Mathematics

Class XII

Sample Paper – 9

Time: 3 hours **Total Marks: 100**

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B, C and D. Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of two marks each, section C comprises of 11 questions of four marks each and section D comprises of 6 questions of six marks each.
3. Use of calculators is not permitted.

SECTION – A

1. Write the element which is denoted by a_{32} in the given matrix

$$\begin{bmatrix} 1 & 16 & 8 & 9 \\ 7 & 5 & 3 & 2 \\ 4 & 10 & 6 & 11 \end{bmatrix}$$

2. Differentiate sin (cos x) w.r.t. x

3. Is the differential equation given by $x+\left(\frac{d}{dx}\right)=\sqrt{1+\left(\frac{dy}{dx}\right)^2}$, linear or nonlinear. Give reason.

4. Find the angle between following pairs of line

$$\frac{x+4}{3}=\frac{y-1}{5}=\frac{z+3}{4} \text{ and } \frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$$

OR

Find the angle between following pairs of line

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3} \text{ and } \frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$$

SECTION – B

5. Consider the function f: $R_+ \rightarrow [4, \infty)$ defined by $f(x) = x^2 + 4$, where R_+ is the set of all non-negative real numbers. Show that f is invertible. Also find the inverse of f.

6. If A = diag (1 -1 2) and B = diag (2 3 -1), find A + B, 3A + 4B.

7. Evaluate:

$$\int \frac{e^x}{\sqrt{5 - 4e^x - e^{2x}}} dx.$$

8. Evaluate: $\int \frac{(x-4)e^x}{(x-2)^3} dx.$

OR

Evaluate: $\int \frac{x^2}{1+x^3} dx$

9. Form the differential equation $y^2 = m(a^2 - x^2)$ by eliminating parameters m and a

10. Find p, if the points (1, 1, p) and (-3, 0, 1) are equidistant from the plane whose equation is

$\vec{r}.\ 3\hat{i} + 4\hat{j} - 12\hat{k}\ + 13 = 0$

OR

Prove that: $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}\ \vec{b}\ \vec{c}]$

11. The probability that a student entering a university graduates is 0.4. Find the probability that out of 3 students of the university:

i. None will graduate

ii. Only one will graduate

12. A bag contains 5 white and 3 black balls and another bag contains 3 white and 4 black balls. A ball is drawn from the first bag and without seeing its colour, is put in the second bag. Find the probability that if now a ball is drawn from the second bag, it is black in colour.

OR

A doctor is to visit a patient. Form past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively. But if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that the doctor came by train?

SECTION – C

13.

(i) If f: N$\rightarrow$ Z s.t f(x) = x and g: Z$\rightarrow$ Z s.t g(x) = |x|.
Show that gof is injective but g is not.

(ii) If f: N$\rightarrow$ N s.t f(x) = x + 1 and g: N$\rightarrow$ N s.t g(x) = $\begin{cases} x-1 & \text{if } x>1 \\ 1 & \text{if } x=1 \end{cases}$.

Show that g of is subjective but f is not.

OR

Show that the function $f: R \rightarrow R$ defined by $f(x) = \frac{2x-1}{3}$, $x \in R$ is one-one and onto function. Also find the inverse of the function f.

14. Solve the equation.

$\sin^{-1}(1 - x) - 2\sin^{-1} x = \frac{\pi}{2}$

15. If x, y, and z are different and $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$; show that xyz = -1

16. Differentiate x^{x^x} w.r.t. x

OR

If $y = (\sin x)^{\tan x} + (\cos x)^{\sec x}$, find $\frac{dy}{dx}$

17. Differentiate $x^{\cot x} + \frac{2x^2-3}{x^2+x+2}$ w.r.t. x

18. Find the interval in which the function $y = \frac{4\sin\theta}{2+\cos\theta} - \theta, 0 \le \theta \le \pi$, is an increasing function of θ.

19. Evaluate:

$$\int \frac{x^2}{x^4+x^2-2}dx$$

20. Evaluate: $\int_0^2 \left(x^2+e^x\right) dx$ using integral as limit of sums.

21. Solve the initial value problem: $(x + y + 1)^2dy = dx$, $y(-1) = 0$

OR

Solve the initial value problem: $(x - y)(dy + dx) = dx - dy$, $y(0) = -1$

22. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$.

23. Find the value of λ so that the lines,

$$\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2} \text{ and } \frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$$

are perpendicular to each other.

SECTION – D

24. If $A = \begin{bmatrix} -1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, show that $(A + B)^T = A^T + B^T$

OR

If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & -3 \\ -1 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -2 & -1 \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}$, find AB and BA.

25. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

26. Find the smaller of the two areas in which the circle $x^2 + y^2 = 2a^2$ is divided by the parabola $y^2 = ax$, $a > 0$.

OR

Find the area of the region $\{(x, y): y^2 \geq 6x, 4x^2 + 4y^2 \leq 64\}$

27. Find a point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$, which is at the distance of $3\sqrt{2}$ units, from the point (1, 2, 3).

OR

Find the value of p, so that the lines $l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2}$ and $l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular to each other. Also find the equations of a line passing through a point (3, 2, − 4) and parallel to line l_1.

28. A nutritionist has to develop a special diet using two foods P and Q. Each packet (containing 30g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, atleast 460 units of iron and atmost 300 units of cholesterol. How many packet of each food should be used to minimise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

29. A bag contains 25 balls of which 10 are purple and the remaining are pink. A ball is drawn at random, its colour is noted and it is replaced. 6 balls are drawn in this way, find the probability that

i. All balls were purple
ii. Not more than 2 were pink
iii. An equal number of purple and pink balls were drawn.
iv. Atleast one ball was pink