CBSE Test Paper 04

CH-05 Complex & Quadratic

1. If $\left(-1+\sqrt{3}i\right)^n+\left(-1-\sqrt{3}i\right)^n=2^n\alpha$ then α is equal to :
   a. -1
   b. $(\omega)^n+\left(\omega^2\right)^n$
   c. 2
   d. none of these
2. Distance of the representative of the number 1 +i from the origin (in Argand's diagram) is
   a. 1
   b. $\sqrt{2}$
   c. 2
   d. none of these
3. If z is purely real and Re (z) $>$ 0 , then Amp. (z) is
   a. 0
   b. - π
   c. π
   d. none of these
4. If z = x + iy ; x, y $\in$ R then :
   a. $z\bar{z}<|z|^2$
   b. $z\bar{z}=|z|^2$

c. $z\bar{z} > |z|^2$

d. none of these

5. The complex numbers sinx + i cos2x and cosx – i sin2x are conjugate to each other, for

   a. x = nπ

   b. no value of x

   c. x = 0

   d. none of these

6. Fill in the blanks:

   The real and imaginary parts of complex number 7 is Re(z) = 7 and Im(z) = 0.

7. Fill in the blanks:

   A purely imaginary number ib is represented by the point (0, b) on Y-axis, so Y-axis is called the ________ axis.

8. Show that $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0, \forall n \in N$.

9. Find the value of $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9}$

10. Find the value of i^{-30}

11. Find the modulus and argument of the following complex number 1 + i tan α.

12. Find the multiplicative inverse of 4 - 5i

13. Solve $x^2 + 3x + 9 = 0$

14. Find the conjugate and modulus of the following complex number $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$.

15. Solve the equation $z^2 = \overline{z}$.

Solution

1. (b) $(\omega)^n + \left(\omega^2\right)^n$

   Explanation:

   We have $-1 + i\sqrt{3} = 2\omega \quad and \quad -1 - i\sqrt{3} = 2\omega^2$

   Now $-1 + i\sqrt{3} = 2\omega \quad and \quad -1 - i\sqrt{3} = 2\omega^2$
   $$\left(-1+\sqrt{3}i\right)^n + \left(-1-\sqrt{3}i\right)^n = (2\omega)^n + \left(2\omega^2\right)^n = 2^n\left[(\omega)^n + \left(\omega^2\right)^n\right]$$

2. (b) $\sqrt{2}$

   Explanation:

   Let P(x,y) represent the complex number Z=x+iy in the complex plane(Argand plane) and let O(0,0) be the origin

   Then we have OP=$\sqrt{x^2+y^2}$

   Here Z=1+i so that P(x,y)=P(1,1)

   Hence OP=$\sqrt{1+1} = \sqrt{2}$

3. (a) 0

   Explanation:

   To find the amplitude of a complex number Z=x+iy first find a value of θ which satisfy the equation $\theta = \tan^{-1}\left|\frac{y}{x}\right|, 0 \leq \theta \leq \frac{\Pi}{2}$

   Now depending on the complex number lies in the first,second ,third or fourth quadrants the amplitudes will be $\theta, (\Pi - \theta), -(\Pi - \theta), -(\theta)$respectively.

   Given z is purely real and Re (z) > 0

   $\therefore \quad Z = x + 0i, \quad x > 0$

Now $\tan^{-1}\left|\frac{y}{x}\right| = \tan^{-1}\left|\frac{0}{1}\right| = \tan^{-1} 0 = 0$

Also $\quad Z = x + 0i, \quad x > 0$ lies in the first quadrant

Hence Amp(Z)=0

4. (b) $z\bar{z} = |z|^2$

   Explanation:

   If z = x + iy then $\bar{Z} = x - iy$

   Now
   $z\bar{z} = (x + iy).(x - iy) = x^2 + y^2 = |z|^2 \qquad \left[\because |z| = \sqrt{x^2 + y^2}\right]$

5. (b) no value of x
   Explanation:
   Let $z_1 = sinx + icos2x$ and $\Rightarrow \bar{z_1} = sinx - icos2x \quad , \bar{z_2} = cosx + isin2x$
   Given that $z_1 = \bar{z_2} \quad and \quad z_2 = \bar{z_1}$
   $\Rightarrow \sin x + i\cos 2x = \cos x + i\sin 2x$ and $\cos x - i\sin 2x = \sin x - i\cos 2x$
   $\Rightarrow \sin x = \cos x \ldots\ldots(i)$ and $\cos 2x = \sin 2x \ldots\ldots(ii)$
   Now $\cos 2x = \sin 2x \Rightarrow 2\cos^2 x - 1 = 2\sin x\cos x$
   $\Rightarrow 2\sin^2 x - 2\sin^2 x = 1$
   $\Rightarrow 0 = 1$
   which is not possible.
   Hence we can say there is no solution for the system of equations

6. True

7. imaginary

8. Given, **LHS** $= i^n + i^{n+1} + i^{n+2} + i^{n+3}$
   $= i^n + i^n . i + i^n . i^2 + i^n . i^3 = i^n (1 + i + i^2 + i^3)$
   = i^n (1 + i - 1 - i) [$\because i^2 = -1, i^3 = i^2 . i = -i$]
   = i^n (0) = 0 = RHS
   Hence proved.

9. $\sqrt{-25} + 3\sqrt{-4} + 2\sqrt{-9}$

= $\sqrt{25}\sqrt{-1}+3\sqrt{4}\sqrt{-1}+2\sqrt{9}\sqrt{-1}$

= 5 $\times$ i + 3 $\times$ 2 $\times$ i + 2 $\times$ 3 $\times$ i [$\because$ $\sqrt{-1}$ = i]

= 5i + 6i + 6i = 17i

10. $i^{-30} = \frac{1}{i^{30}}$

Now, $i^{30} = (i)^{4\times 7+2} = (i^{4\times 7})\, i^2 = (i^4)^7(-1)$

= $(1)^7(-1) = -1$ [$\because$ $i^4 = 1$ and $i^2 = -1$]

11. Let z = 1 + i tanα = 1 + $i\frac{\sin\alpha}{\cos\alpha}$

= 1 + i sinα . secα

= secα (cosα + i sinα)

On comparing with z = r (cosθ + i sinθ), we get

Modulus = r = secα

and argument = θ = α

12. Let z = 4 - 5i, then $\overline{z}$ = 4 + 5i

[conjugate of a complex number is obtained by changing sign of imaginary part]

$\therefore |z| = \sqrt{4^2+(-5)^2}$

= $\sqrt{16+25} = \sqrt{41}$

$\Rightarrow |z|^2$ = 41

$\therefore$ Multiplicative inverse of z is

$z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{4+5i}{41} = \frac{4}{41}+\frac{5}{41}i$

13. Here $x^2 + 3x + 9 = 0$

Comparing the given quadratic equation with $ax^2 + bx + c = 0$ we have

a = 1 , b= 3 and c = 9

$\therefore x = \frac{-3\pm\sqrt{(3)^2-4\times 9}}{2\times 1}$

$= \frac{-3\pm\sqrt{-27}}{2} = \frac{-3\pm 3\sqrt{3}i}{2}$

Thus $x = \frac{-3+3\sqrt{3}i}{2}$ and $x = \frac{-3-3\sqrt{3}i}{2}$

14. Let z = $\frac{3+2i}{2-5i}+\frac{3-2i}{2+5i}$

= $\frac{(3+2i)(2+5i)+(3-2i)(2-5i)}{(2-5i)(2+5i)}$

= $\frac{6+15i+4i+10i^2+6-15i-4i+10i^2}{(2)^2-(5i)^2}$

= $\frac{6+20i^2+6}{4+25}$

= $\frac{12-20}{29} = \frac{-8}{29}$ + 0i

$\overline{z}$ = $-\frac{8}{29} + 0i$ = - $\frac{8}{29}$ - 0i

Then, $|z|$ = $\sqrt{\left(\frac{-8}{29}\right)^2 + 0^2}$

= $\sqrt{\frac{64}{841}}$ = $\frac{8}{29}$

15. Let z = x + i y.

Now, $z^2 = \overline{z}$

$\Rightarrow$ $(x + iy)^2 = x - iy$

$\Rightarrow$ $x^2 + 2ixy + (iy)^2 = x - iy$

$\Rightarrow$ $(x^2 - y^2) + 2ixy = x - iy$

$\Rightarrow$ $x^2 - y^2 = x$ ...(i) and 2 xy = - y ...(ii)

Now, 2xy = - y $\Rightarrow$ (2x + 1) y = 0 $\Rightarrow$ 2x +1 = 0 or y = 0 $\Rightarrow$ x = - $\frac{1}{2}$ or y = 0

Following cases arise:

CASE 1: When y = 0

Putting y = 0 in (i), we obtain

$x^2 = x \Rightarrow x(x-1) = 0 \Rightarrow x = 0$ or x = 1

Thus, we obtain (x = 0 and y = 0) or (x = 1 and y = 0)

$\therefore$ z = 0 + i0 = 0 or z = 1 + i0

CASE 2: When x = - $\frac{1}{2}$

Putting x = - $\frac{1}{2}$ in (i), we obtain

$\frac{1}{4} - y^2 = -\frac{1}{2} \Rightarrow y^2 = \frac{3}{4} \Rightarrow y = \pm\frac{\sqrt{3}}{2}$

Thus, we obtain $\left(x = -\frac{1}{2} \text{ and } y = \frac{\sqrt{3}}{2}\right)$ or $\left(x = -\frac{1}{2} \text{ and } y = -\frac{\sqrt{3}}{2}\right)$

$\therefore$ z = - $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ or $z = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Hence the values of z satisfying the given equation are, z = 0, 1, - $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and - $\frac{1}{2} - \frac{i\sqrt{3}}{2}$