***Geometric Progression in Nature***

***Bacteria such as Shewanella Oneidensis multiply by doubling their population in size after as little as 40 minutes. A geometric progression such as this, where each number is double the previous number produces a rapid increase in the population in a very short time.***

# 9

# Geometric Progression

**Contents :**



## 9.1 Meaning

A sequence is an ordered arrangement of numbers according to some definite rule. There are different types of sequences. We have already learnt the Arithmetic Progression (A.P.). A sequence in which the difference between any two consecutive terms is a non-zero constant is called an arithmetic progression. e.g. 3, 6, 9, 12....

We shall now learn about another sequence called Geometric Progression (G.P.).

Let us take a sequence as 3, 6, 12, 24,...

The first term of the sequence is 3. If the first term 3 is multiplied by 2, we get the number $3 \times 2 = 6$ which is the second term of the sequence. If the second term 6 is multiplied by 2, we get the number $6 \times 2 = 12$, which is the third term of the sequence. In other words, the following ratios of consecutive terms of the sequence remain constant:

$$\frac{\text{second term}}{\text{first term}} = 2, \ \frac{\text{third term}}{\text{second term}} = 2, \frac{\text{fourth term}}{\text{third term}} = 2, \text{ and so on}$$

Consider another sequence 4, -12, 36, -108,...

The first term of the sequence is 4. If the first term 4 is multiplied by –3, we get the number $4 \times (-3) = -12$, which is the second term of the sequence. If the second term (–12) is multiplied by (–3), we get the number $(-12) \times (-3) = 36$, which is the third term of the sequence. In other words, the following ratios of consecutive terms of the sequence remain constant :

$$\frac{\text{second term}}{\text{first term}} = -3, \quad \frac{\text{third term}}{\text{second term}} = -3, \quad \frac{\text{fourth term}}{\text{third term}} = -3, \text{ and so on.}$$

Consider one more sequence, $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

The first term of the sequence is 2. If the first term 2 is multiplied by $\frac{1}{2}$, we get the number $2 \times \frac{1}{2} = 1$, which is the second term of the sequence. If the second term 1 is multiplied by $\frac{1}{2}$, we get the number $1 \times \frac{1}{2} = \frac{1}{2}$, which is the third term of the sequence. In other words, the following ratios of conseutive terms of the sequence remain constant :

$$\frac{\text{second term}}{\text{first term}} = \frac{1}{2}, \quad \frac{\text{third term}}{\text{second term}} = \frac{1}{2}, \quad \frac{\text{fourth term}}{\text{third term}} = \frac{1}{2}, \text{ and so on}$$

In each of the above three sequences, it can be seen that for any $n \geq 1$, the ratio of the $(n+1)$th term to the $n$ th term of the sequence is a non-zero constant. Such a sequence is called **Geometric Progression.**

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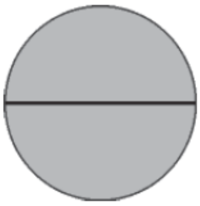
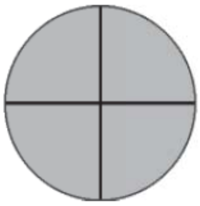
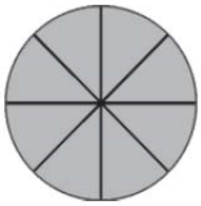
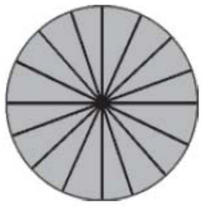
| |  |  |  |  |
| **No. of diameters** | $1 = 2^0$ | $2 = 2^1$ | $4 = 2^2$ | $8 = 2^3$ |
| **No. of parts of circle** | $2 = 2^1$ | $4 = 2^2$ | $8 = 2^3$ | $16 = 2^4$ |

In the above figure, the number of diameters are 1, 2, 4, 8... respectively. They form a G.P.

Also, the number of parts of circle are 2, 4, 8, 16... respectvely. They also form a G.P.

**Let us understand few practical examples of geometric progression.**

**Assume that a rich person's wealth is estimated to be ₹ 200 crore and it doubles every five years.**

The current wealth is ₹ 200 crore, it doubles every five years. So, the wealth after first five years will be $200 \times 2 = 400$ crore rupees. Now it again doubles at the end of the next five years, so the wealth becomes $400 \times 2 = 800$ crore rupees. Thus, his wealth respectively will be 200, 400, 800 (Crore) which is a geometric progression with ratio 2.


**A person deposits an amount of ₹ 10,000 with a bank. The bank offers him an annual return of 10 % compound interest on his investment, i.e. the deposited sum will increase by an interest of 10 % at the end of each year.**

The amount deposited is ₹ 10,000. Bank offers 10% interest, so that amount after 1 year will be 10000 + 1000 (10% of 10000) = 11000. Now the bank counts 10% interest on ₹ 11,000 The amount after the 2nd year will be 11,000 + 1100 (10 % of 11000) = 12,100 ₹ The amount at the end of 3rd year will be 12100 + 1210 (10% of 12100) = 13310 ₹. Thus 10000, 11000, 12100, 13310... forms a geometric progression with ratio 1.1.

**A crude oil reserve at a particular place is 5 million metric tonnes and is diminishing by 10 % each year.**

The current oil reserve is 50,00,000 metric tonnes. Oil reserve diminishes by 10% every year, so that oil reserve at the end of first year is 5000000 – 500000 (10% of 5000000) = 4500000 metric tonnes. The oil reserve after 2nd year 4500000 – 450000 (10 % of 4500000) = 4050000 metric tonnes. Thus oil reserves respectively will 5000000, 4500000, 4050000... which is a geometric progression with ratio 0.9.


## 9.2 Formula for obtaining *n* th term

If $a$ and $r$ are non-zero real numbers, the sequence whose $n$ th term is $T_n = ar^{n-1}$ for an integer $n \geq 1$ is called the **geometric progression.** The real numbers $a$ and $r$ are called the first term and the common ratio respectively of the geometric progression.

From the definition of the G.P., it can be seen that the consecutive terms of G.P. are $a$, $ar$, $ar^2$ $ar^3$, ... and its $n$ th term, $T_n = ar^{n-1}$ for $n \geq 1$ is called the general formula (or term) of the G.P.

**Illustration 1 : If the first term and the common ratio of a G.P. are 7 and 2 respectively, find its sixth term.**

In a given G.P., the first term $a = 7$, common ratio $r = 2$ and we require the sixth term of G.P. i.e. $n = 6$.

Putting values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$, we get,

$$\begin{aligned} T_6 &= 7 \times (2)^{6-1} \\ &= 7 \times (2)^5 \\ &= 7 \times 32 \\ &= 224 \end{aligned}$$

Hence, the sixth term of the G.P. is 224.

**Illustration 2 : The common ratio and the fifth term of a G.P. are 3 and 324 respectively, find the first term of the G.P.**

In the given G.P., the common ratio $r = 3$, the fifth term is 324 i.e. $T_5 = 324$ and we require the first term of the G.P. i.e. $a$.

Here, $T_5 = 324$

$\therefore \quad ar^{5-1} = 324 \qquad (\because T_n = ar^{n-1})$

$\therefore \quad a\,(3)^4 = 324 \qquad (\because r = 3)$

$\therefore \quad a \times 81 = 324$

$\therefore \quad a = \frac{324}{81} = 4$

Hence, the first term of the G.P. is 4.

**Illustration 3 : The first term and the fourth term of a G.P. are 5 and 40 respectively; find the common ratio of a G.P.**

In this G.P., the first term $a = 5$, the fourth term is 40 i.e. $T_4 = 40$ and we want to find the common ratio of the G.P. i.e. $r$

Here, $T_4 = 40$

$\therefore\ ar^{4-1} = 40 \qquad (\because T_n = ar^{n-1})$

$\therefore\ 5 \times r^3 = 40 \qquad (\because a = 5)$

$\therefore\ r^3 = \frac{40}{5} = 8$

$\therefore\ r = 2$

Hence, the common ratio of the G.P. is 2.

**Illustration 4 : The first term and the common ratio of a G.P. are 4 and –2 respectively. If its *n*th term is –128, find the value of *n*.**

For the given G.P. first term $a = 4$, common ratio $r = -2$ and the $n$-th term is –128 i.e. $T_n = -128$.

Here, $T_n = -128$

$\therefore \quad ar^{n-1} = -128 \qquad (\because T_n = ar^{n-1})$

$\therefore \quad 4 \times (-2)^{n-1} = -128 \qquad (\because a = 4$ અને $r = -2)$

$\therefore \quad (-2)^{n-1} = \frac{-128}{4} = -32$

$\therefore \quad (-2)^{n-1} = (-2)^5$

Equating the powers on both the sides, we get

$n - 1 = 5$

$\therefore \quad n = 6$

**Illustration 5 : If the fourth term and the seventh term are $\frac{3}{4}$ and $\frac{3}{32}$ respectively for a G.P., find its tenth term.**

Let $a$ and $r$ be the first term and common ratio respectively of the given G.P.

We are given $T_4 = \frac{3}{4}$ and $T_7 = \frac{3}{32}$

Hence, $\frac{T_7}{T_4} = \frac{ar^6}{ar^3} = r^3$ and $\frac{T_7}{T_4} = \frac{\frac{3}{32}}{\frac{3}{4}} = \frac{3}{32} \times \frac{4}{3} = \frac{1}{8}$

$\therefore \quad r^3 = \frac{1}{8} = \left(\frac{1}{2}\right)^3$

$\therefore \quad r = \frac{1}{2}$

Now putting $r = \frac{1}{2}$ in $T_4 = ar^3 = \frac{3}{4}$, we get

$\therefore \quad a\left(\frac{1}{2}\right)^3 = \frac{3}{4}$

$\therefore \quad a \times \frac{1}{8} = \frac{3}{4}$

$\therefore \quad a = \frac{3}{4} \times 8$

$\therefore \quad a = 6$

We now have to find the tenth term, so $n = 10$.

Putting values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$, we get

$$T_{10} = 6 \times \left(\frac{1}{2}\right)^{10-1}$$

$$= 6 \times \left(\frac{1}{2}\right)^9$$

$$= 6 \times \frac{1}{512}$$

$$= \frac{3}{256}$$

Hence, the tenth term of the G. P. is $\frac{3}{256}$

**Illustration 6 : Find the 5th term of a G.P. 9, –6, 4, ...**

Here, $a = 9$ and $r = \frac{-6}{9} = \frac{-2}{3}$. The 5th term is to be found, so $n = 5$

Putting values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$, we get

$$T_5 = 9 \times \left(\frac{-2}{3}\right)^{5-1}$$

$$= 9 \times \left(\frac{-2}{3}\right)^4$$

$$= 9 \times \frac{16}{81}$$

$$= \frac{16}{9}$$

Hence, the 5th term of the G. P. is $\frac{16}{9}$.

**Illustration 7 : Find the 8th term of the G. P. $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, .....**

Here $a = \frac{1}{8}$ and $r = \frac{\frac{1}{4}}{\frac{1}{8}} = 2$. Now, 8th term is to be found, so $n = 8$

Putting the values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$, we get

$$T_8 = \frac{1}{8} \times (2)^{8-1}$$

$$= \frac{1}{8} \times (2)^7$$

$$= \frac{1}{8} \times 128$$

$$= 16$$

Hence, the 8th term of the G. P. is 16.

**Illustration 8 : If the second term of a G.P. is 4 then find the product of the first three terms of the G.P.**

Let $a$ and $r$ be the first term and common ratio.

Here, the second term is 4. i.e. $T_2 = 4$.

Now putting $n = 1$, $n = 2$ and $n = 3$ in the general term $T_n = ar^{n-1}$, we get

$T_1 = a$, $T_2 = ar$ and $T_3 = ar^2$

Hence, $T_1 \times T_2 \times T_3 = a \times ar \times ar^2$

$= a^3 \times r^3$

$= (ar)^3$

$= (4)^3$ $\quad (\because\ T_2 = ar = 4)$

$= 64$

Hence, the product of the first three terms of the G.P. is 64.

**Illustration 9 : The first term and the product of the first three terms of a G.P. are 3 and 216 respectively. Find the 7th term of the G.P.**

The first term $a = 3$.

Now, putting $n = 1$, $n = 2$ and $n = 3$ in the general term $T_n = ar^{n-1}$, we get $T_1 = a$, $T_2 = ar$ and $T_3 = ar^2$

It is given that product of first three terms is 216.

i.e. $T_1 \times T_2 \times T_3 = 216$

Now, $T_1 \times T_2 \times T_3 = 216$

$\therefore\ a \times ar \times ar^2 = 216$

$\therefore\ a^3 \times r^3 = 216$

$\therefore\ 3^3 \times r^3 = 216$ $\quad (\because\ a = 3)$

$\therefore\ 27 \times r^3 = 216$

$\therefore\ r^3 = \frac{216}{27} = 8$

$\therefore\ r = 2$

The 7th term of the G.P. is $T_7 = ar^6$

$\therefore$ $T_7 = 3 \times (2)^6$

$= 3 \times 64$

$= 192$

Hence, the 7th term of the G. P. is 192.

**Illustration 10 : The numbers 2, G, 50 are in G.P. Find the value of G.**

Here $T_1 = 2$, $T_2 = G$ and $T_3 = 50$

The common ratio of the G. P. is $= \frac{T_2}{T_1} = \frac{T_3}{T_2}$

$\therefore \quad \frac{G}{2} = \frac{50}{G}$

$\therefore \quad G^2 = 100$

$\therefore \quad G = \pm 10$

$\therefore \quad G = 10$ or $-10$

**Illustration 11 : For the numbers *a, b, c, d,* if $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$ , show that the numbers *a, b, c, d* are in G.P.**

Suppose $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = r$, where $r$ is a non-zero constant.

$\therefore \quad b = ar,\ c = br,\ d = cr$

$\therefore \quad c = (ar)\ r = ar^2$ and $d = (ar^2)\ r = ar^3$

If we take $T_1 = a$, $T_2 = b = ar$, $T_3 = c = ar^2$, $T_4 = d = ar^3$

then we see that $T_1$, $T_2$, $T_3$ and $T_4$ are in G.P. Hence $a$, $b$, $c$, $d$ are in G.P.

**Illustration 12 : Which term of the G.P. 0.008, 0.016, 0.032, ... is 4.096 ?**

Here, the first term $a = 0.008$ and the common ratio $r = \frac{0.016}{0.008} = 2$

Now, $T_n = 4.096$

$\therefore \quad ar^{n-1} = 4.096$

$\therefore \quad 0.008 \times (2)^{n-1} = 4.096$

$\therefore \quad 2^{n-1} = \frac{4.096}{0.008}$

$\therefore \quad 2^{n-1} = 512$

$\therefore \quad 2^{n-1} = 2^9$

Equating the powers on both the sides, we get

$n - 1 = 9$

$\therefore \quad n = 10$

Hence, 4.096 is the 10th term of the given G.P.

**Illustration 13 : If the third term of a G.P. is the square of the first term and the fourth term is 243, find the sequence.**

Let $a$ and r be the first term and common ratio respectively of the given G. P. Here, the third term is square of the first i.e. $T_3 = a^2$

$\therefore \quad T_3 = ar^2 = a^2$

$\therefore \quad r^2 = a$

Also, the fourth term is 243 i.e. $T_4 = 243$

$\therefore\ T_4 = ar^3 = 243$

$\therefore\quad r^2 \times r^3 = 243\ (\because\ r^2 = a)$

$\therefore\quad r^5 = 243$

$\therefore\quad r^5 = 3^5$

$\therefore\quad r = 3$

Now, $a = r^2$

$\therefore\quad a = 3^2$

$\therefore\quad a = 9$

Taking $a = 9$ and $r = 3$ the sequence is 9, 27, 81, 243,...

**Illustration 14 : A person deposits ₹ 10,000 in a bank in the year 2009, ₹ 20,000 in the year 2010, ₹ 40,000 in the year 2011. The amount of deposits in any given year is twice the amount of deposit of the previous year. What would be the amount of deposit in the year 2014 ?**

A person deposits ₹ 10,000 in the first year i.e. 2009, so the first term $a = 10000$. Every year the amount of deposits is twice the previous year's amount, so the common ratio ($r$) = 2. We want to find the amount deposited in the year 2014 i.e. the sixth year, so $n = 6$.

Putting the values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$, we get

$T_6 = 10000\ (2)^{6-1}$

$= 10,000\ (2)^5$

$= 10,000 \times 32$

$= 3,20,000$

Thus the amount deposited in the year 2014 is ₹ 3,20,000.

**Illustration 15 : A water tank of a capacity of 50,000 litres is fully filled with water. Every week, the water level reduces to half of the previous level due to leakage. What will be the level of water after five weeks ? (No new water is added in the tank)**

A water tank of a capacity of 50,000 litres is fully filled with water $\therefore\ a = 50000 = T_1$.

Every week, the water level reduces to half of the previous level $\therefore$ common ratio $r = \frac{1}{2}$.

Water level after first week = $50000 \times \frac{1}{2} = 25000 = T_2$. We want the water level after five weeks, so $n = 6$.

Putting the values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$, we get

$T_6 = 50,000 \left(\frac{1}{2}\right)^{6-1}$

$= 50,000 \left(\frac{1}{2}\right)^5$

$= 50,000 \times \frac{1}{32} = 1562.5$

Thus the level of water after five weeks will be 1562.5 litres.

**Illustration 16 : Population of a city is 20 lakhs. If the population increases at the rate of 3% every year, find the population of the city after 6 years.**

Current population of the city is 20,00,000 i.e. $a = 2000000 = T_1$. Population increases at the rate of 3% every year

i.e. $r = 1.03$

| |
|---|
| As the population increases at the rate of 3%, $\therefore\quad r = \frac{100+3}{100} = 1.03$ |

Population after the first year = 2000000 × 1.03 = 2060000 = $T_2$ . We need to find the population after 6 years, so $n = 7$.

Putting the values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$, we get

$T_7 = 20{,}00000\ (1.03)^{7-1}$

$= 20{,}00000\ (1.03)^{6}$

$\therefore \quad T_7 = 2388104.5930$

Thus, population after 6 years will be 23,88,105.

**Illustration 17 : Government decides to fix the depreciation rate of a machine to 15 % per year. If the purchase price of a machine is ₹ 50,000. Find the value of the machine after 7 years.**

Here purchase price of a machine is ₹ 50,000 i.e. $a = 50{,}000 = T_1$. Machine depreciates at the rate of 15% every year i.e. r = 0.85

| As the machine depreciates, its price decreases at the rate of 15% $\therefore r = \frac{100-15}{100} = 0.85$ |
|---|

Value of machine after the first year = 50,000 × 0.85

= 42,500 = $T_2$

We need to find the value of machine after 7 years, so $n = 8$.

Putting the values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$,

$T_8 = 50{,}000\ (0.85)^{8-1}$

$= 50{,}000\ (0.85)^{7}$

$= 16028.8544$

$\approx 16028.85$

Thus, the value of machine after 7 years will be ₹ 16,028.85

## 9.3 Meaning of Series

If the geometric progression is taken as $a, ar, ar^2, ar^3, \ldots$, then the geometric series is given as $a + ar + ar^2 + ar^3 + \ldots$,

e.g. If the geometric progression is 2, 6, 18, 54, ... then its corresponding geometric series is 2 + 6 + 18 + 54 + ...

The sum of the first $n$ terms of the G.P. is denoted by the symbol $S_n$.

Then $S_n = T_1 + T_2 + T_3 + \ldots + T_n$

where $T_n = ar^{n-1}$, $n = 1, 2, 3, \ldots$

Hence, $S_1 = T_1$, $S_2 = T_1 + T_2$, $S_3 = T_1 + T_2 + T_3$ and so on.

If we write all the terms of $S_n$ in terms of $a$ and $r$ then we have

$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

**We shall accept the following results relating to the geometric progression without proof :**

(1) $\frac{T_{n+1}}{T_n} = r$ = common ratio for any positive integer $n$.

(2) $T_{n+1} = S_{n+1} - S_n$ where $n = 1, 2, 3, \ldots$

(3) $S_n = na$ where $r = 1$

(4) $S_n = \frac{a(r^n - 1)}{(r-1)}$ where $r \neq 1$

(5) $S_n = \frac{rT_n - a}{(r-1)}$ where $r \neq 1$

**Illustration 18 : Find the sum of the first five terms of a G.P. 5, 15, 45, ...**

Here, the first term $a = 5$ and the common ratio $r = \frac{15}{5} = 3$.

Sum of the first five terms is required i.e. $n = 5$.

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n - 1)}{(r-1)}$, we get

$$S_5 = \frac{5(3^5 - 1)}{(3-1)}$$

$$= \frac{5(243-1)}{2}$$

$$= \frac{5(242)}{2}$$

$$= 605$$

Thus, the sum of the first five terms of the G. P. is 605.

**Illustration 19 : Find the sum of the first six terms of a G.P. 8, 4, 2, ...**

Here, the first term $a = 8$ and the common ratio $r = \frac{4}{8} = \frac{1}{2}$

Sum of the first six terms is required i.e. $n = 6$.

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n - 1)}{(r-1)}$, we get

$$\therefore \quad S_6 = \frac{8\left[\left(\frac{1}{2}\right)^6 - 1\right]}{\left(\frac{1}{2} - 1\right)}$$

$$= \frac{8\left[\frac{1}{64} - 1\right]}{\left(-\frac{1}{2}\right)}$$

$$= \frac{8\left[-\frac{63}{64}\right]}{\left(-\frac{1}{2}\right)}$$

$$= 8 \times \left(-\frac{63}{64}\right) \times \left(\frac{2}{-1}\right)$$

$$= \frac{63}{4}$$

Thus, the sum of the first six terms of the G. P. is $\frac{63}{4}$.

**Illustration 20 : Find the sum of the first four terms of the G.P. whose first term is 3 and common ratio is 2.**

Here, the first term $a = 3$ and the common ratio $r = 2$. Sum of the first four terms is required i.e. $n = 4$.

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n - 1)}{(r-1)}$, we get

$$S_4 = \frac{3[2^4-1]}{(2-1)}$$

$$= \frac{3(16-1)}{1}$$

$$= 3 \times 15$$

$$= 45$$

Thus, the sum of the first four terms of the G. P. is 45.

**Illustration 21 : The sum of the first three terms of a G.P. with common ratio 0.2 is 0.496. Find the first term of the G.P.**

Here, the common ratio $r = 0.2$ and sum of the first three term is 0.496. i.e. $S_3 = 0.496$

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n-1)}{(r-1)}$ , we get

$$0.496 = \frac{a[0.2^3-1]}{0.2-1}$$

$$\therefore \quad 0.496 = \frac{a(0.008-1)}{(-0.8)}$$

$$\therefore \quad 0.496 = \frac{a(-0.992)}{(-0.8)}$$

$$\therefore \quad a = \frac{0.496 \times (-0.8)}{(-0.992)}$$

$$\therefore \quad a = 0.4$$

Thus, the first term of the G. P. is 0.4.

**Illustration 22 : Sum of how many terms of a G.P. 800,400,200, ... is 1500 ?**

Here, the first term $a = 800$ and the common ratio $r = \frac{400}{800} = 0.5$. Sum of $n$ terms is 1500

i.e. $S_n = 1500$

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n-1)}{(r-1)}$, we get

$$S_n = \frac{800[(0.5)^n-1]}{(0.5-1)}$$

$$\therefore \quad 1500 = \frac{800[(0.5)^n-1]}{(-0.5)}$$

$$\therefore \quad (0.5)^n - 1 = \frac{1500 \times (-0.5)}{800}$$

$$\therefore \quad (0.5)^n = 1 - 0.9375$$

$$\therefore \quad (0.5)^n = 0.0625$$

$$(0.5)^n = (0.5)^4$$

Equating the powers on both the sides, we get

$n = 4$

Thus, the sum of the first four terms of the G. P. is 1500.

**Illustration 23 : In a G.P., the first term is 27 and sum of the first three terms is 189. Find the common ratio of the G.P.**

Here the first term $a = 27$ and sum of the first three terms is 189 i.e. $S_3 = 189$.

Putting the values in $S_n = \frac{a(r^n - 1)}{(r-1)}$ , we get

$$S_3 = \frac{27(r^3 - 1)}{(r-1)}$$

$$\therefore \quad 189 = \frac{27\ (r-1)\ (r^2 + r + 1)}{(r-1)}$$

$\therefore \quad 189 = 27\ (r^2 + r + 1)$

$\therefore \quad r^2 + r + 1 = 7$

$\therefore \quad r^2 + r - 6 = 0$

$\therefore \quad (r + 3)\ (r - 2) = 0$

$\therefore \quad r = -3$ OR $r = 2$

Thus, the common ratio of the G. P. is –3 or 2.

**Illustration 24 : The first term and the sum of the first five terms of the G.P. are equal to 1 each. Find the common ratio of the G. P.**

Here, the first term $a = 1$ and sum of the first five terms is 1 i.e. $S_5 = 1$.

Putting the values in $S_n = \frac{a(r^n - 1)}{(r-1)}$, we get

$$S_5 = \frac{1(r^5 - 1)}{(r-1)}$$

$$\therefore \quad 1 = \frac{(r^5 - 1)}{(r-1)}$$

$\therefore \quad r^5 - 1 = r - 1$

$\therefore \quad r^5 = r$

$\therefore \quad r^4 = 1$

$\therefore \quad r = \pm 1$

If we take $r = 1$ and $a = 1$ then the first five terms of the G. P. will be 1, 1, 1, 1, 1 whose sum is 5.

If we take $r = -1$ and $a = 1$ then the first five terms of the G. P. will be 1, –1, 1, –1, 1 whose sum is 1.

But, in the data, the sum of the first five terms is given as 1, so the only possible value of r is –1.

Thus the common ratio of the G. P. is –1.

**Illustration 25 : Find the maximum value of *n* such that the sum of the first *n* terms of a G. P. 2, 4, 8, 16...... does not exceed 5000.**

Here, the first term $a = 2$ and the common ratio $r = \frac{4}{2} = 2$.

Sum of the first $n$ terms should not exceed 5000 i.e. $S_n \leq 5000$.

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n - 1)}{(r-1)}$, we get

$$S_n = \frac{2\ (2^n - 1)}{(2-1)}$$

$$= 2\ (2^n - 1)$$

Since $S_n \leq 5000$, we have

$2\ (2^n - 1) \leq 5000$

$\therefore \quad 2^n - 1 \leq 2500$

$\therefore \quad 2^n \leq 2501$

We now tabulate values of $2^n$ for different positive values of $n$ as shown in the following table and we shall take the maximum value of $n$ for which $2^n \leq 2501$

| $n$ | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|
| $2^n$ | 16 | 32 | 64 | 128 | 256 | 512 | 1024 | 2048 | 4096 |

We can see from the table that for $n = 10$, $2^n = 1024$, for $n = 11$, $2^n = 2048$ and for $n = 12$. $2^n = 4096$ which exceeds 2501 so the maximum value of $n$ for which $2^n$ does not exceed 2501 is 11.

$\therefore \quad n = 11$

**Illustration 26 : Find the minimum value of $n$ such that the sum of the first $n$ terms of a G. P. 1, 3, $3^2$, $3^3$, ... is greater than or equal to 3000.**

Here, the first term $a = 1$ and the common ratio $r = \frac{3}{1} = 3$. Sum of the first $n$ terms should be greater than or equal to 3000 i.e. $S_n \geq 3000$.

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n - 1)}{(r-1)}$, we get,

$$S_n = \frac{1(3^n - 1)}{(3-1)}$$

$$= \frac{3^n - 1}{2}$$

Since, $S_n \geq 3000$, we have

$$\frac{3^n - 1}{2} \geq 3000$$

$\therefore \quad 3^n - 1 \geq 6000$

$\therefore \quad 3^n \geq 6001$

We now tabulate values of $3^n$ for different positive values of $n$ as shown in the following table and we shall take minimum value of $n$ for which $3^n \geq 6001$.

| $n$ | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|
| $3^n$ | 81 | 243 | 729 | 2187 | 6561 | 19683 |

We can see from the table that when we take $n = 8, 9, ..., 3^n$ exceeds 6001

Thus, the minimum value of $n$ for which $3^n$ is greater than or equal to 6001 is 8.

$\therefore \quad n = 8$

**Illustration 27 : If for a G.P., $S_8 = 10S_4$ find '$r$'.**

Here, $S_8 = 10S_4$

$$\therefore \quad \frac{a(r^8-1)}{(r-1)} = 10\left[\frac{a(r^4-1)}{(r-1)}\right]$$

$$\therefore \quad r^8 - 1 = 10(r^4 - 1)$$

$$\therefore \quad (r^4 - 1)(r^4 + 1) = 10(r^4 - 1)$$

$$\therefore \quad r^4 + 1 = 10$$

$$\therefore \quad r^4 = 9$$

$$\therefore \quad r^2 = 3$$

$$\therefore \quad r = \pm\sqrt{3}$$

**Illustration 28 : A person gives ₹ 5 to his son on 1st March, ₹ 10 on 2nd March, ₹ 20 on 3rd March and so on. Thus each day he gives double the amount than that of the previous day. Find the total amount he has given to his son upto 10th of March.**

The person gives ₹ 5 on 1st March, so the first term $a = 5$. Every day, the amount given to his son is twice the previous day's amount, so the common ratio $r = 2$.

We want to find the total amount given by him to his son upto 10th March i.e. $S_{10}$, so $n = 10$.

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n-1)}{(r-1)}$, we get,

$$S_{10} = \frac{5(2^{10}-1)}{(2-1)}$$

$$= \frac{5(1024-1)}{1}$$

$$= 5 \times 1023$$

$$= 5115$$

Hence, the total amount the person gives to his son upto 10th March is ₹ 5115.

**Illustration 29 : A person wants to donate ₹ 2,42,000 in five months such that every month he donates one-third of the amount he donated in the previous month. Find the amount he donated in the first month.**

The person wants to donate ₹ 242000 in five months i.e. $S_5 = 242000$ and $n = 5$. Every month, he donates one-third of the amount he donated in the previous month, so the common ratio $r = \frac{1}{3}$. We want to find the amount donated in the first month i.e. $a$.

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n-1)}{(r-1)}$, we get,

$$S_5 = \frac{a\left[\left(\frac{1}{3}\right)^5 - 1\right]}{\left(\frac{1}{3}-1\right)}$$

$\therefore \quad 242000 = \dfrac{a\left(\frac{1}{243}-1\right)}{\left(-\frac{2}{3}\right)}$

$\therefore \quad 242000 = \dfrac{a\left(-\frac{242}{243}\right)}{\left(-\frac{2}{3}\right)}$

$\therefore \quad 242000 = a \times \left(\frac{-242}{243}\right)\times\left(\frac{3}{-2}\right)$

$\therefore \quad 242000 = a \times \frac{121}{81}$

$\therefore \quad a = \dfrac{242000 \times 81}{121}$

$\therefore \quad a = 162000$

Thus, the amount donated in the first month is ₹ 1,62,000.

**Illustration 30 : A person deposits ₹ 20,000 in a bank at the compound interest rate of 8% per annum. Find the amount the person receives after 5 years.**

The person deposits ₹ 20,000 i.e. $a = 20{,}000 = T_1$

Amount received after first year $20{,}000 \times 1.08 = 21{,}600 = T_2$

The rate of interest is 8% so the common ratio $r = 1.08$

We want to find the amount received after 5 years,

so $n = 6$

> As the amount increases at the rate of 8%,
> $r = \frac{100+8}{100} = 1.08$

Putting the values of $a$, $r$ and $n$ in the general term $T_n = ar^{n-1}$, we get,

$T_6 = 20{,}000 \times (1.08)^{6-1}$

$= 20{,}000 \times (1.08)^5$

$= 29386.5615$

$\approx 29386.56$

Thus, amount received after 5 years will be ₹ 29,386.56.

**Illustration 31 : If three positive numbers $k + 1$, $3k - 1$, $5k + 1$ are in G.P., find the value of $k$.**

Here, $k + 1$, $3k - 1$, $5k + 1$ are in G. P., so $\frac{T_2}{T_1} = \frac{T_3}{T_2}$ = common ratio $r$

$\therefore \quad \frac{3k-1}{k+1} = \frac{5k+1}{3k-1}$

$\therefore \quad (3k - 1)^2 = (5k + 1)(k + 1)$

$\therefore \quad 9k^2 - 6k + 1 = 5k^2 + 6k + 1$

$\therefore \quad 4k^2 - 12k = 0$

$\therefore \quad 4k(k - 3) = 0$

$\therefore \quad 4k = 0$ or $k - 3 = 0$

$\therefore \quad k = 0$ or $k = 3$

But $k = 0$ is not possible because for $k = 0$, the value of $(3k - 1)$ will be $-1$ which a negative number.

Hence, $k = 3$

**Illustration 32 : If 15, $x$, 240, $y$ are in G.P., find the values of $x$ and $y$.**

Here, 15, $x$, 240, $y$ are in G. P. so $\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$ = common ratio ($r$)

Let us take $\frac{T_2}{T_1} = \frac{T_3}{T_2}$,

$\therefore \quad \frac{x}{15} = \frac{240}{x}$

$\therefore \quad x^2 = 15 \times 240$

$= 3600$

$\therefore \quad x = \pm 60$

$\therefore \quad x = 60$ or $-60$

Also, $\frac{T_3}{T_2} = \frac{T_4}{T_3}$

$\therefore \quad \frac{240}{x} = \frac{y}{240}$

$\therefore \quad xy = 240 \times 240$

$\therefore \quad xy = 57600$

| Now, if we take $x = 60$ | and | if we take $x = -60$ |
|---|---|---|
| $60y = 57600$ | | $-60y = 57600$ |
| $\therefore \quad y = \frac{57600}{60}$ | | $\therefore \quad y = \frac{57600}{-60}$ |
| $\therefore \quad y = 960$ | | $\therefore \quad y = -960$ |

Thus, $x = 60$ and $y = 960$ or $x = -60$ and $y = -960$.

**Illustration 33 : If for a G.P., $T_n = 80$, $S_n = 157.5$ and $r = 2$, find $a$ and $n$.**

Here, values of $T_n$ and $S_n$ are given, so we shall use the formula

$$S_n = \frac{rT_n - a}{r-1}$$

Putting the values of $T_n$, r and $S_n$ in $S_n = \frac{rT_n - a}{r-1}$

$\therefore \quad 157.5 = \frac{2 \times 80 - a}{2-1}$

$\therefore \quad 157.5 = 160 - a$

$\therefore \quad a = 160 - 157.5$

$\therefore \quad a = 2.5$

Now $T_n = ar^{n-1}$

$\therefore \quad 80 = 2.5 \times (2)^{n-1}$

$\therefore \quad 2^{n-1} = 32$

$\therefore \quad 2^{n-1} = 2^5$

Equating the powers on both the sides, we get

$n - 1 = 5$

$\therefore \quad n = 6$

Thus, $a = 2.5$ and $n = 6$

**Illustration 34 : If for a G.P., $T_n = 2^{n+1}$, obtain $S_4$.**

Here, $T_n = 2^{n+1}$

$\therefore \quad T_1 = 2^{1+1} = 4,\ T_2 = 2^{2+1} = 8,\ T_3 = 2^{3+1} = 16,...$

Here, the first term $a = 4$ and the common ratio $r = \frac{8}{4} = 2$. Sum of the first four terms is required i.e. $n = 4$.

Putting the values of $a$, $r$ and $n$ in $S_n = \frac{a(r^n - 1)}{(r-1)}$, we get

$$S_4 = \frac{4(2^4 - 1)}{(2-1)}$$

$$= \frac{4(16-1)}{1}$$

$$= 4 \times 15$$

$$= 60$$

Thus, the sum of the first four terms of the G. P. is 60.

**Illustration 35 : If in a G. P., $S_n = \frac{2}{3}(4^n - 1)$, obtain $T_{n+1}$.**

We know $T_{n+1} = S_{n+1} - S_n$

$$= \frac{2}{3}[4^{n+1} - 1] - \frac{2}{3}[4^n - 1]$$

$$= \frac{2}{3}[(4^{n+1} - 1) - (4^n - 1)]$$

$$= \frac{2}{3}[4^{n+1} - 1 - 4^n + 1]$$

$$= \frac{2}{3}[4^{n+1} - 4^n]$$

$$= \frac{2}{3} \times 4^n [4 - 1]$$

$\therefore \quad T_{n+1} = 2\ (4^n)$

**Illustration 36 : If in a G. P., $S_n = \frac{4}{3}(3^n - 1)$, find $T_3$.**

We know $T_{n+1} = S_{n+1} - S_n$

$\therefore \quad T_3 = S_3 - S_2$

$$= \frac{4}{3}(3^3 - 1) - \frac{4}{3}(3^2 - 1)$$

$$= \frac{4}{3}[(27 - 1) - (9 - 1)]$$

$$= \frac{4}{3}[26 - 8]$$

$$= \frac{4}{3}(18)$$

$\therefore \quad T_3 = 24$

## 9.4 Three consecutive terms of geometric progression

At times, the sum and the product of some consecutive terms of a G.P. are given and these consecutive terms are to be found. These terms can be found if we take the terms of the G.P. as $a$, $ar$, $ar^2$, $ar^3$, .... The calculations of the terms selected in this way may, most likely, become cumbersome. The mathematical forms of the terms of the G.P. are to be assumed in such a way that the calculation of the terms becomes simple. The assumptions of such forms of the terms of the G.P., for some selected values of n, are given below:

For $n = 3$, three consecutive terms are : $\frac{a}{r}, a, ar$

For $n = 4$, four consecutive terms are : $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$

For $n = 5$, five consecutive terms are : $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

**Note :** We shall restrict our study of finding the unknown terms of the G.P. for three terms only.

**Illustration 37 : The sum and the product of the three consecutive terms of a G.P. are 26 and 216 respectively. Find the three terms of the G.P.**

Let us assume three consecutive terms of G.P. as $\frac{a}{r}, a, ar$

Here, the product of the terms = 216

$\therefore \quad \frac{a}{r} \times a \times ar = 216$

$\therefore \quad a^3 = 216$

$\therefore \quad a = 6$

Now, the sum of the terms = 26

$\therefore \quad \frac{a}{r} + a + ar = 26$

$\therefore \quad a\left(\frac{1}{r} + 1 + r\right) = 26$

$\therefore \quad 6\left(\frac{1+r+r^2}{r}\right) = 26 \qquad (\because a = 6)$

$\therefore \quad 3(1 + r + r^2) = 13r$

$\therefore \quad 3 + 3r + 3r^2 = 13r$

$\therefore \quad 3r^2 - 10r + 3 = 0$

$\therefore \quad (r - 3)(3r - 1) = 0$

$\therefore \quad r = 3 \text{ or } r = \frac{1}{3}$

Now, if we take $a = 6$ and $r = 3$ then three consecutive terms will be $\frac{6}{3} = 2$, 6, $6 \times 3 = 18$.

Hence, three consecutive terms are 2, 6, 18.

Now, if we take $a = 6$ and $r = \frac{1}{3}$ then three consecutive terms will be $\frac{6}{1/3} = 18$, 6, $6 \times \frac{1}{3} = 2$

Hence, three consecutive terms are 18, 6 and 2.

**Illustration 38 : The sum and the product of the three consecutive terms of a G.P. are 9.5 and 27 respectiveiy. Find the three terms of the G.P.**

Let us assume three consecutive terms of G.P. as $\frac{a}{r}$, $a$, $ar$

Here the product of the terms = 27

$\therefore \quad \frac{a}{r} \times a \times ar = 27$

$\therefore \quad a^3 = 27$

$\therefore \quad a = 3$

Now the sum of the terms = 9.5

$\therefore \quad \frac{a}{r} + a + ar = 9.5$

$\therefore \quad a\left(\frac{1}{r} + 1 + r\right) = 9.5$

$\therefore \quad 3\left(\frac{1+r+r^2}{r}\right) = 9.5 \qquad (\because \ a = 3)$

$\therefore \quad 3 + 3r + 3r^2 = 9.5r$

$\therefore \quad 3r^2 - 6.5\, r + 3 = 0$

Multiplying by 2 on both the sides, we get

$\therefore \quad 6r^2 - 13r + 6 = 0$

$\therefore \quad (2r - 3)(3r - 2) = 0$

$\therefore \quad r = \frac{3}{2}$ or $r = \frac{2}{3}$

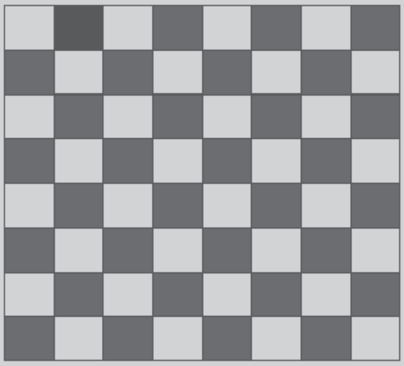
Now, if we take $a = 3$ and $r = \frac{3}{2}$ then three consecutive terms will be $\frac{3}{\left(\frac{3}{2}\right)} = 2$, 3, $3 \times \frac{3}{2} = \frac{9}{2}$

Hence, three consecutive terms are 2, 3, $\frac{9}{2}$

Now, if we take $a = 3$ and $r = \frac{2}{3}$ then three consecutive terms will be $\frac{3}{\frac{2}{3}} = \frac{9}{2}$, 3, $3 \times \frac{2}{3} = 2$

Hence, three consecutive terms are $\frac{9}{2}$, 3, 2.

## Activity



**(1)** Place one grain on 1st square of the chess board, 2 grains on 2nd square of the chess board, 4 grains on 3rd square, 8 grains on 4th square and so on. How many grains will have to be kept on the last square of the chess board ?

**(2)**

$$1 + 0.1 + 0.01 + 0.001 + 0.0001 = 1.1111$$

Verify using Geometric progression.

**Hint :** Here, the sequence is 1, 0.1, 0.01, 0.001, 0.0001

## Summary and Formulae

- For any $n \geq 1$ the ratio of the $(n+1)$th term to the n-th term of the sequence is a non-zero constant then such a sequence is called **Geometric Progression.**
- If the geometric progression is given as $a$, $ar$, $ar^2$, $ar^3$, ...then the **geometric series** is given as $a + ar + ar^2 + ar^3 + ...$
- $n$th term $T_n = ar^{n-1}$ for $n \geq 1$ is called the general formula (or term) of the G.P.
- $\frac{T_{n+1}}{T_n} = r$ = common ratio for any positive integer $n$
- $T_{n+1} = S_{n+1} - S_n$ ; $n = 1, 2, 3, ...$
- When $r = 1$ then $S_n = na$
- When $r \neq 1$ then $S_n = \frac{a(r^n - 1)}{(r - 1)}$
- When $r \neq 1$ then $S_n = \frac{rT_n - a}{r - 1}$
- Three consecutive terms in G. P. are : $\frac{a}{r}$, $a$, $ar$

## EXERCISE 9

### Section A

**For the following multiple choice questions choose the correct option :**

**1.** Find the 6th term of a G. P. 0.2, 1, 5, ...

(a) 25 (b) 0.5 (c) 0.1 (d) 625

**2.** If the first term of a G. P. is '$a$' and the common ratio is '$b$'. Find the $(n + 1)$th term.

(a) $ab^n$ (b) $ar^n$ (c) $ab^{n-1}$ (d) $ar^{n-1}$

**3.** For a G. P. 1, $\sqrt{3}$, 3, $3\sqrt{3}$, ..., find the 5th term ?

(a) 9 (b) $9\sqrt{3}$ (c) 27 (d) $\frac{(\sqrt{3})^5-1}{(\sqrt{3}-1)}$

**4.** For a G. P., $T_1 = a$ and $T_5 = \frac{1}{a}$, where $a > 0$ then obtain its third term.

(a) $a^2$ (b) 1 (c) $\frac{1}{a^2}$ (d) $a$

**5.** For a G. P. $\frac{1}{9}$, $\frac{1}{3}$, 1..., find the seventh term.

(a) 6561 (b) 243 (c) 81 (d) $\frac{1}{81}$

**6.** Find the common ratio of a G.P. whose $n$th term is $3(2^{n-1})$.

(a) 3 (b) 2 (c) 6 (d) 1

**7.** For a G. P 0.4, 0.04, 0.004, ..., find the common ratio.

(a) 10 (b) 0.4 (c) 4 (d) 0.1

**8.** If $x$, 10, –25 are in G.P. then find the value of $x$.

(a) 4 (b) –25 (c) –4 (d) 2

**9.** The common ratio of a G.P. is –1 and its first term is –1 then find the sum of the first six terms of the G.P.

(a) 0 (b) –1 (c) 1 (d) 6

**10.** The common ratio of a G.P. is 1 and $S_{10} = 40$ then find the first term.

(a) 0 (b) 10 (c) 4 (d) 400

### Section B

**Give answer in one sentence for the following questions :**

**1.** What is the $n$th term of the G.P. $ar, ar^2, ar^3, ...$?

**2.** Find the common ratio of the G.P. 0.1, 0.01, 0.001, ...

**3.** Find the sum of twenty terms of the G.P. 7, 7, 7, ...

**4.** If in a G.P. the $n$th term is given as $T_n = 2^{n+1}$ find the common ratio.

**5.** The numbers 4, 1, $y$ are in G.P. Find the value of $y$.

**6.** For a G.P., sum of any two consecutive terms is zero then what will be the common ratio?

**7.** For a G.P., if $S_7 = 15$ and $S_6 = 11$ then find the seventh term of the G.P.

**8.** State whether the statement "if $a$, $b$, $c$, $d$ are in G.P. then $ad = bc$" is true or false.

**9.** State whether the statement "$T_1 = S_1$" is true or false in G.P.

## Section C

**Answer the following questions :**

**1.** Define Geometric progression.

**2.** Define Geometric series.

**3.** If in a G.P., common ratio is 1 and $S_8 = 24$, find the first term of the G.P.

**4.** For a G.P., $T_1 = 2$ and the product of the first three terms is 1000. Find the common ratio.

**5.** For a G.P., $a = 2$ and $r = 3$ then find the sum of first four terms.

**6.** Which term of the G.P. 4, 12, 36, ... is 324 ?

**7.** For a G.P., $a = \frac{4}{9}$ and $r = \frac{-3}{2}$. Find $T_3$.

**8.** If the common ratio of a geometric progression is 2, find the ratio of its 7th and 3rd terms.

**9.** Find the required term of the following sequence using sequence formula :

(1) 2, 10, 50, ... (6th term) (2) 100, 50, 25, ... (7th term)

(3) $\frac{1}{3}, \frac{2}{9}, \frac{4}{27}$ ... (8th term) (4) $2, 2\sqrt{2}, 4$, ... (5th term)

## Section D

**Solve the following questions :**

**1.** For a given G.P., if $T_5 = 405$ and $T_7 = 3645$ then find $T_4$.

**2.** Find $T_5$ and $S_4$ of the geometric progression if the first term is $\frac{27}{16}$ and common ratio is $\frac{2}{3}$.

**3.** For a given G.P., $a = 4$ and $T_5 = \frac{1}{4}$ then find $T_7$.

**4.** For a given G.P., if $T_2 = 9$ and $T_5 = 243$ then find $S_4$.

**5.** The first term of a G.P. is 10 and $T_4 = 0.08$. Find sum of first three terms.

**6.** For a geometric progression, $T_1^2 = T_2$ and $T_3 = 64$. Write the sequence.

**7.** If 5, $m$, 20, $t$ are in geometric progression find $m$ and $t$.

**8.** For a given G.P., if $a = 10$, $r = 0.1$ and $T_n = 0.01$ then find $n$.

**9.** For a given G.P., if $a = 1$, $r = 3$ and $S_n = 121$ then find $n$.

**10.** If $S_n = \frac{2}{3}(2^n - 1)$, find $T_4$.

**11.** If $S_n = 4(3^n - 1)$, find $T_{n+1}$.

**12.** Find the product of the first three terms of a G.P. whose second term is 5.

**13.** How many terms of a geometric progression 2, 4, 8, 16, ...would add to 126 ?

**14.** If for a G.P., $T_n = 324$, $S_n = 484$ and $r = 3$, find $a$ and $n$.

**15.** Find the sum of required terms for the following sequence using series formula :

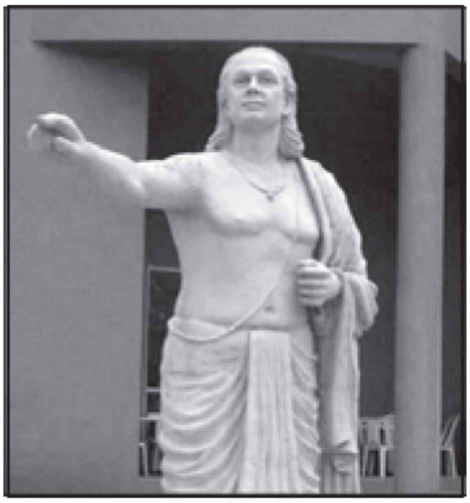
(1) 4, 16, 64, ... (first 4 terms) (3) 100, 20, 4, ... (first 5 terms)

(2) 2, 3, $\frac{9}{2}$, ... (first 5 terms) (4) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$, ... (first 10 terms)

**Section E**

**Solve the following :**

1. If the three positive numbers $k + 4$, $4k - 2$ and $7k + 1$ are in G.P., find $k$.
2. Find the maximum value of $n$ such that the sum of the first $n$ terms of a G.P. $1, 3, 3^2, 3^3 ...$ does not exceed 365.
3. Find the minimum value of $n$ such that the sum of the first $n$ terms of a G.P. $1, 2, 2^2, 2^3, ...$ .is greater than or equal to 2000.
4. The sum of the first five terms of the G.P. $y, \frac{y}{3}, \frac{y}{9}, ...$ (where $y > 0$) is 121. Find $y$.
5. For a geometric progression, $S_4 = 10\ S_2$. Find the common ratio.
6. For a geometric progression, the ratio of sum of the fifth and the third term to the difference of the fifth and the third term is 5:3. Find r.
7. The sum and the product of the three concecutive numbers in a geometric progression are 31 and 125 respectively. Find the three numbers of the G.P.
8. The sum and the product of the three consecutive terms of G.P. are 6 and – 64 respectively. Find the three terms of the G.P.
9. A construction company offers a scheme on a flat to attract customers. In this scheme, customer has to pay ₹ 10,000 as the first installment and has to pay double the amount of the preceeding installment in the subsequent annual installments. What is the total amount that the customer has to pay upto 10 installments ?
10. A banker counts 128 notes in the first minute and there after he counts half the number of notes he counted in the previous minute. How many notes he would count in five minutes?
11. Population of a village is 5000. Population increases at the rate of 2% every year. What will be the population of the village after 10 years ?
12. A car depreciates at the rate of 10% every year. If the cost price of the car is ₹ 5,00,000, what will be the value of the car after 6 years ?



**Aryabhatta**
**(476 – 550)**

**Aryabhatta** was a famous Indian mathematician and astronomer. His notable contributions to the world of science and mathematics includes the theory that the earth rotates on its axis, explanations of the solar and lunar eclipses, solving of quadratic equations, place value system with zero, and approximation of pie ($\pi$). Aryabhatta had defined sine, cosine, and inverse sine back in his era, influencing the birth of trigonometry. His calendar calculation has been in continuous use in India, on which the present day Panchangam is based. His studies are also base for the national calendars of Iran and Afghanistan today.

The ISRO (Indian Space Research Organization) named its first satellite after the genius mathematician and astronomer.

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