2. If the sum and product of the zeroes of the polynomial $ax^2 - 5x + c$ are equal to 10 each, find the value of 'a' and 'c'.

A **[Board Term-1, 2011, Set–25]**

Sol. Given, polynomial $f(x) = ax^2 - 5x + c$

Let the zeroes of $f(x)$ be α and β, then according to the question

Sum of zeroes = (α + β) and product of zeroes = (αβ) = 10

Since, $$\alpha + \beta = -\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2} \Rightarrow -\frac{-5}{a} \quad \textbf{1}$$

So, $$10 = \frac{+5}{a}$$

∴ $$a = \frac{1}{2}$$

and $$\alpha\beta = \frac{\text{Constant term}}{\text{Coeff. of } x^2} \quad \textbf{1}$$

$$\Rightarrow \frac{c}{a} = 10$$

$$\Rightarrow \frac{c}{\frac{1}{2}} = 10$$

$$2c = 10$$

∴ $$c = 5$$

Hence, $$a = \frac{1}{2} \text{ and } c = 5 \quad \textbf{1}$$

Q. 3. If one of the zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

C + U **[Board Term-1, 2011, Set– 40]**

Sol. Let α and β be the zeroes of the polynomial.

Given, $$\beta = 7\alpha$$

∴ $$\alpha + 7\alpha = -\left(-\frac{8}{3}\right) \quad \textbf{½}$$

∴ $$8\alpha = \frac{8}{3}$$

So, $$\alpha = \frac{1}{3} \quad \textbf{½}$$

and $$\alpha \times 7\alpha = \frac{2k+1}{3}$$

$$\Rightarrow 7\alpha^2 = \frac{2k+1}{3}$$

$$\Rightarrow 7\left(\frac{1}{3}\right)^2 = \frac{2k+1}{3} \quad \textbf{1}$$

$$\Rightarrow 7 \times \frac{1}{9} = \frac{2k+1}{3}$$

or, $$\frac{7}{3} - 1 = 2k \quad \text{or, } 2k = \frac{4}{3}$$

∴ $$\frac{2}{3} = k \quad \textbf{1}$$

Q. 4. Quadratic polynomial $2x^2 - 3x + 1$ has zeroes as α and β. Now form a quadratic polynomial whose zeroes are 3α and 3β.

A [Board Term-2, 2015, Set–DDE–E]

Sol. If α and β are the zeroes of $2x^2 - 3x + 1$, then

$$\alpha + \beta = \frac{-b}{a}$$

$$\Rightarrow \quad \alpha + \beta = \frac{3}{2}$$

and $$\alpha\beta = \frac{c}{a}$$

$$\Rightarrow \quad \alpha\beta = \frac{1}{2} \qquad 1$$

New quadratic polynomial whose zeroes are 3α and 3β is :

x^2 – (Sum of the roots) x + Product of the roots 1

$$= x^2 - (3\alpha + 3\beta)x + 3\alpha \times 3\beta$$
$$= x^2 - 3(\alpha + \beta)x + 9\alpha\beta$$
$$= x^2 - 3\left(\frac{3}{2}\right)x + 9\left(\frac{1}{2}\right)$$
$$= x^2 - \frac{9}{2}x + \frac{9}{2}$$
$$= \frac{1}{2}(2x^2 - 9x + 9)$$

Hence, required quadratic polynomial is

$$\frac{1}{2}(2x^2 - 9x + 9). \qquad 1$$

Q. 5. If α and β are the zeroes of the polynomial $6y^2 - 7y + 2$, find a quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. A [Board Term-1, 2011, Set–39]

Sol. $$p(y) = 6y^2 - 7y + 2$$

Then $$\alpha + \beta = -\left(-\frac{7}{6}\right) = \frac{7}{6}$$

and $$\alpha\beta = \frac{2}{6} = \frac{1}{3}$$

Now, $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7/6}{2/6} = \frac{7}{2} \qquad 1$$

and $$\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{1/3} = 3 \qquad 1$$

Hence, the required polynomial is given by,

y^2 – (Sum of the roots)y + Product of the roots.

$$y^2 - \frac{7}{2}y + 3 = \frac{1}{2}[2y^2 - 7y + 6]. \qquad 1$$

Q. 6. Show that $\frac{1}{2}$ and $\frac{-3}{2}$ are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of the polynomial. U [Board Term-1, 2011, Set–21]

Sol. Let $$f(x) = 4x^2 + 4x - 3$$

Thus, $$f\left(\frac{1}{2}\right) = 4\left(\frac{1}{4}\right) + 4\left(\frac{1}{2}\right) - 3$$
$$= 1 + 2 - 3 = 0$$

and $$f\left(-\frac{3}{2}\right) = 4\left(\frac{9}{4}\right) + 4\left(-\frac{3}{2}\right) - 3$$
$$= 9 - 6 - 3 = 0$$

$\therefore \frac{1}{2}$ and $-\frac{3}{2}$ are zeroes of polynomial $4x^2 + 4x - 3$. **1**

$$\text{Sum of zeroes} = \frac{1}{2} - \frac{3}{2} = -1 \Rightarrow \frac{-4}{4}$$
$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \qquad 1$$

$$\text{Product of zeroes} = \left(\frac{1}{2}\right)\left(-\frac{3}{2}\right) = \frac{-3}{4}$$
$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$ **Hence verified. 1**

Q. 7. Find the zeroes of the quadratic polynomial $x^2 - 2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients.

U [Board Term-1, 2015, Set–FHN8MG0]

Sol. Let $p(x) = 0 \Rightarrow x^2 - 2\sqrt{2}\,x = 0$.

Then, $x(x - 2\sqrt{2}) = 0$

Thus, zeroes are 0 and $2\sqrt{2}$

$$\text{Sum of zeroes} = 2\sqrt{2} \Rightarrow -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{and Product of zeroes} = 0 \Rightarrow \frac{\text{Constant term}}{\text{Coefficient of } x^2} \qquad 3$$

[CBSE Marking Scheme, 2015]

Q. 8. If α and β are the zeroes of a quadratic polynomial such that $\alpha + \beta = 24$ and $\alpha - \beta = 8$. Find the quadratic polynomial having α and β as its zeroes.

A [Board Term-1, 2011, Set– 44]

Sol. Given, $\alpha + \beta = 24$ ...(i)

and $\alpha - \beta = 8$ ...(ii)

Adding equations (i) and (ii), **1**

$$2\alpha = 32$$
$$\Rightarrow \quad \alpha = 16$$

Putting the value of α in equation (i),

$$16 + \beta = 24$$
$$\Rightarrow \quad \beta = 24 - 16 = 8 \qquad 1$$

Hence, the quadratic polynomial

x^2 – (Sum of the roots) x + Product of the roots

$$= x^2 - (\alpha + \beta)x + \alpha\beta$$
$$= x^2 - (16 + 8)x + (16)(8)$$
$$= x^2 - 24x + 128 \qquad 1$$

Q. 9. If α, β and γ are zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$

A [KVS practice Test 2017, CBSE Board 2010]

Sol. Given, α, β and γ are zeroes of polynomial $6x^3 + 3x^2 - 5x + 1$.

Then, $$\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-3}{6}$$

or $$\alpha + \beta + \gamma = \frac{1}{2}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = -\frac{5}{6} \quad \textbf{1}$$

and $$\alpha\beta\gamma = -\frac{d}{a} = -\frac{1}{6}$$

$$\therefore \quad \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} \quad \textbf{1}$$

$$= \frac{-5/6}{-1/6} = \frac{-5}{6} \times \frac{6}{-1}$$

Hence, $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = 5$. **1**

Q. 10. Find the zeroes of the following polynomial:

$$5\sqrt{5}x^2 + 30x + 8\sqrt{5}$$ U **[CBSE SQP, 2018]**

Sol. $$5\sqrt{5}x^2 + 30x + 8\sqrt{5} \quad \textbf{1}$$
$$= 5\sqrt{5}x^2 + 20x + 10x + 8\sqrt{5}$$
$$= 5x(\sqrt{5}x + 4) + 2\sqrt{5}(\sqrt{5}x + 4)$$
$$= (\sqrt{5}x + 4)(5x + 2\sqrt{5}) \quad \textbf{1}$$

Thus, zeroes are $\frac{-4}{\sqrt{5}} = \frac{-4\sqrt{5}}{5}$ and $\frac{-2\sqrt{5}}{5}$ **1**

[CBSE Marking Scheme, 2018]

Commonly Made Error

- Candidates commit error in simplifying the equation $5\sqrt{5}x^2 + 30x + 8\sqrt{5}$.

Answering Tip

- Adequate Practice is necessary for factorization problems.

Long Answer Type Questions

(4 marks each)

Q. 1. Polynomial $x^4 + 7x^3 + 7x^2 + px + q$ is exactly divisible by $x^2 + 7x + 12$, then find the value of p and q. A **[Board Term-1, 2015, Set–DDE-M]**

Sol. Factors of $x^2 + 7x + 12$:

$$x^2 + 7x + 12 = 0$$
$$\Rightarrow \quad x^2 + 4x + 3x + 12 = 0$$
$$\Rightarrow \quad x(x + 4) + 3(x + 4) = 0$$
$$\Rightarrow \quad (x + 4)(x + 3) = 0$$
$$\Rightarrow \quad x = -4 \text{ or } -3 \quad \text{...(i) } \textbf{1}$$

Since, $p'(x) = x^4 + 7x^3 + 7x^2 + px + q$

If $p'(x)$ is exactly divisible by $x^2 + 7x + 12$, then $x = -4$ and $x = -3$ are its zeroes. So, putting $x = -4$ and $x = -3$.

$$p'(-4) = (-4)^4 + 7(-4)^3 + 7(-4)^2 + p(-4) + q$$
but $p'(-4) = 0$
$$\therefore \quad 0 = 256 - 448 + 112 - 4p + q$$
$$0 = -4p + q - 80$$
$$4p - q = -80 \quad \text{...(i)}$$
and $$p'(-3) = (-3)^4 + 7(-3)^3 + 7(-3)^2 + p(-3) + q$$
but $p'(-3) = 0$
$$\therefore \quad 0 = 81 - 189 + 63 - 3p + q$$
$$0 = -3p + q - 45 \quad \textbf{1}$$
$$3p - q = -45 \quad \text{...(ii)}$$

Subtracting equation (ii) from equation (i)

$$4p - q = -80$$
$$3p - q = -45$$
$$- \quad + \quad +$$
$$p = -35$$

On putting the value of p in eq. (i), **1**

$$4(-35) - q = -80$$
$$-140 - q = -80$$
$$-q = 140 - 80$$
$$\Rightarrow \quad -q = 60$$
$$\therefore \quad q = -60$$

Hence, $p = -35$ and $q = -60$ **1**

Q. 2. If α and β are the zeroes of the polynomial $p(x) = 2x^2 + 5x + k$ satisfying the relation, $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k.

C + A **[Board Term-1, 2012, Set–50]**

Sol. Given, $$p(x) = 2x^2 + 5x + k$$

Then, $$\text{Sum of zeroes} = -\frac{\text{Coeff. of } x}{\text{Coeff. of } x^2}$$

$$\Rightarrow \quad \alpha + \beta = \frac{-5}{2} \quad \textbf{1}$$

and $$\text{product of zeroes} = \frac{\text{Constant term}}{\text{Coeff. of } x^2}$$

$$\Rightarrow \quad \alpha\beta = \frac{k}{2}$$

According to the question,

$$\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4} \quad \textbf{1}$$

or, $$(\alpha + \beta)^2 - 2\alpha\beta + \alpha\beta = \frac{21}{4}$$

$$\Rightarrow \quad \left(\frac{-5}{2}\right)^2 - \frac{k}{2} = \frac{21}{4}$$

$$\Rightarrow \quad \frac{k}{2} = \frac{25}{4} - \frac{21}{4} \quad \textbf{1}$$

or, $$\frac{k}{2} = \frac{4}{4}$$

Hence, $k = 2$ **1**

AI **Q. 3. If α and β are the zeroes of the polynomial $p(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\frac{1-\alpha}{1+\alpha}$ and $\frac{1-\beta}{1+\beta}$.**

A **[Board Term-1, 2012, Set– 45, 62, 2010, Set-15]**

Sol. Since, α and β are the zeroes of polynomial $3x^2 + 2x + 1$.

Hence, $\alpha + \beta = -\frac{2}{3}$

and $\alpha\beta = \frac{1}{3}$ **1**

Now for the new polynomial,

$$\text{Sum of the zeroes} = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$

$$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$\therefore$ Sum of zeroes $= \frac{4/3}{2/3} \Rightarrow 2$ **1**

$$\text{and product of zeroes} = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$

$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta} \Rightarrow \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$\therefore$ Product of zeroes $= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} \Rightarrow \frac{\frac{6}{3}}{\frac{2}{3}} = 3$ **1**

Hence, required polynomial $= x^2 -$ (Sum of zeroes) x + Product of zeroes

$= x^2 - 2x + 3$ **1**

Q. 4. If α and β are the zeroes of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeroes are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$. A **[Board Term-1 2013 LK59]**

Sol. Since, α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$

So, $\alpha + \beta = -4$

and $\alpha\beta = 3$ **1**

Sum of zeroes of new polynomial

$$= 1+\frac{\beta}{\alpha}+1+\frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta+\beta^2+\alpha\beta+\alpha^2}{\alpha\beta}$$

$$= \frac{\alpha^2+\beta^2+2\alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta} \Rightarrow \frac{(-4)^2}{3} = \frac{16}{3}$$ **1**

Product of zeroes of new polynomial

$$= \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)$$

$$= \left(\frac{\alpha+\beta}{\alpha}\right)\left(\frac{\beta+\alpha}{\beta}\right)$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$ **1**

So, required polynomial $= x^2 -$ (Sum of the zeroes) x + Product of the zeroes

$$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$$

$$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$$

$$= \frac{1}{3}(3x^2 - 16x + 16)$$ **1**

TOPIC-2
Problems on Polynomials

Revision Notes

- **Degree of a Polynomial :** The exponent of the highest degree term in a polynomial is known as its degree. In other words, the highest power of x in a polynomial $f(x)$ is called the degree of the polynomial $f(x)$. *e.g.*,

  (i) $f(x) = 5x + \frac{1}{3}$ is a polynomial in variable x of degree 1.

  (ii) $g(y) = 3y^2 - \frac{5}{2}y + 7$ is a polynomial in variable y of degree 2.

- **Division Algorithm for Polynomials :** If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$, such that

$$p(x) = g(x) \times q(x) + r(x),$$

where, degree of $r(x)$ < degree of $g(x)$ and $r(x)$ is denoted for remainder.

> **Note :**
> (i) If $r(x) = 0$, then $g(x)$ is a factor of $p(x)$.
> (ii) Dividend = Divisor × Quotient + Remainder.

To Divide Quadratic Polynomial by Linear Polynomial :

Let, $p(x) = ax^2 + bx + c$ and $g(x) = mx + n$

$$\frac{a}{m}x + \frac{1}{m}\left(b - \frac{an}{m}\right) \longrightarrow \text{Quotient}$$

$$\text{Divisor} \longrightarrow mx + n\overline{)\,ax^2 + bx + c} \longrightarrow \text{Dividend}$$

$$\begin{array}{l} +ax^2 + \dfrac{an}{m}x \\ - \qquad - \\ \hline \left(b - \dfrac{an}{m}\right)x + c \\ \left(b - \dfrac{an}{m}\right)x + \dfrac{n}{m}\left(b - \dfrac{an}{m}\right) \\ - \qquad\qquad - \\ \hline \qquad c - \dfrac{n}{m}\left(b - \dfrac{an}{m}\right) \end{array}$$

Remainder = Constant term

Step I : To obtain the first term of the quotient, divide the highest degree term of the dividend (*i.e.*, ax^2) by the highest degree term of the divisor (*i.e.*, mx). *i.e.*, $\frac{a}{m}x$. Then, carry out the division process.

What remains is $\left(b - \frac{an}{m}\right)x + c$.

Step II : Now, to obtain the second term of the quotient, divide the highest degree term of the new dividend $\left\{i.e., \left(b - \frac{an}{m}x\right)\right\}$ by the highest degree term of the divisor (*i.e.*, mx). *i.e.*, $\frac{1}{m}\left(b - \frac{an}{m}\right)$. Then, carry on the division process.

What remain as remainder, $c - \frac{n}{m}\left(b - \frac{an}{m}\right)$ which is a constant term.

How it is done on GREENBOARD ?

Q. What should be added to $x^2 - 8x + 10$ to make it divisible by $x - 3$:

Sol. : **Step I :** Let k be added to $x^2 - 8x + 10$ to make it divisible by $x - 3$.

$\Rightarrow \quad p(x) = x^2 - 8x + 10 + k$

Step II : Applying Long division

$$\begin{array}{r} x - 3\overline{)\,x^2 - 8x + 10 + k\,(}\; x - 5 \\ \underset{-}{+}x^2 \underset{+}{-} 3x \qquad\qquad \\ \hline -5x + 10 + k \\ \underset{+}{-}5x \underset{-}{+} 15 \quad \\ \hline -5 + k \end{array}$$

Step III : Here, remainder is $-5 + k$ which must be zero.

Then, $\quad -5 + k = 0$

$\therefore \quad k = 5$

Short Answer Type Questions-I (2 marks each)

Q. 1. On dividing $x^3 - 5x^2 + 6x + 4$ by a polynomial $g(x)$, the quotient and the remainder were $x - 3$ and 4 respectively. Find $g(x)$.
U [Board Term-1, 2016 Set–O4YP6G7]

Sol. Given, $x^3 - 5x^2 + 6x + 4 = g(x)(x - 3) + 4$

$$g(x) = \frac{x^3 - 5x^2 + 6x + 4 - 4}{x - 3}$$

$$\Rightarrow \quad g(x) = \frac{x^3 - 5x^2 + 6x}{x - 3} \qquad 1$$

$$x - 3\,)\; x^3 - 5x^2 + 6x\; (\, x^2 - 2x$$
$$+x^3 - 3x^2$$
$$- \quad +$$
$$-2x^2 + 6x$$
$$-2x^2 + 6x$$
$$+ \quad -$$
$$\times$$

Hence, $g(x) = x^2 - 2x$. 1

[CBSE Marking Scheme, 2016]

Q. 2. Find the quotient and remainder on dividing $p(x)$ by $g(x)$:
$p(x) = 4x^3 + 8x^2 + 8x + 7$ and $g(x) = 2x^2 - x + 1$
U [Board Term-1, 2012, Set–55]

Sol.

$$2x + 5$$
$$2x^2 - x + 1\,)\; 4x^3 + 8x^2 + 8x + 7$$
$$4x^3 - 2x^2 + 2x$$
$$- \quad + \quad -$$
$$+10x^2 + 6x + 7$$
$$+10x^2 - 5x + 5$$
$$- \quad + \quad -$$
$$+11x + 2$$

Thus, quotient $= 2x + 5$ 1

and remainder $= 11x + 2$ 1

[CBSE Marking Scheme, 2012]

Q. 3. Check whether the polynomial $g(x) = x^2 + 3x + 1$ is a factor of the polynomial $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$.
U [Board Term-1, 2012, Set– 48]

Sol. Given, $g(x) = x^2 + 3x + 1$ and $f(x) = 3x^4 + 5x^3 - 7x^2 + 2x + 4$ ½

$$3x^2 - 4x + 2$$
$$x^2 + 3x + 1\,)\; 3x^4 + 5x^3 - 7x^2 + 2x + 4$$
$$3x^4 + 9x^3 + 3x^2$$
$$- \quad - \quad -$$
$$-4x^3 - 10x^2 + 2x$$
$$-4x^3 - 12x^2 - 4x$$
$$+ \quad + \quad +$$
$$2x^2 + 6x + 4$$
$$2x^2 + 6x + 2$$
$$- \quad - \quad -$$
$$2$$

Since, remainder, $r(x) = 2$ 1½

i.e. $r(x) \neq 0$. Hence, $g(x)$ is not a factor of $p(x)$.

[CBSE Marking Scheme, 2012]

Q. 4. What should be added in the polynomial $x^3 - 6x^2 + 11x + 8$ so that it is completely divisible by $x^2 - 3x + 2$?
U [Board Term-1, Set, 2015]

Sol.

$$x - 3$$
$$x^2 - 3x + 2\,)\; x^3 - 6x^2 + 11x + 8$$
$$x^3 - 3x^2 + 2x$$
$$- \quad + \quad -$$
$$-3x^2 + 9x + 8$$
$$-3x^2 + 9x - 6$$
$$+ \quad - \quad +$$
$$14$$

Since, remainder = 14.

Hence, – 14 should be added to make it zero. 2

[CBSE Marking Scheme, 2015]

Q. 5. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find the values of a and b.
U [Board Term-1, Set FHN8MG1, 2015]

Sol.

$$3x^2 + 4x + 1\,)\; 6x^4 + 8x^3 + 17x^2 + 21x + 7\; (\, 2x^2 + 5$$
$$6x^4 + 8x^3 + 2x^2$$
$$- \quad - \quad -$$
$$15x^2 + 21x + 7$$
$$15x^2 + 20x + 5$$
$$- \quad - \quad -$$
$$x + 2$$

$\therefore \quad ax + b = x + 2$

On comparing both the sides, we get

$a = 1$ and $b = 2$. [CBSE Marking Scheme, 2015] 2

Q. 6. If $x^3 - 6x^2 + 6x + k$ is completely divisible by $x - 3$, then find the value of k.
C + U [Board Term-1, 2015, Set-WJQZQBN]

Sol.

$$x^2 - 3x - 3$$
$$x - 3\,)\; x^3 - 6x^2 + 6x + k$$
$$x^3 - 3x^2$$
$$- \quad +$$
$$-3x^2 + 6x + k$$
$$-3x^2 + 9x$$
$$+ \quad -$$
$$-3x + k$$
$$-3x + 9$$
$$+ \quad -$$
$$k - 9$$

Since, the remainder should be zero.

$\therefore \quad k - 9 = 0$

So, $\quad k = 9$

[CBSE Marking Scheme, 2015]

Q. 7. Divide the polynomial $p(x) = x^3 - 4x + 6$ by the polynomial $g(x) = 2 - x^2$ and find the quotient and the remainder.
U [Board Term-1, 2015, Set-1E]

Sol. Try yourself, Similar to Q. 2. of Short Answer Type Question-I.

Q. 8. Divide the polynomial $p(x) = x^2 - 5x + 16$ by the polynomial $g(x) = x - 2$ and find the quotient and the remainder.

U [Board Term-1, 2015, Set-WJQZQBN]

Sol. Try yourself, Similar to Q. 2. of Short Answer Type Question-I.

Q. 9. Mr. Kulkarni has asked his friends to do carpooling for commuting to office because their offices are located in the same tower of the city. For this, they calculated the total expense of the fuel and other charges which together is represented by $x^3 + 8x^2 + 16x + 9$. If there are $x + 1$ members, who are sharing, find the share of each member. A; E

Sol. To get share of each member, we have to divide $x^3 + 8x^2 + 16x + 9$ by $x + 1$.

$$
\begin{array}{r|l}
x + 1 & x^3 + 8x^2 + 16x + 9 \;(x^2 + 7x + 9 \\
 & x^3 + x^2 \\
 & \underline{- \quad -} \\
 & 7x^2 + 16x \\
 & 7x^2 + 7x \\
 & \underline{- \quad -} \\
 & 9x + 9 \\
 & 9x + 9 \\
 & \underline{- \quad -} \\
 & 0
\end{array}
$$ 2

Hence, share to each member is $x^2 + 7x + 9$.

Short Answer Type Questions-II

(3 marks each)

Q. 1. What should be added to $x^3 + 5x^2 + 7x + 3$ so that it is completely divisible by $x^2 + 2x$.

U [Board Term-1, 2016 Set–MV98HN3]

Sol. Try yourself, Similar to Q. 4. of Short Answer Type Question-I.

Q. 2. Divide $6x^3 + 2x^2 - 4x + 3$ by $3x^2 - 2x + 1$ and verify the division algorithm.

U [Board Term-1, 2011, Set–74]

Sol.

$$
\begin{array}{r|l}
 & 2x + 2 \\
3x^2 - 2x + 1 & 6x^3 + 2x^2 - 4x + 3 \\
 & 6x^3 - 4x^2 + 2x \\
 & \underline{- \quad + \quad -} \\
 & 6x^2 - 6x + 3 \\
 & 6x^2 - 4x + 2 \\
 & \underline{- \quad + \quad -} \\
 & -2x + 1
\end{array}
$$

Quotient $= 2x + 2$ and Remainder $= -2x + 1$ 1

Then, $p(x) = g(x)\, q(x) + r(x)$

$= (3x^2 - 2x + 1)(2x + 2) + (-2x + 1)$

$= 6x^3 - 4x^2 + 2x + 6x^2 - 4x + 2 - 2x + 1$ 1

$= 6x^3 + 2x^2 - 4x + 3$ (Hence verified.) 1

Q. 3. Find the value of a and b so that $8x^4 + 14x^3 - 2x^2 + ax + b$ is exactly divisible by $4x^2 + 3x - 2$.

U [Board Term-1, 2011, Set–66]

Sol.

$$
\begin{array}{r|l}
 & 2x^2 + 2x - 1 \\
4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + ax + b \\
 & 8x^4 + 6x^3 - 4x^2 \\
 & \underline{- \quad - \quad +} \\
 & 8x^3 + 2x^2 + ax + b \\
 & 8x^3 + 6x^2 - 4x \\
 & \underline{- \quad - \quad +} \\
 & -4x^2 + (a + 4)x + b \\
 & -4x^2 - 3x + 2 \\
 & \underline{+ \quad + \quad -} \\
 & (a + 7)x + b - 2
\end{array}
$$

For exact division, remainder should be zero, so

$(a + 7)\,x + b - 2 = 0$ 2

$\Rightarrow$ $a + 7 = 0, b - 2 = 0$

Thus, $a = -7$ and $b = 2$. 1

Q. 4 On dividing a polynomial $3x^3 + 4x^2 + 5x - 13$ by a polynomial $g(x)$, the quotient and the remainder are $(3x + 10)$ and $(16x - 43)$ respectively. Find $g(x)$.

A [Board Term-1, 2011, Set– 40]

Sol. Try yourself, Similar to Q. 1. of Short Answer Type Question-I.

Q. 5. Check by division, algorithm, whether $x^2 - 2$ is a factor of $x^4 + x^3 + x^2 - 2x - 3$.

U [Board Term-1, 2011, Set–39]

Sol.

$$
\begin{array}{r|l}
 & x^2 + x + 3 \\
x^2 - 2 & x^4 + x^3 + x^2 - 2x - 3 \\
 & x^4 \quad\quad - 2x^2 \\
 & \underline{- \quad\quad +} \\
 & x^3 + 3x^2 - 2x - 3 \\
 & x^3 \quad\quad - 2x \\
 & \underline{- \quad\quad +} \\
 & 3x^2 - 3 \\
 & 3x^2 - 6 \\
 & \underline{- \quad +} \\
 & +3
\end{array}
$$ 1 1

Since, remainder is 3 *i.e.*, reminder $\neq 0$.

Hence, $x^2 - 2$ is not a factor of the given polynomial. 1

AI **Q. 6. On dividing $x^4 - x^3 - 3x^2 + 3x + 2$ by a polynomial $g(x)$, the quotient and the remainder are $x^2 - x - 2$ and $2x$ respectively. Find $g(x)$.**

A [Board Term-1, 2015, Set–CJTOQ]

[Board Term-I, 2011, Set–40]

Sol. Try yourself, Similar to Q. 1. of Short Answer Type Question-I.

Q. 7. What should be added in the polynomial $x^3 + 2x^2 - 9x + 1$ so that it is completely divisible by $x + 4$?

A [Board Term-1, 2015, Set–DDE-M]

Sol. Try yourself, Similar to Q. 4. of Short Answer Type Question-I.

Q. 8. If the polynomial $f(x) = 3x^4 + 3x^3 - 11x^2 - 5x + 10$ is completely divisible by $3x^2 - 5$, find all its zeroes.

A [Board Term-1, 2013, FFC; 2011, Set–13]

Sol. Since, $3x^2 - 5$ divides $f(x)$ completely.

$\therefore$ $(3x^2 - 5)$ is a factor of $f(x)$

$$\therefore \quad 3x^2 - 5 = 0$$

$$\Rightarrow \quad x^2 = \frac{5}{3}$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{5}{3}} \qquad 1$$

$$\begin{array}{r|l} & x^2 + x - 2 \\ 3x^2 - 5 & 3x^4 + 3x^3 - 11x^2 - 5x + 10 \\ & 3x^4 \quad\quad - 5x^2 \\ & - \quad\quad + \\ \hline & 3x^3 - 6x^2 - 5x + 10 \\ & 3x^3 \quad\quad - 5x \\ & - \quad\quad + \\ \hline & -6x^2 + 10 \\ & -6x^2 + 10 \\ & + \quad - \\ \hline & 0 \end{array} \qquad 1$$

Since, $(x^2 + x - 2)$ is a factor of $p(x)$.
On factorising it, we get

$$x = -2 \text{ or } 1$$

$\therefore$ -2 and 1 are zeroes of $p(x)$. ½

Hence, all the zeroes of $p(x) = \sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, -2$ and 1. ½

Q. 9. When $p(x) = x^2 + 7x + 9$ is divided by $g(x)$, we get $(x + 2)$ and -1 as the quotient and remainder respectively, find $g(x)$.

A **[Board Term-1, 2011, Set–74]**

Sol. Given,

$$p(x) = x^2 + 7x + 9$$
$$q(x) = x + 2$$
$$r(x) = -1$$
$$g(x) = ?$$

Since, $p(x) = g(x)\, q(x) + r(x)$ 1

Then $x^2 + 7x + 9 = g(x)(x + 2) - 1$

$$\Rightarrow \quad g(x) = \frac{x^2+7x+10}{x+2} \qquad 1$$

$$= \frac{(x+2)(x+5)}{(x+2)}$$

$$\therefore \quad g(x) = x + 5 \qquad 1$$

Q. 10. Find all zeroes of the polynomial $(2x^4 - 9x^3 + 5x^2 + 3x - 1)$ if two of its zeroes are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

A **[CBSE Delhi, O.D-2018]**

Sol. Let $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

and $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of $p(x)$.

$$\therefore \quad p(x) = (x-2-\sqrt{3})(x-2+\sqrt{3}) \times g(x)$$

$$= (x^2 - 4x + 1)\, g(x) \qquad 1$$

$(2x^4 - 9x^3 + 5x^2 + 3x - 1) \div (x^2 - 4x + 1) = 2x^2 - x - 1$

$$\therefore \quad g(x) = 2x^2 - x - 1$$
$$= (2x + 1)(x - 1) \qquad 1$$

Therefore, other zeroes are $x = -\frac{1}{2}$ and $x = 1$ 1

$\therefore$ Therefore, all zeroes are $2+\sqrt{3}, 2-\sqrt{3}, \frac{-1}{2}$ and 1

[CBSE Marking Scheme, 2018]

Detailed Answer :

Since, $(2+\sqrt{3})$ and $(2-\sqrt{3})$ are two zeroes of given polynomial.

So, $(x-2-\sqrt{3})$, $(x-2+\sqrt{3})$ will be its two factors.

$\therefore$ $(x-2-\sqrt{3})$, and $(x-2+\sqrt{3}) = x^2 - 4x + 1$

is a factor of the given polynomial.

Now, dividing it by $x^2 - 4x + 1$ 1

$$\begin{array}{r|l} & 2x^2 - x - 1 \\ x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \\ & 2x^4 - 8x^3 + 2x^2 \\ & - \quad + \quad - \\ \hline & -x^3 + 3x^2 + 3x \\ & -x^3 + 4x^2 - x \\ & + \quad - \quad + \\ \hline & -x^2 + 4x - 1 \\ & -x^2 + 4x - 1 \\ & + \quad - \quad + \\ \hline & 0 \end{array} \qquad 1$$

$$2x^2 - x - 1 = (2x + 1)(x - 1)$$

Two other zeroes $= -\frac{1}{2}$ and 1

Therefore, all zeroes are

$(2+\sqrt{3}), (2-\sqrt{3}), -\frac{1}{2}$ and 1. 1

Commonly Made Error

- Many candidates makes mistake in dividing the polynomial. A few candidates do not write all four zeroes.

Answering Tip

- Adequate practice is necessary for division of polynomials.

Q. 11. Ram's mother has given him money to buy some boxes from the market at the rate of $4x^2 + 3x - 2$. The total amount of money is represented by $8x^4 + 14x^3 - 2x^2 + 7x - 8$. Out of this money he donated some amount to a child who was studying in the light of street lamp. Find how much amount of money he donated and purchased how many boxes from the market ?

A; E **[Board Term-1, 2015, Set–WJQZQBN]**

Sol.

$$p(x) = 8x^4 + 14x^3 - 2x^2 + 7x - 8$$

$g(x)$ = rate of the each box $= 4x^2 + 3x - 2$

$q(x)$ = number of boxes

$r(x)$ = amount of money he donated to child

and by using long division method

$$\begin{array}{r|l} & 2x^2 + 2x - 1 \\ 4x^2 + 3x - 2 & 8x^4 + 14x^3 - 2x^2 + 7x - 8 \\ & 8x^4 + 6x^3 - 4x^2 \\ & - \quad - \quad + \\ \hline & 8x^3 + 2x^2 + 7x \\ & 8x^3 + 6x^2 - 4x \\ & - \quad - \quad + \\ \hline & -4x^2 + 11x - 8 \\ & -4x^2 - 3x + 2 \\ & + \quad + \quad - \\ \hline & 14x - 10 \end{array}$$

$\because$ $q(x) = 2x^2 + 2x - 1$ = number of Boxes and $r(x) = 14x - 10$ **[CBSE Marking Scheme, 2015] 3**

Q. 12. Rehman's mother has given him money to buy some boxes from the market at the rate of $x^2 + 2x - 3$. The total amount of money given by his mother is represented by $4x^4 + 2x^3 - 2x^2 + x - 1$. Out of this money, he donated some amount to a child who was studying in the light of a street lamp. Find how much amount of money he must have so that he is able to buy exact and maximum number of boxes from the market.

A ; E **[Board Term-1, 2015, Set–DDE-M]**

Sol.
$p(x) = 4x^4 + 2x^3 - 2x^2 + x - 1$
$g(x)$ = rate of the each box $= x^2 + 2x - 3$
$q(x)$ = number of boxes
$r(x)$ = amount of money he donated to child.

By using long division method
We need to find $q(x)$ and $r(x)$
On dividing $p(x)$ by $g(x)$

$$
\begin{array}{r|l}
x^2 + 2x - 3 & 4x^4 + 2x^3 - 2x^2 + x - 1 \;(4x^2 - 6x + 22 \\
 & 4x^4 + 8x^3 - 12x^2 \\
 & -\quad -\quad + \\
\hline
 & -6x^3 + 10x^2 + x - 1 \\
 & -6x^3 - 12x^2 + 18x \\
 & +\quad +\quad - \\
\hline
 & 22x^2 - 17x - 1 \\
 & 22x^2 + 44x - 66 \\
 & -\quad -\quad + \\
\hline
 & -61x + 65
\end{array}
$$

Number of boxes $q(x) = 4x^2 - 6x + 22$
Amount of money he donated to child $= r(x)$
$= -61x + 65$

Long Answer Type Questions

(4 marks each)

Q. 1. If two zeroes of a polynomial $x^3 + 5x^2 + 7x + 3$ are -1 and -3, then find the third zero.

A **[Board Term-1, 2016 Set MV98HN3]**

Sol. Given, $x = -1$ and $x = -3$ are zeroes. **1**

$(x + 1)(x + 3) = x^2 + 4x + 3$

$$
\begin{array}{r|l}
x^2 + 4x + 3 & x^3 + 5x^2 + 7x + 3 \;(x + 1 \\
 & x^3 + 4x^2 + 3x \\
 & -\quad -\quad - \\
\hline
 & x^2 + 4x + 3 \\
 & x^2 + 4x + 3 \\
 & -\quad -\quad - \\
\hline
 & 0
\end{array}
$$

1

1

Since, remainder is 0.
$\therefore$ $x + 1 = 0$
$\Rightarrow$ $x = -1$
$\therefore$ The third zero is -1. **1**

[CBSE Marking Scheme, 2015]

AI **Q. 2. Given that $x - \sqrt{5}$ is a factor of the polynomial $x^3 - 3\sqrt{5}\,x^2 - 5x + 15\sqrt{5}$, find all the zeroes of the polynomial.** A **[Board Term-I, 2014]**
[Board Term-1, 2012, Set–39]

Sol.

$$
\begin{array}{r|l}
 & x^2 - 2\sqrt{5}\,x - 15 \\
\hline
x - \sqrt{5} & x^3 - 3\sqrt{5}\,x^2 - 5x + 15\sqrt{5} \\
 & x^3 - \sqrt{5}\,x^2 \\
 & -\quad + \\
\hline
 & -2\sqrt{5}\,x^2 - 5x + 15\sqrt{5} \\
 & -2\sqrt{5}\,x^2 + 10x \\
 & +\qquad - \\
\hline
 & -15x + 15\sqrt{5} \\
 & -15x + 15\sqrt{5} \\
 & +\quad - \\
\hline
 & 0
\end{array}
$$

2

On factorising the quotient, we get

$x^2 - 2\sqrt{5}\,x - 15 = x^2 - 3\sqrt{5}\,x + \sqrt{5}\,x - 15$
$= x(x - 3\sqrt{5}) + \sqrt{5}(x - 3\sqrt{5})$
$= (x + \sqrt{5})(x - 3\sqrt{5})$ **1**

$(x + \sqrt{5})(x - 3\sqrt{5}) = 0$

$x = -\sqrt{5}, 3\sqrt{5}$

Thus all the zeroes are $\sqrt{5}, -\sqrt{5}$ and $3\sqrt{5}$. **1**

[CBSE Marking Scheme, 2014, 2012]

Q. 3. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $(x^2 - 2x + k)$, the remainder comes out to be $x + a$, find k and a.

A **[Board Term-1, 2012, Set–35]**

Sol.

$$
\begin{array}{r|l}
 & x^2 - 4x + (8 - k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 \\
 & x^4 - 2x^3 + kx^2 \\
 & -\quad +\quad - \\
\hline
 & -4x^3 + (16 - k)x^2 - 25x + 10 \\
 & -4x^3 + 8x^2 - 4kx \\
 & +\quad -\quad + \\
\hline
 & (8 - k)x^2 - (25 - 4k)x + 10 \\
 & (8 - k)x^2 - (16 - 2k)x + (8k - k^2) \\
 & -\quad +\quad - \\
\hline
 & (2k - 9)x + (10 - 8k + k^2)
\end{array}
$$

1

1

Given, remainder $= x + a$
On comparing the multiples of x we get
$(2k - 9)x = 1 \times x$
$\Rightarrow$ $2k - 9 = 1$ or $k = \dfrac{10}{2} \Rightarrow 5$ **1**

On putting this value of k into other portion of remainder, we get
$a = 10 - 8k + k^2 \Rightarrow 10 - 40 + 25$
$= -5$ **1**

[CBSE Marking Scheme, 2012]

AI **Q. 4. Obtain all other zeroes of the polynomial $4x^4 + x^3 - 72x^2 - 18x$, if two of its zeroes are $3\sqrt{2}$ and $-3\sqrt{2}$.** A **[Board Term-1, 2015, Set–C3TOQ]** **[Board Term-I, 2017, Set–35]**

Sol. Try yourself, Similar to Q. 8. of Short Answer Type Question-II.

AI **Q. 5. Obtain all other zeroes of the polynomial $9x^4 - 6x^3 - 35x^2 + 24x - 4$, if two of its zeroes are 2 and – 2.** A **[Board Term-1, 2015, Set–DDE-M]** **[Board Term-I, 2015, Set–DDE]**

Sol. Try yourself, Similar to Q. 8. of Short Answer Type Question-II.

AI **Q. 6. Obtain all zeroes of $3x^4 - 15x^3 + 13x^3 + 25x - 30$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.**

[CBSE Compt Set- I/II/III 2018] **SQP 2017]**

Sol. Let, $p(x) = 3x^2 - 15x^3 + 13x^2 + 25x - 30$ and

$x - \sqrt{\frac{5}{3}}$ and $x + \sqrt{\frac{5}{3}}$ are factors of $p(x)$.

$\Rightarrow$ $x^2 - \frac{5}{3}$ or $\frac{(3x^2 - 5)}{3}$ is also a factor of $p(x)$. **1**

$$\Rightarrow \quad p(x) = \frac{(3x^2 - 5)}{3}(x^2 - 5x + 6) \qquad \mathbf{2}$$

$$= \frac{1}{3}(3x^2 - 5)(x - 3)(x - 2)$$

$\therefore$ Zeroes of $p(x)$ are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, 2$ and 3. **1**

[CBSE Marking Scheme, 2018]

Detailed Answer :

Since, $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeroes of the given polynomial.

So, $\left(x - \sqrt{\frac{5}{3}}\right)$ and $\left(x + \sqrt{\frac{5}{3}}\right)$ will be its two factors.

$$\therefore \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right) = \frac{1}{3}(3x^2 - 5)$$

is also a factor of given polynomial.

Now dividing it by $3x^2 - 5$.

$$\begin{array}{r|l} 3x^2 - 5 & 3x^4 - 15x^3 + 13x^2 + 25x - 30 \,(x^2 - 5x + 6 \\ & 3x^4 \qquad\quad - 5x^2 \\ & - \quad + \quad + \\ \hline & -15x^3 + 18x^2 + 25x - 30 \\ & -15x^3 \qquad\quad + 25x \\ & + \qquad\qquad - \\ \hline & \qquad 18x^2 - 30 \\ & \qquad 18x^2 - 30 \\ & \qquad - \quad + \\ \hline & \qquad\quad 0 \end{array}$$

On factorising the quotient, we get

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

Thus, two other zeroes = 2 and 3

Hence, all the zeros of given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}, 2$ and 3.

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