CHAPTER 12
Composition, Recursion, and Exponential Functions

12-1 Composition of Functions

Given the two functions f and g, the **composite function**, denoted by $f \circ g$, is defined as $(f \circ g)(x) = f \circ g(x) = f(g(x))$, read " f of g of x."

In order for a value of x to be in the domain of $f \circ g$, two conditions must be satisfied:
1) x must be in the domain of g.
2) $g(x)$ must be in the domain of f.

Example 1 □ If $f(x) = x^2 + 1$ and $g(x) = x - 2$, find the following.

a. $(f \circ g)(x)$ b. $(g \circ f)(x)$ c. $(f \circ g)(3)$

Solution □ a. $(f \circ g)(x) = f(g(x))$

$= f(x-2)$ Substitute $x-2$ for $g(x)$.

$= (x-2)^2 + 1$ Evaluate f when x is $x-2$.

$= x^2 - 4x + 5$ Simplify.

b. $(g \circ f)(x) = g(f(x))$

$= g(x^2+1)$ Substitute x^2+1 for $g(x)$.

$= (x^2+1) - 2$ Evaluate g when x is x^2+1.

$= x^2 - 1$ Simplify.

c. $(f \circ g)(3) = f(g(3))$

$= f(3-2)$ $g(3) = 3-2$.

$= f(1)$ Simplify.

$= 1^2 + 1$ $f(1) = 1^2 + 1$

$= 2$ Simplify.

Example 2 □ If $f = \{(-1,-3),\ (2,5),\ (4,1)\}$ and $g = \{(-2,2),\ (1,3),\ (6,-1)\}$, find the following.

a. $(f \circ g)(-2)$ b. $(g \circ f)(4)$ c. $(f \circ g)(6)$

Solution □ a. $(f \circ g)(-2) = f(g(-2))$

$= f(2)$ $g(-2) = 2$

$= 5$ Simplify.

b. $(g \circ f)(4) = g(f(4))$

$= g(1)$ $f(4) = 1$

$= 3$ Simplify.

c. $(f \circ g)(6) = f(g(6))$

$= f(-1)$ $g(6) = -1$

$= -3$ Simplify.

Exercises - Operations on Functions and Composition of Functions

1

If $f(x) = x^2 - 3x - 1$ and $g(x) = 1 - x$, what is the value of $f \circ g(-2)$?

A) -3

B) -1

C) 1

D) 3

2

If $f = \{(-4,12),\ (-2,4),\ (2,0),\ (3,\frac{3}{2})\}$ and $g = \{(-2,5),\ (0,1),\ (4,-7),\ (5,-9)\}$, what is the value of $g \circ f(2)$?

A) -9

B) -7

C) 1

D) 5

3

A function f satisfies $f(-1) = 8$ and $f(1) = -2$. A function g satisfies $g(2) = 5$ and $g(-1) = 1$. What is the value of $f(g(-1))$?

A) -2

B) 1

C) 5

D) 8

4

If $f(x) = \dfrac{1-5x}{2}$ and $g(x) = 2 - x$, what is the value of $f(g(3))$?

A) -7

B) -2

C) 2

D) 3

Questions 5 and 6 refer to the following information.

x	$f(x)$	$g(x)$
-2	-5	0
0	6	4
3	0	-5

The table above gives values of f and g at selected values of x.

5

What is the value of $f(g(-2))$?

6

What is the value of $g(f(3))$?

12-2 Recursive Formula

A **recursive formula** for a sequence describes how to find the nth term from the term(s) before it.

A recursive formula consists of two parts:
1. An initial condition that shows where the sequence starts.
2. A recursion equation that shows how to find each term from the term(s) before it.

The process of composing a function from itself repeatedly is a special type of recursion.
For example, the composition of function $f \circ f(x)$ is a recursion.
The compound interest formula also involves recursion.

Example 1 □ A sequence is recursively defined by $a_n = a_{n-1} + \frac{2}{n}$. If $a_0 = 3$, what is the value of a_3?

Solution □

$a_1 = a_0 + \frac{2}{1} = 3 + 2 = 5$ Substitute 1 for n and 3 for a_0.

$a_2 = a_1 + \frac{2}{2} = 5 + 1 = 6$ Substitute 2 for n and 5 for a_1.

$a_3 = a_2 + \frac{2}{3} = 6 + \frac{2}{3} = \frac{20}{3}$ Substitute 3 for n and 6 for a_2.

Example 2 □ Let $f(x) = \sqrt{x^2 + 5}$, find $f \circ f \circ f(1)$.

Solution □

$f(1) = \sqrt{(1)^2 + 5} = \sqrt{6}$

$f \circ f(1) = f(\sqrt{6}) = \sqrt{(\sqrt{6})^2 + 5} = \sqrt{11}$

$f \circ f \circ f(1) = f(\sqrt{11}) = \sqrt{(\sqrt{11})^2 + 5} = \sqrt{16} = 4$

Example 3 □ For next year's vacation, Cabrera deposited \$2,000 into a savings account that pays 0.5% compounded monthly. In addition to this initial deposit, on the first day of each month, he deposits \$200 into the account. The amount of money n months after he opened the account can be calculated by the equation, $A_n = (1 + 0.005) \cdot A_{n-1} + 200$.
According to the formula, what will be the amount in Cabrera's savings account three months after he started it?

Solution □ One month after, the amount will be:
$A_1 = (1 + 0.005) \cdot A_0 + 200 = (1.005) \cdot 2000 + 200 = 2210$
Two months after, the amount will be:
$A_2 = (1 + 0.005) \cdot A_1 + 200 = (1.005) \cdot 2210 + 200 = 2421.05$
Three months after, the amount will be:
$A_3 = (1 + 0.005) \cdot A_2 + 200 = (1.005) \cdot 2421.05 + 200 = 2633.16$

Exercises - Recursive Formula

1

A sequence is recursively defined by $a_n = \sqrt{(a_{n-1})^2 + 2}$. If $a_0 = \sqrt{2}$, what is the value of a_2?

A) $\sqrt{5}$

B) $\sqrt{6}$

C) $\sqrt{8}$

D) 3

2

A sequence is recursively defined by $a_{n+1} = a_n - \dfrac{f(a_n)}{g(a_n)}$. If $a_0 = 1$, $f(x) = x^2 - 3x$, and $g(x) = 2x - 3$, what is the value of a_2?

A) -3

B) $-\dfrac{1}{5}$

C) 2

D) $\dfrac{3}{2}$

3

If $f(x) = \sqrt{2x^2 - 1}$, what is the value of $f \circ f \circ f(2)$?

A) $\sqrt{10}$

B) $\sqrt{15}$

C) $\sqrt{21}$

D) 5

4

If A_0 is the initial amount deposited into a savings account that earns at a fixed rate of r percent per year, and a constant amount of $12b$ is added to the account each year, then amount A_n of the savings n years after the initial deposit is made is given by the equation $A_n = (1 + \dfrac{r}{100}) \cdot A_{n-1} + 12b$. What is A_3, the amount you have in the savings three years after you made the initial deposit, if $r = 5$, $A_0 = 12{,}000$, and $b = 400$?

A) $\$23{,}070.00$

B) $\$26{,}048.00$

C) $\$29{,}023.50$

D) $\$35{,}274.68$

5

The number of gallons, P_n, of a pollutant in a lake at the end of each month is given by the recursively defined formula $P_n = 0.85P_{n-1} + 20$. If the initial amount P_0 of a pollutant in the lake is 400 gallons, what is P_3, the amount of pollutant in the lake at the end of the third month, to the nearest gallon?

A) 297

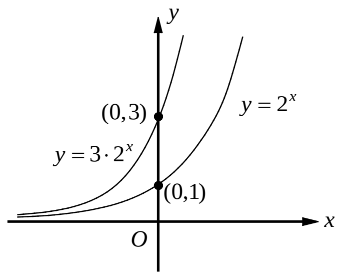
B) 285

C) 273

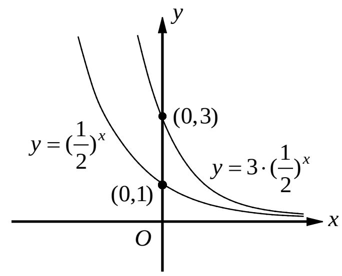
D) 262

12-3. Exponential Functions and Graphs

An **exponential function** is a function of the form $f(x) = ab^x$, in which $a \neq 0$, $b > 0$, and $b \neq 1$.



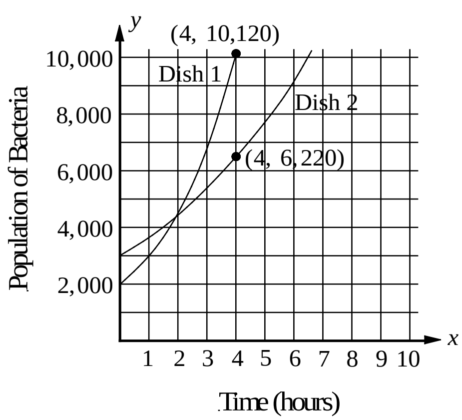
If $b > 1$, the graph rises as x increases.
The graph shows exponential growth.



If $0 < b < 1$, the graph falls as x increases.
The graph shows exponential decay.

Any function of the form $f(x) = ab^x$, in which $a > 0$, $b > 0$, and $b \neq 1$, the domain is the set of all real numbers and the range is the set of positive real numbers.

Example 1 □ In the diagram below, each exponential curve represents the population of bacteria in a petri dish as a function of time, in hours. At time $t = 0$, the population of Dish 1 is 2,000 and the population of Dish 2 is 3,000.



a. At time $t = 0$, the number of bacteria in Dish 2 is what percent more than the number of bacteria in Dish 1?

b. Find the average growth rate of bacteria in Dish 1 and in Dish 2 from time $t = 0$ to time $t = 4$.

Solution □ a. $\dfrac{\text{number of bacteria in Dish 2 } - \text{number of bacteria in Dish 1 at time } t = 0}{\text{number of bacteria in Dish 2 at time } t = 0}$

$$= \frac{3,000 - 2,000}{3,000} = \frac{1}{3} = 33\frac{1}{3}\%$$

At time $t = 0$, the number of bacteria in Dish 2 is $33\frac{1}{3}\%$ more than the number of bacteria in Dish 1.

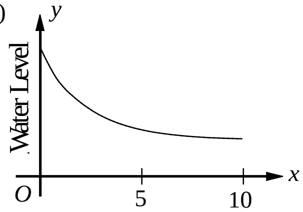
b. Average groth rate of Dish 1 $= \dfrac{10,120 - 2,000}{4 - 0} = 2,030$ bacteria per hour

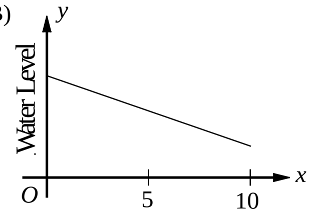
Average groth rate of Dish 2 $= \dfrac{6,220 - 3,000}{4 - 0} = 805$ bacteria per hour

Exercises - Exponential Functions and Graphs

1

During a decade of continuous drought, the water level of a lake has decreased by 10 percent each year. Which of the following graphs could model the water level of the lake as a function of time?

A)


B)


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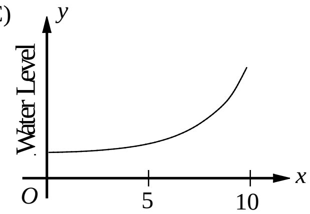
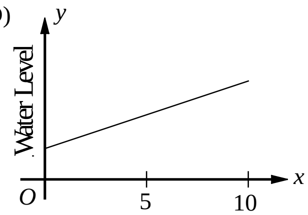


D)


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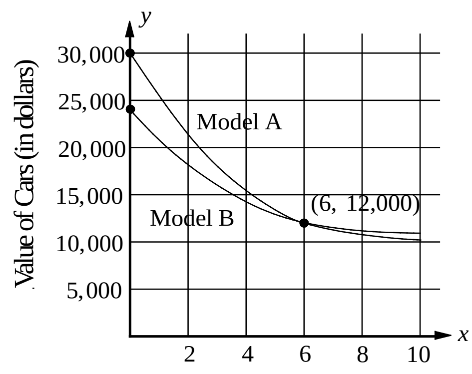


In the graph above, each exponential curve represents the values, in dollars, of two different cars as a function of time in years. At time $t = 0$, the price of model A was \$30,000 and the price of model B was \$24,000. At time $t = 6$, the price of both models were \$12,000.
Based on the graphs above, which of the following must be true?

I. At time $t = 0$, the price of model A was 25% more than the price of model B.

II. At time $t = 0$, the price of model B was 20% less than the price of model A.

III. From time $t = 0$ to $t = 6$, the average rate of decrease in the value of model A was 1.5 times the average rate of decrease in the value of model B.

A) I and II only

B) I and III only

C) II and III only

D) I, II, and III

3

If $f(x) = 12{,}000(0.9)^x$ and $g(x) = 14{,}000(0.85)^x$, what is the value of $g(2) - f(2)$?

12-4. Exponential Growth and Decay

Compound Interest Formulas

If initial amount P is invested at annual interest rate r, the investment will grow to final amount A in t years. $A = P(1+r)^t$

We can use the same formula for the population or value of goods that is increasing or decreasing.

Exponential Growth and **Doubling-Time Growth Formula**

If a population is increasing at a constant rate r each year, the population at the end of t years would be $A = P(1+r)^t$.

If an initial population of size P doubles every d years (or any other unit of time), the final number A in t years is given by $A = P(2)^{t/d}$.

Exponential Decay and **Half-Life Decay Formula**

If a population is decreasing at a constant rate r each year, the population at the end of t years would be $A = P(1-r)^t$.

The **half-life** of a substance is the amount of time it takes for half of the substance to decay.

If an initial population of size P has a half-life of d years (or any other unit of time), the final number A in t years is given by $A = P(\frac{1}{2})^{t/d}$.

Example 1 □ a. Mark invests $1,500 at a rate of 6% interest compounded annually. How much is the investment worth after 5 years?

b. The price of a new automobile is $28,000. If the value of the automobile decreases 12% per year, what will be the price of the automobile after 5 years?

c. The population of a western town doubles in size every 12 years. If the population of town is 8,000, what will the population be 18 years from now?

d. The half-life of carbon-14 is approximately 6000 years. How much of 800 g of this substance will remain after 30,000 years?

Solution □ a. $A = P(1+r)^t$ — Compound Interest Formula

$= 1,500(1+0.06)^5$ — Substitute $P = 1500$, $r = 0.06$, and $t = 5$.

$= 2,007.34$ — Use a calculator.

b. $A = P(1-r)^t$ — Exponential Decay Formula

$= 28,000(1-0.12)^5$ — Substitute $P = 28,000$, $r = 0.12$, and $t = 5$.

$\approx 14,776.49$ — Use a calculator.

c. $A = P(2)^{t/d}$ — Doubling-Time Growth Formula

$= 8,000 \cdot 2^{18/12}$ — Substitute $P = 8,000$, $t = 18$, and $d = 12$.

$\approx 22,627$ — Use a calculator.

d. $A = P(\frac{1}{2})^{t/d}$ — Half-Life Decay Formula

$A = 800(\frac{1}{2})^{30,000/6,000}$ — $P = 800$, $t = 30,000$, and $d = 6,000$.

$= 25$ — Use a calculator.

Exercises - Exponential Growth and Decay

1

The number of rabbits in a certain population doubles every 40 days. If the population starts with 12 rabbits, which of the following gives the total number of rabbits in the population after t days?

A) $12(2)(\frac{t}{40})$

B) $12(2)(\frac{40}{t})$

C) $12(2)^{\frac{40}{t}}$

D) $12(2)^{\frac{t}{40}}$

2

Population P of a town is 80,000 this year. If the population of the town decreases at a rate of 4 percent each year, which of the following expressions gives population P after t years?

A) $80,000(0.6)^t$

B) $80,000(0.96)^t$

C) $80,000(0.96t)$

D) $80,000(1-0.04t)$

3

A house bought ten years ago for $150,000 was sold for $240,000 this year. Which of the following equations can be used to solve the annual growth rate r of the value of the house?

A) $240,000 = 150,000(1+\frac{r}{10})$

B) $240,000 = 150,000(1+10r)$

C) $240,000 = 150,000(1+r)^{10}$

D) $240,000 = 150,000(r)^{10}$

4

A certain radioactive substance has a half-life of 12 days. This means that every 12 days, half of the original amount of the substance decays. If there are 128 milligrams of the radioactive substance today, how many milligrams will be left after 48 days?

A) 4

B) 8

C) 16

D) 32

Questions 5 and 6 refer to the following information.

Evelyn deposited $3,000 into her bank account, which earns 4 percent interest compounded annually. She uses the expression $\$3,000(x)^t$ to find the value of the account after t years.

5

What is the value of x in the expression?

6

Evelyn deposited the same amount into an account that earns 5 percent interest rate compounded annually. How much more money than her original deposit in the account with 4 percent interest rate compounded annually will she have earned in 10 years?
(Round your answer to the nearest dollar.)

Chapter 12 Practice Test

1

If $f(x)=\sqrt{2x}$ and $g(x)=2x^2$, what is the value of $f(g(1))-g(f(1))$?

A) –4

B) –2

C) 2

D) 4

2

If $f(x)=\sqrt{625-x^2}$ and $g(x)=\sqrt{225-x^2}$, what is the value of $f(f(5))-g((g5))$?

A) 0

B) 5

C) 10

D) 20

3

The population of a certain town doubles every 25 years. If the population of the town was 51,200 in 1980, in what year was the population 6,400?

A) 1855

B) 1880

C) 1905

D) 1930

4

The half-life of a radioactive substance is the amount of time it takes for half of the substance to decay. The table below shows the time (in years) and the amount of substance left for a certain radioactive substance.

Time (years)	Amount (grams)
0	1,200
14	850
28	600
42	425
56	300

How much of the original amount of the substance, to the nearest whole gram, will remain after 140 years?

A) 85

B) 75

C) 53

D) 38

5

A radioactive substance decays at a rate of 18% per year. If the initial amount of the substance is 100 grams, which of the following functions models the remaining amount of the substance, in grams, after t years?

A) $f(t)=100(0.18)^t$

B) $f(t)=100(0.82)^t$

C) $f(t)=100-100(0.18)^t$

D) $f(t)=100-100(0.82)^t$

6

$$5{,}000\left(1+\frac{r}{100}\right)^t$$

The expression above gives the value of an investment, in dollars, that pays an annual interest rate of r % compounded yearly. 5,000 is the initial amount and t is the number of years after the initial amount was deposited. Which of the following expressions shows the difference between the value of a 15 year investment at 6% annual compound interest and a 12 year investment at 6% annual compound interest?

A) $5{,}000\left[(1.06)^{\frac{15}{12}}\right]$

B) $5{,}000\left[\frac{(1.06)^{15}}{(1.06)^{12}}\right]$

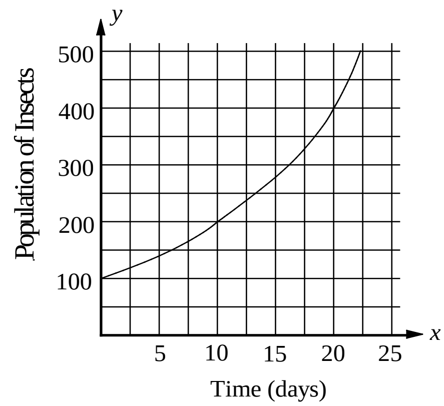
C) $5{,}000\left[(1.06)^{15}-(1.06)^{12}\right]$

D) $5{,}000\left[(1.06)^{15-12}\right]$

7

The price P, in dollars, of a truck t years after it was purchased is given by the function $P(t) = 24{,}000\left(\frac{1}{2}\right)^{\frac{t}{6}}$. To the nearest dollar, what is the price of the truck 9 years after it was purchased?

Questions 8 and 9 refer to the following information.



The graph above shows the size of a certain insect population over 25 days. The population at time $t = 0$ was 100. A biologist used the equation $f(t) = 100(2)^{\frac{t}{d}}$ to model the population.

8

What is the value of d in the equation?

9

What was the population of the insect after 15 days, to the nearest whole number?

Answer Key

Section 12-1

1. B 2. C 3. A 4. D 5. 6
6. 4

Section 12-2

1. B 2. B 3. D 4. C 5. A

Section 12-3

1. A 2. D 3. 395

Section 12-4

1. D 2. B 3. C 4. B 5. 1.04
6. 446

Chapter 12 Practice Test

1. B 2. A 3.C 4. D 5. B
6. C 7. 8485 8. 10 9. 283

Answers and Explanations

Section 12-1

1. B

$g(x) = 1 - x$

$g(-2) = 1 - (-2)$ Substitute -2 for x.

$= 3$

$f(x) = x^2 - 3x - 1$

$f \circ g(-2) = f(g(-2))$

$= f(3)$ $g(-2) = 3$

$= (3)^2 - 3(3) - 1$ Substitute 3 for x.

$= -1$

2. C

$f = \{(-4,12),\ (-2,4),\ (2,0),\ (3,\frac{3}{2})\}\ \Rightarrow$

$f(-4) = 12$, $f(-2) = 4$, $f(2) = 0$ and $f(3) = \frac{3}{2}$

$g = \{(-2,5),\ (0,1),\ (4,-7),\ (5,-9)\}\ \Rightarrow$

$g(-2) = 5$, $g(0) = 1$, $g(4) = -7$, $g(5) = -9$

$g \circ f(2) = g(f(2))$

$= g(0)$ $f(2) = 0$

$= 1$ $g(0) = 1$

3. A

$f(g(-1))$

$= f(1)$ $g(-1) = 1$

$= -2$ $f(1) = -2$

4. D

$g(x) = 2 - x$

$g(3) = 2 - 3$ Substitute 3 for x.

$= -1$

$f(g(3))$

$= f(-1)$ $g(3) = -1$

$= \frac{1 - 5(-1)}{2}$ Substitute -1 for x.

$= 3$

5. 6

x	$f(x)$	$g(x)$
-2	-5	0
0	6	4
3	0	-5

Based on the table, $g(-2) = 0$.

$f(g(-2))$

$= f(0)$ $g(-2) = 0$

$= 6$

6. 4

Based on the table, $f(3) = 0$.

$g(f(3))$

$= g(0)$ $f(3) = 0$

$= 4$

Section 12-2

1. B

$a_n = \sqrt{(a_{n-1})^2 + 2}$

$a_1 = \sqrt{(a_0)^2 + 2}$ $n = 1$

$= \sqrt{(\sqrt{2})^2 + 2}$ $a_0 = \sqrt{2}$

$= \sqrt{4} = 2$

$$a_2 = \sqrt{(a_1)^2 + 2} \qquad n = 2$$
$$= \sqrt{(2)^2 + 2} \qquad a_1 = 2$$
$$= \sqrt{6}$$

2. B

$$a_{n+1} = a_n - \frac{f(a_n)}{g(a_n)}$$
$$a_1 = a_0 - \frac{f(a_0)}{g(a_0)} \qquad n = 0$$
$$= 1 - \frac{f(1)}{g(1)} \qquad a_0 = 1$$

Since $f(x) = x^2 - 3x$ and $g(x) = 2x - 3$,
$f(1) = (1)^2 - 3(1) = -2$ and $g(1) = 2(1) - 3 = -1$.

Thus, $a_1 = 1 - \frac{f(1)}{g(1)} = 1 - \frac{-2}{-1} = -1$.

$$a_2 = a_1 - \frac{f(a_1)}{g(a_1)} \qquad n = 1$$
$$= -1 - \frac{f(-1)}{g(-1)} \qquad a_1 = -1$$

$f(-1) = (-1)^2 - 3(-1) = 4$ and
$g(-1) = 2(-1) - 3 = -5$.

Thus, $a_2 = -1 - \frac{f(-1)}{g(-1)} = -1 - \frac{4}{-5} = -\frac{1}{5}$

3. D

$$f(x) = \sqrt{2x^2 - 1}$$
$$f \circ f \circ f(2)$$
$$= f(f(f(2)) = f(f(\sqrt{2(2)^2 - 1}))$$
$$= f(f(\sqrt{7})) = f(\sqrt{2(\sqrt{7})^2 - 1})$$
$$= f(\sqrt{13}) = \sqrt{2(\sqrt{13})^2 - 1}$$
$$= \sqrt{25} = 5$$

4. C

$$A_n = (1 + \frac{r}{100}) \cdot A_{n-1} + 12b$$
$$A_1 = (1 + \frac{r}{100}) \cdot A_0 + 12b \qquad n = 1$$
$$= (1 + \frac{5}{100}) \cdot 12,000 + 12(400)$$
$$= 17,400$$

$$A_2 = (1 + \frac{r}{100}) \cdot A_1 + 12b \qquad n = 2$$
$$= (1 + \frac{5}{100}) \cdot 17,400 + 12(400) \qquad A_1 = 17,400$$
$$= 23,070$$
$$A_3 = (1 + \frac{r}{100}) \cdot A_2 + 12b \qquad n = 3$$
$$= (1 + \frac{5}{100}) \cdot 23,070 + 12(400) \qquad A_2 = 23,070$$
$$= 29,023.50$$

5. A

$$P_n = 0.85P_{n-1} + 20$$
$$P_1 = 0.85P_0 + 20 \qquad n = 1$$
$$= 0.85(400) + 20 \qquad P_0 = 400$$
$$= 360$$
$$P_2 = 0.85P_1 + 20 \qquad n = 2$$
$$= 0.85(360) + 20 \qquad P_1 = 360$$
$$= 326$$
$$P_3 = 0.85P_2 + 20 \qquad n = 2$$
$$= 0.85(326) + 20 \qquad P_2 = 326$$
$$= 297.1$$

Section 12-3

1. A

Suppose the initial water level was 100 units. If the water level decreases by 10 percent each year, the water level will be $100(1 - 0.1)^n$, or $100(0.9)^n$, n years later. The water level decreases exponentially, not linearly. Of the graphs shown, only choice A would appropriately model exponential decrease.

2. D

I. At time $t = 0$, the price of model A was \$30,000 and the price of model B was \$24,000. To find out what percent the price of model A was higher than the price of model B, use the following equation.

$$30,000 = 24,000(1 \underbrace{+ \frac{x}{100}}_{x\% \text{ more than}})$$
$$\frac{30,000}{24,000} = 1 + \frac{x}{100}$$
$$\Rightarrow 1.25 = 1 + \frac{x}{100} \Rightarrow 0.25 = \frac{x}{100}$$
$$\Rightarrow 25 = x$$

Therefore the price of model A was 25% higher than and the price of model B.

Roman numeral I is true.

To find out what percent the price of model B was less than the price of model A, use the following equation.

$$24,000 = 30,000(1 \underbrace{- \frac{x}{100}}_{x\% \text{ less than}})$$

$$\frac{24,000}{30,000} = 1 - \frac{x}{100}$$

$$0.8 = 1 - \frac{x}{100} \quad \Rightarrow \quad 0.2 = \frac{x}{100}$$

$$\Rightarrow \quad 20 = x$$

Therefore the price of model B was 20% less than the price of model A.

Roman numeral II is true.

From time $t = 0$ to $t = 6$, the average rate of decrease in the value of model A

$$= \frac{\text{amount of decrease}}{\text{change in years}} = \frac{30,000 - 12,000}{6}$$

$$= 3,000$$

From time $t = 0$ to $t = 6$, the average rate of decrease in the value of model B

$$= \frac{\text{amount of decrease}}{\text{change in years}} = \frac{24,000 - 12,000}{6}$$

$$= 2,000$$

Therefore, from time $t = 0$ to $t = 6$, the average rate of decrease in the value of model A was 1.5 times the average rate of decrease in the value of model B.

Roman numeral III is also true.

Choice D is correct.

3. 395

$f(x) = 12,000(0.9)^x$ and $g(x) = 14,000(0.85)^x$

$g(2) - f(2) = 14,000(0.85)^2 - 12,000(0.9)^2$

$= 10,115 - 9720 = 395$

Section 12-4

1. D

The present population must be multiplied by a factor of 2 to double. If a certain population doubles every 40 days, the population grows by a multiple of $(2)^{\frac{1}{40}}$ each day. After t days, the population will be multiplied by $(2)^{\frac{t}{40}}$. If the population starts with 12 rabbits, after t days, the population will be $12 \times (2)^{\frac{t}{40}}$.

2. B

For the present population to decrease by 4%, the initial population must be multiplied by a factor of 0.96. If population P is 80,000 this year, it will be

$80,000(0.96)$ one year later,

$80,000(0.96)(0.96)$ two years later,

$80,000(0.96)(0.96)(0.96)$ three years later,

and so on. After t years, the population will be $80,000(0.96)^t$.

3. C

For the price of a house to increase at an annual growth rate of r, it must be multiplied by a factor of $(1+r)$ each year. If the price of the house is \$150,000 this year, it will be

$150,000(1+r)$ one year later,

$150,000(1+r)(1+r)$ two years later,

$150,000(1+r)(1+r)(1+r)$ three years later,

and so on. Thus, 10 years later, the price of the house will be $150,000(1+r)^{10}$.

4. B

If the half-life of a substance is 12 days, half of the substance decays every 12 days.

Make a chart.

Amount	Days
128	0
$128 \times \frac{1}{2}$	12 days after
$128 \times \frac{1}{2} \times \frac{1}{2}$	24 days after
$128 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$	36 days after
$128 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$	48 days after

Therefore, after 48 days, there will be $128 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$, or 8 milligrams, of the radioactive substance left.

5. 1.04

The initial deposit earns 4 percent interest compounded annually. Thus at the end of one year, the new value of the account is the initial deposit of \$3,000 plus 4 percent of the initial deposit: $\$3,000 + 0.04(\$3,000) = \$3,000(1+0.04)$.
Since the interest is compounded annually, the value at the end of each succeeding year is the previous year's value plus 4 percent of the previous year's value. Thus after 2 years, the value will be $\$3,000(1.04)(1.04)$. After 3 years, the value will be $\$3,000(1.04)(1.04)(1.04)$.

After t years, the value will be $\$3,000(1.04)^t$.
Therefore, the value of x in the expression $\$3,000(x)^t$ is 1.04.

6. 446

The difference in the amount after 10 years will be
$\$3,000(1.05)^{10} - \$3,000(1.04)^{10}$
$\approx \$445.95$.
To the nearest dollar the difference in the amount will be \$446.

Chapter 12 Practice Test

1. B

$f(x) = \sqrt{2x}$ and $g(x) = 2x^2$
$g(1) = 2(1)^2 = 2$ and $f(1) = \sqrt{2(1)} = \sqrt{2}$
$f(g(1)) - g(f(1))$
$= f(2) - g(\sqrt{2})$
$= \sqrt{2(2)} - 2(\sqrt{2})^2$
$= \sqrt{4} - 2(2) = 2 - 4 = -2$

2. A

$f(x) = \sqrt{625 - x^2}$ and $g(x) = \sqrt{225 - x^2}$
$f(5) = \sqrt{625 - 5^2} = \sqrt{600}$
$g(5) = \sqrt{225 - 5^2} = \sqrt{200}$
$f(f(5)) - g((g5))$
$= f(\sqrt{600}) - g(\sqrt{200})$
$= (\sqrt{625 - (\sqrt{600})^2}) - (\sqrt{225 - (\sqrt{200})^2})$
$= \sqrt{625 - 600} - \sqrt{225 - 200}$
$= \sqrt{25} - \sqrt{25} = 0$

3. C

Method I:

You can keep dividing by 2 until you get to a population of 6,400.

Year	Population
1980	51,200
1955	25,600
1930	12,800
1905	6,400

Method II:

Use the half-life formula, $A = P(\frac{1}{2})^{t/d}$.

$6,400 = 51,200(\frac{1}{2})^{t/25}$

$\frac{6,400}{51,200} = (\frac{1}{2})^{t/25}$ Divide each side by 51,200.

$\frac{1}{8} = (\frac{1}{2})^{t/25}$ Simplify.

$(\frac{1}{2})^3 = (\frac{1}{2})^{t/25}$ $\frac{1}{8} = (\frac{1}{2})^3$

$3 = \frac{t}{25}$ If $b^x = b^y$, then $x = y$.

$75 = t$

Therefore, in year $1980 - 75$, or 1905, the population of the town was 6,400.

4. D

The table shows that one-half of the substance decays every 28 years. Therefore, the half-life of the radioactive substance is 28 years. Use the half-life formula, $A = P(\frac{1}{2})^{t/d}$, to find out how much of the original amount of the substance will remain after 140 years. P is the initial amount, t is the number of years and d is the half-life.

$A = 1,200(\frac{1}{2})^{140/28}$
$= 37.5$ Use a calculator.

To the nearest gram, 38 grams of the substance will remain after 140 years.

5. B

If the substance decays at a rate of 18% per year the amount of substance remaining each year will be multiplied by $(1 - 0.18)$, or 0.82.
The initial amount of 100 grams will become

$100(1-0.18)$ one year later,

$100(1-0.18)(1-0.18)$ two years later,

$100(1-0.18)(1-0.18)(1-0.18)$ three years later,

and so on. Thus, t years later, the remaining amount of the substance, in grams, is $f(t)=100(0.82)^t$.

6. C

$$5,000(1+\frac{r}{100})^t$$

The value of the 15 year investment at 6% annual compound interest

$$=5,000(1+\frac{6}{100})^{15}=5,000(1.06)^{15}.$$

The value of the 12 year investment at 6% annual compound interest

$$=5,000(1+\frac{6}{100})^{12}=5,000(1.06)^{12}.$$

The difference is

$$=5,000(1.06)^{15}-5,000(1.06)^{12}$$

$$=5,000\left[(1.06)^{15}-(1.06)^{12}\right]$$

7. 8485

$$P(t)=24,000(\frac{1}{2})^{\frac{t}{6}}$$

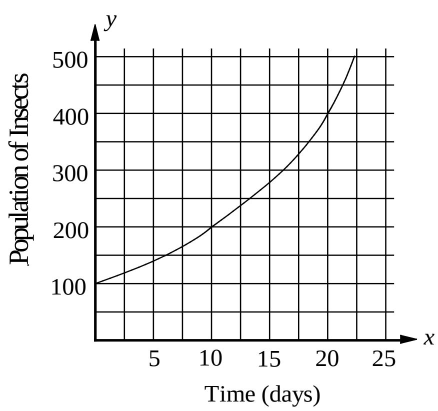
$$P(9)=24,000(\frac{1}{2})^{\frac{9}{6}}$$ Substitute 9 for t.

$$=24,000(\frac{1}{2})^{\frac{3}{2}}$$

$$\approx 8,485.28$$ Use a calculator.

To the nearest dollar, the price of the truck 9 years after it was purchased is \$8,485.

8. 10



$$f(t)=100(2)^{\frac{t}{d}}$$

In the equation, d represents the amount of time it takes to double the population. The graph shows that the population was 100 at $t=0$, 200 at $t=10$, and 400 at $t=20$. Therefore, the value of doubling time d is 10 days.

9. 283

$$f(t)=100(2)^{\frac{t}{d}}$$

$$f(15)=100(2)^{\frac{15}{10}}=100(2)^{1.5}$$

$$\approx 282.84$$ Use a calculator.

The population of the insect after 15 days was 283, to the nearest whole number.